Computer algebra independent integration tests

4-Trig-functions/4.7-Miscellaneous/4.7.3-c+d-x-^m-trig^n-trig^p

Nasser M. Abbasi

July 17, 2021 Compiled on July 17, 2021 at 10:38pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [397]. This is test number [137].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (397)	% 0.00 (0)
Mathematica	% 99.75 (396)	% 0.25 (1)
Maple	% 90.43 (359)	% 9.57 (38)
Maxima	% 76.07 (302)	% 23.93 (95)
Fricas	% 91.94 (365)	% 8.06 (32)
Sympy	% 30.48 (121)	% 69.52 (276)
Giac	% 55.16 (219)	% 44.84 (178)
Mupad	% 39.04 (155)	% 60.96 (242)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

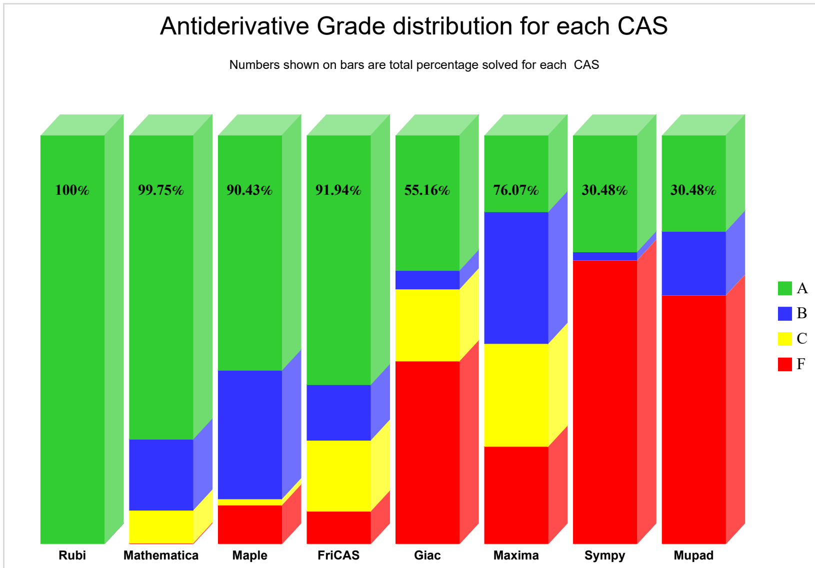
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

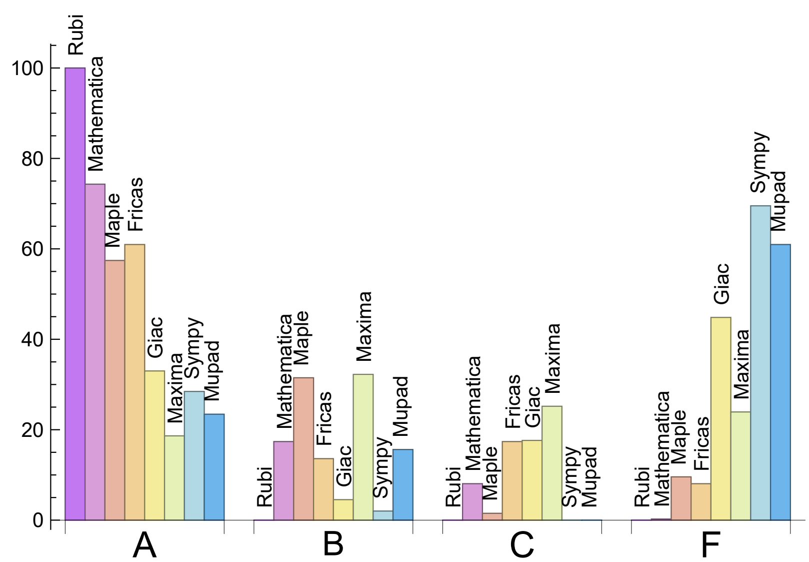
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	74.31	17.38	8.06	0.25
Maple	57.43	31.49	1.51	9.57
Maxima	18.64	32.24	25.19	23.93
Fricas	60.96	13.60	17.38	8.06
Sympy	28.46	2.02	0.00	69.52
Giac	33.00	4.53	17.63	44.84
Mupad	23.43	15.62	0.00	60.96

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	1	0.00 %	100.00 %	0.00 %
Maple	38	100.00 %	0.00 %	0.00 %
Maxima	95	48.42 %	48.42 %	3.16 %
Fricas	32	0.00 %	0.00 %	100.00 %
Sympy	276	60.51 %	34.06 %	5.43 %
Giac	178	81.46 %	18.54 %	0.00 %
Mupad	242	85.54 %	14.46 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

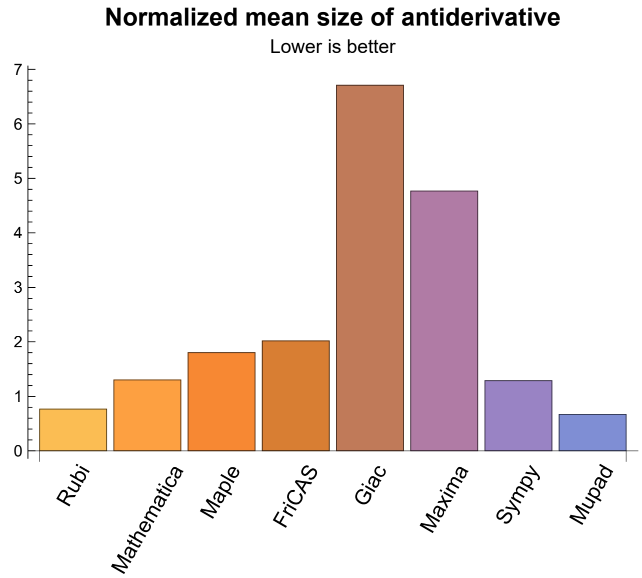
1.3 Performance

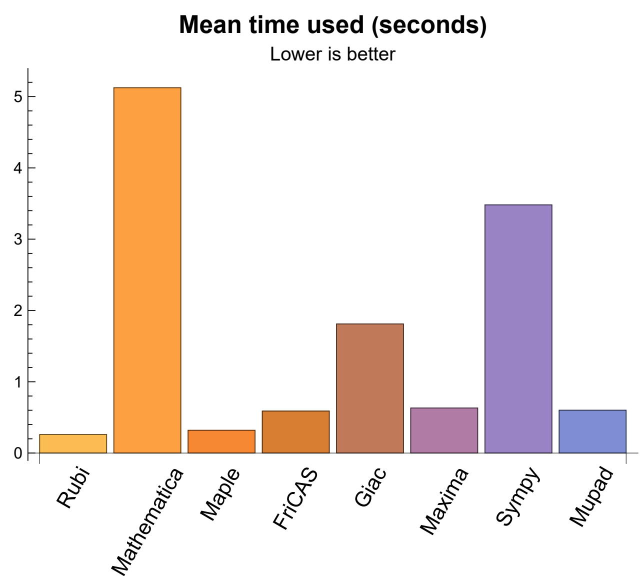
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.26	149.21	0.77	115.00	1.00
Mathematica	5.12	277.59	1.30	132.00	0.91
Maple	0.32	343.87	1.80	242.00	1.40
Maxima	0.63	908.93	4.77	405.50	1.67
Fricas	0.59	405.49	2.02	235.00	1.21
Sympy	3.48	177.93	1.29	0.00	0.00
Giac	1.81	946.22	6.71	145.00	1.23
Mupad	0.60	84.90	0.67	-1.00	-0.01

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {32, 33, 34, 35, 39, 41, 46, 47, 58, 63, 99, 105, 106, 107, 130, 131, 134, 135, 164, 165, 166, 171, 173, 178, 179, 180, 181, 190, 195, 222, 223, 236, 237, 241, 242, 249, 254, 255, 261, 267, 268, 280, 285, 291, 292, 300, 304, 305, 306, 310, 311, 312, 318, 319, 331, 376, 382, 383, 384, 385, 390}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at `https://`

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

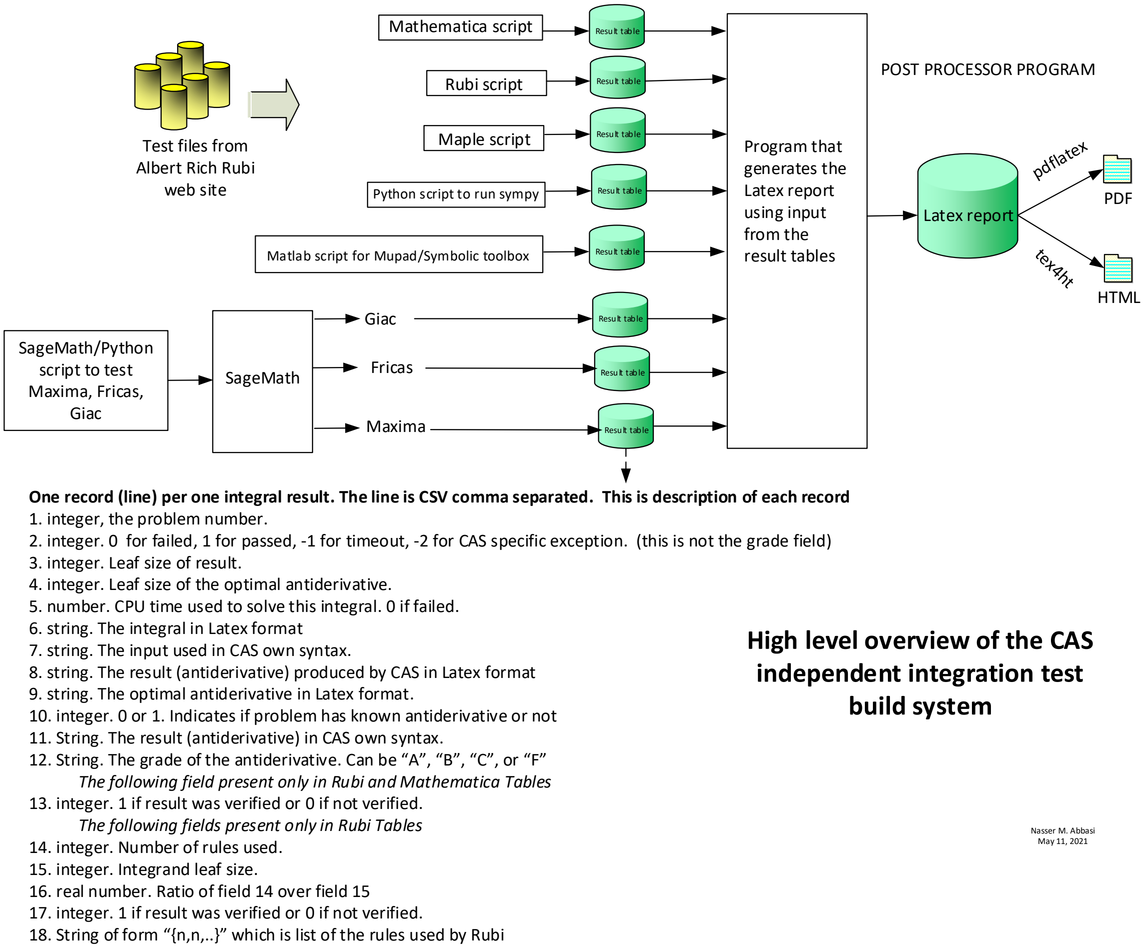
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 36, 37, 38, 39, 43, 44, 45, 49, 50, 51, 52, 53, 54, 55, 56, 57, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 99, 100, 101, 102, 103, 104, 109, 110, 111, 113, 116, 117, 124, 125, 126, 127, 128, 129, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 167, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 217, 219, 220, 221, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 275, 276, 277, 278, 279, 282, 283, 284, 285, 288, 289, 290, 293, 294, 295, 296, 298, 301, 302, 303, 307, 308, 309, 313, 314, 315, 316, 317, 320, 321, 322, 323, 326, 327, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 343, 344, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 32, 33, 34, 35, 40, 41, 42, 46, 47, 98, 105, 106, 107, 112, 114, 115, 164, 165, 166, 171, 172, 173, 178, 179, 180, 181, 216, 218, 222, 223, 228, 235, 236, 241, 242, 250, 254, 255, 260, 261, 273, 274, 280, 281, 286, 287, 291, 292, 299, 300, 304, 305, 306, 310, 311, 312, 318, 324, 325, 331, 340, 342, 382, 383, 384, 385, 389, 390, 397 }

C grade: { 48, 58, 59, 60, 61, 62, 63, 108, 118, 119, 120, 121, 122, 123, 130, 131, 132, 133, 134, 135, 190, 191, 192, 193, 194, 195, 237, 319, 345, 347, 356, 358 }

F grade: { 297 }

2.1.3 Maple

A grade: { 5, 6, 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 26, 27, 28, 29, 30, 31, 36, 37, 38, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 84, 85, 86, 87, 92, 93, 94, 95, 96, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 168, 169, 170, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 208, 213, 214, 215, 219, 220, 221, 224, 225, 226, 227, 232, 233, 234, 237, 238, 239, 240, 244, 245, 246, 250, 251, 252, 253, 256, 257, 258, 259, 263, 264, 265, 269, 270, 271, 272, 276, 277, 278, 281, 282, 283, 284, 287, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 362, 363, 364, 365, 366, 367, 371, 372, 373, 374, 375, 379, 380, 381, 385, 386, 387, 388, 391, 392, 393, 394, 395, 396 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 46, 47, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 158, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 204, 209, 210, 211, 212, 216, 217, 218, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 273, 274, 275, 279, 280, 286, 291, 292, 298, 299, 300, 304, 305, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 361, 368, 369, 370, 376, 377, 378, 382, 383, 384, 389, 390, 397 }

C grade: { 174, 262, 331, 340, 347, 356 }

F grade: { 1, 13, 22, 70, 79, 88, 136, 145, 154, 285, 328, 329, 330, 332, 333, 334, 335, 336, 337, 338, 339, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 357, 358, 359 }

2.1.4 Maxima

A grade: { 5, 17, 26, 31, 36, 37, 38, 43, 45, 74, 92, 97, 102, 103, 104, 109, 111, 116, 117, 140, 149, 158, 163, 168, 169, 170, 175, 177, 208, 213, 214, 215, 221, 224, 225, 226, 227, 232, 233, 234, 240, 246, 253, 257, 259, 265, 272, 278, 284, 290, 297, 303, 307, 308, 309, 314, 315, 316, 322, 360, 361, 362, 363, 364, 368, 369, 370, 371, 379, 380, 384, 386, 387, 388 }

B grade: { 2, 3, 4, 14, 15, 16, 23, 24, 25, 32, 33, 34, 35, 39, 40, 41, 42, 46, 47, 48, 49, 71, 72, 73, 80, 81, 82, 83, 89, 90, 91, 98, 99, 100, 101, 105, 106, 107, 108, 112, 113, 114, 115, 137, 138, 139, 146, 147, 148, 155, 156, 157, 164, 165, 166, 167, 172, 173, 174, 178, 179, 180, 181, 205, 206, 207, 209, 210, 211, 212, 216, 217, 222, 223, 228, 229, 230, 231, 235, 236, 241, 242, 243, 247, 248, 250, 254, 255, 256, 260, 262, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 285, 286, 287, 291, 292, 293, 294, 298, 299, 304, 305, 306, 310, 311, 312, 313, 317, 318, 323, 324, 325, 376, 377, 382, 383, 391, 396 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 18, 19, 20, 21, 27, 28, 29, 30, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 76, 77, 78, 84, 85, 86, 87, 93, 94, 95, 96, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 142, 143, 144, 150, 151, 152, 153, 159, 160, 161, 162, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 365, 366, 367, 372, 373, 374, 375 }

F grade: { 1, 13, 22, 44, 50, 51, 70, 79, 88, 110, 136, 145, 154, 171, 176, 182, 183, 202, 203, 204, 218, 219, 220, 237, 238, 239, 244, 245, 249, 251, 252, 258, 261, 263, 264, 270, 271, 276, 277, 282, 283, 288, 289, 295, 296, 300, 301, 302, 319, 320, 321, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 378, 381, 385, 389, 390, 392, 393, 394, 395, 397 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 31, 36, 37, 38, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 83, 84, 85, 88, 89, 90, 91, 92, 93, 94, 95, 97, 102, 103, 104, 109, 110, 111, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 154, 157, 158, 159, 160, 161, 163, 168, 169, 170, 174, 175, 176, 177, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 262, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 293, 294, 295, 296, 297, 301, 302, 303, 306, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 379, 380, 381, 386, 387, 388, 391, 392, 393, 394, 396 }

B grade: { 8, 9, 21, 30, 35, 41, 42, 47, 78, 80, 81, 82, 86, 87, 96, 101, 107, 108, 115, 144, 153, 155, 156, 162, 167, 173, 181, 203, 207, 212, 218, 224, 231, 237, 243, 249, 250, 255, 261, 269, 274, 275, 281, 287, 292, 300, 313, 319, 325, 378, 385, 390, 395, 397 }

C grade: { 32, 33, 34, 39, 40, 46, 98, 99, 100, 105, 106, 112, 113, 114, 164, 165, 166, 171, 172, 178, 179, 180, 202, 205, 206, 209, 210, 211, 216, 217, 222, 223, 228, 229, 230, 235, 236, 241, 242, 247, 248, 254, 260, 266, 267, 268, 273, 279, 280, 285, 286, 291, 298, 299, 304, 305, 310, 311, 312, 317, 318, 323, 324, 376, 377, 382, 383, 384, 389 }

F grade: { 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 50, 51, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 108, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 219, 220, 227, 232, 233, 238, 239, 244, 245, 246, 251, 252, 253, 256, 257, 258, 259, 263, 264, 270, 271, 276, 277, 282, 283, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 314, 315, 320, 321, 326, 327, 360, 364 }

B grade: { 53, 54, 55, 56, 361, 362, 363, 396 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 42, 45, 46, 47, 48, 49, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 215, 216, 217, 218, 221, 222, 223, 224, 225, 226, 228, 229, 230, 231, 234, 235, 236, 237, 240, 241, 242, 243, 247, 248, 249, 250, 254, 255, 260, 261, 262, 265, 266, 267, 268, 269, 272, 273, 274, 275, 278, 279, 280, 281, 284, 285, 286, 287, 291, 292, 293, 294, 298, 299, 300, 304, 305, 306, 309, 310, 311, 312, 313, 316, 317, 318, 319, 322, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397 }

2.1.7 Giac

A grade: { 2, 3, 4, 5, 10, 11, 12, 14, 15, 16, 17, 23, 24, 25, 26, 31, 36, 37, 38, 43, 44, 45, 50, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 97, 102, 103, 104, 109, 110, 111, 116, 117, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 316, 322, 326, 360, 361, 362, 363, 364, 365, 366, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 42, 48, 49, 108, 174, 204, 250, 256, 262, 293, 294, 367, 368, 369, 370, 371, 391, 396 }

C grade: { 6, 7, 8, 9, 18, 27, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 75, 84, 85, 86, 87, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 141, 159, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 372, 373, 374 }

F grade: { 1, 13, 19, 20, 21, 22, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 51, 70, 76, 77, 78, 79, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 136, 142, 143, 144, 145, 150, 151, 152, 153, 154, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 202, 203, 205, 206, 207, 209, 210, 211, 212, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 245, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 275, 279, 280, 281, 282, 283, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 315, 317, 318, 319, 320, 321, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 395, 397 }

2.1.8 Mupad

A grade: { 31, 36, 37, 38, 43, 44, 45, 50, 51, 97, 102, 103, 104, 109, 110, 111, 116, 117, 163, 168, 169, 170, 175, 176, 177, 182, 183, 208, 213, 214, 215, 219, 220, 221, 225, 226, 227, 232, 233, 234, 238, 239, 240, 244, 245, 246, 251, 252, 253, 257, 258, 259, 263, 264, 265, 270, 271, 272, 276, 277, 278, 282, 283, 284, 288, 289, 290, 295, 296, 297, 301, 302, 303, 307, 308, 309, 314, 315, 316, 320, 321, 322, 326, 327, 379, 380, 381, 386, 387, 388, 392, 393, 394 }

B grade: { 2, 3, 4, 5, 14, 15, 16, 17, 23, 24, 25, 26, 42, 48, 49, 71, 72, 73, 74, 80, 81, 82, 83, 89, 90, 91, 92, 108, 137, 138, 139, 140, 146, 147, 148, 149, 155, 156, 157, 158, 174, 204, 212, 250, 256, 262, 275, 293, 294, 360, 361, 362, 363, 364, 368, 369, 370, 371, 391, 395, 396, 397 }

C grade: { }

F grade: { 1, 6, 7, 8, 9, 10, 11, 12, 13, 18, 19, 20, 21, 22, 27, 28, 29, 30, 32, 33, 34, 35, 39, 40, 41, 46, 47, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 84, 85, 86, 87, 88, 93, 94, 95, 96, 98, 99, 100, 101, 105, 106, 107, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 142, 143, 144, 145, 150, 151, 152, 153, 154, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 205, 206, 207, 209, 210, 211, 216, 217, 218, 222, 223, 224, 228, 229, 230, 231, 235, 236, 237, 241, 242, 243, 247, 248, 249, 254, 255, 260, 261, 266, 267, 268, 269, 273, 274, 279, 280, 281, 285, 286, 287, 291, 292, 298, 299, 300, 304, 305, 306, 310, 311, 312, 313, 317, 318, 319, 323, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 365, 366, 367, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	138	0	0	94	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.69	0.00	0.00	-0.01
time (sec)	N/A	0.156	0.082	0.353	0.000	0.479	0.000	0.000	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	86	853	586	255	502	181	245
normalized size	1	1.00	0.55	5.47	3.76	1.63	3.22	1.16	1.57
time (sec)	N/A	0.107	0.479	0.056	0.385	0.656	4.252	0.196	0.491

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	71	466	342	166	342	121	165
normalized size	1	1.00	0.59	3.88	2.85	1.38	2.85	1.01	1.38
time (sec)	N/A	0.083	0.298	0.009	0.364	0.440	2.385	0.221	0.854

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	50	215	171	92	175	73	100
normalized size	1	1.00	0.56	2.42	1.92	1.03	1.97	0.82	1.12
time (sec)	N/A	0.054	0.229	0.007	0.363	0.495	1.050	0.205	0.159

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	34	74	65	42	80	38	47
normalized size	1	1.00	0.68	1.48	1.30	0.84	1.60	0.76	0.94
time (sec)	N/A	0.026	0.100	0.007	0.335	0.634	0.473	1.720	0.699

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	60	84	141	80	0	569	-1
normalized size	1	1.00	0.92	1.29	2.17	1.23	0.00	8.75	-0.02
time (sec)	N/A	0.139	0.126	0.017	0.408	0.611	0.000	0.847	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	80	124	164	132	0	2870	-1
normalized size	1	1.00	0.94	1.46	1.93	1.55	0.00	33.76	-0.01
time (sec)	N/A	0.149	0.308	0.010	0.436	0.714	0.000	0.564	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	102	162	199	230	0	5398	-1
normalized size	1	1.00	0.89	1.42	1.75	2.02	0.00	47.35	-0.01
time (sec)	N/A	0.175	1.084	0.012	0.506	0.805	0.000	0.534	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	164	200	249	320	0	7592	-1
normalized size	1	1.00	1.14	1.39	1.73	2.22	0.00	52.72	-0.01
time (sec)	N/A	0.198	0.667	0.010	0.636	0.609	0.000	0.612	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	13	6	5	6	-1
normalized size	1	1.00	1.00	0.88	1.62	0.75	0.62	0.75	-0.12
time (sec)	N/A	0.029	0.006	0.022	0.374	0.721	0.859	0.174	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	15	24	22	19	-1
normalized size	1	1.00	1.00	0.94	0.94	1.50	1.38	1.19	-0.06
time (sec)	N/A	0.046	0.006	0.018	0.385	0.763	1.572	0.145	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	26	15	30	24	26	-1
normalized size	1	1.00	1.00	0.90	0.52	1.03	0.83	0.90	-0.03
time (sec)	N/A	0.059	0.008	0.017	0.394	0.506	1.161	1.601	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	237	0	0	186	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.330	0.692	0.348	0.000	0.615	0.000	0.000	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	385	835	880	352	646	350	448
normalized size	1	1.00	1.88	4.07	4.29	1.72	3.15	1.71	2.19
time (sec)	N/A	0.200	1.449	0.091	0.416	0.544	7.476	0.230	1.409

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	121	447	499	227	391	231	289
normalized size	1	1.00	0.80	2.96	3.30	1.50	2.59	1.53	1.91
time (sec)	N/A	0.135	0.941	0.011	0.371	0.494	4.010	0.224	1.149

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	93	204	240	130	216	137	161
normalized size	1	1.00	0.90	1.98	2.33	1.26	2.10	1.33	1.56
time (sec)	N/A	0.077	0.580	0.010	0.373	0.485	2.094	0.212	0.868

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	44	71	85	59	85	69	59
normalized size	1	1.00	0.86	1.39	1.67	1.16	1.67	1.35	1.16
time (sec)	N/A	0.033	0.169	0.009	0.340	0.801	0.872	0.188	0.152

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	274	153	0	6059	-1
normalized size	1	1.00	0.84	1.37	2.26	1.26	0.00	50.07	-0.01
time (sec)	N/A	0.270	0.316	0.012	0.447	0.493	0.000	0.540	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	242	302	236	0	0	-1
normalized size	1	1.00	0.83	1.44	1.80	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.301	1.370	0.013	0.518	0.597	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	183	311	337	399	0	0	-1
normalized size	1	1.00	0.83	1.41	1.52	1.81	0.00	0.00	-0.00
time (sec)	N/A	0.358	2.156	0.015	0.676	0.546	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	298	384	387	564	0	0	-1
normalized size	1	1.00	1.10	1.42	1.43	2.09	0.00	0.00	-0.00
time (sec)	N/A	0.420	1.682	0.012	0.871	0.726	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	246	0	0	184	0	0	-1
normalized size	1	1.00	0.91	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.331	0.290	0.241	0.000	0.495	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1143	967	434	935	361	576
normalized size	1	1.00	0.61	4.40	3.72	1.67	3.60	1.39	2.22
time (sec)	N/A	0.241	1.699	0.120	0.421	0.594	13.311	2.893	1.944

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	135	594	549	283	602	241	366
normalized size	1	1.00	0.69	3.03	2.80	1.44	3.07	1.23	1.87
time (sec)	N/A	0.165	0.893	0.019	0.380	0.486	7.281	3.966	1.714

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	91	260	263	159	320	145	202
normalized size	1	1.00	0.68	1.94	1.96	1.19	2.39	1.08	1.51
time (sec)	N/A	0.092	0.491	0.019	0.362	0.455	3.625	0.199	1.249

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	92	76	138	75	94
normalized size	1	1.00	1.04	1.18	1.28	1.06	1.92	1.04	1.31
time (sec)	N/A	0.045	0.109	0.017	0.337	0.439	1.832	3.714	0.248

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	274	156	0	6046	-1
normalized size	1	1.00	0.85	1.38	2.12	1.21	0.00	46.87	-0.01
time (sec)	N/A	0.232	0.392	0.019	0.471	0.487	0.000	2.114	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	301	245	0	0	-1
normalized size	1	1.00	0.84	1.43	1.68	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.280	1.229	0.023	0.527	0.513	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	199	329	336	423	0	0	-1
normalized size	1	1.00	0.87	1.44	1.47	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.344	2.760	0.022	0.683	0.649	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	386	588	0	0	-1
normalized size	1	1.00	1.10	1.41	1.34	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.389	2.248	0.023	0.933	0.695	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.018	2.575	0.160	0.000	0.479	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	799	1150	1262	1204	0	0	-1
normalized size	1	1.00	5.29	7.62	8.36	7.97	0.00	0.00	-0.01
time (sec)	N/A	0.220	6.115	0.172	0.619	0.602	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	560	783	747	814	0	0	-1
normalized size	1	1.00	4.41	6.17	5.88	6.41	0.00	0.00	-0.01
time (sec)	N/A	0.192	2.667	0.095	0.516	0.580	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	356	468	404	498	0	0	-1
normalized size	1	1.00	3.83	5.03	4.34	5.35	0.00	0.00	-0.01
time (sec)	N/A	0.166	1.407	0.074	0.469	0.496	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	188	215	189	250	0	0	-1
normalized size	1	1.00	2.89	3.31	2.91	3.85	0.00	0.00	-0.02
time (sec)	N/A	0.096	5.112	0.073	0.459	0.505	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	3.662	0.174	0.000	0.513	0.000	0.000	0.000

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.021	6.983	0.200	0.000	0.441	0.000	0.000	0.000

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.204	2.930	0.080	0.000	0.451	0.000	0.000	0.000

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	308	716	2944	1021	0	0	-1
normalized size	1	1.00	1.48	3.44	14.15	4.91	0.00	0.00	-0.00
time (sec)	N/A	0.172	1.358	0.125	0.839	0.553	0.000	0.000	0.000

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	311	433	1770	669	0	0	-1
normalized size	1	1.00	2.13	2.97	12.12	4.58	0.00	0.00	-0.01
time (sec)	N/A	0.116	1.168	0.089	0.561	0.547	0.000	0.000	0.000

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	234	233	556	375	0	0	-1
normalized size	1	1.00	2.60	2.59	6.18	4.17	0.00	0.00	-0.01
time (sec)	N/A	0.062	2.037	0.028	0.525	0.532	0.000	0.000	0.000

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	131	52	259	62	0	801	88
normalized size	1	1.00	4.37	1.73	8.63	2.07	0.00	26.70	2.93
time (sec)	N/A	0.020	0.055	0.021	0.378	0.473	0.000	0.752	2.281

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.115	16.759	0.218	0.000	0.493	0.000	0.000	0.000

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.152	20.304	0.379	0.000	0.535	0.000	0.000	0.000

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.217	6.314	0.099	0.000	0.492	0.000	0.000	0.000

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	504	716	4540	1071	0	0	-1
normalized size	1	1.00	3.68	5.23	33.14	7.82	0.00	0.00	-0.01
time (sec)	N/A	0.256	6.608	0.133	0.893	0.699	0.000	0.000	0.000

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	277	409	1044	587	0	0	-1
normalized size	1	1.00	2.41	3.56	9.08	5.10	0.00	0.00	-0.01
time (sec)	N/A	0.174	6.414	0.105	0.697	0.746	0.000	0.000	0.000

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	94	95	1130	102	0	3482	147
normalized size	1	1.00	1.74	1.76	20.93	1.89	0.00	64.48	2.72
time (sec)	N/A	0.065	0.892	0.033	0.356	0.717	0.000	2.881	2.559

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	287	44	0	526	53
normalized size	1	1.00	1.37	1.74	8.20	1.26	0.00	15.03	1.51
time (sec)	N/A	0.031	0.073	0.029	0.393	0.584	0.000	0.358	1.716

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.137	11.329	0.366	0.000	1.245	0.000	0.000	0.000

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.166	10.641	0.539	0.000	0.879	0.000	0.000	0.000

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	275	222	0	1198	-1
normalized size	1	1.00	0.91	1.19	1.40	1.13	0.00	6.11	-0.01
time (sec)	N/A	0.448	2.239	0.028	0.548	0.805	0.000	3.063	0.000

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	256	167	665	743	-1
normalized size	1	1.00	0.93	1.11	1.52	0.99	3.96	4.42	-0.01
time (sec)	N/A	0.295	0.842	0.025	0.556	0.803	41.479	0.509	0.000

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	209	125	389	402	-1
normalized size	1	1.00	0.94	1.00	1.47	0.88	2.74	2.83	-0.01
time (sec)	N/A	0.231	0.258	0.023	0.452	0.749	6.139	0.560	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	134	142	209	125	389	402	-1
normalized size	1	1.00	0.94	1.00	1.47	0.88	2.74	2.83	-0.01
time (sec)	N/A	0.221	0.024	0.000	0.601	0.469	6.213	0.403	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	B	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	157	187	256	167	665	743	-1
normalized size	1	1.00	0.93	1.11	1.52	0.99	3.96	4.42	-0.01
time (sec)	N/A	0.275	0.040	0.000	0.547	0.501	42.222	1.025	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	179	234	275	222	0	1198	-1
normalized size	1	1.00	0.91	1.19	1.40	1.13	0.00	6.11	-0.01
time (sec)	N/A	0.334	1.056	0.000	0.470	0.677	0.000	0.678	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	543	370	0	2453	-1
normalized size	1	1.00	2.88	1.17	1.34	0.91	0.00	6.04	-0.00
time (sec)	N/A	1.137	15.071	0.049	0.596	0.591	0.000	3.345	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	495	298	0	1529	-1
normalized size	1	1.00	1.92	1.09	1.40	0.84	0.00	4.33	-0.00
time (sec)	N/A	0.684	9.140	0.042	0.592	0.632	0.000	2.766	0.000

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	264	294	422	245	0	838	-1
normalized size	1	1.00	0.87	0.97	1.39	0.81	0.00	2.76	-0.00
time (sec)	N/A	0.470	5.364	0.042	0.569	0.561	0.000	2.852	0.000

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	264	294	422	245	0	838	-1
normalized size	1	1.00	0.87	0.97	1.39	0.81	0.00	2.76	-0.00
time (sec)	N/A	0.468	3.049	0.000	0.579	0.629	0.000	3.872	0.000

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	677	386	495	298	0	1529	-1
normalized size	1	1.00	1.92	1.09	1.40	0.84	0.00	4.33	-0.00
time (sec)	N/A	0.571	8.949	0.000	0.607	0.824	0.000	6.939	0.000

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1171	474	543	370	0	2453	-1
normalized size	1	1.00	2.88	1.17	1.34	0.91	0.00	6.04	-0.00
time (sec)	N/A	0.668	13.628	0.000	0.603	0.692	0.000	2.458	0.000

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	547	406	0	2418	-1
normalized size	1	1.00	1.35	1.15	1.34	1.00	0.00	5.94	-0.00
time (sec)	N/A	1.051	14.246	0.039	0.539	0.880	0.000	3.276	0.000

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	316	0	1503	-1
normalized size	1	1.00	1.12	1.07	1.43	0.90	0.00	4.28	-0.00
time (sec)	N/A	0.674	3.076	0.037	0.514	0.583	0.000	4.752	0.000

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	425	244	0	818	-1
normalized size	1	1.00	0.88	0.96	1.42	0.82	0.00	2.74	-0.00
time (sec)	N/A	0.499	0.811	0.033	0.492	0.807	0.000	2.828	0.000

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	425	244	0	818	-1
normalized size	1	1.00	0.88	0.96	1.42	0.82	0.00	2.74	-0.00
time (sec)	N/A	0.460	0.287	0.000	0.484	0.847	0.000	1.073	0.000

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	316	0	1503	-1
normalized size	1	1.00	1.12	1.07	1.43	0.90	0.00	4.28	-0.00
time (sec)	N/A	0.566	2.367	0.000	0.497	0.679	0.000	6.767	0.000

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	547	406	0	2418	-1
normalized size	1	1.00	1.35	1.15	1.34	1.00	0.00	5.94	-0.00
time (sec)	N/A	0.697	9.966	0.000	0.634	0.911	0.000	4.041	0.000

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	250	0	0	184	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.69	0.00	0.00	-0.00
time (sec)	N/A	0.283	0.455	0.184	0.000	0.683	0.000	0.000	0.000

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	150	835	889	294	646	350	448
normalized size	1	1.00	0.73	4.07	4.34	1.43	3.15	1.71	2.19
time (sec)	N/A	0.203	1.550	0.057	0.471	0.487	7.187	0.247	1.898

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	127	447	505	183	391	231	290
normalized size	1	1.00	0.84	2.96	3.34	1.21	2.59	1.53	1.92
time (sec)	N/A	0.133	0.890	0.010	0.370	0.702	3.951	1.910	1.338

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	86	204	243	100	216	137	145
normalized size	1	1.00	0.83	1.98	2.36	0.97	2.10	1.33	1.41
time (sec)	N/A	0.079	0.490	0.010	0.374	0.719	2.034	4.947	1.131

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	71	71	86	46	85	69	58
normalized size	1	1.00	1.39	1.39	1.69	0.90	1.67	1.35	1.14
time (sec)	N/A	0.034	0.145	0.010	0.386	0.681	0.867	0.171	0.949

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	167	273	152	0	6279	-1
normalized size	1	1.00	0.83	1.38	2.26	1.26	0.00	51.89	-0.01
time (sec)	N/A	0.224	0.296	0.010	0.432	0.617	0.000	3.832	0.000

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	139	240	300	233	0	0	-1
normalized size	1	1.00	0.83	1.43	1.79	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.266	1.087	0.012	0.523	0.827	0.000	0.000	0.000

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	181	313	335	393	0	0	-1
normalized size	1	1.00	0.82	1.42	1.52	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.324	2.558	0.013	0.650	0.526	0.000	0.000	0.000

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	300	381	385	558	0	0	-1
normalized size	1	1.00	1.11	1.41	1.43	2.07	0.00	0.00	-0.00
time (sec)	N/A	0.377	1.823	0.013	0.877	0.666	0.000	0.000	0.000

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	213	0	0	134	0	0	-1
normalized size	1	1.00	1.31	0.00	0.00	0.83	0.00	0.00	-0.01
time (sec)	N/A	0.205	1.038	0.151	0.000	0.591	0.000	0.000	0.000

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	132	1915	735	466	1231	224	349
normalized size	1	1.00	1.01	14.62	5.61	3.56	9.40	1.71	2.66
time (sec)	N/A	0.164	1.270	0.092	0.373	0.618	13.619	1.113	1.684

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	106	1074	442	308	835	153	329
normalized size	1	1.00	1.01	10.23	4.21	2.93	7.95	1.46	3.13
time (sec)	N/A	0.130	0.655	0.025	0.364	0.500	7.943	0.227	1.691

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	77	519	232	180	493	94	179
normalized size	1	1.00	0.97	6.57	2.94	2.28	6.24	1.19	2.27
time (sec)	N/A	0.123	0.425	0.021	0.341	0.436	3.978	0.203	1.306

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	54	194	96	85	238	48	57
normalized size	1	1.00	1.02	3.66	1.81	1.60	4.49	0.91	1.08
time (sec)	N/A	0.054	0.286	0.023	0.326	0.594	1.998	1.363	1.040

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	65	105	160	88	0	669	-1
normalized size	1	1.00	0.83	1.35	2.05	1.13	0.00	8.58	-0.01
time (sec)	N/A	0.140	0.160	0.026	0.410	0.494	0.000	0.223	0.000

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	81	156	171	138	0	3218	-1
normalized size	1	1.00	0.78	1.50	1.64	1.33	0.00	30.94	-0.01
time (sec)	N/A	0.169	0.432	0.026	0.444	0.548	0.000	0.975	0.000

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	105	193	206	255	0	5600	-1
normalized size	1	1.00	0.83	1.52	1.62	2.01	0.00	44.09	-0.01
time (sec)	N/A	0.198	0.846	0.026	0.491	0.471	0.000	0.548	0.000

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	123	230	256	406	0	8508	-1
normalized size	1	1.00	0.78	1.46	1.62	2.57	0.00	53.85	-0.01
time (sec)	N/A	0.228	1.651	0.027	0.610	0.543	0.000	0.640	0.000

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	376	0	0	276	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.68	0.00	0.00	-0.00
time (sec)	N/A	0.403	0.628	0.211	0.000	0.721	0.000	0.000	0.000

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	238	1812	1339	471	1098	531	816
normalized size	1	1.00	0.72	5.49	4.06	1.43	3.33	1.61	2.47
time (sec)	N/A	0.391	3.172	0.115	0.420	0.811	20.156	0.871	4.606

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	369	992	766	296	690	351	516
normalized size	1	1.00	1.42	3.83	2.96	1.14	2.66	1.36	1.99
time (sec)	N/A	0.279	1.453	0.023	0.379	0.533	11.140	0.280	2.573

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	127	466	375	166	382	209	249
normalized size	1	1.00	0.69	2.53	2.04	0.90	2.08	1.14	1.35
time (sec)	N/A	0.197	0.879	0.023	0.348	0.471	5.959	0.251	0.809

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	94	163	139	76	163	106	99
normalized size	1	1.00	0.86	1.50	1.28	0.70	1.50	0.97	0.91
time (sec)	N/A	0.097	0.289	0.023	0.337	0.931	3.051	1.171	1.240

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	154	253	407	228	0	0	-1
normalized size	1	1.00	0.83	1.37	2.20	1.23	0.00	0.00	-0.01
time (sec)	N/A	0.339	0.490	0.024	0.471	0.647	0.000	0.000	0.000

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	213	365	438	347	0	0	-1
normalized size	1	1.00	0.83	1.42	1.70	1.35	0.00	0.00	-0.00
time (sec)	N/A	0.416	1.468	0.027	0.584	0.910	0.000	0.000	0.000

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	279	475	473	585	0	0	-1
normalized size	1	1.00	0.83	1.41	1.40	1.73	0.00	0.00	-0.00
time (sec)	N/A	0.505	3.905	0.027	0.779	0.826	0.000	0.000	0.000

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	457	580	523	824	0	0	-1
normalized size	1	1.00	1.11	1.40	1.27	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.590	2.896	0.027	1.104	0.618	0.000	0.000	0.000

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	6.383	0.241	0.000	0.532	0.000	0.000	0.000

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	837	1295	1538	1367	0	0	-1
normalized size	1	1.00	2.51	3.89	4.62	4.11	0.00	0.00	-0.00
time (sec)	N/A	0.284	1.325	0.227	0.638	0.750	0.000	0.000	0.000

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	330	847	919	921	0	0	-1
normalized size	1	1.00	1.30	3.33	3.62	3.63	0.00	0.00	-0.00
time (sec)	N/A	0.198	0.938	0.151	0.517	0.785	0.000	0.000	0.000

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	221	479	507	558	0	0	-1
normalized size	1	1.00	1.29	2.80	2.96	3.26	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.847	0.128	0.470	1.544	0.000	0.000	0.000

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	176	199	199	277	0	0	-1
normalized size	1	1.00	1.87	2.12	2.12	2.95	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.171	0.076	0.454	0.648	0.000	0.000	0.000

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.107	8.354	0.237	0.000	0.539	0.000	0.000	0.000

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.131	4.020	0.471	0.000	0.691	0.000	0.000	0.000

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.035	1.198	0.089	0.000	0.713	0.000	0.000	0.000

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	795	913	3229	856	0	0	-1
normalized size	1	1.00	5.13	5.89	20.83	5.52	0.00	0.00	-0.01
time (sec)	N/A	0.229	6.720	0.130	1.205	0.948	0.000	0.000	0.000

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	374	573	1945	599	0	0	-1
normalized size	1	1.00	2.94	4.51	15.31	4.72	0.00	0.00	-0.01
time (sec)	N/A	0.197	6.154	0.108	0.742	0.782	0.000	0.000	0.000

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	198	297	646	384	0	0	-1
normalized size	1	1.00	2.04	3.06	6.66	3.96	0.00	0.00	-0.01
time (sec)	N/A	0.130	6.026	0.093	0.742	0.724	0.000	0.000	0.000

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	82	49	292	97	104	1375	67
normalized size	1	1.00	2.00	1.20	7.12	2.37	2.54	33.54	1.63
time (sec)	N/A	0.026	0.479	0.053	0.584	0.587	0.480	2.528	1.573

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.036	4.521	0.223	0.000	0.797	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.034	2.349	0.372	0.000	0.561	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.075	38.567	0.104	0.000	0.636	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	966	1673	6952	2762	0	0	-1
normalized size	1	1.00	2.32	4.02	16.71	6.64	0.00	0.00	-0.00
time (sec)	N/A	0.502	8.295	0.187	5.492	2.058	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	528	1056	3872	1734	0	0	-1
normalized size	1	1.00	1.71	3.43	12.57	5.63	0.00	0.00	-0.00
time (sec)	N/A	0.344	4.744	0.144	1.920	1.190	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	471	546	1932	966	0	0	-1
normalized size	1	1.00	2.63	3.05	10.79	5.40	0.00	0.00	-0.01
time (sec)	N/A	0.223	7.578	0.120	0.809	0.706	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	260	246	770	454	0	0	-1
normalized size	1	1.00	2.41	2.28	7.13	4.20	0.00	0.00	-0.01
time (sec)	N/A	0.107	1.593	0.099	0.536	0.573	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.081	35.887	2.760	0.000	0.733	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.080	41.721	4.191	0.000	0.557	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	543	341	0	2465	-1
normalized size	1	1.00	2.88	1.17	1.34	0.84	0.00	6.07	-0.00
time (sec)	N/A	0.667	15.946	0.043	0.533	0.544	0.000	3.084	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	499	280	0	1538	-1
normalized size	1	1.00	1.92	1.09	1.41	0.79	0.00	4.36	-0.00
time (sec)	N/A	0.526	9.036	0.038	0.510	0.695	0.000	4.780	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	278	296	422	235	0	842	-1
normalized size	1	1.00	0.91	0.97	1.39	0.77	0.00	2.77	-0.00
time (sec)	N/A	0.416	6.616	0.034	0.499	0.727	0.000	2.783	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	264	296	422	235	0	842	-1
normalized size	1	1.00	0.87	0.97	1.39	0.77	0.00	2.77	-0.00
time (sec)	N/A	0.419	6.389	0.000	0.497	0.720	0.000	4.918	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	676	384	499	280	0	1538	-1
normalized size	1	1.00	1.92	1.09	1.41	0.79	0.00	4.36	-0.00
time (sec)	N/A	0.528	8.929	0.000	0.530	0.724	0.000	1.932	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	1168	476	543	341	0	2465	-1
normalized size	1	1.00	2.88	1.17	1.34	0.84	0.00	6.07	-0.00
time (sec)	N/A	0.627	15.313	0.000	0.521	0.523	0.000	4.904	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	285	347	0	1358	-1
normalized size	1	1.00	0.90	1.10	1.25	1.52	0.00	5.96	-0.00
time (sec)	N/A	0.398	3.419	0.049	0.473	0.508	0.000	3.297	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	264	249	0	842	-1
normalized size	1	1.00	0.94	1.03	1.32	1.24	0.00	4.21	-0.00
time (sec)	N/A	0.329	2.797	0.049	0.480	0.745	0.000	1.995	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	219	175	0	452	-1
normalized size	1	1.00	0.93	0.91	1.26	1.01	0.00	2.60	-0.01
time (sec)	N/A	0.270	0.822	0.046	0.462	0.519	0.000	0.991	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	161	159	219	175	0	452	-1
normalized size	1	1.00	0.93	0.91	1.26	1.01	0.00	2.60	-0.01
time (sec)	N/A	0.249	0.112	0.000	0.462	0.751	0.000	1.006	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	187	206	264	249	0	842	-1
normalized size	1	1.00	0.94	1.03	1.32	1.24	0.00	4.21	-0.00
time (sec)	N/A	0.318	1.268	0.000	0.782	0.516	0.000	4.123	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	206	251	285	347	0	1358	-1
normalized size	1	1.00	0.90	1.10	1.25	1.52	0.00	5.96	-0.00
time (sec)	N/A	0.378	2.325	0.000	2.022	0.608	0.000	4.929	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	820	521	0	3689	-1
normalized size	1	1.00	5.44	1.17	1.33	0.85	0.00	6.00	-0.00
time (sec)	N/A	1.153	24.098	0.047	0.902	1.013	0.000	15.988	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	760	427	0	2300	-1
normalized size	1	1.00	1.95	1.09	1.42	0.80	0.00	4.31	-0.00
time (sec)	N/A	0.879	11.684	0.045	0.924	0.992	0.000	4.627	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	674	356	0	1258	-1
normalized size	1	1.00	0.94	0.97	1.47	0.78	0.00	2.74	-0.00
time (sec)	N/A	0.671	7.338	0.045	0.836	1.013	0.000	2.743	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	432	447	674	356	0	1258	-1
normalized size	1	1.00	0.94	0.97	1.47	0.78	0.00	2.74	-0.00
time (sec)	N/A	0.658	7.259	0.000	0.570	0.671	0.000	2.849	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1041	580	760	427	0	2300	-1
normalized size	1	1.00	1.95	1.09	1.42	0.80	0.00	4.31	-0.00
time (sec)	N/A	0.801	11.455	0.000	0.798	0.626	0.000	4.214	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	3348	719	820	521	0	3689	-1
normalized size	1	1.00	5.44	1.17	1.33	0.85	0.00	6.00	-0.00
time (sec)	N/A	0.953	22.966	0.000	1.787	0.822	0.000	7.576	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	245	0	0	184	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.67	0.00	0.00	-0.00
time (sec)	N/A	0.288	0.237	0.176	0.000	0.601	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	158	1150	967	378	935	361	576
normalized size	1	1.00	0.61	4.42	3.72	1.45	3.60	1.39	2.22
time (sec)	N/A	0.234	1.849	0.089	0.392	0.699	13.230	0.261	1.335

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	135	594	549	238	602	241	366
normalized size	1	1.00	0.69	3.03	2.80	1.21	3.07	1.23	1.87
time (sec)	N/A	0.161	0.925	0.020	0.361	0.670	7.633	4.509	2.057

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	89	260	263	130	320	145	202
normalized size	1	1.00	0.66	1.94	1.96	0.97	2.39	1.08	1.51
time (sec)	N/A	0.087	0.462	0.018	0.340	0.640	3.792	2.963	1.634

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	75	85	92	58	138	75	94
normalized size	1	1.00	1.04	1.18	1.28	0.81	1.92	1.04	1.31
time (sec)	N/A	0.047	0.138	0.020	0.321	0.616	1.936	0.202	0.313

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	274	155	0	6046	-1
normalized size	1	1.00	0.85	1.38	2.12	1.20	0.00	46.87	-0.01
time (sec)	N/A	0.212	0.341	0.022	0.494	0.423	0.000	1.922	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	151	256	301	235	0	0	-1
normalized size	1	1.00	0.84	1.43	1.68	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.268	1.631	0.022	0.559	0.744	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	197	329	336	397	0	0	-1
normalized size	1	1.00	0.85	1.42	1.45	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.329	3.754	0.022	0.687	0.752	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	316	404	386	568	0	0	-1
normalized size	1	1.00	1.10	1.41	1.34	1.98	0.00	0.00	-0.00
time (sec)	N/A	0.451	2.490	0.020	1.896	0.705	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	409	0	0	276	0	0	-1
normalized size	1	1.00	0.98	0.00	0.00	0.66	0.00	0.00	-0.00
time (sec)	N/A	0.435	0.573	0.316	0.000	0.872	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	563	1842	1339	527	1098	531	816
normalized size	1	1.00	1.71	5.58	4.06	1.60	3.33	1.61	2.47
time (sec)	N/A	0.368	3.480	0.073	0.427	1.199	20.952	0.294	4.382

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	195	1016	766	342	690	351	516
normalized size	1	1.00	0.75	3.92	2.96	1.32	2.66	1.36	1.99
time (sec)	N/A	0.273	2.181	0.022	0.852	0.765	11.354	0.360	2.426

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	252	484	375	193	382	209	295
normalized size	1	1.00	1.37	2.63	2.04	1.05	2.08	1.14	1.60
time (sec)	N/A	0.190	0.960	0.020	0.600	0.507	6.228	2.110	0.845

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	110	175	139	91	163	106	119
normalized size	1	1.00	1.01	1.61	1.28	0.83	1.50	0.97	1.09
time (sec)	N/A	0.094	0.357	0.022	0.947	0.760	3.101	2.966	0.469

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	154	252	408	229	0	0	-1
normalized size	1	1.00	0.83	1.36	2.21	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.507	0.021	0.507	0.607	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	212	367	439	339	0	0	-1
normalized size	1	1.00	0.82	1.43	1.71	1.32	0.00	0.00	-0.00
time (sec)	N/A	0.345	2.105	0.023	0.566	0.508	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	283	473	474	567	0	0	-1
normalized size	1	1.00	0.84	1.40	1.40	1.68	0.00	0.00	-0.00
time (sec)	N/A	0.438	3.373	0.023	1.372	0.541	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	413	451	583	524	811	0	0	-1
normalized size	1	1.00	1.09	1.41	1.27	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.538	3.402	0.023	1.272	0.628	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	255	0	0	184	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.65	0.00	0.00	-0.00
time (sec)	N/A	0.317	3.338	0.252	0.000	0.497	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	153	2061	1033	546	1334	359	576
normalized size	1	1.00	0.66	8.85	4.43	2.34	5.73	1.54	2.47
time (sec)	N/A	0.266	1.537	0.132	0.391	0.503	31.302	1.126	2.585

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	132	1100	602	349	857	241	366
normalized size	1	1.00	0.73	6.08	3.33	1.93	4.73	1.33	2.02
time (sec)	N/A	0.219	2.313	0.023	0.349	0.458	18.632	0.379	1.227

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	91	498	303	194	461	145	202
normalized size	1	1.00	0.71	3.86	2.35	1.50	3.57	1.12	1.57
time (sec)	N/A	0.144	0.563	0.020	0.407	0.468	9.992	0.410	0.806

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	176	119	87	201	75	84
normalized size	1	1.00	0.82	2.29	1.55	1.13	2.61	0.97	1.09
time (sec)	N/A	0.074	0.219	0.020	0.796	0.441	5.332	0.241	0.709

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	110	178	274	156	0	6046	-1
normalized size	1	1.00	0.85	1.38	2.12	1.21	0.00	46.87	-0.01
time (sec)	N/A	0.246	0.308	0.023	0.724	0.440	0.000	0.568	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	189	256	301	248	0	0	-1
normalized size	1	1.00	1.06	1.43	1.68	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.297	0.955	0.025	0.485	0.483	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	239	329	336	434	0	0	-1
normalized size	1	1.00	1.02	1.40	1.43	1.85	0.00	0.00	-0.00
time (sec)	N/A	0.353	1.039	0.025	0.686	0.539	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	554	404	386	638	0	0	-1
normalized size	1	1.00	1.93	1.41	1.34	2.22	0.00	0.00	-0.00
time (sec)	N/A	0.419	4.944	0.024	0.859	0.561	0.000	0.000	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	7.774	0.125	0.000	0.459	0.000	0.000	0.000

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	2828	1326	1635	1453	0	0	-1
normalized size	1	1.00	9.21	4.32	5.33	4.73	0.00	0.00	-0.00
time (sec)	N/A	0.340	6.521	0.536	1.062	0.661	0.000	0.000	0.000

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	1918	899	967	984	0	0	-1
normalized size	1	1.00	7.80	3.65	3.93	4.00	0.00	0.00	-0.00
time (sec)	N/A	0.278	6.406	0.463	0.798	0.585	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	564	535	522	594	0	0	-1
normalized size	1	1.00	3.12	2.96	2.88	3.28	0.00	0.00	-0.01
time (sec)	N/A	0.227	2.904	0.493	0.460	0.562	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	131	249	222	292	0	0	-1
normalized size	1	1.00	1.15	2.18	1.95	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.346	0.411	0.787	0.588	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.813	0.302	0.000	0.462	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.170	2.493	0.461	0.000	0.458	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.239	9.893	0.096	0.000	0.427	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	798	1056	0	1233	0	0	-1
normalized size	1	1.00	2.67	3.53	0.00	4.12	0.00	0.00	-0.00
time (sec)	N/A	0.293	1.713	0.161	0.000	0.634	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	539	649	11018	797	0	0	-1
normalized size	1	1.00	2.50	3.00	51.01	3.69	0.00	0.00	-0.00
time (sec)	N/A	0.222	1.424	0.129	1.962	0.559	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	310	332	3284	448	0	0	-1
normalized size	1	1.00	2.23	2.39	23.63	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.149	3.915	0.131	1.665	0.497	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	104	124	2110	95	0	1967	162
normalized size	1	1.00	1.79	2.14	36.38	1.64	0.00	33.91	2.79
time (sec)	N/A	0.063	0.679	0.141	0.394	0.464	0.000	5.159	2.305

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.214	3.717	0.273	0.000	0.442	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.256	4.041	0.435	0.000	0.450	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.036	11.481	0.115	0.000	0.439	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	1534	1868	7111	1747	0	0	-1
normalized size	1	1.00	5.08	6.19	23.55	5.78	0.00	0.00	-0.00
time (sec)	N/A	0.462	7.120	0.204	4.852	0.583	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	994	1194	3952	1135	0	0	-1
normalized size	1	1.00	3.88	4.66	15.44	4.43	0.00	0.00	-0.00
time (sec)	N/A	0.368	6.893	0.139	1.737	0.550	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	540	635	1966	655	0	0	-1
normalized size	1	1.00	3.21	3.78	11.70	3.90	0.00	0.00	-0.01
time (sec)	N/A	0.266	6.681	0.112	0.709	0.474	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	240	281	839	339	0	0	-1
normalized size	1	1.00	2.20	2.58	7.70	3.11	0.00	0.00	-0.01
time (sec)	N/A	0.128	6.166	0.086	0.528	0.459	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.040	8.080	2.646	0.000	0.438	0.000	0.000	0.000

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	9.289	4.187	0.000	0.471	0.000	0.000	0.000

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	547	376	0	2418	-1
normalized size	1	1.00	1.35	1.15	1.34	0.92	0.00	5.94	-0.00
time (sec)	N/A	0.797	12.148	0.045	0.563	0.567	0.000	3.319	0.000

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	294	0	1503	-1
normalized size	1	1.00	1.12	1.07	1.43	0.84	0.00	4.28	-0.00
time (sec)	N/A	0.556	2.983	0.039	0.511	0.503	0.000	2.647	0.000

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	425	233	0	818	-1
normalized size	1	1.00	0.88	0.96	1.42	0.78	0.00	2.74	-0.00
time (sec)	N/A	0.447	0.590	0.038	0.537	0.510	0.000	2.818	0.000

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	286	425	233	0	818	-1
normalized size	1	1.00	0.88	0.96	1.42	0.78	0.00	2.74	-0.00
time (sec)	N/A	0.455	0.159	0.000	0.511	0.546	0.000	0.981	0.000

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	393	376	503	294	0	1503	-1
normalized size	1	1.00	1.12	1.07	1.43	0.84	0.00	4.28	-0.00
time (sec)	N/A	0.572	1.199	0.001	0.535	0.507	0.000	3.867	0.000

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	470	547	376	0	2418	-1
normalized size	1	1.00	1.35	1.15	1.34	0.92	0.00	5.94	-0.00
time (sec)	N/A	0.672	8.395	0.000	0.519	0.535	0.000	3.104	0.000

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	820	548	0	3677	-1
normalized size	1	1.00	2.92	1.16	1.33	0.89	0.00	5.98	-0.00
time (sec)	N/A	1.145	22.421	0.068	0.566	0.579	0.000	18.657	0.000

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	754	446	0	2293	-1
normalized size	1	1.00	1.95	1.09	1.41	0.84	0.00	4.29	-0.00
time (sec)	N/A	0.838	12.002	0.065	0.584	0.560	0.000	7.722	0.000

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	674	365	0	1258	-1
normalized size	1	1.00	0.95	0.97	1.47	0.80	0.00	2.74	-0.00
time (sec)	N/A	0.693	6.942	0.062	0.519	0.541	0.000	2.148	0.000

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	459	459	435	444	674	365	0	1258	-1
normalized size	1	1.00	0.95	0.97	1.47	0.80	0.00	2.74	-0.00
time (sec)	N/A	0.672	6.874	0.000	0.513	0.538	0.000	4.485	0.000

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	534	534	1043	583	754	446	0	2293	-1
normalized size	1	1.00	1.95	1.09	1.41	0.84	0.00	4.29	-0.00
time (sec)	N/A	0.858	11.710	0.000	0.535	0.585	0.000	7.042	0.000

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	C	A	F(-1)	C	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	615	615	1795	716	820	548	0	3677	-1
normalized size	1	1.00	2.92	1.16	1.33	0.89	0.00	5.98	-0.00
time (sec)	N/A	1.015	21.748	0.002	0.577	0.571	0.000	12.742	0.000

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	557	445	0	2417	-1
normalized size	1	1.00	1.35	1.17	1.37	1.09	0.00	5.94	-0.00
time (sec)	N/A	0.897	5.073	0.046	0.534	0.594	0.000	13.771	0.000

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	513	326	0	1502	-1
normalized size	1	1.00	1.11	1.09	1.46	0.93	0.00	4.28	-0.00
time (sec)	N/A	0.628	2.831	0.045	0.514	0.559	0.000	12.684	0.000

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	435	242	0	818	-1
normalized size	1	1.00	0.88	0.98	1.45	0.81	0.00	2.74	-0.00
time (sec)	N/A	0.458	1.250	0.043	0.525	0.511	0.000	7.825	0.000

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	264	293	435	242	0	818	-1
normalized size	1	1.00	0.88	0.98	1.45	0.81	0.00	2.74	-0.00
time (sec)	N/A	0.448	0.523	0.000	0.512	0.564	0.000	3.740	0.000

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	391	383	513	326	0	1502	-1
normalized size	1	1.00	1.11	1.09	1.46	0.93	0.00	4.28	-0.00
time (sec)	N/A	0.559	0.202	0.000	0.542	0.580	0.000	13.119	0.000

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	550	477	557	445	0	2417	-1
normalized size	1	1.00	1.35	1.17	1.37	1.09	0.00	5.94	-0.00
time (sec)	N/A	0.669	2.738	0.000	0.564	0.606	0.000	16.884	0.000

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	104	150	0	244	0	0	-1
normalized size	1	1.00	0.93	1.34	0.00	2.18	0.00	0.00	-0.01
time (sec)	N/A	0.185	0.171	0.122	0.000	0.510	0.000	0.000	0.000

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	112	0	162	0	0	-1
normalized size	1	1.00	0.87	1.35	0.00	1.95	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.100	0.111	0.000	0.484	0.000	0.000	0.000

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	76	0	45	0	206	56
normalized size	1	1.00	1.00	2.30	0.00	1.36	0.00	6.24	1.70
time (sec)	N/A	0.055	0.024	0.115	0.000	0.461	0.000	0.224	1.234

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	159	240	3719	508	0	0	-1
normalized size	1	1.00	0.88	1.33	20.66	2.82	0.00	0.00	-0.01
time (sec)	N/A	0.401	0.408	0.157	0.945	0.510	0.000	0.000	0.000

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	108	170	2855	370	0	0	-1
normalized size	1	1.00	1.02	1.60	26.93	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.278	0.332	0.164	0.752	0.536	0.000	0.000	0.000

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	62	109	1739	203	0	0	-1
normalized size	1	1.00	0.85	1.49	23.82	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.111	0.138	0.588	0.482	0.000	0.000	0.000

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.018	2.554	0.131	0.000	0.424	0.000	0.000	0.000

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	157	616	792	1402	0	0	-1
normalized size	1	1.00	0.99	3.90	5.01	8.87	0.00	0.00	-0.01
time (sec)	N/A	0.210	0.068	0.109	0.604	0.569	0.000	0.000	0.000

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	126	423	490	970	0	0	-1
normalized size	1	1.00	0.95	3.20	3.71	7.35	0.00	0.00	-0.01
time (sec)	N/A	0.183	0.085	0.079	0.573	0.537	0.000	0.000	0.000

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	100	257	280	594	0	0	-1
normalized size	1	1.00	1.04	2.68	2.92	6.19	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.041	0.064	0.529	0.496	0.000	0.000	0.000

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	70	123	114	310	0	0	148
normalized size	1	1.00	1.06	1.86	1.73	4.70	0.00	0.00	2.24
time (sec)	N/A	0.093	0.014	0.071	0.525	0.494	0.000	0.000	1.572

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.020	3.532	0.128	0.000	0.438	0.000	0.000	0.000

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.020	5.299	0.135	0.000	0.440	0.000	0.000	0.000

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	6.617	0.270	0.000	0.431	0.000	0.000	0.000

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	557	901	924	1071	0	0	-1
normalized size	1	1.00	2.03	3.28	3.36	3.89	0.00	0.00	-0.00
time (sec)	N/A	0.215	1.470	0.362	0.673	0.555	0.000	0.000	0.000

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	315	512	510	656	0	0	-1
normalized size	1	1.00	1.69	2.75	2.74	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.832	0.260	0.597	0.518	0.000	0.000	0.000

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	213	209	0	331	0	0	-1
normalized size	1	1.00	2.07	2.03	0.00	3.21	0.00	0.00	-0.01
time (sec)	N/A	0.068	0.402	0.057	0.000	1.036	0.000	0.000	0.000

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	6.038	0.269	0.000	0.434	0.000	0.000	0.000

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.139	7.080	0.422	0.000	0.428	0.000	0.000	0.000

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	152	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.171	7.810	0.252	0.000	0.444	0.000	0.000	0.000

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	1720	641	685	1134	0	0	-1
normalized size	1	1.00	6.85	2.55	2.73	4.52	0.00	0.00	-0.00
time (sec)	N/A	0.298	7.120	0.218	0.561	0.615	0.000	0.000	0.000

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	518	379	379	688	0	0	-1
normalized size	1	1.00	2.82	2.06	2.06	3.74	0.00	0.00	-0.01
time (sec)	N/A	0.228	6.469	0.327	0.531	0.543	0.000	0.000	0.000

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	134	179	145	346	0	0	-1
normalized size	1	1.00	1.17	1.56	1.26	3.01	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.300	0.339	0.516	0.495	0.000	0.000	0.000

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.151	0.763	0.453	0.000	0.438	0.000	0.000	0.000

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.176	2.401	0.693	0.000	0.422	0.000	0.000	0.000

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.183	5.892	0.060	0.000	0.431	0.000	0.000	0.000

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	578	1242	1779	2600	0	0	-1
normalized size	1	1.00	2.34	5.03	7.20	10.53	0.00	0.00	-0.00
time (sec)	N/A	0.229	1.285	0.170	0.714	0.658	0.000	0.000	0.000

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	350	816	1063	1778	0	0	-1
normalized size	1	1.00	1.78	4.14	5.40	9.03	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.946	0.119	0.610	0.604	0.000	0.000	0.000

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	213	469	590	1090	0	0	-1
normalized size	1	1.00	1.68	3.69	4.65	8.58	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.589	0.093	0.596	0.559	0.000	0.000	0.000

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	141	208	267	554	0	0	-1
normalized size	1	1.00	1.99	2.93	3.76	7.80	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.115	0.081	0.697	0.517	0.000	0.000	0.000

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.045	4.533	0.099	0.000	0.441	0.000	0.000	0.000

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.041	5.715	0.099	0.000	0.446	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.197	17.786	0.082	0.000	0.459	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	739	1158	3240	1753	0	0	-1
normalized size	1	1.00	2.11	3.31	9.26	5.01	0.00	0.00	-0.00
time (sec)	N/A	0.640	6.410	0.501	1.411	0.651	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	593	556	1632	1067	0	0	-1
normalized size	1	1.00	2.62	2.46	7.22	4.72	0.00	0.00	-0.00
time (sec)	N/A	0.384	6.275	0.277	0.774	0.560	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	517	235	0	434	0	0	-1
normalized size	1	1.00	3.95	1.79	0.00	3.31	0.00	0.00	-0.01
time (sec)	N/A	0.135	2.908	0.129	0.000	0.504	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.160	11.546	2.634	0.000	0.462	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.190	11.077	3.124	0.000	0.463	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.237	18.288	0.111	0.000	0.422	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	1477	1223	5140	3459	0	0	-1
normalized size	1	1.00	4.54	3.76	15.82	10.64	0.00	0.00	-0.00
time (sec)	N/A	0.823	6.975	0.187	2.137	0.772	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	872	632	2522	1987	0	0	-1
normalized size	1	1.00	4.34	3.14	12.55	9.89	0.00	0.00	-0.00
time (sec)	N/A	0.443	6.757	0.164	0.823	0.642	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	210	270	1035	942	0	0	-1
normalized size	1	1.00	1.49	1.91	7.34	6.68	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.898	0.125	0.625	0.541	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.147	14.068	3.382	0.000	0.530	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.192	16.231	5.592	0.000	0.638	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.142	2.700	0.080	0.000	0.439	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	428	767	2944	1186	0	0	-1
normalized size	1	1.00	1.89	3.38	12.97	5.22	0.00	0.00	-0.00
time (sec)	N/A	0.186	1.168	0.196	0.792	0.575	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	256	463	1774	779	0	0	-1
normalized size	1	1.00	1.61	2.91	11.16	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.807	0.128	0.621	0.530	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	174	239	0	446	0	0	-1
normalized size	1	1.00	1.79	2.46	0.00	4.60	0.00	0.00	-0.01
time (sec)	N/A	0.068	1.655	0.023	0.000	0.504	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	93	49	259	60	0	1537	78
normalized size	1	1.00	3.21	1.69	8.93	2.07	0.00	53.00	2.69
time (sec)	N/A	0.019	0.045	0.021	0.449	0.475	0.000	1.308	2.583

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	11.050	0.169	0.000	0.441	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.115	19.124	0.237	0.000	0.433	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.036	2.877	0.083	0.000	0.430	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	424	348	1363	373	0	0	-1
normalized size	1	1.00	3.31	2.72	10.65	2.91	0.00	0.00	-0.01
time (sec)	N/A	0.210	6.545	0.131	0.630	0.462	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	276	191	417	210	0	0	-1
normalized size	1	1.00	2.88	1.99	4.34	2.19	0.00	0.00	-0.01
time (sec)	N/A	0.141	6.368	0.089	0.602	0.465	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	76	47	237	53	65	223	52
normalized size	1	1.00	1.90	1.18	5.92	1.32	1.62	5.58	1.30
time (sec)	N/A	0.028	0.276	0.053	0.457	0.436	0.246	0.561	1.436

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.040	3.833	0.168	0.000	0.439	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	5.071	0.235	0.000	0.458	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	150	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.157	23.697	0.090	0.000	0.431	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	532	677	11054	892	0	0	-1
normalized size	1	1.00	2.33	2.97	48.48	3.91	0.00	0.00	-0.00
time (sec)	N/A	0.211	1.545	0.125	2.377	0.566	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	362	345	0	511	0	0	-1
normalized size	1	1.00	2.50	2.38	0.00	3.52	0.00	0.00	-0.01
time (sec)	N/A	0.131	3.118	0.126	0.000	0.533	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	107	123	2123	93	0	2762	151
normalized size	1	1.00	1.91	2.20	37.91	1.66	0.00	49.32	2.70
time (sec)	N/A	0.054	0.299	0.158	0.483	0.480	0.000	7.901	1.161

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	3.778	0.186	0.000	0.457	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	4.069	0.280	0.000	0.491	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.248	24.569	0.082	0.000	0.435	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	694	1866	5695	2507	0	0	-1
normalized size	1	1.00	1.48	3.98	12.14	5.35	0.00	0.00	-0.00
time (sec)	N/A	0.795	3.365	0.599	2.568	0.815	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	473	1152	3205	1697	0	0	-1
normalized size	1	1.00	1.38	3.36	9.34	4.95	0.00	0.00	-0.00
time (sec)	N/A	0.573	1.342	0.467	1.143	0.650	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	317	568	1598	1031	0	0	-1
normalized size	1	1.00	1.45	2.59	7.30	4.71	0.00	0.00	-0.00
time (sec)	N/A	0.377	2.472	0.266	0.687	0.569	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	122	212	160	806	366	0	0	-1
normalized size	1	1.08	1.88	1.42	7.13	3.24	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.485	0.106	0.619	0.471	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.153	9.652	2.332	0.000	0.438	0.000	0.000	0.000

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.180	9.758	2.560	0.000	0.446	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.190	2.633	0.076	0.000	0.427	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	285	687	2355	1627	0	0	-1
normalized size	1	1.00	2.42	5.82	19.96	13.79	0.00	0.00	-0.01
time (sec)	N/A	0.280	2.131	0.133	0.745	0.619	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	277	351	777	950	0	0	-1
normalized size	1	1.00	3.15	3.99	8.83	10.80	0.00	0.00	-0.01
time (sec)	N/A	0.191	1.716	0.119	0.686	0.558	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	32	182	308	75	0	0	55
normalized size	1	1.00	0.91	5.20	8.80	2.14	0.00	0.00	1.57
time (sec)	N/A	0.059	0.203	0.088	0.477	0.448	0.000	0.000	1.656

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.094	7.065	0.431	0.000	0.431	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.086	7.326	0.776	0.000	0.425	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.215	25.514	0.136	0.000	0.432	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	601	601	907	1613	8043	3173	0	0	-1
normalized size	1	1.00	1.51	2.68	13.38	5.28	0.00	0.00	-0.00
time (sec)	N/A	2.313	8.346	0.540	6.030	0.813	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	889	802	3820	1801	0	0	-1
normalized size	1	1.00	2.91	2.63	12.52	5.90	0.00	0.00	-0.00
time (sec)	N/A	0.865	7.971	0.307	1.616	0.656	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	174	520	267	1503	621	0	0	-1
normalized size	1	1.13	3.38	1.73	9.76	4.03	0.00	0.00	-0.01
time (sec)	N/A	0.191	4.923	0.158	0.900	0.517	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.201	22.219	2.670	0.000	0.506	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.224	30.370	5.195	0.000	0.584	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.924	39.651	0.113	0.000	0.452	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	672	0	3989	1735	0	0	-1
normalized size	1	1.00	1.74	0.00	10.31	4.48	0.00	0.00	-0.00
time (sec)	N/A	0.960	7.210	1.624	1.241	0.619	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	613	429	2219	1229	0	0	-1
normalized size	1	1.00	2.61	1.83	9.44	5.23	0.00	0.00	-0.00
time (sec)	N/A	0.537	6.623	0.224	0.743	0.562	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	282	182	1184	527	0	0	-1
normalized size	1	1.00	2.24	1.44	9.40	4.18	0.00	0.00	-0.01
time (sec)	N/A	0.168	2.409	0.141	0.685	0.512	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.500	47.009	0.779	0.000	0.439	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.513	19.784	0.882	0.000	0.452	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.152	5.405	0.095	0.000	0.448	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	418	489	3438	888	0	0	-1
normalized size	1	1.00	3.01	3.52	24.73	6.39	0.00	0.00	-0.01
time (sec)	N/A	0.258	6.561	0.111	0.684	0.572	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	286	301	667	540	0	0	-1
normalized size	1	1.00	2.49	2.62	5.80	4.70	0.00	0.00	-0.01
time (sec)	N/A	0.174	6.378	0.098	0.629	0.533	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	66	95	988	86	0	4474	150
normalized size	1	1.00	1.20	1.73	17.96	1.56	0.00	81.35	2.73
time (sec)	N/A	0.062	0.510	0.030	0.523	0.451	0.000	1.815	3.063

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	48	61	283	36	0	571	53
normalized size	1	1.00	1.37	1.74	8.09	1.03	0.00	16.31	1.51
time (sec)	N/A	0.032	0.061	0.030	0.351	0.409	0.000	1.844	2.187

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.106	7.244	0.278	0.000	0.488	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.128	10.385	0.292	0.000	0.448	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.084	180.018	0.090	0.000	0.422	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	530	1127	3828	1311	0	0	-1
normalized size	1	1.00	1.57	3.34	11.36	3.89	0.00	0.00	-0.00
time (sec)	N/A	0.408	3.400	0.208	2.009	0.639	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	526	584	1893	791	0	0	-1
normalized size	1	1.00	2.73	3.03	9.81	4.10	0.00	0.00	-0.01
time (sec)	N/A	0.271	7.183	0.174	0.851	0.548	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	555	267	0	435	0	0	-1
normalized size	1	1.00	4.74	2.28	0.00	3.72	0.00	0.00	-0.01
time (sec)	N/A	0.129	6.504	0.116	0.000	0.525	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.091	26.390	1.595	0.000	0.456	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.089	29.120	2.605	0.000	0.449	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.035	9.704	0.118	0.000	0.422	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	803	720	2405	590	0	0	-1
normalized size	1	1.00	3.10	2.78	9.29	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.356	6.886	0.127	1.186	0.454	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	454	400	1226	352	0	0	-1
normalized size	1	1.00	2.69	2.37	7.25	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.222	6.656	0.098	0.673	0.455	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	240	183	519	168	0	0	-1
normalized size	1	1.00	2.22	1.69	4.81	1.56	0.00	0.00	-0.01
time (sec)	N/A	0.117	6.153	0.076	0.583	0.430	0.000	0.000	0.000

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	6.531	1.759	0.000	0.425	0.000	0.000	0.000

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.037	6.961	2.733	0.000	0.417	0.000	0.000	0.000

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.269	13.368	0.105	0.000	0.415	0.000	0.000	0.000

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	2090	1729	8770	3308	0	0	-1
normalized size	1	1.00	5.24	4.33	21.98	8.29	0.00	0.00	-0.00
time (sec)	N/A	0.971	7.470	0.247	6.051	1.046	0.000	0.000	0.000

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	1486	1115	4918	2260	0	0	-1
normalized size	1	1.00	4.57	3.43	15.13	6.95	0.00	0.00	-0.00
time (sec)	N/A	0.640	6.909	0.162	2.019	0.830	0.000	0.000	0.000

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	875	614	2442	1396	0	0	-1
normalized size	1	1.00	4.35	3.05	12.15	6.95	0.00	0.00	-0.00
time (sec)	N/A	0.414	6.757	0.132	0.815	0.654	0.000	0.000	0.000

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	141	212	270	1035	760	0	0	-1
normalized size	1	1.01	1.53	1.94	7.45	5.47	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.564	0.109	0.633	0.558	0.000	0.000	0.000

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.189	9.076	3.352	0.000	0.525	0.000	0.000	0.000

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.209	6.651	5.624	0.000	0.629	0.000	0.000	0.000

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.233	30.513	0.139	0.000	0.462	0.000	0.000	0.000

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	486	486	819	1629	8032	2218	0	0	-1
normalized size	1	1.00	1.69	3.35	16.53	4.56	0.00	0.00	-0.00
time (sec)	N/A	1.206	7.897	0.560	6.459	0.837	0.000	0.000	0.000

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	889	770	3819	1362	0	0	-1
normalized size	1	1.00	2.61	2.26	11.20	3.99	0.00	0.00	-0.00
time (sec)	N/A	0.648	7.530	0.316	1.622	0.649	0.000	0.000	0.000

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	162	182	669	344	0	592	0	0	-1
normalized size	1	1.12	4.13	2.12	0.00	3.65	0.00	0.00	-0.01
time (sec)	N/A	0.196	6.588	0.164	0.000	0.537	0.000	0.000	0.000

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.187	20.743	3.249	0.000	0.527	0.000	0.000	0.000

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.197	25.884	4.286	0.000	0.578	0.000	0.000	0.000

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.265	32.372	0.118	0.000	0.430	0.000	0.000	0.000

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	483	1329	5610	4193	0	0	-1
normalized size	1	1.00	1.52	4.18	17.64	13.19	0.00	0.00	-0.00
time (sec)	N/A	0.321	8.626	0.218	3.734	0.945	0.000	0.000	0.000

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	381	716	2722	2387	0	0	-1
normalized size	1	1.00	2.01	3.77	14.33	12.56	0.00	0.00	-0.01
time (sec)	N/A	0.213	8.177	0.157	1.102	0.700	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	236	325	1078	1193	0	0	-1
normalized size	1	1.00	2.15	2.95	9.80	10.85	0.00	0.00	-0.01
time (sec)	N/A	0.106	2.088	0.171	0.748	0.596	0.000	0.000	0.000

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.087	28.703	1.049	0.000	0.512	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.088	32.466	3.790	0.000	0.568	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.330	0.080	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	51	0	0	0	0	0	-1
normalized size	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.040	0.375	0.042	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	52	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.039	0.152	0.039	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	181	310	0	0	0	0	-1
normalized size	1	1.00	5.48	9.39	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	1.759	0.156	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.026	0.160	0.039	0.000	0.000	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.185	0.039	0.000	0.000	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.038	0.230	0.039	0.000	0.000	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	65	0	0	0	0	0	-1
normalized size	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.048	0.256	0.043	0.000	0.000	0.000	0.000	0.000

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	65	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.280	0.060	0.000	0.000	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	61	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.049	0.231	0.048	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	54	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.053	0.184	0.049	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	42	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.036	0.127	0.049	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	132	310	0	0	0	0	-1
normalized size	1	1.00	2.49	5.85	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.037	2.154	0.083	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	63	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.236	0.048	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	212	0	0	0	0	0	-1
normalized size	1	1.00	2.65	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.047	8.104	0.046	0.000	0.000	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	89	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.338	0.045	0.000	0.000	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.541	0.056	0.000	0.000	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	108	0	0	0	0	0	-1
normalized size	1	1.00	1.66	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.903	0.046	0.000	0.000	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.189	0.041	0.000	0.000	0.000	0.000	0.000

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	86	308	0	0	0	0	-1
normalized size	1	1.00	2.26	8.11	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.022	1.159	0.115	0.000	0.000	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	37	0	0	0	0	0	-1
normalized size	1	1.00	0.97	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.023	0.177	0.040	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	56	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.179	0.042	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	57	0	0	0	0	0	-1
normalized size	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.228	0.043	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	73	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.288	0.043	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	73	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	0.266	0.076	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.292	0.060	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	56	0	0	0	0	0	-1
normalized size	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	0.178	0.064	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	46	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.137	0.061	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	106	308	0	0	0	0	-1
normalized size	1	1.00	1.83	5.31	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.032	0.724	0.125	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	65	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.225	0.053	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	114	0	0	0	0	0	-1
normalized size	1	1.00	1.34	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.044	0.983	0.052	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	93	0	0	0	0	0	-1
normalized size	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.394	0.053	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	22	26	18	17	37	18	18
normalized size	1	1.00	0.71	0.84	0.58	0.55	1.19	0.58	0.58
time (sec)	N/A	0.041	0.018	0.055	0.318	0.449	1.884	0.150	0.092

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	154	260	146	200	440	167	212
normalized size	1	1.00	1.18	1.98	1.11	1.53	3.36	1.27	1.62
time (sec)	N/A	0.188	0.224	0.078	0.346	0.447	25.349	2.643	2.261

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	109	179	101	127	289	112	136
normalized size	1	1.00	0.95	1.56	0.88	1.10	2.51	0.97	1.18
time (sec)	N/A	0.141	0.163	0.058	0.338	0.471	13.049	0.294	0.343

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	107	60	70	155	64	73
normalized size	1	1.00	0.82	1.47	0.82	0.96	2.12	0.88	1.00
time (sec)	N/A	0.103	0.109	0.056	0.333	0.455	6.972	0.163	1.815

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	52	27	27	56	27	30
normalized size	1	1.00	0.83	1.27	0.66	0.66	1.37	0.66	0.73
time (sec)	N/A	0.056	0.018	0.056	0.342	0.439	3.852	1.506	1.715

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	49	58	95	62	0	51	-1
normalized size	1	1.00	0.86	1.02	1.67	1.09	0.00	0.89	-0.02
time (sec)	N/A	0.252	0.061	0.056	0.382	0.440	0.000	2.954	0.000

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	61	82	324	95	0	111	-1
normalized size	1	1.00	0.78	1.05	4.15	1.22	0.00	1.42	-0.01
time (sec)	N/A	0.238	0.138	0.054	0.401	0.479	0.000	0.196	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	77	104	362	158	0	201	-1
normalized size	1	1.00	0.78	1.05	3.66	1.60	0.00	2.03	-0.01
time (sec)	N/A	0.329	0.243	0.058	0.422	0.443	0.000	2.727	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	128	1000	244	283	0	4684	344
normalized size	1	1.00	0.65	5.05	1.23	1.43	0.00	23.66	1.74
time (sec)	N/A	0.252	0.683	0.056	0.401	0.449	0.000	0.553	0.650

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	105	580	173	188	0	3139	216
normalized size	1	1.00	0.61	3.39	1.01	1.10	0.00	18.36	1.26
time (sec)	N/A	0.184	0.422	0.036	0.372	0.449	0.000	4.428	2.182

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	73	294	108	111	0	1880	121
normalized size	1	1.00	0.65	2.62	0.96	0.99	0.00	16.79	1.08
time (sec)	N/A	0.136	0.400	0.033	0.353	0.446	0.000	5.766	0.313

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	46	119	55	54	0	920	53
normalized size	1	1.00	0.70	1.80	0.83	0.82	0.00	13.94	0.80
time (sec)	N/A	0.068	0.143	0.032	0.345	0.436	0.000	1.992	0.205

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	63	116	117	85	0	1118	-1
normalized size	1	1.00	0.89	1.63	1.65	1.20	0.00	15.75	-0.01
time (sec)	N/A	0.281	0.148	0.040	0.406	0.457	0.000	0.323	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	81	169	118	131	0	5381	-1
normalized size	1	1.00	0.79	1.66	1.16	1.28	0.00	52.75	-0.01
time (sec)	N/A	0.276	0.532	0.043	0.421	0.446	0.000	10.367	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	C	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	104	207	130	225	0	9416	-1
normalized size	1	1.00	0.76	1.52	0.96	1.65	0.00	69.24	-0.01
time (sec)	N/A	0.374	0.989	0.041	0.430	0.460	0.000	10.030	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	C	A	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	125	243	141	343	0	0	-1
normalized size	1	1.00	0.61	1.19	0.69	1.67	0.00	0.00	-0.00
time (sec)	N/A	0.380	1.142	0.040	0.447	0.495	0.000	0.000	0.000

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	459	849	602	925	0	0	-1
normalized size	1	1.00	1.80	3.33	2.36	3.63	0.00	0.00	-0.00
time (sec)	N/A	0.347	1.537	0.121	0.543	0.558	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	223	481	409	562	0	0	-1
normalized size	1	1.00	1.30	2.80	2.38	3.27	0.00	0.00	-0.01
time (sec)	N/A	0.229	1.096	0.204	0.491	0.541	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	171	204	0	281	0	0	-1
normalized size	1	1.00	1.80	2.15	0.00	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.304	0.068	0.000	0.478	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.210	6.230	0.280	0.000	0.435	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.283	6.714	0.471	0.000	0.420	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	115	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.328	7.140	0.753	0.000	0.438	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	2482	956	607	1644	0	0	-1
normalized size	1	1.00	8.30	3.20	2.03	5.50	0.00	0.00	-0.00
time (sec)	N/A	0.503	6.746	0.128	0.517	0.644	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	1719	639	442	1122	0	0	-1
normalized size	1	1.00	7.10	2.64	1.83	4.64	0.00	0.00	-0.00
time (sec)	N/A	0.446	6.638	0.115	0.480	0.603	0.000	0.000	0.000

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	516	377	301	677	0	0	-1
normalized size	1	1.00	2.98	2.18	1.74	3.91	0.00	0.00	-0.01
time (sec)	N/A	0.328	6.563	0.133	0.447	0.554	0.000	0.000	0.000

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	254	177	0	340	0	0	-1
normalized size	1	1.00	2.37	1.65	0.00	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.180	5.574	0.310	0.000	0.528	0.000	0.000	0.000

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	80	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.299	3.226	0.305	0.000	0.436	0.000	0.000	0.000

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-2)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	103	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.342	4.208	0.493	0.000	0.442	0.000	0.000	0.000

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	129	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.398	6.019	0.643	0.000	0.443	0.000	0.000	0.000

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	607	677	0	896	0	0	-1
normalized size	1	1.00	2.64	2.94	0.00	3.90	0.00	0.00	-0.00
time (sec)	N/A	0.330	2.599	0.180	0.000	0.555	0.000	0.000	0.000

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	B	F(-2)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	364	328	0	513	0	0	-1
normalized size	1	1.00	2.48	2.23	0.00	3.49	0.00	0.00	-0.01
time (sec)	N/A	0.212	3.829	0.073	0.000	0.535	0.000	0.000	0.000

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	105	87	3330	93	0	2876	150
normalized size	1	1.00	1.84	1.53	58.42	1.63	0.00	50.46	2.63
time (sec)	N/A	0.091	0.483	0.041	1.121	0.444	0.000	1.789	1.305

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.266	13.726	0.338	0.000	0.454	0.000	0.000	0.000

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	98	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.344	16.576	0.442	0.000	1.607	0.000	0.000	0.000

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.436	19.134	0.769	0.000	0.514	0.000	0.000	0.000

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	77	66	0	106	0	0	46
normalized size	1	1.00	1.35	1.16	0.00	1.86	0.00	0.00	0.81
time (sec)	N/A	0.067	0.031	0.059	0.000	1.227	0.000	0.000	2.214

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	15	111	26	144	118	31
normalized size	1	1.00	1.00	1.07	7.93	1.86	10.29	8.43	2.21
time (sec)	N/A	0.033	0.020	0.056	0.418	0.564	5.739	1.553	2.347

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	146	102	0	144	0	0	63
normalized size	1	1.00	2.18	1.52	0.00	2.15	0.00	0.00	0.94
time (sec)	N/A	0.137	0.281	0.209	0.000	2.067	0.000	0.000	2.317

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [205] had the largest ratio of [1.250]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	20	0.200
2	A	5	4	1.00	20	0.200
3	A	5	5	1.00	20	0.250
4	A	3	2	1.00	20	0.100
5	A	3	3	1.00	18	0.167
6	A	5	5	1.00	20	0.250
7	A	6	6	1.00	20	0.300
8	A	7	6	1.00	20	0.300
9	A	8	6	1.00	20	0.300
10	A	3	3	1.00	8	0.375
11	A	4	4	1.00	8	0.500
12	A	5	4	1.00	8	0.500
13	A	8	3	1.00	22	0.136
14	A	9	5	1.00	22	0.227
15	A	7	5	1.00	22	0.227
16	A	4	4	1.00	22	0.182
17	A	3	2	1.00	20	0.100
18	A	8	4	1.00	22	0.182
19	A	10	5	1.00	22	0.227
20	A	12	5	1.00	22	0.227
21	A	14	5	1.00	22	0.227
22	A	8	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	9	4	1.00	22	0.182
24	A	9	5	1.00	22	0.227
25	A	4	2	1.00	22	0.091
26	A	4	3	1.00	20	0.150
27	A	8	4	1.00	22	0.182
28	A	10	5	1.00	22	0.227
29	A	12	5	1.00	22	0.227
30	A	14	5	1.00	22	0.227
31	A	0	0	0.00	0	0.000
32	A	7	6	1.00	14	0.429
33	A	6	6	1.00	14	0.429
34	A	5	5	1.00	14	0.357
35	A	4	4	1.00	12	0.333
36	A	0	0	0.00	0	0.000
37	A	0	0	0.00	0	0.000
38	A	0	0	0.00	0	0.000
39	A	10	6	1.00	20	0.300
40	A	8	5	1.00	20	0.250
41	A	6	4	1.00	20	0.200
42	A	2	2	1.00	18	0.111
43	A	0	0	0.00	0	0.000
44	A	0	0	0.00	0	0.000
45	A	0	0	0.00	0	0.000
46	A	7	7	1.00	22	0.318
47	A	6	6	1.00	22	0.273
48	A	3	3	1.00	22	0.136

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
49	A	3	3	1.00	20	0.150
50	A	0	0	0.00	0	0.000
51	A	0	0	0.00	0	0.000
52	A	10	8	1.00	22	0.364
53	A	9	8	1.00	22	0.364
54	A	8	8	1.00	22	0.364
55	A	8	8	1.00	22	0.364
56	A	9	8	1.00	22	0.364
57	A	10	8	1.00	22	0.364
58	A	18	7	1.00	24	0.292
59	A	16	7	1.00	24	0.292
60	A	14	7	1.00	24	0.292
61	A	14	7	1.00	24	0.292
62	A	16	7	1.00	24	0.292
63	A	18	7	1.00	24	0.292
64	A	18	7	1.00	24	0.292
65	A	16	7	1.00	24	0.292
66	A	14	7	1.00	24	0.292
67	A	14	7	1.00	24	0.292
68	A	16	7	1.00	24	0.292
69	A	18	7	1.00	24	0.292
70	A	8	3	1.00	22	0.136
71	A	9	5	1.00	22	0.227
72	A	7	5	1.00	22	0.227
73	A	4	4	1.00	22	0.182
74	A	3	2	1.00	20	0.100

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
75	A	8	4	1.00	22	0.182
76	A	10	5	1.00	22	0.227
77	A	12	5	1.00	22	0.227
78	A	14	5	1.00	22	0.227
79	A	5	3	1.00	24	0.125
80	A	7	3	1.00	24	0.125
81	A	6	3	1.00	24	0.125
82	A	5	3	1.00	24	0.125
83	A	4	3	1.00	22	0.136
84	A	5	4	1.00	24	0.167
85	A	6	5	1.00	24	0.208
86	A	7	5	1.00	24	0.208
87	A	8	5	1.00	24	0.208
88	A	11	3	1.00	24	0.125
89	A	17	3	1.00	24	0.125
90	A	14	3	1.00	24	0.125
91	A	11	3	1.00	24	0.125
92	A	8	3	1.00	22	0.136
93	A	11	4	1.00	24	0.167
94	A	14	5	1.00	24	0.208
95	A	17	5	1.00	24	0.208
96	A	20	5	1.00	24	0.208
97	A	0	0	0.00	0	0.000
98	A	17	8	1.00	20	0.400
99	A	14	8	1.00	20	0.400
100	A	11	7	1.00	20	0.350

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
101	A	8	6	1.00	18	0.333
102	A	0	0	0.00	0	0.000
103	A	0	0	0.00	0	0.000
104	A	0	0	0.00	0	0.000
105	A	8	8	1.00	16	0.500
106	A	7	7	1.00	16	0.438
107	A	6	6	1.00	16	0.375
108	A	3	2	1.00	14	0.143
109	A	0	0	0.00	0	0.000
110	A	0	0	0.00	0	0.000
111	A	0	0	0.00	0	0.000
112	A	31	7	1.00	22	0.318
113	A	25	9	1.00	22	0.409
114	A	17	7	1.00	22	0.318
115	A	12	5	1.00	20	0.250
116	A	0	0	0.00	0	0.000
117	A	0	0	0.00	0	0.000
118	A	18	7	1.00	24	0.292
119	A	16	7	1.00	24	0.292
120	A	14	7	1.00	24	0.292
121	A	14	7	1.00	24	0.292
122	A	16	7	1.00	24	0.292
123	A	18	7	1.00	24	0.292
124	A	10	7	1.00	26	0.269
125	A	9	7	1.00	26	0.269
126	A	8	7	1.00	26	0.269

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
127	A	8	7	1.00	26	0.269
128	A	9	7	1.00	26	0.269
129	A	10	7	1.00	26	0.269
130	A	26	7	1.00	26	0.269
131	A	23	7	1.00	26	0.269
132	A	20	7	1.00	26	0.269
133	A	20	7	1.00	26	0.269
134	A	23	7	1.00	26	0.269
135	A	26	7	1.00	26	0.269
136	A	8	3	1.00	22	0.136
137	A	9	4	1.00	22	0.182
138	A	9	5	1.00	22	0.227
139	A	4	2	1.00	22	0.091
140	A	4	3	1.00	20	0.150
141	A	8	4	1.00	22	0.182
142	A	10	5	1.00	22	0.227
143	A	12	5	1.00	22	0.227
144	A	14	5	1.00	22	0.227
145	A	11	3	1.00	24	0.125
146	A	17	3	1.00	24	0.125
147	A	14	3	1.00	24	0.125
148	A	11	3	1.00	24	0.125
149	A	8	3	1.00	22	0.136
150	A	11	4	1.00	24	0.167
151	A	14	5	1.00	24	0.208
152	A	17	5	1.00	24	0.208

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
153	A	20	5	1.00	24	0.208
154	A	8	3	1.00	24	0.125
155	A	12	3	1.00	24	0.125
156	A	10	3	1.00	24	0.125
157	A	8	3	1.00	24	0.125
158	A	6	3	1.00	22	0.136
159	A	8	4	1.00	24	0.167
160	A	10	5	1.00	24	0.208
161	A	12	5	1.00	24	0.208
162	A	14	5	1.00	24	0.208
163	A	0	0	0.00	0	0.000
164	A	13	11	1.00	22	0.500
165	A	12	12	1.00	22	0.546
166	A	9	8	1.00	22	0.364
167	A	8	8	1.00	20	0.400
168	A	0	0	0.00	0	0.000
169	A	0	0	0.00	0	0.000
170	A	0	0	0.00	0	0.000
171	A	16	9	1.00	22	0.409
172	A	13	8	1.00	22	0.364
173	A	10	7	1.00	22	0.318
174	A	5	5	1.00	20	0.250
175	A	0	0	0.00	0	0.000
176	A	0	0	0.00	0	0.000
177	A	0	0	0.00	0	0.000
178	A	15	8	1.00	16	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
179	A	13	10	1.00	16	0.625
180	A	9	7	1.00	16	0.438
181	A	7	7	1.00	14	0.500
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000
184	A	18	7	1.00	24	0.292
185	A	16	7	1.00	24	0.292
186	A	14	7	1.00	24	0.292
187	A	14	7	1.00	24	0.292
188	A	16	7	1.00	24	0.292
189	A	18	7	1.00	24	0.292
190	A	26	7	1.00	26	0.269
191	A	23	7	1.00	26	0.269
192	A	20	7	1.00	26	0.269
193	A	20	7	1.00	26	0.269
194	A	23	7	1.00	26	0.269
195	A	26	7	1.00	26	0.269
196	A	18	7	1.00	26	0.269
197	A	16	7	1.00	26	0.269
198	A	14	7	1.00	26	0.269
199	A	14	7	1.00	26	0.269
200	A	16	7	1.00	26	0.269
201	A	18	7	1.00	26	0.269
202	A	12	10	1.00	12	0.833
203	A	11	10	1.00	12	0.833
204	A	6	5	1.00	10	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
205	A	26	15	1.00	12	1.250
206	A	19	11	1.00	12	0.917
207	A	16	10	1.00	10	1.000
208	A	0	0	0.00	0	0.000
209	A	7	6	1.00	14	0.429
210	A	6	6	1.00	14	0.429
211	A	5	5	1.00	14	0.357
212	A	4	4	1.00	12	0.333
213	A	0	0	0.00	0	0.000
214	A	0	0	0.00	0	0.000
215	A	0	0	0.00	0	0.000
216	A	14	8	1.00	20	0.400
217	A	11	7	1.00	20	0.350
218	A	8	6	1.00	18	0.333
219	A	0	0	0.00	0	0.000
220	A	0	0	0.00	0	0.000
221	A	0	0	0.00	0	0.000
222	A	12	12	1.00	22	0.546
223	A	9	8	1.00	22	0.364
224	A	8	8	1.00	20	0.400
225	A	0	0	0.00	0	0.000
226	A	0	0	0.00	0	0.000
227	A	0	0	0.00	0	0.000
228	A	12	6	1.00	20	0.300
229	A	10	6	1.00	20	0.300
230	A	8	5	1.00	20	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
231	A	6	4	1.00	18	0.222
232	A	0	0	0.00	0	0.000
233	A	0	0	0.00	0	0.000
234	A	0	0	0.00	0	0.000
235	A	23	14	1.00	22	0.636
236	A	19	15	1.00	22	0.682
237	A	10	10	1.00	20	0.500
238	A	0	0	0.00	0	0.000
239	A	0	0	0.00	0	0.000
240	A	0	0	0.00	0	0.000
241	A	22	18	1.00	22	0.818
242	A	17	13	1.00	22	0.591
243	A	11	10	1.00	20	0.500
244	A	0	0	0.00	0	0.000
245	A	0	0	0.00	0	0.000
246	A	0	0	0.00	0	0.000
247	A	10	6	1.00	20	0.300
248	A	8	5	1.00	20	0.250
249	A	6	4	1.00	20	0.200
250	A	2	2	1.00	18	0.111
251	A	0	0	0.00	0	0.000
252	A	0	0	0.00	0	0.000
253	A	0	0	0.00	0	0.000
254	A	7	7	1.00	16	0.438
255	A	6	6	1.00	16	0.375
256	A	3	2	1.00	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
257	A	0	0	0.00	0	0.000
258	A	0	0	0.00	0	0.000
259	A	0	0	0.00	0	0.000
260	A	13	8	1.00	22	0.364
261	A	10	7	1.00	22	0.318
262	A	5	5	1.00	20	0.250
263	A	0	0	0.00	0	0.000
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	27	14	1.00	22	0.636
267	A	23	14	1.00	22	0.636
268	A	19	15	1.00	22	0.682
269	A	10	10	1.08	20	0.500
270	A	0	0	0.00	0	0.000
271	A	0	0	0.00	0	0.000
272	A	0	0	0.00	0	0.000
273	A	7	7	1.00	24	0.292
274	A	6	6	1.00	24	0.250
275	A	3	3	1.00	22	0.136
276	A	0	0	0.00	0	0.000
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	64	24	1.00	24	1.000
280	A	36	22	1.00	24	0.917
281	A	13	12	1.13	22	0.546
282	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
283	A	0	0	0.00	0	0.000
284	A	0	0	0.00	0	0.000
285	A	40	18	1.00	20	0.900
286	A	29	19	1.00	20	0.950
287	A	13	12	1.00	18	0.667
288	A	0	0	0.00	0	0.000
289	A	0	0	0.00	0	0.000
290	A	0	0	0.00	0	0.000
291	A	7	7	1.00	22	0.318
292	A	6	6	1.00	22	0.273
293	A	3	3	1.00	22	0.136
294	A	3	3	1.00	20	0.150
295	A	0	0	0.00	0	0.000
296	A	0	0	0.00	0	0.000
297	A	0	0	0.00	0	0.000
298	A	25	9	1.00	22	0.409
299	A	17	7	1.00	22	0.318
300	A	12	5	1.00	20	0.250
301	A	0	0	0.00	0	0.000
302	A	0	0	0.00	0	0.000
303	A	0	0	0.00	0	0.000
304	A	13	10	1.00	16	0.625
305	A	9	7	1.00	16	0.438
306	A	7	7	1.00	14	0.500
307	A	0	0	0.00	0	0.000
308	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	0	0	0.00	0	0.000
310	A	25	16	1.00	22	0.727
311	A	22	18	1.00	22	0.818
312	A	17	13	1.00	22	0.591
313	A	11	10	1.01	20	0.500
314	A	0	0	0.00	0	0.000
315	A	0	0	0.00	0	0.000
316	A	0	0	0.00	0	0.000
317	A	44	19	1.00	24	0.792
318	A	31	19	1.00	24	0.792
319	A	13	12	1.12	22	0.546
320	A	0	0	0.00	0	0.000
321	A	0	0	0.00	0	0.000
322	A	0	0	0.00	0	0.000
323	A	16	9	1.00	24	0.375
324	A	10	7	1.00	24	0.292
325	A	7	5	1.00	22	0.227
326	A	0	0	0.00	0	0.000
327	A	0	0	0.00	0	0.000
328	A	4	3	1.00	18	0.167
329	A	3	3	1.00	18	0.167
330	A	3	3	1.00	18	0.167
331	A	2	2	1.00	18	0.111
332	A	2	2	1.00	18	0.111
333	A	3	3	1.00	18	0.167
334	A	3	3	1.00	18	0.167

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
335	A	4	3	1.00	18	0.167
336	A	5	4	1.00	18	0.222
337	A	4	4	1.00	18	0.222
338	A	4	4	1.00	18	0.222
339	A	3	3	1.00	18	0.167
340	A	3	3	1.00	18	0.167
341	A	4	4	1.00	18	0.222
342	A	4	4	1.00	18	0.222
343	A	5	4	1.00	18	0.222
344	A	4	3	1.00	18	0.167
345	A	3	3	1.00	18	0.167
346	A	3	3	1.00	18	0.167
347	A	2	2	1.00	18	0.111
348	A	2	2	1.00	18	0.111
349	A	3	3	1.00	18	0.167
350	A	3	3	1.00	18	0.167
351	A	4	3	1.00	18	0.167
352	A	5	4	1.00	18	0.222
353	A	4	4	1.00	18	0.222
354	A	4	4	1.00	18	0.222
355	A	3	3	1.00	18	0.167
356	A	3	3	1.00	18	0.167
357	A	4	4	1.00	18	0.222
358	A	4	4	1.00	18	0.222
359	A	5	4	1.00	18	0.222
360	A	6	3	1.00	8	0.375

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	14	5	1.00	14	0.357
362	A	10	4	1.00	14	0.286
363	A	10	5	1.00	14	0.357
364	A	6	2	1.00	12	0.167
365	A	12	5	1.00	14	0.357
366	A	12	6	1.00	14	0.429
367	A	16	7	1.00	14	0.500
368	A	14	5	1.00	23	0.217
369	A	10	4	1.00	23	0.174
370	A	10	5	1.00	23	0.217
371	A	6	2	1.00	21	0.095
372	A	12	5	1.00	23	0.217
373	A	12	6	1.00	23	0.261
374	A	16	7	1.00	23	0.304
375	A	16	8	1.00	23	0.348
376	A	20	9	1.00	25	0.360
377	A	16	8	1.00	25	0.320
378	A	12	7	1.00	23	0.304
379	A	0	0	0.00	0	0.000
380	A	0	0	0.00	0	0.000
381	A	0	0	0.00	0	0.000
382	A	20	12	1.00	23	0.522
383	A	19	13	1.00	23	0.565
384	A	14	9	1.00	23	0.391
385	A	13	9	1.00	21	0.429
386	A	0	0	0.00	0	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
387	A	0	0	0.00	0	0.000
388	A	0	0	0.00	0	0.000
389	A	19	9	1.00	25	0.360
390	A	15	8	1.00	25	0.320
391	A	9	6	1.00	23	0.261
392	A	0	0	0.00	0	0.000
393	A	0	0	0.00	0	0.000
394	A	0	0	0.00	0	0.000
395	A	12	7	1.00	8	0.875
396	A	5	4	1.00	10	0.400
397	A	19	6	1.00	10	0.600

Chapter 3

Listing of integrals

3.1 $\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=137

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2ib(c\right.}{d}}{b}$$

```
[Out] -2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.16, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4406, 12, 3308, 2181}

$$-\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamm}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-3 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx &= \int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx\\ &= \frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx\\ &= \frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx - \frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx\\ &= -\frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2i}}{}\end{aligned}$$

Mathematica [A] time = 0.08, size = 138, normalized size = 1.01

$$-\frac{2^{-m-3}e^{-\frac{2i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{4ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)+e^{\frac{4ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((2^(-3 - m)*(c + d*x)^m*(E^((4*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, (
(-2*I)*b*(c + d*x))/d] + E^(((4*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1
 + m, ((2*I)*b*(c + d*x))/d]))/(b*E^(((2*I)*(b*c + a*d))/d)*((b^2*(c + d*x)
^2)/d^2)^m))
```

fricas [A] time = 0.48, size = 94, normalized size = 0.69

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)+e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(m+1,\frac{-2ibdx-2ibc}{d}\right)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m + 1, (2*I*b*d*x
 + 2*I*b*c)/d) + e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)/d)*gamma(m + 1
, (-2*I*b*d*x - 2*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,\left(c+d\,x\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x), x)
```

3.2 $\int (c+dx)^4 \cos(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=156

$$\frac{3d^4\sin^2(a+bx)}{4b^5}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}+\frac{d(c+dx)^3\sin(a+bx)\cos(a}{b^2}$$

```
[Out] 3/2*c*d^3*x/b^3+3/4*d^4*x^2/b^3-1/4*(d*x+c)^4/b-3/2*d^3*(d*x+c)*cos(b*x+a)*
sin(b*x+a)/b^4+d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3/4*d^4*sin(b*x+a)^2/b
^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+1/2*(d*x+c)^4*sin(b*x+a)^2/b
```

Rubi [A] time = 0.11, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{d(c+dx)^3\sin(a+bx)\cos(a+bx)}{b^2}+\frac{3d^4\sin^2(}{4b^5}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*c*d^3*x)/(2*b^3) + (3*d^4*x^2)/(4*b^3) - (c + d*x)^4/(4*b) - (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 + (3*d^4*Sin[a + b*x]^2)/(4*b^5) - (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) + ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx &= \frac{(c+dx)^4\sin^2(a+bx)}{2b}-\frac{(2d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b}\\ &= \frac{d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2}-\frac{3d^2(c+dx)^2\sin^2(a+bx)}{2b^3}+\frac{(c+d\\ &= -\frac{(c+dx)^4}{4b}-\frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}+\frac{d(c+dx)^3\cos(a+b}{b^2}\\ &= \frac{3cd^3x}{2b^3}+\frac{3d^4x^2}{4b^3}-\frac{(c+dx)^4}{4b}-\frac{3d^3(c+dx)\cos(a+bx)\sin(a+bx)}{2b^4}+\frac{d(c}{}\end{aligned}$$

Mathematica [A] time = 0.48, size = 86, normalized size = 0.55

$$\frac{4bd(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-2\cos(2(a+bx))\left(2b^4(c+dx)^4-6b^2d^2(c+dx)^2+3d^4\right)}{16b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
4*b*d*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^5)
```

fricas [A] time = 0.66, size = 255, normalized size = 1.63

$$\frac{b^4d^4x^4+4\,b^4cd^3x^3+3\left(2\,b^4c^2d^2-b^2d^4\right)x^2-\left(2\,b^4d^4x^4+8\,b^4cd^3x^3+2\,b^4c^4-6\,b^2c^2d^2+3\,d^4+6\left(2\,b^4c^2d^2-b^2d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b
^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4
*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 +
 2*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^
2*d^2 - b*d^4)*x)*cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)
*x)/b^5
```

giac [A] time = 0.20, size = 181, normalized size = 1.16

$$-\frac{\left(2\,b^4d^4x^4+8\,b^4cd^3x^3+12\,b^4c^2d^2x^2+8\,b^4c^3dx+2\,b^4c^4-6\,b^2d^4x^2-12\,b^2cd^3x-6\,b^2c^2d^2+3\,d^4\right)\cos\left(2\,bx+2\right.}{8\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x
+ 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x - 6*b^2*c^2*d^2 + 3*d^4)*cos(2
*b*x + 2*a)/b^5 + 1/4*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

maple [B] time = 0.06, size = 853, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/2*(b*x+a)^4*cos(b*x+a)^2+2*(b*x+a)^3*(1/2*cos(b*x+a)*sin
(b*x+a)+1/2*b*x+1/2*a)+3/2*(b*x+a)^2*cos(b*x+a)^2-3*(b*x+a)*(1/2*cos(b*x+a)
*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)^2+3/4*sin(b*x+a)^2-3/4*(b*x+a)^4)-4/
b^4*a*d^4*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)-3/8*
b*x-3/8*a-1/2*(b*x+a)^3)+4/b^3*c*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+
a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8
*cos(b*x+a)*sin(b*x+a)-3/8*b*x-3/8*a-1/2*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/2*(b*
x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(
b*x+a)^2-1/4*sin(b*x+a)^2)-12/b^3*a*c*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^
2)+6/b^2*c^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b
*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-4/b^4*a^3*d^4*(-1/2*(b
*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+12/b^3*a^2*c*d^
3*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-12/b^
2*a*c^2*d^2*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/
4*a)+4/b*c^3*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x
```

```
+1/4*a)-1/2/b^4*a^4*d^4*cos(b*x+a)^2+2/b^3*a^3*c*d^3*cos(b*x+a)^2-3/b^2*a^2
*c^2*d^2*cos(b*x+a)^2+2/b*a*c^3*d*cos(b*x+a)^2-1/2*c^4*cos(b*x+a)^2)
```

maxima [B] time = 0.39, size = 586, normalized size = 3.76

$$-\frac{4\,c^4\cos\left(bx+a\right)^2-\frac{16\,ac^3d\cos(bx+a)^2}{b}+\frac{24\,a^2c^2d^2\cos(bx+a)^2}{b^2}-\frac{16\,a^3cd^3\cos(bx+a)^2}{b^3}+\frac{4\,a^4d^4\cos(bx+a)^2}{b^4}+\frac{4\,(2\,(bx+a)\cos(2\,bx+2\,a)-}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^4*cos(b*x + a)^2 - 16*a*c^3*d*cos(b*x + a)^2/b + 24*a^2*c^2*d^2*c
os(b*x + a)^2/b^2 - 16*a^3*c*d^3*cos(b*x + a)^2/b^3 + 4*a^4*d^4*cos(b*x + a
)^2/b^4 + 4*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c^3*d/b - 12*
(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 12*(2*(b*
x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^2*c*d^3/b^3 - 4*(2*(b*x + a)*
cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a^3*d^4/b^4 + 6*((2*(b*x + a)^2 - 1)*c
os(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 12*((2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 6
*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(2*(b*x +
 a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 2*(2*(2*(b*x + a)^3 - 3*b*x - 3*a)
*cos(2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((2
*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 2*(2*(b*x + a)^3 - 3*b
*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

mupad [B] time = 0.49, size = 245, normalized size = 1.57

$$\frac{3\,x^2\,\cos\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\,d^2\right)}{4\,b^3}-\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{b^4\,c^4}{2}-\frac{3\,b^2\,c^2\,d^2}{2}+\frac{3\,d^4}{4}\right)}{2\,b^5}-\frac{3\,x\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^4-2\,b^2\,c^2\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (3*x^2*cos(2*a + 2*b*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^3) - (cos(2*a + 2*b*x)*
((3*d^4)/4 + (b^4*c^4)/2 - (3*b^2*c^2*d^2)/2))/(2*b^5) - (3*x*sin(2*a + 2*b
*x)*(d^4 - 2*b^2*c^2*d^2))/(4*b^4) - (d^4*x^4*cos(2*a + 2*b*x))/(4*b) - (si
n(2*a + 2*b*x)*(3*c*d^3 - 2*b^2*c^3*d))/(4*b^4) + (x*cos(2*a + 2*b*x)*(3*c*
d^3 - 2*b^2*c^3*d))/(2*b^3) + (d^4*x^3*sin(2*a + 2*b*x))/(2*b^2) - (c*d^3*x
^3*cos(2*a + 2*b*x))/b + (3*c*d^3*x^2*sin(2*a + 2*b*x))/(2*b^2)
```

sympy [A] time = 4.25, size = 502, normalized size = 3.22

$$\begin{cases} -\frac{c^4\cos^2(a+bx)}{2b} + \frac{c^3dx\sin^2(a+bx)}{b} - \frac{c^3dx\cos^2(a+bx)}{b} + \frac{3c^2d^2x^2\sin^2(a+bx)}{2b} - \frac{3c^2d^2x^2\cos^2(a+bx)}{2b} + \frac{cd^3x^3\sin^2(a+bx)}{b} - \frac{cd^3x^3\cos^2(a+ \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**2/(2*b) + c**3*d*x*sin(a + b*x)**2/b - c**3*
d*x*cos(a + b*x)**2/b + 3*c**2*d**2*x**2*sin(a + b*x)**2/(2*b) - 3*c**2*d**
2*x**2*cos(a + b*x)**2/(2*b) + c*d**3*x**3*sin(a + b*x)**2/b - c*d**3*x**3*
cos(a + b*x)**2/b + d**4*x**4*sin(a + b*x)**2/(4*b) - d**4*x**4*cos(a + b*x
)**2/(4*b) + c**3*d*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*x*sin(a +
b*x)*cos(a + b*x)/b**2 + 3*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)/b**2 + d**
4*x**3*sin(a + b*x)*cos(a + b*x)/b**2 + 3*c**2*d**2*cos(a + b*x)**2/(2*b**3
) - 3*c*d**3*x*sin(a + b*x)**2/(2*b**3) + 3*c*d**3*x*cos(a + b*x)**2/(2*b**
3) - 3*d**4*x**2*sin(a + b*x)**2/(4*b**3) + 3*d**4*x**2*cos(a + b*x)**2/(4*
b**3) - 3*c*d**3*sin(a + b*x)*cos(a + b*x)/(2*b**4) - 3*d**4*x*sin(a + b*x)
*cos(a + b*x)/(2*b**4) - 3*d**4*cos(a + b*x)**2/(4*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*c
os(a), True))
```

3.3 $\int (c+dx)^3 \cos(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=120

$$-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)^3\sin^2(a+$$

```
[Out] 3/8*d^3*x/b^3-1/4*(d*x+c)^3/b-3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3/4*d*(d*x+
c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3+1/2*(d*x+c)
^3*sin(b*x+a)^2/b
```

Rubi [A] time = 0.08, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{(c+dx)^3\sin^2(a+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (3*d^3*x)/(8*b^3) - (c + d*x)^3/(4*b) - (3*d^3*Cos[a + b*x]*Sin[a + b*x])/(
8*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) - (3*d^2*(c +
d*x)*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^3*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b}$$
$$= \frac{3d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3} + \frac{(c+$$
$$= -\frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+bx)\sin}{4b^2}$$
$$= \frac{3d^3x}{8b^3} - \frac{(c+dx)^3}{4b} - \frac{3d^3\cos(a+bx)\sin(a+bx)}{8b^4} + \frac{3d(c+dx)^2\cos(a+}{4b^2}$$

Mathematica [A] time = 0.30, size = 71, normalized size = 0.59

$$\frac{3d\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)-2b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 3*d*(-d^2 +
 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(16*b^4)
```

fricas [A] time = 0.44, size = 166, normalized size = 1.38

$$\frac{2\,b^3d^3x^3+6\,b^3cd^2x^2-2\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+2\,b^3c^3-3\,bcd^2+3\left(2\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^2+3\left(2\,b^2d^3x\right.}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b
^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) +
 3*(2*b^3*c^2*d - b*d^3)*x)/b^4
```

giac [A] time = 0.22, size = 121, normalized size = 1.01

$$-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+2\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(2\,bx+2\,a\right)}{8\,b^4}+\frac{3\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-\right.}{16\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3
*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^4 + 3/16*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
+ 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a)/b^4
```

maple [B] time = 0.01, size = 466, normalized size = 3.88

$$\frac{\frac{d^3\left(-\frac{(bx+a)^3\left(\cos^2(bx+a)\right)}{2}+\frac{3(bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{2}+\frac{3(bx+a)\left(\cos^2(bx+a)\right)}{4}-\frac{3\cos(bx+a)\sin(bx+a)}{8}-\frac{3bx}{8}-\frac{3a}{8}-\frac{(bx+a)^3}{2}\right)}{b^3}-\frac{3a\,d^3\left(-\frac{(bx+a)^2\left(\cos^2(bx+a)\right)}{2}+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/2*(b*x+a)^3*cos(b*x+a)^2+3/2*(b*x+a)^2*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)+3/4*(b*x+a)*cos(b*x+a)^2-3/8*cos(b*x+a)*sin(b*x+a)
-3/8*b*x-3/8*a-1/2*(b*x+a)^3)-3/b^3*a*d^3*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x
+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^
2)+3/b^2*c*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+3/b^3*a^2*d^3*(-1/2*(b*x
+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-6/b^2*a*c*d^2*(-1
/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+3/b*c^2*d*
(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+1/2/b^3
*a^3*d^3*cos(b*x+a)^2-3/2/b^2*a^2*c*d^2*cos(b*x+a)^2+3/2/b*a*c^2*d*cos(b*x+
a)^2-1/2*c^3*cos(b*x+a)^2)
```

maxima [B] time = 0.36, size = 342, normalized size = 2.85

$$-\frac{8\,c^3\cos\left(bx+a\right)^2-\frac{24\,ac^2d\cos(bx+a)^2}{b}+\frac{24\,a^2cd^2\cos(bx+a)^2}{b^2}-\frac{8\,a^3d^3\cos(bx+a)^2}{b^3}+\frac{6\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))c^2d}{b}-\frac{12\,(2}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*c^3*cos(b*x + a)^2 - 24*a*c^2*d*cos(b*x + a)^2/b + 24*a^2*c*d^2*co
s(b*x + a)^2/b^2 - 8*a^3*d^3*cos(b*x + a)^2/b^3 + 6*(2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*c^2*d/b - 12*(2*(b*x + a)*cos(2*b*x + 2*a) - sin
(2*b*x + 2*a))*a*c*d^2/b^2 + 6*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x +
2*a))*a^2*d^3/b^3 + 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*s
in(2*b*x + 2*a))*c*d^2/b^2 - 6*((2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 2*(b
*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (2*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(
2*b*x + 2*a) - 3*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

mupad [B] time = 0.85, size = 165, normalized size = 1.38

$$\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{3\,c\,d^2}{4}-\frac{b^2\,c^3}{2}\right)}{2\,b^3}-\frac{3\,\sin\left(2\,a+2\,b\,x\right)\,\left(d^3-2\,b^2\,c^2\,d\right)}{16\,b^4}-\frac{d^3\,x^3\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{3\,d^3\,x^2\,\sin\left(2\,a+2\,b\right.}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(2*a + 2*b*x)*((3*c*d^2)/4 - (b^2*c^3)/2))/(2*b^3) - (3*sin(2*a + 2*b*x
)*(d^3 - 2*b^2*c^2*d))/(16*b^4) - (d^3*x^3*cos(2*a + 2*b*x))/(4*b) + (3*d^3
*x^2*sin(2*a + 2*b*x))/(8*b^2) + (3*x*cos(2*a + 2*b*x)*(d^3 - 2*b^2*c^2*d))
/(8*b^3) + (3*c*d^2*x*sin(2*a + 2*b*x))/(4*b^2) - (3*c*d^2*x^2*cos(2*a + 2*
b*x))/(4*b)
```

sympy [A] time = 2.39, size = 342, normalized size = 2.85

$$\begin{cases} -\frac{c^3\cos^2(a+bx)}{2b}+\frac{3c^2dx\sin^2(a+bx)}{4b}-\frac{3c^2dx\cos^2(a+bx)}{4b}+\frac{3cd^2x^2\sin^2(a+bx)}{4b}-\frac{3cd^2x^2\cos^2(a+bx)}{4b}+\frac{d^3x^3\sin^2(a+bx)}{4b}-\frac{d^3x^3\cos^2(a+bx)}{4b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**2/(2*b) + 3*c**2*d*x*sin(a + b*x)**2/(4*b) -
 3*c**2*d*x*cos(a + b*x)**2/(4*b) + 3*c*d**2*x**2*sin(a + b*x)**2/(4*b) - 3
*c*d**2*x**2*cos(a + b*x)**2/(4*b) + d**3*x**3*sin(a + b*x)**2/(4*b) - d**3
*x**3*cos(a + b*x)**2/(4*b) + 3*c**2*d*sin(a + b*x)*cos(a + b*x)/(4*b**2) +
 3*c*d**2*x*sin(a + b*x)*cos(a + b*x)/(2*b**2) + 3*d**3*x**2*sin(a + b*x)*c
os(a + b*x)/(4*b**2) + 3*c*d**2*cos(a + b*x)**2/(4*b**3) - 3*d**3*x*sin(a +
 b*x)**2/(8*b**3) + 3*d**3*x*cos(a + b*x)**2/(8*b**3) - 3*d**3*sin(a + b*x)
```

```
*cos(a + b*x)/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sin(a)*cos(a), True))
```

3.4 $\int (c+dx)^2 \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=89

$$-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

```
[Out] -1/2*c*d*x/b-1/4*d^2*x^2/b+1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-1/4*d^2*
sin(b*x+a)^2/b^3+1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4404, 3310}

$$\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{(c+dx)^2\sin^2(a+bx)}{2b}-\frac{cdx}{2b}-\frac{d^2x^2}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(c*d*x)/(2*b) - (d^2*x^2)/(4*b) + (d*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/
(2*b^2) - (d^2*Sin[a + b*x]^2)/(4*b^3) + ((c + d*x)^2*Sin[a + b*x]^2)/(2*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)^2\sin^2(a+bx)}{2b} - \frac{d\int (c+dx)\sin^2(a+bx)\,dx}{b}$$
$$= \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{d^2\sin^2(a+bx)}{4b^3} + \frac{(c+dx)^2\sin^2(a+}{2b}$$
$$= -\frac{cdx}{2b} - \frac{d^2x^2}{4b} + \frac{d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2} - \frac{d^2\sin^2(a+bx)}{4b^3} + \frac{(c}{}$$

Mathematica [A] time = 0.23, size = 50, normalized size = 0.56

$$\frac{\cos(2(a+bx))\left(d^2-2b^2(c+dx)^2\right)+2bd(c+dx)\sin(2(a+bx))}{8b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] ((d^2 - 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 2*b*d*(c + d*x)*Sin[2*(a + b*
x)])/(8*b^3)
```

fricas [A] time = 0.50, size = 92, normalized size = 1.03

$$\frac{b^2d^2x^2+2\,b^2cdx-\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(bx+a\right)^2+2\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 -
 d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/b^3
```

giac [A] time = 0.20, size = 73, normalized size = 0.82

$$-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{8\,b^3}+\frac{\left(bd^2x+bcd\right)\sin\left(2\,bx+2\,a\right)}{4\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^3 +
 1/4*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a)/b^3
```

maple [B] time = 0.01, size = 215, normalized size = 2.42

$$\frac{\frac{d^2\left(-\frac{(bx+a)^2\left(\cos^2(bx+a)\right)}{2}+(bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\left(\sin^2(bx+a)\right)}{4}\right)}{b^2}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}+\frac{\cos(bx+a)\sin(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}+\frac{2cd\left(-\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/2*(b*x+a)^2*cos(b*x+a)^2+(b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-2/b^2*a*d^2*(-1/2*(b*x+a
)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)+2/b*c*d*(-1/2*(b*x+
a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4*a)-1/2/b^2*a^2*d^2*co
s(b*x+a)^2+1/b*a*c*d*cos(b*x+a)^2-1/2*c^2*cos(b*x+a)^2)
```

maxima [B] time = 0.36, size = 171, normalized size = 1.92

$$-\frac{4\,c^2\cos\left(bx+a\right)^2-\frac{8\,acd\cos(bx+a)^2}{b}+\frac{4\,a^2d^2\cos(bx+a)^2}{b^2}+\frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(2\,bx+2\,a))cd}{b}-\frac{2\,(2\,(bx+a)\cos(2\,bx+2\,a)-\sin(}{b^2}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c^2*cos(b*x + a)^2 - 8*a*c*d*cos(b*x + a)^2/b + 4*a^2*d^2*cos(b*x +
 a)^2/b^2 + 2*(2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*c*d/b - 2*(
2*(b*x + a)*cos(2*b*x + 2*a) - sin(2*b*x + 2*a))*a*d^2/b^2 + ((2*(b*x + a)^
2 - 1)*cos(2*b*x + 2*a) - 2*(b*x + a)*sin(2*b*x + 2*a))*d^2/b^2)/b
```

mupad [B] time = 0.16, size = 100, normalized size = 1.12

$$\frac{\cos\left(2\,a+2\,b\,x\right)\,\left(\frac{d^2}{4}-\frac{b^2\,c^2}{2}\right)}{2\,b^3}+\frac{d^2\,x\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{d^2\,x^2\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{c\,d\,\sin\left(2\,a+2\,b\,x\right)}{4\,b^2}-\frac{c\,d\,x\,\cos\left(2\,a+\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (cos(2*a + 2*b*x)*(d^2/4 - (b^2*c^2)/2))/(2*b^3) + (d^2*x*sin(2*a + 2*b*x))
/(4*b^2) - (d^2*x^2*cos(2*a + 2*b*x))/(4*b) + (c*d*sin(2*a + 2*b*x))/(4*b^2
) - (c*d*x*cos(2*a + 2*b*x))/(2*b)
```

sympy [A] time = 1.05, size = 175, normalized size = 1.97

$$\begin{cases} -\frac{c^2\cos^2(a+bx)}{2b} + \frac{cdx\sin^2(a+bx)}{2b} - \frac{cdx\cos^2(a+bx)}{2b} + \frac{d^2x^2\sin^2(a+bx)}{4b} - \frac{d^2x^2\cos^2(a+bx)}{4b} + \frac{cd\sin(a+bx)\cos(a+bx)}{2b^2} + \frac{d^2x\sin(a+bx)\cos}{2b^2} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**2/(2*b) + c*d*x*sin(a + b*x)**2/(2*b) - c*d*
x*cos(a + b*x)**2/(2*b) + d**2*x**2*sin(a + b*x)**2/(4*b) - d**2*x**2*cos(a
 + b*x)**2/(4*b) + c*d*sin(a + b*x)*cos(a + b*x)/(2*b**2) + d**2*x*sin(a +
b*x)*cos(a + b*x)/(2*b**2) + d**2*cos(a + b*x)**2/(4*b**3), Ne(b, 0)), ((c*
*2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a), True))
```

3.5 $\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=50

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

```
[Out] -1/4*d*x/b+1/4*d*cos(b*x+a)*sin(b*x+a)/b^2+1/2*(d*x+c)*sin(b*x+a)^2/b
```

Rubi [A] time = 0.03, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4404, 2635, 8}

$$\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\sin^2(a+bx)}{2b}-\frac{dx}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(d*x)/(4*b) + (d*Cos[a + b*x]*Sin[a + b*x])/(4*b^2) + ((c + d*x)*Sin[a + b
*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx = \frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int \sin^2(a+bx)\,dx}{2b}$$
$$= \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b} - \frac{d\int 1\,dx}{4b}$$
$$= -\frac{dx}{4b} + \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{(c+dx)\sin^2(a+bx)}{2b}$$

Mathematica [**A**] time = 0.10, size = 34, normalized size = 0.68

$$\frac{d\sin(2(a+bx)) - 2b(c+dx)\cos(2(a+bx))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)])/(8*b^2)
```

fricas [**A**] time = 0.63, size = 42, normalized size = 0.84

$$\frac{bdx - 2\left(bdx + bc\right)\cos\left(bx+a\right)^2 + d\cos\left(bx+a\right)\sin\left(bx+a\right)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a))/
b^2
```

giac [**A**] time = 1.72, size = 38, normalized size = 0.76

$$-\frac{(bdx + bc)\cos\left(2\,bx + 2\,a\right)}{4\,b^2} + \frac{d\sin\left(2\,bx + 2\,a\right)}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/4*(b*d*x + b*c)*cos(2*b*x + 2*a)/b^2 + 1/8*d*sin(2*b*x + 2*a)/b^2
```

maple [**A**] time = 0.01, size = 74, normalized size = 1.48

$$\frac{\frac{d\left(-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2} + \frac{\cos(bx+a)\sin(bx+a)}{4} + \frac{bx}{4} + \frac{a}{4}\right)}{b} + \frac{da\left(\cos^2(bx+a)\right)}{2b} - \frac{c\left(\cos^2(bx+a)\right)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(-1/2*(b*x+a)*cos(b*x+a)^2+1/4*cos(b*x+a)*sin(b*x+a)+1/4*b*x+1/4
*a)+1/2/b*d*a*cos(b*x+a)^2-1/2*c*cos(b*x+a)^2)
```

maxima [A] time = 0.33, size = 65, normalized size = 1.30

$$-\frac{4\,c\cos\left(bx+a\right)^2-\frac{4\,ad\cos\left(bx+a\right)^2}{b}+\frac{\left(2\,\left(bx+a\right)\cos\left(2\,bx+2\,a\right)-\sin\left(2\,bx+2\,a\right)\right)d}{b}}{8\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8*(4*c*cos(b*x + a)^2 - 4*a*d*cos(b*x + a)^2/b + (2*(b*x + a)*cos(2*b*x
+ 2*a) - sin(2*b*x + 2*a))*d/b)/b
```

mupad [B] time = 0.70, size = 47, normalized size = 0.94

$$\frac{d\,\sin\left(2\,a+2\,b\,x\right)}{8\,b^2}-\frac{c\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}-\frac{d\,x\,\cos\left(2\,a+2\,b\,x\right)}{4\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x),x)
```

```
[Out] (d*sin(2*a + 2*b*x))/(8*b^2) - (c*cos(2*a + 2*b*x))/(4*b) - (d*x*cos(2*a +
2*b*x))/(4*b)
```

sympy [A] time = 0.47, size = 80, normalized size = 1.60

$$\begin{cases} -\frac{c\cos^2\left(a+bx\right)}{2b}+\frac{dx\sin^2\left(a+bx\right)}{4b}-\frac{dx\cos^2\left(a+bx\right)}{4b}+\frac{d\sin\left(a+bx\right)\cos\left(a+bx\right)}{4b^2} & \text{for}\ b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\sin\left(a\right)\cos\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**2/(2*b) + d*x*sin(a + b*x)**2/(4*b) - d*x*cos(a
 + b*x)**2/(4*b) + d*sin(a + b*x)*cos(a + b*x)/(4*b**2), Ne(b, 0)), ((c*x +
 d*x**2/2)*sin(a)*cos(a), True))
```

3.6 $\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=65

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*
c/d)/d
```

Rubi [A] time = 0.14, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4406, 12, 3303, 3299, 3302}

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) + (Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\\ &= \frac{\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 60, normalized size = 0.92

$$\frac{\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)+\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d] + Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

fricas [A] time = 0.61, size = 80, normalized size = 1.23

$$\frac{\left(\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*s
in(-2*(b*c - a*d)/d) + 2*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)
/d))/d
```

giac [C] time = 0.85, size = 569, normalized size = 8.75

$$\Im\left(\mathrm{Ci}\left(2\,bx+\frac{2\,bc}{d}\right)\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2-\Im\left(\mathrm{Ci}\left(-2\,bx-\frac{2\,bc}{d}\right)\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2+2\,\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)\tan(a)^2\tan\left(\frac{bc}{d}\right)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)^2*tan(b*c/d) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d)^2 - 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d
)^2 - imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*sin_integral(2*(b*d*x + b*c)/d)*tan(a
)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) - imag_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*c/d)^2 + imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 -
2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a) + 2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
) - 2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(2*b*x + 2*
b*c/d)) - imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 2*sin_integral(2*(b*d
*x + b*c)/d))/(d*tan(a)^2*tan(b*c/d)^2 + d*tan(a)^2 + d*tan(b*c/d)^2 + d)
```

maple [A] time = 0.02, size = 84, normalized size = 1.29

$$\frac{\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{2d}-\frac{\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-1/2*Ci(2*b*x+2*a+2*(
-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d
```

maxima [C] time = 0.41, size = 141, normalized size = 2.17

$$-\frac{b\left(iE_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-iE_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_1\left(-\frac{2i\,bc-}{}\right.\right.}{4\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I*e
xp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c - a
*d)/d) + b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) + ex
p_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c - a*
d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x), x)
```

3.7 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=85

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/
d)/d^2-1/2*sin(2*b*x+2*a)/d/(d*x+c)
```

Rubi [A] time = 0.15, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\sin(2a+2bx)}{2d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - Sin[2*a + 2*b
*x]/(2*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x
])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}\\ &= -\frac{\sin(2a+2bx)}{2d(c+dx)} + \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin}{}}{d}\\ &= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{\sin(2a+2bx)}{2d(c+dx)} - \frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2b\right.}{d^2}\end{aligned}$$

Mathematica [A] time = 0.31, size = 80, normalized size = 0.94

$$\frac{2b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)-2b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)-\frac{d\sin(2(a+bx))}{c+dx}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - (d*Sin[2*(a + b*
x)])/(c + d*x) - 2*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/(
2*d^2)
```

fricas [A] time = 0.71, size = 132, normalized size = 1.55

$$-\frac{2\,d\cos\left(bx+a\right)\sin\left(bx+a\right)+2\left(bdx+bc\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(bdx+bc\right)}{d}\right)-\left(\left(bdx+bc\right)\mathrm{Ci}\left(\frac{2\left(bdx+bc\right)}{d}\right)+\left(bdx+bc\right)\right.}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a)*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b
*c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)
/d))/(d^3*x + c*d^2)
```

giac [C] time = 0.56, size = 2870, normalized size = 33.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan
(b*c/d)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - b*d*x*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b*d*x*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 2*b*c*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b*c*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - b*d*x*real_
```

```
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b*c*imag_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b*c*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 4*b
*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b*d*x*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 4*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 - b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2 + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(b*c/d) + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c
/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(
b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d) - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(
b*c/d) + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c
/d) - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) - b*c*rea
l_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - b*c*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b*d*x*imag_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b*d*x*imag_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b*d*x*sin_integral(
2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + b*c*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(a)^2*tan(b*c/d)^2 + b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2 + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 -
 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b*c*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 4*b*c*sin_inte
gral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - b*d*x*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*
c/d) - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d
) + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 4*b*d*x*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 4*b*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 2*b*c*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b*c*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*b*c*sin_integral(2*(b*d*x + b
*c)/d)*tan(a)^2*tan(b*c/d) - b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(b*c/d)^2 - b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^
2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*
b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 2*d*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 + 2*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*c*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2 + b*c*real_part(cos_integral(-2*b*x - 2*b*
```

```
c/d))*tan(b*x)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
+ 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b*d*x*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a) - b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2 - b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 + 2*
b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b*d*x*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*c/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*ta
n(b*c/d) - b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - b*c*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d)) + b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/
d)) - 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b*c*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 4*b*c*sin_integral(2*(b*d*x + b
*c)/d)*tan(a) + 2*d*tan(b*x)^2*tan(a) + 2*d*tan(b*x)*tan(a)^2 + 2*b*c*imag_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b*c*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*ta
n(b*c/d) - 2*d*tan(b*x)*tan(b*c/d)^2 - 2*d*tan(a)*tan(b*c/d)^2 + b*c*real_p
art(cos_integral(2*b*x + 2*b*c/d)) + b*c*real_part(cos_integral(-2*b*x - 2*
b*c/d)) - 2*d*tan(b*x) - 2*d*tan(a))/(d^3*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^
2 + c*d^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^3*x*tan(b*x)^2*tan(a)^2 + d^
3*x*tan(b*x)^2*tan(b*c/d)^2 + d^3*x*tan(a)^2*tan(b*c/d)^2 + c*d^2*tan(b*x)^
2*tan(a)^2 + c*d^2*tan(b*x)^2*tan(b*c/d)^2 + c*d^2*tan(a)^2*tan(b*c/d)^2 +
d^3*x*tan(b*x)^2 + d^3*x*tan(a)^2 + d^3*x*tan(b*c/d)^2 + c*d^2*tan(b*x)^2 +
 c*d^2*tan(a)^2 + c*d^2*tan(b*c/d)^2 + d^3*x + c*d^2)
```

maple [A] time = 0.01, size = 124, normalized size = 1.46

$$\frac{b\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/4*b*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*
c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)
/d)/d)/d)
```

maxima [C] time = 0.44, size = 164, normalized size = 1.93

$$-\frac{b^2\left(i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_2\left(-\frac{2i\,bc}{}\right.\right.}{4\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**2, x)
```

3.8 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=114

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

```
[Out] -1/2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^
3-b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-1/4*sin(2*b*x+2*a)/d/(d*x+c)^2
```

Rubi [A] time = 0.17, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cos(2a+2bx)}{2d^2(c+dx)}-\frac{\sin(2a+2bx)}{4d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(2*d^2*(c + d*x)) - (b^2*CosIntegral[(2*b*c)/d + 2*b*
x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(4*d*(c + d*x)^2) - (b^2*Co
s[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$
$$= -\frac{\sin(2a+2bx)}{4d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2}$$
$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d^2} - \frac{\left(b^2\right.}{}$$
$$= -\frac{b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{b^2\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{4d(c+dx)^2} - \frac{b^2\cos\left(\right.}{}$$

Mathematica [A] time = 1.08, size = 102, normalized size = 0.89

$$-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right)+4b^2\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}}{4d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/4*(4*b^2*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(
c + d*x)*Cos[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*
a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

fricas [B] time = 0.81, size = 230, normalized size = 2.02

$$\frac{bd^2x - d^2\cos(bx+a)\sin(bx+a) + bcd - 2\left(bd^2x + bcd\right)\cos(bx+a)^2 - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\cos\left(-\frac{2\,(bc-}{d}\right.}{2\left(d^5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b*d^2*x - d^2*cos(b*x + a)*sin(b*x + a) + b*c*d - 2*(b*d^2*x + b*c*d)*
cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)
/d)*sin_integral(2*(b*d*x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*c
os_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

giac [C] time = 0.53, size = 5398, normalized size = 47.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a
)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^
2 - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 4*b^2*c*d*x*sin_integral(2*(b*
d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*sin_integra
```

```
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 + 4*b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 8*b^2*d^2
*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d) + 4*b^2*c
*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d
) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)
^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*d^2*x^2*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2*x^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^2*d^2
*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*d^2
*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*
d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 4*b^2*c
*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 2*b^2*d^2*x^
2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c
*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)
- 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)*
tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^
2*tan(b*c/d) + 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^2*c*d*x*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - 2*b^2*c^2*real_part(
cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^2*c*d*
x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*b^2*c*
d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2
*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*d^2*x*tan(
b*x)^2*tan(a)^2*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2 - b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
```

```
*tan(b*x)^2 + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*
b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^
2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - b^2*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*d^2*x^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2 - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(a)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 - 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)^2 - 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(b*c/d) - 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(b*c/d) + 4*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)*tan(b*c/d) - 4*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a)*tan(b*c/d) + 8*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan
(b*c/d) + 4*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)*tan(b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
x)^2*tan(a)*tan(b*c/d) + 8*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)
^2*tan(a)*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))
*tan(a)^2*tan(b*c/d) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(a)^2*tan(b*c/d) - b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*c/d)^2 + b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*c/d)^2 - 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b^2
*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + b^2
*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*
b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 - 4*b^2*c*d
*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^2*c*d
*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + b^2*c^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^2*c^2*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 + b*c*d*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2 - 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2 + 4*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 2*b^2*d^2*x^2*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 2*b^2*d^2*x^2*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a) + 2*b^2*c^2*real_part(cos_integral(2*
b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 2*b^2*c^2*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*x)^2*tan(a) - 2*b^2*c*d*x*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2 + 2*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(a)^2 - 4*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + b*d^2*x*
tan(b*x)^2*tan(a)^2 - 2*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*c/d) - 2*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*c/d) - 2*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
b*c/d) - 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(b*c/d) + 8*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b
*c/d) - 8*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*
c/d) + 16*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + 2*b
```

```
^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) + 2*b^2
*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 2*b^2*
c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 2*b^2*c*d*x*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*b^2*c*d*x*sin_int
egral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b*d^2*x*tan(b*x)^2*tan(b*c/d)^2 - 2
*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b*
d^2*x*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*d^2*x*tan(a)^2*tan(b*c/d)^2 + b^2*d^
2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d)) - b^2*d^2*x^2*imag_part(cos_
integral(-2*b*x - 2*b*c/d)) + 2*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)
 + b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - b^2*c^2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 2*b^2*c^2*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2 + 4*b^2*c*d*x*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a) + 4*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a) - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + b^2*c^2
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 2*b^2*c^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2 + b*c*d*tan(b*x)^2*tan(a)^2 - 4*b^2*c*d*x*rea
l_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^2*c*d*x*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) + 4*b^2*c^2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 4*b^2*c^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^2*c^2*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)*tan(b*c/d) - b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*c/d)^2 + b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2
- 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 - b*c*d*tan(b*x)^2
*tan(b*c/d)^2 - 4*b*c*d*tan(b*x)*tan(a)*tan(b*c/d)^2 - d^2*tan(b*x)^2*tan(a
)*tan(b*c/d)^2 - b*c*d*tan(a)^2*tan(b*c/d)^2 - d^2*tan(b*x)*tan(a)^2*tan(b*
c/d)^2 + 2*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) - 2*b^2*c*d*x
*imag_part(cos_integral(-2*b*x - 2*b*c/d)) + 4*b^2*c*d*x*sin_integral(2*(b*
d*x + b*c)/d) - b*d^2*x*tan(b*x)^2 + 2*b^2*c^2*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a) + 2*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(a) - 4*b*d^2*x*tan(b*x)*tan(a) - b*d^2*x*tan(a)^2 - 2*b^2*c^2*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*b^2*c^2*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(b*c/d) + b*d^2*x*tan(b*c/d)^2 + b^2*c^2*imag_part(
cos_integral(2*b*x + 2*b*c/d)) - b^2*c^2*imag_part(cos_integral(-2*b*x - 2*
b*c/d)) + 2*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d) - b*c*d*tan(b*x)^2 - 4*
b*c*d*tan(b*x)*tan(a) - d^2*tan(b*x)^2*tan(a) - b*c*d*tan(a)^2 - d^2*tan(b*
x)*tan(a)^2 + b*c*d*tan(b*c/d)^2 + d^2*tan(b*x)*tan(b*c/d)^2 + d^2*tan(a)*t
an(b*c/d)^2 + b*d^2*x + b*c*d + d^2*tan(b*x) + d^2*tan(a))/(d^5*x^2*tan(b*x
)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^
5*x^2*tan(b*x)^2*tan(a)^2 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + d^5*x^2*tan(a
)^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan
(b*x)^2*tan(a)^2 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(a)^2*t
an(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + d^5*x^2*tan(a)^2 + c^2*d^3*tan(b*x)^2*ta
n(a)^2 + d^5*x^2*tan(b*c/d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + c^2*d^3*t
an(a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2 + 2*c*d^4*x*tan(a)^2 + 2*c*d^4*
```

```
x*tan(b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(b*x)^2 + c^2*d^3*tan(a)^2 + c^2*d^3*
tan(b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

maple [A] time = 0.01, size = 162, normalized size = 1.42

$$\frac{b^2\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{2\cos(2bx+2a)}{((bx+a)d-da+cb)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}\right)}{d}}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/4*b^2*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a
)*d-d*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci
(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)
```

maxima [C] time = 0.51, size = 199, normalized size = 1.75

$$-\frac{b^3\left(i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_3\left(-\frac{2i\,bc}{}\right.\right.}{4\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**3, x)
```

3.9 $\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=144

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2}-\frac{\sin(2a}{6d(c}$$

```
[Out] -2/3*b^3*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^4-1/6*b*cos(2*b*x+2*a)/d^2/(d
*x+c)^2+2/3*b^3*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/6*sin(2*b*x+2*a)/d
/(d*x+c)^3+1/3*b^2*sin(2*b*x+2*a)/d^3/(d*x+c)
```

Rubi [A] time = 0.20, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4406, 12, 3297, 3303, 3299, 3302}

$$-\frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{b^2\sin(2a+2bx)}{3d^3(c+dx)}-\frac{b\cos(2a+2b}{6d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(6*d^2*(c + d*x)^2) - (2*b^3*Cos[2*a - (2*b*c)/d]*Cos
Integral[(2*b*c)/d + 2*b*x])/(3*d^4) - Sin[2*a + 2*b*x]/(6*d*(c + d*x)^3) +
 (b^2*Sin[2*a + 2*b*x])/(3*d^3*(c + d*x)) + (2*b^3*Sin[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \frac{\sin(2a+2bx)}{2(c+dx)^4}\,dx$$

$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{3d}$$

$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} - \frac{b^2\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} - \frac{\left(2b^3\right)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{3d^3}$$

$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{3d^3(c+dx)} - \frac{\left(2b^3\cos\left(2a-\frac{2bc}{d}\right)\right)\int}{3d^3}$$

$$= -\frac{b\cos(2a+2bx)}{6d^2(c+dx)^2} - \frac{2b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} - \frac{\sin(2a+2bx)}{6d(c+dx)^3} + \frac{b^2\sin}{3d^3}$$

Mathematica [A] time = 0.67, size = 164, normalized size = 1.14

$$\frac{-4b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)\right)-d\cos(2bx)\left(\sin(2a)\left(d^2-2b^2(c+dx)^2\right.\right.}{6d^4(c+dx)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*a
])) + d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a])*Sin[
2*b*x] - 4*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x)
)/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]))/(6*d^4*(c + d*
x)^3)
```

fricas [B] time = 0.61, size = 320, normalized size = 2.22

$$\frac{bd^3x+bcd^2-2\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2+2\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + b*c*d^2 - 2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 2*(2*b^2*d^
3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 4*(b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/
d)*sin_integral(2*(b*d*x + b*c)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*c
os(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

giac [C] time = 0.61, size = 7592, normalized size = 52.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/6*(2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan
(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x
```

```
^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*d
^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 2*b^3*d^3*x^3*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 8*b^3*d^3*x
^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) +
8*b^3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*t
an(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b
*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(
2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b^3*d^3*x^3*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d
^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d
)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2
*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 - 4*b^3
*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3
*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^
3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x
^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c*d
^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 4*b^
3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/
d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) +
24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)
*tan(b*c/d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2*tan(a)*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(a)^2*tan(b*c/d) + 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(a)^2*tan(b*c/d) - 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c
)/d)*tan(a)^2*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_
integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^
2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*
c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2
 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)
```

```
^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c
/d)^2 + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 +
 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*
tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 2*b^
3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*d^3*x
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c*d^2*x
^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c*d
^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 2*b^3*d^3*x^3*re
al_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*d^3*x^3*real_part(c
os_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(c
os_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 24*b^3*c*d^2*
x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) + 8*b^3*d^3*x^3*r
eal_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*d^3*x^3*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) + 24*b^3*c^2*d*x
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a)*tan(b*c/d) + 24
*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a)*ta
n(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
^2*tan(b*c/d) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(b*c/d) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(
a)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(a)^2*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(a)^2*tan(b*c/d) - 2*b^3*d^3*x^3*real_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*d^3*x^3*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c*d^2*x^2*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)^2*ta
n(a)*tan(b*c/d)^2 + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*sin_integral(2*(b*d*x + b*
c)/d)*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c^2*d*x*real_part(cos_inte
```

```
gral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(
a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*x)^2 - 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) +
4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*d^3*
x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a) - 12*b^3*c^2*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 12*b^3*c^2*d*x*imag_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 24*b^3*c^2*d*x*sin_integral
(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(a) - 6*b^3*c*d^2*x^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(a)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(b*x)^2*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(a)^2 + 4*b^3*d^3*x^3*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(b*c/d) - 4*b^3*d^3*x^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(b*c/d) + 8*b^3*d^3*x^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 12*b^
3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) -
12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)
+ 24*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/
d) + 24*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(
b*c/d) + 8*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
a)*tan(b*c/d) + 8*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x
)^2*tan(a)*tan(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c
/d))*tan(a)^2*tan(b*c/d) + 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(a)^2*tan(b*c/d) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/
d)*tan(a)^2*tan(b*c/d) - 6*b^3*c*d^2*x^2*real_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*c/d)^2 - 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(b*x)^2*tan(b*c/d)^2 + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(a)*tan(b*c/d)^2 - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(a)*tan(b*c/d)^2 + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)
/d)*tan(a)*tan(b*c/d)^2 + 8*b^2*c*d^2*x*tan(b*x)^2*tan(a)*tan(b*c/d)^2 + 2*
b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*
b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 8
*b^2*c*d^2*x*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*d^3*x*tan(b*x)^2*tan(a)^2*t
an(b*c/d)^2 + 2*b^3*d^3*x^3*real_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b^
3*d^3*x^3*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 6*b^3*c^2*d*x*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 6*b^3*c^2*d*x*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(a) + 12*b^3*c*d^2*x^2*imag_part(cos_integral(-2
*b*x - 2*b*c/d))*tan(a) - 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*
tan(a) + 4*b^2*d^3*x^2*tan(b*x)^2*tan(a) - 4*b^3*c^3*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(a) + 4*b^3*c^3*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(b*x)^2*tan(a) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/
```

```
d)*tan(b*x)^2*tan(a) - 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(a)^2 - 6*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
)^2 + 4*b^2*d^3*x^2*tan(b*x)*tan(a)^2 + 12*b^3*c*d^2*x^2*imag_part(cos_inte
gral(2*b*x + 2*b*c/d))*tan(b*c/d) - 12*b^3*c*d^2*x^2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*c/d) + 24*b^3*c*d^2*x^2*sin_integral(2*(b*d*x + b
*c)/d)*tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
b*x)^2*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*t
an(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*
tan(b*c/d) + 24*b^3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a
)*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(b*c/d) + 4*b^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*t
an(b*c/d) - 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d) -
 6*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 6*b^
3*c^2*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*b^2*d^
3*x^2*tan(b*x)*tan(b*c/d)^2 - 4*b^2*d^3*x^2*tan(a)*tan(b*c/d)^2 + 4*b^3*c^3
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*b^3*c^3*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 8*b^3*c^3*si
n_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*d*tan(b*x)^2*
tan(a)*tan(b*c/d)^2 + 4*b^2*c^2*d*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + b*c*d^2*
tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 6*b^3*c*d^2*x^2*real_part(cos_integral(2
*b*x + 2*b*c/d)) + 6*b^3*c*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d)
) + 2*b^3*c^3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b^3*c
^3*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 12*b^3*c^2*d*x*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 12*b^3*c^2*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(a) - 24*b^3*c^2*d*x*sin_integral(2*(b*d*x
 + b*c)/d)*tan(a) + 8*b^2*c*d^2*x*tan(b*x)^2*tan(a) - 2*b^3*c^3*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*b^3*c^3*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2 + 8*b^2*c*d^2*x*tan(b*x)*tan(a)^2 + b*d^3*x*tan(
b*x)^2*tan(a)^2 + 12*b^3*c^2*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*c/d) - 12*b^3*c^2*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*
c/d) + 24*b^3*c^2*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 8*b^3*c^
3*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*b^3*c^3*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 2*b^3*c^3*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*b^3*c^3*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(b*x)*tan(b*c/d
)^2 - b*d^3*x*tan(b*x)^2*tan(b*c/d)^2 - 8*b^2*c*d^2*x*tan(a)*tan(b*c/d)^2 -
 4*b*d^3*x*tan(b*x)*tan(a)*tan(b*c/d)^2 - b*d^3*x*tan(a)^2*tan(b*c/d)^2 + 6
*b^3*c^2*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) + 6*b^3*c^2*d*x*real_
part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*d^3*x^2*tan(b*x) - 4*b^2*d^3*x
^2*tan(a) - 4*b^3*c^3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*b
^3*c^3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a) - 8*b^3*c^3*sin_int
egral(2*(b*d*x + b*c)/d)*tan(a) + 4*b^2*c^2*d*tan(b*x)^2*tan(a) + 4*b^2*c^2
*d*tan(b*x)*tan(a)^2 + b*c*d^2*tan(b*x)^2*tan(a)^2 + 4*b^3*c^3*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*b^3*c^3*imag_part(cos_integral(
```

```
-2*b*x - 2*b*c/d))*tan(b*c/d) + 8*b^3*c^3*sin_integral(2*(b*d*x + b*c)/d)*t
an(b*c/d) - 4*b^2*c^2*d*tan(b*x)*tan(b*c/d)^2 - b*c*d^2*tan(b*x)^2*tan(b*c/
d)^2 - 4*b^2*c^2*d*tan(a)*tan(b*c/d)^2 - 4*b*c*d^2*tan(b*x)*tan(a)*tan(b*c/
d)^2 - 2*d^3*tan(b*x)^2*tan(a)*tan(b*c/d)^2 - b*c*d^2*tan(a)^2*tan(b*c/d)^2
 - 2*d^3*tan(b*x)*tan(a)^2*tan(b*c/d)^2 + 2*b^3*c^3*real_part(cos_integral(
2*b*x + 2*b*c/d)) + 2*b^3*c^3*real_part(cos_integral(-2*b*x - 2*b*c/d)) - 8
*b^2*c*d^2*x*tan(b*x) - b*d^3*x*tan(b*x)^2 - 8*b^2*c*d^2*x*tan(a) - 4*b*d^3
*x*tan(b*x)*tan(a) - b*d^3*x*tan(a)^2 + b*d^3*x*tan(b*c/d)^2 - 4*b^2*c^2*d*
tan(b*x) - b*c*d^2*tan(b*x)^2 - 4*b^2*c^2*d*tan(a) - 4*b*c*d^2*tan(b*x)*tan
(a) - 2*d^3*tan(b*x)^2*tan(a) - b*c*d^2*tan(a)^2 - 2*d^3*tan(b*x)*tan(a)^2
+ b*c*d^2*tan(b*c/d)^2 + 2*d^3*tan(b*x)*tan(b*c/d)^2 + 2*d^3*tan(a)*tan(b*c
/d)^2 + b*d^3*x + b*c*d^2 + 2*d^3*tan(b*x) + 2*d^3*tan(a))/(d^7*x^3*tan(b*x
)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 +
d^7*x^3*tan(b*x)^2*tan(a)^2 + d^7*x^3*tan(b*x)^2*tan(b*c/d)^2 + d^7*x^3*tan
(a)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + 3*c*d^6
*x^2*tan(b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*x)^2*tan(b*c/d)^2 + 3*c*d^6*x^
2*tan(a)^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(a)^2*tan(b*c/d)^2 + d^7*x^
3*tan(b*x)^2 + d^7*x^3*tan(a)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(a)^2 + d^7*x^3
*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(b*x)^2*tan(b*c/d)^2 + 3*c^2*d^5*x*tan(a)^2*
tan(b*c/d)^2 + 3*c*d^6*x^2*tan(b*x)^2 + 3*c*d^6*x^2*tan(a)^2 + c^3*d^4*tan(
b*x)^2*tan(a)^2 + 3*c*d^6*x^2*tan(b*c/d)^2 + c^3*d^4*tan(b*x)^2*tan(b*c/d)^
2 + c^3*d^4*tan(a)^2*tan(b*c/d)^2 + d^7*x^3 + 3*c^2*d^5*x*tan(b*x)^2 + 3*c^
2*d^5*x*tan(a)^2 + 3*c^2*d^5*x*tan(b*c/d)^2 + 3*c*d^6*x^2 + c^3*d^4*tan(b*x
)^2 + c^3*d^4*tan(a)^2 + c^3*d^4*tan(b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
```

maple [A] time = 0.01, size = 200, normalized size = 1.39

$$\frac{b^3\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{2\cos(2bx+2a)}{3((bx+a)d-da+cb)^2 d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/4*b^3*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+2*a)/(
(b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b
*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)
*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
```

maxima [C] time = 0.64, size = 249, normalized size = 1.73

$$-\frac{b^4\left(iE_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+E_4\left(-\frac{2i\,bc}{}\right.\right.}{4\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\,abcd^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) - I
*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*c -
 a*d)/d) + b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2*(b*c
- a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4
- a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x))/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)/(c + d*x)**4, x)
```

3.10 $\int \frac{\cos(x)\sin(x)}{x}\,dx$

Optimal. Leaf size=8

$$\frac{\mathrm{Si}(2x)}{2}$$

[Out] 1/2*Si(2*x)

Rubi [A] time = 0.03, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4406, 12, 3299}

$$\frac{\mathrm{Si}(2x)}{2}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/x,x]

[Out] SinIntegral[2*x]/2

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin(x)}{x}\,dx = \int \frac{\sin(2x)}{2x}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2x)}{x}\,dx$$
$$= \frac{\mathrm{Si}(2x)}{2}$$

Mathematica [**A**] time = 0.01, size = 8, normalized size = 1.00

$$\frac{\mathrm{Si}(2x)}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x,x]
```

```
[Out] SinIntegral[2*x]/2
```

fricas [**A**] time = 0.72, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\mathrm{Si}\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="fricas")
```

```
[Out] 1/2*sin_integral(2*x)
```

giac [**A**] time = 0.17, size = 6, normalized size = 0.75

$$\frac{1}{2}\,\mathrm{Si}\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="giac")
```

```
[Out] 1/2*sin_integral(2*x)
```

maple [**A**] time = 0.02, size = 7, normalized size = 0.88

$$\frac{\mathrm{Si}\left(2x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x,x)
[Out] 1/2*Si(2*x)
```

maxima [C] time = 0.37, size = 13, normalized size = 1.62

$$-\frac{1}{4} i \operatorname{Ei}(2 i x) + \frac{1}{4} i \operatorname{Ei}(-2 i x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x, algorithm="maxima")
[Out] -1/4*I*Ei(2*I*x) + 1/4*I*Ei(-2*I*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.12

$$\int \frac{\cos(x)\,\sin(x)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x,x)
[Out] int((cos(x)*sin(x))/x, x)
```

sympy [A] time = 0.86, size = 5, normalized size = 0.62

$$\frac{\operatorname{Si}(2x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x,x)
[Out] Si(2*x)/2
```

3.11 $\int \frac{\cos(x)\sin(x)}{x^2}\,dx$

Optimal. Leaf size=16

$$\text{Ci}(2x) - \frac{\sin(2x)}{2x}$$

[Out] Ci(2*x)-1/2*sin(2*x)/x

Rubi [A] time = 0.05, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4406, 12, 3297, 3302}

$$\text{CosIntegral}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

[In] Int[(Cos[x]*Sin[x])/x^2,x]

[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{x^2}\,dx &= \int \frac{\sin(2x)}{2x^2}\,dx\\ &= \frac{1}{2}\int \frac{\sin(2x)}{x^2}\,dx\\ &= -\frac{\sin(2x)}{2x} + \int \frac{\cos(2x)}{x}\,dx\\ &= \text{Ci}(2x) - \frac{\sin(2x)}{2x}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$\text{Ci}(2x) - \frac{\sin(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^2,x]
[Out] CosIntegral[2*x] - Sin[2*x]/(2*x)
```

fricas [A] time = 0.76, size = 24, normalized size = 1.50

$$\frac{x\,\text{Ci}\left(2\,x\right) + x\,\text{Ci}\left(-2\,x\right) - 2\,\cos(x)\sin(x)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="fricas")
[Out] 1/2*(x*cos_integral(2*x) + x*cos_integral(-2*x) - 2*cos(x)*sin(x))/x
```

giac [A] time = 0.14, size = 19, normalized size = 1.19

$$\frac{2\,x\,\text{Ci}\left(2\,x\right) - \sin\left(2\,x\right)}{2\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="giac")
[Out] 1/2*(2*x*cos_integral(2*x) - sin(2*x))/x
```

maple [A] time = 0.02, size = 15, normalized size = 0.94

$$\mathrm{Ci}\left(2x\right)-\frac{\sin\left(2x\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^2,x)
```

```
[Out] Ci(2*x)-1/2*sin(2*x)/x
```

maxima [C] time = 0.38, size = 15, normalized size = 0.94

$$\frac{1}{2}\,\Gamma\left(-1,2i\,x\right)+\frac{1}{2}\,\Gamma\left(-1,-2i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*gamma(-1, 2*I*x) + 1/2*gamma(-1, -2*I*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int\frac{\cos(x)\,\sin(x)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x^2,x)
```

```
[Out] int((cos(x)*sin(x))/x^2, x)
```

sympy [A] time = 1.57, size = 22, normalized size = 1.38

$$-\log\left(x\right)+\frac{\log\left(x^2\right)}{2}+\mathrm{Ci}\left(2x\right)-\frac{\sin\left(2x\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**2,x)
```

```
[Out] -log(x) + log(x**2)/2 + Ci(2*x) - sin(2*x)/(2*x)
```

3.12 $\int \frac{\cos(x)\sin(x)}{x^3}\,dx$

Optimal. Leaf size=29

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

```
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4406, 12, 3297, 3299}

$$-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -Cos[2*x]/(2*x) - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(x)\sin(x)}{x^3}\,dx &= \int \frac{\sin(2x)}{2x^3}\,dx \\
&= \frac{1}{2}\int \frac{\sin(2x)}{x^3}\,dx \\
&= -\frac{\sin(2x)}{4x^2} + \frac{1}{2}\int \frac{\cos(2x)}{x^2}\,dx \\
&= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \int \frac{\sin(2x)}{x}\,dx \\
&= -\frac{\cos(2x)}{2x} - \frac{\sin(2x)}{4x^2} - \text{Si}(2x)
\end{aligned}
$$

Mathematica [**A**] time = 0.01, size = 29, normalized size = 1.00

$$
-\text{Si}(2x) - \frac{\sin(2x)}{4x^2} - \frac{\cos(2x)}{2x}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/x^3,x]
```

```
[Out] -1/2*Cos[2*x]/x - Sin[2*x]/(4*x^2) - SinIntegral[2*x]
```

fricas [**A**] time = 0.51, size = 30, normalized size = 1.03

$$
-\frac{2\,x\cos(x)^2 + 2\,x^2\,\text{Si}\,(2\,x) + \cos(x)\sin(x) - x}{2\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*x*cos(x)^2 + 2*x^2*sin_integral(2*x) + cos(x)*sin(x) - x)/x^2
```

giac [**A**] time = 1.60, size = 26, normalized size = 0.90

$$
-\frac{4\,x^2\,\text{Si}\,(2\,x) + 2\,x\cos\,(2\,x) + \sin\,(2\,x)}{4\,x^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="giac")
```

```
[Out] -1/4*(4*x^2*sin_integral(2*x) + 2*x*cos(2*x) + sin(2*x))/x^2
```

maple [A] time = 0.02, size = 26, normalized size = 0.90

$$-\frac{\cos\left(2x\right)}{2x}-\mathrm{Si}\left(2x\right)-\frac{\sin\left(2x\right)}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/x^3,x)
[Out] -1/2*cos(2*x)/x-Si(2*x)-1/4*sin(2*x)/x^2
```

maxima [C] time = 0.39, size = 15, normalized size = 0.52

$$i\,\Gamma\left(-2,2i\,x\right)-i\,\Gamma\left(-2,-2i\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x^3,x, algorithm="maxima")
[Out] I*gamma(-2, 2*I*x) - I*gamma(-2, -2*I*x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cos(x)\,\sin(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/x^3,x)
[Out] int((cos(x)*sin(x))/x^3, x)
```

sympy [A] time = 1.16, size = 24, normalized size = 0.83

$$-\mathrm{Si}\left(2x\right)-\frac{\cos\left(2x\right)}{2x}-\frac{\sin\left(2x\right)}{4x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/x**3,x)
[Out] -Si(2*x) - cos(2*x)/(2*x) - sin(2*x)/(4*x**2)
```

3.13 $\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=275

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-$$

```
[Out] -1/8*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c
)/d)^m)+1/8*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(
d*x+c)/d)^m)+1/8*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(
d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(
d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.33, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3307, 2181}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m-\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/8)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])
/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/8)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d
*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/8)*3^(-1 - m)*
E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(
b*(((-I)*b*(c + d*x))/d)^m) - ((I/8)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((
3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
```

```
f, m}, x] && IntegerQ[2*k]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\sin^2(a+bx)\,dx &= \int \left(\frac{1}{4}(c+dx)^m \cos(a+bx) - \frac{1}{4}(c+dx)^m \cos(3a+3bx)\right)dx\\ &= \frac{1}{4}\int (c+dx)^m \cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^m \cos(3a+3bx)\,dx\\ &= \frac{1}{8}\int e^{-i(a+bx)}(c+dx)^m\,dx + \frac{1}{8}\int e^{i(a+bx)}(c+dx)^m\,dx - \frac{1}{8}\int e^{-i(3a+3bx)}\\ &= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} + \frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+}{}\end{aligned}$$

Mathematica [A] time = 0.69, size = 237, normalized size = 0.86

$$-\frac{ie^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\left(\frac{ib(c+dx)}{d}\right)^{-m}\left(3^{-m}\left(e^{\frac{6ibc}{d}}\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^{2m}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+}{d}\right.\right.\right.\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/24*I)*(c + d*x)^m*((3*E^((2*I)*(2*a + (b*c)/d))*Gamma[1 + m, ((-I)*b*(
c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m + (-3*E^((2*I)*a + ((4*I)*b*c)/d)*Ga
mma[1 + m, (I*b*(c + d*x))/d] + (-((E^((6*I)*a)*((I*b*(c + d*x))/d)^(2*m)*G
amma[1 + m, ((-3*I)*b*(c + d*x))/d])/((b^2*(c + d*x)^2)/d^2)^m) + E^(((6*I)
*b*c)/d)*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/3^m)/((I*b*(c + d*x))/d)^m))/
(b*E^(((3*I)*(b*c + a*d))/d))
```

fricas [A] time = 0.61, size = 186, normalized size = 0.68

$$\frac{-i\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,\frac{3i\,bdx+3i\,bc}{d}\right)+3i\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(m+1,\frac{i\,bdx+i\,bc}{d}\right)-3i\,e^{\left(-\frac{dm\log\left(-\frac{i\,b}{d}\right)+i\,bc-i\,ad}{d}\right)}\Gamma\left(\right.}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*
d*x + 3*I*b*c)/d) + 3*I*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1
, (I*b*d*x + I*b*c)/d) - 3*I*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma
(m + 1, (-I*b*d*x - I*b*c)/d) + I*e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*
d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\sin^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)**2*cos(a + b*x), x)
```

3.14 $\int (c+dx)^4 \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=205

$$\frac{8d^4\sin^3(a+bx)}{81b^5}+\frac{160d^4\sin(a+bx)}{27b^5}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a+bx)}{27b^4}-\frac{4d^2(c+d}{}$$

```
[Out] -160/27*d^3*(d*x+c)*cos(b*x+a)/b^4+8/9*d*(d*x+c)^3*cos(b*x+a)/b^2+160/27*d^
4*sin(b*x+a)/b^5-8/3*d^2*(d*x+c)^2*sin(b*x+a)/b^3-8/27*d^3*(d*x+c)*cos(b*x+
a)*sin(b*x+a)^2/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2/b^2+8/81*d^4*si
n(b*x+a)^3/b^5-4/9*d^2*(d*x+c)^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^4*sin(b*x+a)^
3/b
```

Rubi [A] time = 0.20, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 3296, 2637, 3310}

$$-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}-\frac{8d^2(c+dx)^2\sin(a+bx)}{3b^3}-\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin^2(a+bx)\cos(a}{27b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cos[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Cos[a + b*x])
/(9*b^2) + (160*d^4*Sin[a + b*x])/(27*b^5) - (8*d^2*(c + d*x)^2*Sin[a + b*x
])/(3*b^3) - (8*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(27*b^4) + (4*d*
(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) + (8*d^4*Sin[a + b*x]^3)/(
81*b^5) - (4*d^2*(c + d*x)^2*Sin[a + b*x]^3)/(9*b^3) + ((c + d*x)^4*Sin[a +
 b*x]^3)/(3*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)^4\sin^3(a+bx)}{3b}-\frac{(4d)\int (c+dx)^3\sin^3(a+bx)\,dx}{3b}\\
&= \frac{4d(c+dx)^3\cos(a+bx)\sin^2(a+bx)}{9b^2}-\frac{4d^2(c+dx)^2\sin^3(a+bx)}{9b^3}+\frac{(c\\
&= \frac{8d(c+dx)^3\cos(a+bx)}{9b^2}-\frac{8d^3(c+dx)\cos(a+bx)\sin^2(a+bx)}{27b^4}+\frac{4d(\\
&= -\frac{16d^3(c+dx)\cos(a+bx)}{27b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{9b^2}-\frac{8d^2(c+dx)^2 s}{3b^3}\\
&= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{9b^2}+\frac{16d^4\sin(a+}{27b^5}\\
&= -\frac{160d^3(c+dx)\cos(a+bx)}{27b^4}+\frac{8d(c+dx)^3\cos(a+bx)}{9b^2}+\frac{160d^4\sin(a}{27b^5}\end{aligned}$$

Mathematica [A] time = 1.45, size = 385, normalized size = 1.88

$$\frac{81b^4c^4\sin(a+bx)-27b^4c^4\sin(3(a+bx))+324b^4c^3dx\sin(a+bx)-108b^4c^3dx\sin(3(a+bx))+486b^4c^2d^2x^2 s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (324*b*d*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 12*b*d*(c + d*
x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*b^4*c^4*Sin[a + b*x]
- 972*b^2*c^2*d^2*Sin[a + b*x] + 1944*d^4*Sin[a + b*x] + 324*b^4*c^3*d*x*Si
n[a + b*x] - 1944*b^2*c*d^3*x*Sin[a + b*x] + 486*b^4*c^2*d^2*x^2*Sin[a + b*
x] - 972*b^2*d^4*x^2*Sin[a + b*x] + 324*b^4*c*d^3*x^3*Sin[a + b*x] + 81*b^4
*d^4*x^4*Sin[a + b*x] - 27*b^4*c^4*Sin[3*(a + b*x)] + 36*b^2*c^2*d^2*Sin[3*
(a + b*x)] - 8*d^4*Sin[3*(a + b*x)] - 108*b^4*c^3*d*x*Sin[3*(a + b*x)] + 72
*b^2*c*d^3*x*Sin[3*(a + b*x)] - 162*b^4*c^2*d^2*x^2*Sin[3*(a + b*x)] + 36*b
^2*d^4*x^2*Sin[3*(a + b*x)] - 108*b^4*c*d^3*x^3*Sin[3*(a + b*x)] - 27*b^4*d
^4*x^4*Sin[3*(a + b*x)])/(324*b^5)
```

fricas [A] time = 0.54, size = 352, normalized size = 1.72

$$-\frac{12\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d-2\,bcd^3+\left(9\,b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)^3-36\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/81*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (9*b
^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^3 - 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 3*b^3*c^3*d - 14*b*c*d^3 + (9*b^3*c^2*d^2 - 14*b*d^4)*x)*cos(b*x + a)
- (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 252*b^2*c^2*d^2 + 488*
d^4 + 18*(9*b^4*c^2*d^2 - 14*b^2*d^4)*x^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3
*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)
*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)*x)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d -
 14*b^2*c*d^3)*x)*sin(b*x + a))/b^5
```

giac [A] time = 0.23, size = 350, normalized size = 1.71

$$-\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b^3c^2d^2x+3\,b^3c^3d-2\,bd^4x-2\,bcd^3\right)\cos\left(3\,bx+3\,a\right)}{27\,b^5}+\frac{\left(b^3d^4x^3+3\,b^3cd^3x^2+3\,b^3c^2d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*
b*d^4*x - 2*b*c*d^3)*cos(3*b*x + 3*a)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
+ 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1
/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^
3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d
```

```
^4)*sin(3*b*x + 3*a)/b^5 + 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2
*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

maple [B] time = 0.09, size = 835, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*sin(b*x+a)^3+4/9*(b*x+a)^3*(2+sin(b*x+a)^2)*c
os(b*x+a)-8/3*(b*x+a)^2*sin(b*x+a)+160/27*sin(b*x+a)-16/3*(b*x+a)*cos(b*x+a
)-4/9*(b*x+a)^2*sin(b*x+a)^3-8/27*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+8/81*
sin(b*x+a)^3)-4/b^4*a*d^4*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(
b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(
b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b^3*c*d^3*(1/3*(b*x+a)^3*sin(b
*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a
)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/27*(2+sin(b*x+a)^2)*cos(b*x+a))+6/b
^4*a^2*d^4*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x
+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*sin(b*x
+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x
+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)
*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*si
n(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*s
in(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-12/b^2*a*c^2*d^2*(1/3*(b*x+a)*
sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+4/b*c^3*d*(1/3*(b*x+a)*sin(b*
x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^4*a^4*d^4*sin(b*x+a)^3-4/3/b^
3*a^3*c*d^3*sin(b*x+a)^3+2/b^2*a^2*c^2*d^2*sin(b*x+a)^3-4/3/b*a*c^3*d*sin(b
*x+a)^3+1/3*c^4*sin(b*x+a)^3)
```

maxima [B] time = 0.42, size = 880, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/324*(108*c^4*sin(b*x + a)^3 - 432*a*c^3*d*sin(b*x + a)^3/b + 648*a^2*c^2*
d^2*sin(b*x + a)^3/b^2 - 432*a^3*c*d^3*sin(b*x + a)^3/b^3 + 108*a^4*d^4*sin
(b*x + a)^3/b^4 - 36*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x +
a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*c^3*d/b + 108*(3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*
c^2*d^2/b^2 - 108*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*c*d^3/b^3 + 36*(3*(b*x + a)*sin(3*
```

```
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
a^3*d^4/b^4 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
+ (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))
*c^2*d^2/b^2 + 36*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a)
 + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a)
)*a*c*d^3/b^3 - 18*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a
) + (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a
))*a^2*d^4/b^4 - 12*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2
 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*
((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 + 12*((9*(b*x + a)^2 -
2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a) + 3*(3*(b*x + a)^3
- 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
a))*a*d^4/b^4 - (12*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 324*((
b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) + (27*(b*x + a)^4 - 36*(b*x + a)^2 +
 8)*sin(3*b*x + 3*a) - 81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*sin(b*x + a))
*d^4/b^4)/b
```

mupad [B] time = 1.41, size = 448, normalized size = 2.19

$$\frac{\sin(a+b\,x)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5}-\frac{8\cos(a+b\,x)^3\,\left(20\,c\,d^3-3\,b^2\,c^3\,d\right)}{27\,b^4}+\frac{8\cos(a+b\,x)^2\,\sin(a+b\,x}{27\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] (sin(a + b*x)^3*(488*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos
(a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*cos(a + b*x)^2*sin(a +
b*x)*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)*sin(a + b*x)^2*(1
4*c*d^3 - 3*b^2*c^3*d))/(9*b^4) + (8*d^4*x^3*cos(a + b*x)^3)/(9*b^2) - (8*x
*cos(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (d^4*x^4*sin(a + b*x)^
3)/(3*b) - (4*x*sin(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (2*x^2*s
in(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) + (8*c*d^3*x^2*cos(a + b*x)
^3)/(3*b^2) + (4*d^4*x^3*cos(a + b*x)*sin(a + b*x)^2)/(3*b^2) - (8*d^4*x^2*
cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^3*x^3*sin(a + b*x)^3)/(3*b) -
 (4*x*cos(a + b*x)*sin(a + b*x)^2*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^4) + (4*c*
d^3*x^2*cos(a + b*x)*sin(a + b*x)^2)/b^2 - (16*c*d^3*x*cos(a + b*x)^2*sin(a
 + b*x))/(3*b^3)
```

sympy [A] time = 7.48, size = 646, normalized size = 3.15

$$\begin{cases} \frac{c^4\sin^3(a+bx)}{3b}+\frac{4c^3dx\sin^3(a+bx)}{3b}+\frac{2c^2d^2x^2\sin^3(a+bx)}{b}+\frac{4cd^3x^3\sin^3(a+bx)}{3b}+\frac{d^4x^4\sin^3(a+bx)}{3b}+\frac{4c^3d\sin^2(a+bx)\cos(a+bx)}{3b^2}+\frac{8c^3d\cos^{}}{9b} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin^2(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*x*sin(a + b*x)**3/(3*b) +
2*c**2*d**2*x**2*sin(a + b*x)**3/b + 4*c*d**3*x**3*sin(a + b*x)**3/(3*b) +
d**4*x**4*sin(a + b*x)**3/(3*b) + 4*c**3*d*sin(a + b*x)**2*cos(a + b*x)/(3*
b**2) + 8*c**3*d*cos(a + b*x)**3/(9*b**2) + 4*c**2*d**2*x*sin(a + b*x)**2*c
os(a + b*x)/b**2 + 8*c**2*d**2*x*cos(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*s
in(a + b*x)**2*cos(a + b*x)/b**2 + 8*c*d**3*x**2*cos(a + b*x)**3/(3*b**2) +
 4*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)/(3*b**2) + 8*d**4*x**3*cos(a + b*
x)**3/(9*b**2) - 28*c**2*d**2*sin(a + b*x)**3/(9*b**3) - 8*c**2*d**2*sin(a
+ b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**3*x*sin(a + b*x)**3/(9*b**3) - 16
*c*d**3*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 28*d**4*x**2*sin(a + b*x)
**3/(9*b**3) - 8*d**4*x**2*sin(a + b*x)*cos(a + b*x)**2/(3*b**3) - 56*c*d**
3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*c*d**3*cos(a + b*x)**3/(27*b*
*4) - 56*d**4*x*sin(a + b*x)**2*cos(a + b*x)/(9*b**4) - 160*d**4*x*cos(a +
b*x)**3/(27*b**4) + 488*d**4*sin(a + b*x)**3/(81*b**5) + 160*d**4*sin(a + b
*x)*cos(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2
*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a), True))
```

3.15 $\int (c+dx)^3 \cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=151

$$\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{14d^3\cos(a+bx)}{9b^4}-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}$$

```
[Out] -14/9*d^3*cos(b*x+a)/b^4+2/3*d*(d*x+c)^2*cos(b*x+a)/b^2+2/27*d^3*cos(b*x+a)
^3/b^4-4/3*d^2*(d*x+c)*sin(b*x+a)/b^3+1/3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)
^2/b^2-2/9*d^2*(d*x+c)*sin(b*x+a)^3/b^3+1/3*(d*x+c)^3*sin(b*x+a)^3/b
```

Rubi [A] time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 3296, 2638, 2633}

$$-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}-\frac{4d^2(c+dx)\sin(a+bx)}{3b^3}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin^2(a+bx)\cos(a+bx}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (-14*d^3*Cos[a + b*x])/(9*b^4) + (2*d*(c + d*x)^2*Cos[a + b*x])/(3*b^2) + (
2*d^3*Cos[a + b*x]^3)/(27*b^4) - (4*d^2*(c + d*x)*Sin[a + b*x])/(3*b^3) + (
d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2)/(3*b^2) - (2*d^2*(c + d*x)*Sin[a
 + b*x]^3)/(9*b^3) + ((c + d*x)^3*Sin[a + b*x]^3)/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)^3\sin^3(a+bx)}{3b}-\frac{d\int (c+dx)^2\sin^3(a+bx)\,dx}{b}\\ &= \frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2}-\frac{2d^2(c+dx)\sin^3(a+bx)}{9b^3}+\frac{(c+\\ &= \frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{d(c+dx)^2\cos(a+bx)\sin^2(a+bx)}{3b^2}-\frac{2d^2(\\ &= -\frac{2d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{4d\\ &= -\frac{14d^3\cos(a+bx)}{9b^4}+\frac{2d(c+dx)^2\cos(a+bx)}{3b^2}+\frac{2d^3\cos^3(a+bx)}{27b^4}-\frac{4\end{aligned}$$

Mathematica [A] time = 0.94, size = 121, normalized size = 0.80

$$-\frac{-81d\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+d\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+6b(c+dx)\sin(a+bx)\left(\cos(2(a\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -1/108*(-81*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + d*(-2*d^2 + 9*b^2*(
c + d*x)^2)*Cos[3*(a + b*x)] + 6*b*(c + d*x)*(26*d^2 - 3*b^2*(c + d*x)^2 +
(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^4
```

fricas [A] time = 0.49, size = 227, normalized size = 1.50

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3-3\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-14\,d^3\right)\cos\left(bx+a\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^
3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 14*d^3)*cos(b*x + a)
- 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 14*b*c*d^2 - (3*b^3*d^3*
x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*c^2*d - 2*b*d^3)*x)*
cos(b*x + a)^2 + (9*b^3*c^2*d - 14*b*d^3)*x)*sin(b*x + a))/b^4
```

giac [A] time = 0.22, size = 231, normalized size = 1.53

$$-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx+3\,a\right)}{108\,b^4}+\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)}{4\,b^4}-\frac{(3\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(3*b*x + 3
*a)/b^4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a
)/b^4 - 1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 -
 2*b*d^3*x - 2*b*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/4*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

maple [B] time = 0.01, size = 447, normalized size = 2.96

$$\frac{\frac{d^3\left(\frac{(bx+a)^3\left(\sin^3(bx+a)\right)}{3}+\frac{(bx+a)^2\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}-\frac{4\cos(bx+a)}{3}-\frac{4(bx+a)\sin(bx+a)}{3}-\frac{2(bx+a)\left(\sin^3(bx+a)\right)}{9}-\frac{2\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{27}\right)}{b^3}-\frac{3a\,d^3\left(\frac{(bx+a)^2\left(\sin^3(\right.}{3}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*sin(b*x+a)^3+1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*c
os(b*x+a)-4/3*cos(b*x+a)-4/3*(b*x+a)*sin(b*x+a)-2/9*(b*x+a)*sin(b*x+a)^3-2/
27*(2+sin(b*x+a)^2)*cos(b*x+a))-3/b^3*a*d^3*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9
*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^
2*c*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)
```

```
-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))+3/b^3*a^2*d^3*(1/3*(b*x+a)*sin(b*x+a)^3+
1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-6/b^2*a*c*d^2*(1/3*(b*x+a)*sin(b*x+a)^3+1/
9*(2+sin(b*x+a)^2)*cos(b*x+a))+3/b*c^2*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+s
in(b*x+a)^2)*cos(b*x+a))-1/3/b^3*a^3*d^3*sin(b*x+a)^3+1/b^2*a^2*c*d^2*sin(b
*x+a)^3-1/b*a*c^2*d*sin(b*x+a)^3+1/3*c^3*sin(b*x+a)^3)
```

maxima [B] time = 0.37, size = 499, normalized size = 3.30

$$\frac{36\,c^3\sin(bx+a)^3 - \frac{108\,ac^2d\sin(bx+a)^3}{b} + \frac{108\,a^2cd^2\sin(bx+a)^3}{b^2} - \frac{36\,a^3d^3\sin(bx+a)^3}{b^3} - \frac{9\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+c\text{c}}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^3*sin(b*x + a)^3 - 108*a*c^2*d*sin(b*x + a)^3/b + 108*a^2*c*d^2
*sin(b*x + a)^3/b^2 - 36*a^3*d^3*sin(b*x + a)^3/b^3 - 9*(3*(b*x + a)*sin(3*
b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*
c^2*d/b + 18*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos
(3*b*x + 3*a) - 9*cos(b*x + a))*a*c*d^2/b^2 - 9*(3*(b*x + a)*sin(3*b*x + 3*
a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a^2*d^3/
b^3 - 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x
 + a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^2/b^
2 + 3*(6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x +
 a)^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^3/b^3
- ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*cos(b*x + a)
 + 3*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) - 27*((b*x + a)^3 - 6*b
*x - 6*a)*sin(b*x + a))*d^3/b^3)/b
```

mupad [B] time = 1.15, size = 289, normalized size = 1.91

$$\frac{2\,d^3\,x^2\cos(a+b\,x)^3}{3\,b^2} - \frac{\sin(a+b\,x)^3\,\left(14\,c\,d^2-3\,b^2\,c^3\right)}{9\,b^3} - \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2\,\left(14\,d^3-9\,b^2\,c^2\,d\right)}{9\,b^4} - \frac{x\sin(a-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] (2*d^3*x^2*cos(a + b*x)^3)/(3*b^2) - (sin(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3)
)/(9*b^3) - (cos(a + b*x)*sin(a + b*x)^2*(14*d^3 - 9*b^2*c^2*d))/(9*b^4) -
(x*sin(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (2*cos(a + b*x)^3*(20*d
^3 - 9*b^2*c^2*d))/(27*b^4) + (d^3*x^3*sin(a + b*x)^3)/(3*b) - (4*c*d^2*cos
(a + b*x)^2*sin(a + b*x))/(3*b^3) + (4*c*d^2*x*cos(a + b*x)^3)/(3*b^2) - (4
*d^3*x*cos(a + b*x)^2*sin(a + b*x))/(3*b^3) + (d^3*x^2*cos(a + b*x)*sin(a +
 b*x)^2)/b^2 + (c*d^2*x^2*sin(a + b*x)^3)/b + (2*c*d^2*x*cos(a + b*x)*sin(a
 + b*x)^2)/b^2
```

sympy [A] time = 4.01, size = 391, normalized size = 2.59

$$\begin{cases} \frac{c^3\sin^3(a+bx)}{3b} + \frac{c^2dx\sin^3(a+bx)}{b} + \frac{cd^2x^2\sin^3(a+bx)}{b} + \frac{d^3x^3\sin^3(a+bx)}{3b} + \frac{c^2d\sin^2(a+bx)\cos(a+bx)}{b^2} + \frac{2c^2d\cos^3(a+bx)}{3b^2} + \frac{2cd^2x\sin^2(a+b}{b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^2(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**3/(3*b) + c**2*d*x*sin(a + b*x)**3/b + c*d**2
*x**2*sin(a + b*x)**3/b + d**3*x**3*sin(a + b*x)**3/(3*b) + c**2*d*sin(a +
b*x)**2*cos(a + b*x)/b**2 + 2*c**2*d*cos(a + b*x)**3/(3*b**2) + 2*c*d**2*x*
sin(a + b*x)**2*cos(a + b*x)/b**2 + 4*c*d**2*x*cos(a + b*x)**3/(3*b**2) + d
**3*x**2*sin(a + b*x)**2*cos(a + b*x)/b**2 + 2*d**3*x**2*cos(a + b*x)**3/(3
*b**2) - 14*c*d**2*sin(a + b*x)**3/(9*b**3) - 4*c*d**2*sin(a + b*x)*cos(a +
 b*x)**2/(3*b**3) - 14*d**3*x*sin(a + b*x)**3/(9*b**3) - 4*d**3*x*sin(a + b
*x)*cos(a + b*x)**2/(3*b**3) - 14*d**3*sin(a + b*x)**2*cos(a + b*x)/(9*b**4
) - 40*d**3*cos(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/
2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**2*cos(a), True))
```

3.16 $\int (c+dx)^2 \cos(a+bx) \sin^2(a+bx)\, dx$

Optimal. Leaf size=103

$$-\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2}+\frac{(c+dx)^2\sin^3(a+bx)}{3b}$$

```
[Out] 4/9*d*(d*x+c)*cos(b*x+a)/b^2-4/9*d^2*sin(b*x+a)/b^3+2/9*d*(d*x+c)*cos(b*x+a
)*sin(b*x+a)^2/b^2-2/27*d^2*sin(b*x+a)^3/b^3+1/3*(d*x+c)^2*sin(b*x+a)^3/b
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3310, 3296, 2637}

$$\frac{4d(c+dx)\cos(a+bx)}{9b^2}+\frac{2d(c+dx)\sin^2(a+bx)\cos(a+bx)}{9b^2}-\frac{2d^2\sin^3(a+bx)}{27b^3}-\frac{4d^2\sin(a+bx)}{9b^3}+\frac{(c+dx)^2\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (4*d*(c + d*x)*Cos[a + b*x])/(9*b^2) - (4*d^2*Sin[a + b*x])/(9*b^3) + (2*d*
(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2)/(9*b^2) - (2*d^2*Sin[a + b*x]^3)/(27
*b^3) + ((c + d*x)^2*Sin[a + b*x]^3)/(3*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin^2(a+bx)\,dx = \frac{(c+dx)^2\sin^3(a+bx)}{3b} - \frac{(2d)\int (c+dx)\sin^3(a+bx)\,dx}{3b}$$
$$= \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin^3(a+bx)}{27b^3} + \frac{(c+dx)^2\sin}{3b}$$
$$= \frac{4d(c+dx)\cos(a+bx)}{9b^2} + \frac{2d(c+dx)\cos(a+bx)\sin^2(a+bx)}{9b^2} - \frac{2d^2\sin}{2}$$
$$= \frac{4d(c+dx)\cos(a+bx)}{9b^2} - \frac{4d^2\sin(a+bx)}{9b^3} + \frac{2d(c+dx)\cos(a+bx)\sin}{9b^2}$$

Mathematica [A] time = 0.58, size = 93, normalized size = 0.90

$$\frac{-2\sin(a+bx)\left(\cos(2(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-9b^2(c+dx)^2+26d^2\right)+54bd(c+dx)\cos(a+bx)-6bd(c+}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (54*b*d*(c + d*x)*Cos[a + b*x] - 6*b*d*(c + d*x)*Cos[3*(a + b*x)] - 2*(26*d
^2 - 9*b^2*(c + d*x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin
[a + b*x])/(108*b^3)
```

fricas [A] time = 0.49, size = 130, normalized size = 1.26

$$-\frac{6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-18\left(bd^2x+bcd\right)\cos\left(bx+a\right)-\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-\left(9\,b^2d^2x^2+18\,b^2c\right.\right.}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/27*(6*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 18*(b*d^2*x + b*c*d)*cos(b*x +
a) - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - (9*b^2*d^2*x^2 + 18*b^2*c*
d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - 14*d^2)*sin(b*x + a))/b^3
```

giac [A] time = 0.21, size = 137, normalized size = 1.33

$$-\frac{\left(bd^2x+bcd\right)\cos\left(3\,bx+3\,a\right)}{18\,b^3}+\frac{\left(bd^2x+bcd\right)\cos\left(bx+a\right)}{2\,b^3}-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\sin\left(3\,bx+3\,a\right)}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/18*(b*d^2*x + b*c*d)*cos(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*cos(b*
x + a)/b^3 - 1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3
*b*x + 3*a)/b^3 + 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x
 + a)/b^3
```

maple [B] time = 0.01, size = 204, normalized size = 1.98

$$\frac{\frac{d^2\left(\frac{(bx+a)^2\left(\sin^3(bx+a)\right)}{3}+\frac{2(bx+a)\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}-\frac{2\left(\sin^3(bx+a)\right)}{27}-\frac{4\sin(bx+a)}{9}\right)}{b^2}-\frac{2a\,d^2\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b^2}+\frac{2cd\left(\frac{(bx+a)\left(s\right.}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*sin(b*x+a)^3+2/9*(b*x+a)*(2+sin(b*x+a)^2)*cos
(b*x+a)-2/27*sin(b*x+a)^3-4/9*sin(b*x+a))-2/b^2*a*d^2*(1/3*(b*x+a)*sin(b*x+
a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))+2/b*c*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9
*(2+sin(b*x+a)^2)*cos(b*x+a))+1/3/b^2*a^2*d^2*sin(b*x+a)^3-2/3/b*a*c*d*sin(
b*x+a)^3+1/3*c^2*sin(b*x+a)^3)
```

maxima [B] time = 0.37, size = 240, normalized size = 2.33

$$\frac{36\,c^2\sin\left(bx+a\right)^3-\frac{72\,acd\sin(bx+a)^3}{b}+\frac{36\,a^2d^2\sin(bx+a)^3}{b^2}-\frac{6\,(3\,(bx+a)\sin(3\,bx+3\,a)-9\,(bx+a)\sin(bx+a)+\cos(3\,bx+3\,a)-9\,\cos(bx+a))ca}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/108*(36*c^2*sin(b*x + a)^3 - 72*a*c*d*sin(b*x + a)^3/b + 36*a^2*d^2*sin(b
*x + a)^3/b^2 - 6*(3*(b*x + a)*sin(3*b*x + 3*a) - 9*(b*x + a)*sin(b*x + a)
+ cos(3*b*x + 3*a) - 9*cos(b*x + a))*c*d/b + 6*(3*(b*x + a)*sin(3*b*x + 3*a
) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*a*d^2/b^2
 - (6*(b*x + a)*cos(3*b*x + 3*a) - 54*(b*x + a)*cos(b*x + a) + (9*(b*x + a)
^2 - 2)*sin(3*b*x + 3*a) - 27*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

mupad [B] time = 0.87, size = 161, normalized size = 1.56

$$\frac{4\,d^2\,x\cos\left(a+b\,x\right)^3}{9\,b^2}-\frac{4\,d^2\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{9\,b^3}-\frac{\sin\left(a+b\,x\right)^3\,\left(14\,d^2-9\,b^2\,c^2\right)}{27\,b^3}+\frac{d^2\,x^2\sin\left(a+b\,x\right)^3}{3\,b}+\frac{4\,c\,d\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (4*d^2*x*cos(a + b*x)^3)/(9*b^2) - (4*d^2*cos(a + b*x)^2*sin(a + b*x))/(9*b
^3) - (sin(a + b*x)^3*(14*d^2 - 9*b^2*c^2))/(27*b^3) + (d^2*x^2*sin(a + b*x
)^3)/(3*b) + (4*c*d*cos(a + b*x)^3)/(9*b^2) + (2*c*d*cos(a + b*x)*sin(a + b
*x)^2)/(3*b^2) + (2*c*d*x*sin(a + b*x)^3)/(3*b) + (2*d^2*x*cos(a + b*x)*sin
(a + b*x)^2)/(3*b^2)
```

sympy [A] time = 2.09, size = 216, normalized size = 2.10

$$\begin{cases}\frac{c^2\sin^3\left(a+bx\right)}{3b}+\frac{2cdx\sin^3\left(a+bx\right)}{3b}+\frac{d^2x^2\sin^3\left(a+bx\right)}{3b}+\frac{2cd\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b^2}+\frac{4cd\cos^3\left(a+bx\right)}{9b^2}+\frac{2d^2x\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{3b^2}+\frac{4d^2x\,c}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2\left(a\right)\cos\left(a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**3/(3*b) + 2*c*d*x*sin(a + b*x)**3/(3*b) + d**
2*x**2*sin(a + b*x)**3/(3*b) + 2*c*d*sin(a + b*x)**2*cos(a + b*x)/(3*b**2)
+ 4*c*d*cos(a + b*x)**3/(9*b**2) + 2*d**2*x*sin(a + b*x)**2*cos(a + b*x)/(3
*b**2) + 4*d**2*x*cos(a + b*x)**3/(9*b**2) - 14*d**2*sin(a + b*x)**3/(27*b*
*3) - 4*d**2*sin(a + b*x)*cos(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c
*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a), True))
```

3.17 $\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

```
[Out] 1/3*d*cos(b*x+a)/b^2-1/9*d*cos(b*x+a)^3/b^2+1/3*(d*x+c)*sin(b*x+a)^3/b
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4404, 2633}

$$-\frac{d\cos^3(a+bx)}{9b^2}+\frac{d\cos(a+bx)}{3b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(3*b^2) - (d*Cos[a + b*x]^3)/(9*b^2) + ((c + d*x)*Sin[a + b*x]^3)/(3*b)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos(a+bx)\sin^2(a+bx)\,dx &= \frac{(c+dx)\sin^3(a+bx)}{3b}-\frac{d\int \sin^3(a+bx)\,dx}{3b}\\ &= \frac{(c+dx)\sin^3(a+bx)}{3b}+\frac{d\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,\cos(a+bx)\right)}{3b^2}\\ &= \frac{d\cos(a+bx)}{3b^2}-\frac{d\cos^3(a+bx)}{9b^2}+\frac{(c+dx)\sin^3(a+bx)}{3b}\end{aligned}$$

Mathematica [A] time = 0.17, size = 44, normalized size = 0.86

$$\frac{12b(c+dx)\sin^3(a+bx)+9d\cos(a+bx)-d\cos(3(a+bx))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (9*d*Cos[a + b*x] - d*Cos[3*(a + b*x)] + 12*b*(c + d*x)*Sin[a + b*x]^3)/(36
*b^2)
```

fricas [A] time = 0.80, size = 59, normalized size = 1.16

$$-\frac{d\cos(bx+a)^3-3\,d\cos(bx+a)-3\left(bdx-(bdx+bc)\cos(bx+a)^2+bc\right)\sin(bx+a)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/9*(d*cos(b*x + a)^3 - 3*d*cos(b*x + a) - 3*(b*d*x - (b*d*x + b*c)*cos(b*
x + a)^2 + b*c)*sin(b*x + a))/b^2
```

giac [A] time = 0.19, size = 69, normalized size = 1.35

$$-\frac{d\cos(3\,bx+3\,a)}{36\,b^2}+\frac{d\cos(bx+a)}{4\,b^2}-\frac{(bdx+bc)\sin(3\,bx+3\,a)}{12\,b^2}+\frac{(bdx+bc)\sin(bx+a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/36*d*cos(3*b*x + 3*a)/b^2 + 1/4*d*cos(b*x + a)/b^2 - 1/12*(b*d*x + b*c)*
sin(3*b*x + 3*a)/b^2 + 1/4*(b*d*x + b*c)*sin(b*x + a)/b^2
```

maple [A] time = 0.01, size = 71, normalized size = 1.39

$$\frac{\frac{d\left(\frac{(bx+a)\left(\sin^3(bx+a)\right)}{3}+\frac{\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{9}\right)}{b}-\frac{da\left(\sin^3(bx+a)\right)}{3b}+\frac{c\left(\sin^3(bx+a)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*(1/3*(b*x+a)*sin(b*x+a)^3+1/9*(2+sin(b*x+a)^2)*cos(b*x+a))-1/3/b
*d*a*sin(b*x+a)^3+1/3*c*sin(b*x+a)^3)
```

maxima [A] time = 0.34, size = 85, normalized size = 1.67

$$\frac{12\,c\sin\left(bx+a\right)^{3}-\frac{12\,ad\sin\left(bx+a\right)^{3}}{b}-\frac{\left(3\,\left(bx+a\right)\sin\left(3\,bx+3\,a\right)-9\,\left(bx+a\right)\sin\left(bx+a\right)+\cos\left(3\,bx+3\,a\right)-9\,\cos\left(bx+a\right)\right)d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/36*(12*c*sin(b*x + a)^3 - 12*a*d*sin(b*x + a)^3/b - (3*(b*x + a)*sin(3*b*
x + 3*a) - 9*(b*x + a)*sin(b*x + a) + cos(3*b*x + 3*a) - 9*cos(b*x + a))*d/
b)/b
```

mupad [B] time = 0.15, size = 59, normalized size = 1.16

$$\frac{\frac{2\,d\cos(a+b\,x)^3}{9}+b\,\left(\frac{c\sin(a+b\,x)^3}{3}+\frac{d\,x\sin(a+b\,x)^3}{3}\right)+\frac{d\,\cos(a+b\,x)\sin(a+b\,x)^2}{3}}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] ((2*d*cos(a + b*x)^3)/9 + b*((c*sin(a + b*x)^3)/3 + (d*x*sin(a + b*x)^3)/3)
 + (d*cos(a + b*x)*sin(a + b*x)^2)/3)/b^2
```

sympy [A] time = 0.87, size = 85, normalized size = 1.67

$$\begin{cases} \frac{c\sin^{3}{\left(a+bx\right)}}{3b}+\frac{dx\sin^{3}{\left(a+bx\right)}}{3b}+\frac{d\sin^{2}{\left(a+bx\right)}\cos{\left(a+bx\right)}}{3b^{2}}+\frac{2d\cos^{3}{\left(a+bx\right)}}{9b^{2}} & \text{for}\: b \neq 0 \\ \left(cx+\frac{dx^{2}}{2}\right)\sin^{2}{\left(a\right)}\cos{\left(a\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**3/(3*b) + d*x*sin(a + b*x)**3/(3*b) + d*sin(a +
b*x)**2*cos(a + b*x)/(3*b**2) + 2*d*cos(a + b*x)**3/(9*b**2), Ne(b, 0)), ((
c*x + d*x**2/2)*sin(a)**2*cos(a), True))
```

3.18 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

```
[Out] -1/4*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+
1/4*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/4*Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] time = 0.27, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sin\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)} - \frac{\cos(3a+3bx)}{4(c+dx)}\right) dx$$
$$= \frac{1}{4}\int \frac{\cos(a+bx)}{c+dx}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx\right) + \frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}$$
$$= \frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d} - \frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}\right.}{4d}$$

Mathematica [A] time = 0.32, size = 102, normalized size = 0.84

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right) - \sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) + \sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosIntegr
al[(3*b*(c + d*x))/d] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

fricas [A] time = 0.49, size = 153, normalized size = 1.26

$$\frac{\left(\text{Ci}\left(\frac{bdx+bc}{d}\right)+\text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) - \left(\text{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right) + 2\,\sin\left(-\frac{3\,(bc-ad)}{d}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*cos(-
(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*cos(-3*(b*c - a*d)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) - 2*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

giac [C] time = 0.54, size = 6059, normalized size = 50.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/8*(real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))
*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(co
s_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/
2*b*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(
3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b
*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integr
al(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2
*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3
*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
+ 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/
2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_integral(-3*b*
x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_
integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - 4*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3
/2*b*c/d)^2*tan(1/2*b*c/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d)
)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_p
```

```
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*t
an(1/2*b*c/d)^2 + 4*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*ta
n(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + real_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/
d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)
^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c/d))
*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - real_part(cos
_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2
 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x +
 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integ
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*sin_int
egral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 2*imag_
part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) -
2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) + 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b
*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^
2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(b*x + b*c/d))*t
an(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(-b*x
 - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 4*sin_integral((b
*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(c
os_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2 + 2*imag
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
- 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d)^2
- 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/
2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2
*a)^2*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(1
/2*a)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3
/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*
(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part
(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*
```

```
b*c/d)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/
2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(-3*b*x -
3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 4*sin_integral(3*(
b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*imag_part(
cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(
1/2*b*c/d)^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - real_part(
cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + 4*real_part(cos
_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 4*real
_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d
) + real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2
- real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - real
_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + real_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - real_part(
cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(co
s_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + real_part(cos_inte
gral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - real_part(cos_integral(
-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 4*real_part(cos_integral
(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*re
al_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c
/d) - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^
2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + re
al_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(
cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - real_part(cos_in
tegral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + real_part(cos_integra
l(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*real_part(cos_integr
al(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 4*real_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)
^2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + real_part(cos_integral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
^2 + real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^
2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a) - 2*i
mag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 4*sin_integr
al((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a) - 2*imag_part(cos_integral(3*b*
x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 + 2*imag_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)*tan(1/2*a)^2 - 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2
```

```
*tan(3/2*b*c/d) + 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*
tan(3/2*b*c/d) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(3/2*b*c
/d) + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d
) - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)
 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*imag_p
art(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 - 2*imag_par
t(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integ
ral(3*(b*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)^2 + 2*imag_part(cos_integr
al(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 4*sin_integral((b*d*x + b*c)/d)*t
an(1/2*a)*tan(3/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(3/2
*a)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2
*tan(1/2*b*c/d) - 4*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/
d) + 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 4*sin_
integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*imag_part(cos_int
egral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*sin_integral((b*d*x
+ b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*imag_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 4*sin_integral(3*(b*d*x + b*c)/d)*
tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/
2*a)*tan(1/2*b*c/d)^2 + 2*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*
tan(1/2*b*c/d)^2 - 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d
)^2 + 2*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 2*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)*tan(1/2*
b*c/d)^2 + 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^
2 - real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2 - real_part(cos_i
ntegral(b*x + b*c/d))*tan(3/2*a)^2 - real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2 + rea
l_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2 + real_part(cos_integral
(b*x + b*c/d))*tan(1/2*a)^2 + real_part(cos_integral(-b*x - b*c/d))*tan(1/2
*a)^2 + real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 4*real_par
t(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 4*real_part(co
s_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - real_part(cos_int
egral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2 - real_part(cos_integral(b*x + b*c
/d))*tan(3/2*b*c/d)^2 - real_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d
)^2 - real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 4*real_p
art(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*real_part(cos_
integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 + real_part(cos_integral(b*x + b*c/d))*t
an(1/2*b*c/d)^2 + real_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 +
real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 - 2*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a) - 4*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a) + 2*im
```

```
ag_part(cos_integral(b*x + b*c/d))*tan(1/2*a) - 2*imag_part(cos_integral(-b
*x - b*c/d))*tan(1/2*a) + 4*sin_integral((b*d*x + b*c)/d)*tan(1/2*a) + 2*im
ag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*imag_part(cos_int
egral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) + 4*sin_integral(3*(b*d*x + b*c)/d)
*tan(3/2*b*c/d) - 2*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*b*c/d) + 2
*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d) - 4*sin_integral((b*d
*x + b*c)/d)*tan(1/2*b*c/d) + real_part(cos_integral(3*b*x + 3*b*c/d)) - re
al_part(cos_integral(b*x + b*c/d)) - real_part(cos_integral(-b*x - b*c/d))
+ real_part(cos_integral(-3*b*x - 3*b*c/d)))/(d*tan(3/2*a)^2*tan(1/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/
2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan
(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)
^2*tan(1/2*b*c/d)^2 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2
+ d*tan(1/2*a)^2 + d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

maple [A] time = 0.01, size = 166, normalized size = 1.37

$$\frac{\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}\right)}{4}-\frac{b\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/b*(1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d)-1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+
b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

maxima [C] time = 0.45, size = 274, normalized size = 2.26

$$-\frac{b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3i\,}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integra
l_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp_in
tegral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*(-I*exp_
integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + I*exp_integral_e(1, -(I*
```

```
b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integral_e(
1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3*I*b*c
 + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x), x)
```

3.19 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=168

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}\right.}{4d^2}$$

```
[Out] -1/4*cos(b*x+a)/d/(d*x+c)+1/4*cos(3*b*x+3*a)/d/(d*x+c)-1/4*b*cos(a-b*c/d)*S
i(b*c/d+b*x)/d^2+3/4*b*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^2+3/4*b*Ci(3*b*
c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A] time = 0.30, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Cos[a + b*x]/(4*d*(c + d*x)) + Cos[3*a + 3*b*x]/(4*d*(c + d*x)) + (3*b*Cos
Integral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d^2) - (b*CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d^2) - (b*Cos[a - (b*c)/d]*SinIntegral[
(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
 3*b*x])/(4*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\cos(a+bx)}{4(c+dx)^2} - \frac{\cos(3a+3bx)}{4(c+dx)^2}\right)dx \\
&= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx \\
&= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{4d} + \frac{(3b)\int \frac{\sin(3a+3bx)}{c+dx}\,dx}{4d} \\
&= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d} - \frac{\left(b\cos\right.}{}\\
&= -\frac{\cos(a+bx)}{4d(c+dx)} + \frac{\cos(3a+3bx)}{4d(c+dx)} + \frac{3b\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d^2} - \frac{b\text{Ci}\left(\frac{bc}{d}+\right.}{}
\end{aligned}$$

Mathematica [A] time = 1.37, size = 139, normalized size = 0.83

$$-\frac{-3b\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+b\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right.}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/4*((d*Cos[a + b*x])/(c + d*x) - (d*Cos[3*(a + b*x)])/(c + d*x) - 3*b*Cos
Integral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b*CosIntegral[b*(c/d + x
)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 3*b*Cos
[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

fricas [A] time = 0.60, size = 236, normalized size = 1.40

$$\frac{8\,d\cos\left(bx+a\right)^{3}+6\,(bdx+bc)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\,(bdx+bc)\cos\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)-8\,d\cos\left(bx+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*d*cos(b*x + a)^3 + 6*(b*d*x + b*c)*cos(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) - 2*(b*d*x + b*c)*cos(-(b*c - a*d)/d)*sin_integral((b*
d*x + b*c)/d) - 8*d*cos(b*x + a) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c
)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) +
3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 242, normalized size = 1.44

$$\frac{\frac{b^{2}\left(-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}\right)}{4}-\frac{b^{2}\left(-\frac{3\cos(3bx+3a)}{((bx+a)d-da+cb)d}-\frac{3\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\cdots}{}\right.}{d}\right.}{12}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/4*b^2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos
((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/12*b^2*(-
```

```
3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d
))
```

maxima [C] time = 0.52, size = 302, normalized size = 1.80

$$-\frac{8192\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_2\left(-\frac{3i}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^2*(-8192*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^2*(8192*I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**2, x)
```

3.20 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)}{8d^3}$$

```
[Out] 9/8*b^2*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*cos(a-
b*c/d)/d^3-1/8*cos(b*x+a)/d/(d*x+c)^2+1/8*cos(3*b*x+3*a)/d/(d*x+c)^2-9/8*b^
2*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3+1/8*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)
/d^3+1/8*b*sin(b*x+a)/d^2/(d*x+c)-3/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)
```

Rubi [A] time = 0.36, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2
*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^3) + (b*Sin[a + b*x])/(8*d^2*
(c + d*x)) - (3*b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)) + (b^2*Sin[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Sin[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)^3} - \frac{\cos(3a+3bx)}{4(c+dx)^3}\right) dx$$

$$= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{8d} + \frac{(3b)\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx}{8d}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b\sin(a+bx)}{8d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{8d^2(c+dx)} - \frac{b^2\int \frac{\cos(}{c}}{8a}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} + \frac{b\sin(a+bx)}{8d^2(c+dx)} - \frac{3b\sin(3a+3bx)}{8d^2(c+dx)} + \frac{\left(9b^2\cos\right.}{}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)^2} + \frac{\cos(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cos\left(3a-\right.}{}$$

Mathematica [A] time = 2.16, size = 183, normalized size = 0.83

$$\frac{b^2\left(-\cos\left(a-\frac{bc}{d}\right)\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+b^2\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)-9b^2\sin\left(3a\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (-(b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) + 9*b^2*Cos[3*a - (3*b*c)
/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Si
n[a + b*x]))/(c + d*x)^2 + (d*(d*Cos[3*(a + b*x)] - 3*b*(c + d*x)*Sin[3*(a
+ b*x)]))/(c + d*x)^2 + b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b
^2*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(8*d^3)
```

fricas [A] time = 0.55, size = 399, normalized size = 1.81

$$\frac{8\,d^2\cos\left(bx+a\right)^3 - 8\,d^2\cos\left(bx+a\right) - 18\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{3\,(bdx+bc)}{d}\right) + 2\left(b^2d^2x^2 + 2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(8*d^2*cos(b*x + a)^3 - 8*d^2*cos(b*x + a) - 18*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c - a*d)/d)*sin_integral((b*d*
x + b*c)/d) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x +
b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)
/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(b*d^2*x + b*c*d - 3*(b*d
^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 311, normalized size = 1.41

$$\frac{\frac{b^3\left(-\frac{\cos(bx+a)}{2((bx+a)d-da+cb)^2 d}-\frac{-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}\right)}{4}-\frac{b^3\left(-\frac{3\cos(3bx+3a)}{2((bx+a)d-da+cb)^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{((bx+a)d-da+cb}\right.}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/4*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/((b*
x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/12*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)
*d-d*a+c*b)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+
3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*co
s(3*(-a*d+b*c)/d)/d)/d)/d))
```

maxima [C] time = 0.68, size = 337, normalized size = 1.52

$$-\frac{8192\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_3\left(-\frac{3i}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^3*(-8192*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^3*(8192*I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
d)/d) - 8192*I*exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**3, x)
```

3.21 $\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=270

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right.}{8d}$$

```
[Out] -1/12*cos(b*x+a)/d/(d*x+c)^3+1/24*b^2*cos(b*x+a)/d^3/(d*x+c)+1/12*cos(3*b*x
+3*a)/d/(d*x+c)^3-3/8*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/24*b^3*cos(a-b*c/d)*
Si(b*c/d+b*x)/d^4-9/8*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-9/8*b^3*Ci
(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^
4+1/24*b*sin(b*x+a)/d^2/(d*x+c)^2-1/8*b*sin(3*b*x+3*a)/d^2/(d*x+c)^2
```

Rubi [A] time = 0.42, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+b\right.}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(12*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(24*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(12*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(8*d^3*(c +
d*x)) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^4)
 + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(24*d^4) + (b*Sin[a +
b*x])/(24*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(8*d^2*(c + d*x)^2) + (b^
3*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^3*Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\cos(a+bx)}{4(c+dx)^4} - \frac{\cos(3a+3bx)}{4(c+dx)^4}\right)dx$$

$$= \frac{1}{4}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx - \frac{1}{4}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{12d} + \frac{b\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx}{4d}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} + \frac{b\sin(a+bx)}{24d^2(c+dx)^2} - \frac{b\sin(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^2\int \frac{\cos(a}{(c+a}}{24d}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\sin(}{24d^2(}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} + \frac{b\sin(}{24d^2(}$$

$$= -\frac{\cos(a+bx)}{12d(c+dx)^3} + \frac{b^2\cos(a+bx)}{24d^3(c+dx)} + \frac{\cos(3a+3bx)}{12d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{8d^3(c+dx)} - \frac{9b^3\mathrm{Ci}}{}$$

Mathematica [**A**] time = 1.68, size = 298, normalized size = 1.10

$$\frac{b^3(c+dx)^3\left(\sin\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)-27b^3(c+dx)^3\left(\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3b(c+dx)}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] (d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) - d*
Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a
]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x]
- d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Sin[3*a])*Sin[
3*b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]*Sin[a - (b*c)/d] + Cos[a
 - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 27*b^3*(c + d*x)^3*(CosIntegral[(3*
b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a - (3*b*c)/d]*SinIntegral[(3*
b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

fricas [**B**] time = 0.73, size = 564, normalized size = 2.09

$$-\frac{8\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^3+54\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\cos\left(-\frac{3\,(bc}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(8*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a
)^3 + 54*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(
b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x +
b*c)/d) - 8*(7*b^2*d^3*x^2 + 14*b^2*c*d^2*x + 7*b^2*c^2*d - 2*d^3)*cos(b*x
+ a) - 8*(b*d^3*x + b*c*d^2 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x
 + a) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c
)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integr
al(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c
^2*d^5*x + c^3*d^4)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 384, normalized size = 1.42

$$\frac{\frac{b^4\left(-\frac{\cos(bx+a)}{3((bx+a)d-da+cb)^3 d}-\frac{-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2 d}+\frac{-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}}{3d}\right)}{4}-\frac{b^4\left(-\frac{\cos(3bx+3a)}{((bx+a)d-da+cb)^3 d}-\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/4*b^4*(-1/3*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^3/d-1/3*(-1/2*sin(b*x+a)/
((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-
a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)
/d)/d)/d)-1/12*b^4*(-cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^3/d-(-3/2*sin(3*b*x
+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-
3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-
a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)/d))
```

maxima [C] time = 0.87, size = 387, normalized size = 1.43

$$-\frac{8192\,b^4\left(E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-8192\,b^4\left(E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_4\left(-\frac{3i\,bc\cdots}{}\right.\right.}{65536\left(b^3c^3d-3\,ab^2c\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/65536*(8192*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
- 8192*b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + ex
p_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^4*(-8192*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
 8192*I*exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c -
a*d)/d) + b^4*(8192*I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*
```

```
d)/d) - 8192*I*exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)\,\sin(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^2)/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)/(c + d*x)**4, x)
```

3.22 $\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=271

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4\cdots}{}\right)}{b}$$

```
[Out] -2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)+exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)+(d*x+c)^m*GAMMA(1+m,4*I*b*(d*x+
c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.33, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gam}\cdots}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) + (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(2a+2bx) - \frac{1}{8}(c+dx)^m \sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^m \sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^m \sin(2a+2bx)\,dx$$

$$= -\left(\frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx\right) + \frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx + \frac{1}{8}i$$

$$= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{2^{-4-m}e}{}$$

Mathematica [A] time = 0.29, size = 246, normalized size = 0.91

$$\frac{4^{-m-3}e^{-\frac{4i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-2^{m+2}e^{2i\left(a+\frac{3bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)-2^{m+2}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(-(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 2^(2 + m)*E^((2*I)*(a + (
3*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E
^((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^(
((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]
))/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

fricas [A] time = 0.50, size = 184, normalized size = 0.68

$$\frac{e^{\left(-\frac{dm\log\left(\frac{4ib}{d}\right)-4ibc+4iad}{d}\right)}\Gamma\left(m+1,\frac{4ibdx+4ibc}{d}\right)-4e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)-4e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*x
 + 4*I*b*c)/d) - 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
1, (2*I*b*d*x + 2*I*b*c)/d) - 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d)
/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*b
*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

maple [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\sin^3(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\sin(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.23 $\int (c+dx)^4 \cos(a+bx) \sin^3(a+bx)\,dx$

Optimal. Leaf size=260

$$\frac{3d^4\sin^4(a+bx)}{128b^5}+\frac{45d^4\sin^2(a+bx)}{128b^5}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)\cos(a+bx)}{64b^4}$$

```
[Out] 45/64*c*d^3*x/b^3+45/128*d^4*x^2/b^3-3/32*(d*x+c)^4/b-45/64*d^3*(d*x+c)*cos
(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+45/128*d^4
*sin(b*x+a)^2/b^5-9/16*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-3/32*d^3*(d*x+c)*cos(
b*x+a)*sin(b*x+a)^3/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3/b^2+3/128*d
^4*sin(b*x+a)^4/b^5-3/16*d^2*(d*x+c)^2*sin(b*x+a)^4/b^3+1/4*(d*x+c)^4*sin(b
*x+a)^4/b
```

Rubi [A] time = 0.24, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4404, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3}-\frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin^3(a+bx)\cos(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (45*c*d^3*x)/(64*b^3) + (45*d^4*x^2)/(128*b^3) - (3*(c + d*x)^4)/(32*b) - (
45*d^3*(c + d*x)*Cos[a + b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^2) + (45*d^4*Sin[a + b*x]^2)/(128*b^5) - (9*d^
2*(c + d*x)^2*Sin[a + b*x]^2)/(16*b^3) - (3*d^3*(c + d*x)*Cos[a + b*x]*Sin[
a + b*x]^3)/(32*b^4) + (d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3)/(4*b^2)
+ (3*d^4*Sin[a + b*x]^4)/(128*b^5) - (3*d^2*(c + d*x)^2*Sin[a + b*x]^4)/(16
*b^3) + ((c + d*x)^4*Sin[a + b*x]^4)/(4*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^4\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)^3\sin^4(a+bx)\,dx}{b}$$

$$= \frac{d(c+dx)^3\cos(a+bx)\sin^3(a+bx)}{4b^2} - \frac{3d^2(c+dx)^2\sin^4(a+bx)}{16b^3} + \frac{(c$$

$$= \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{8b^2} - \frac{9d^2(c+dx)^2\sin^2(a+bx)}{16b^3} - \frac{3d$$

$$= -\frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx)}{64b^4} + \frac{3d(c+dx)^3\cos$$

$$= \frac{45cd^3x}{64b^3} + \frac{45d^4x^2}{128b^3} - \frac{3(c+dx)^4}{32b} - \frac{45d^3(c+dx)\cos(a+bx)\sin(a+bx}{64b^4}$$

Mathematica [A] time = 1.70, size = 158, normalized size = 0.61

$$\frac{-8bd(c+dx)\sin(2(a+bx))\left(\cos(2(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-16\left(2b^2(c+dx)^2-3d^2\right)\right)-64\cos(2(a+bx))}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)] +
 (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*x)] - 8
```

```
*b*d*(c + d*x)*(-16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c + d*x
)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/(1024*b^5)
```

fricas [A] time = 0.59, size = 434, normalized size = 1.67

$$\frac{20\,b^4d^4x^4 + 80\,b^4cd^3x^3 + \left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\left(8\,b^4c^2d^2 - b^2d^4\right)x^2 + 16\left(8\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(20*b^4*d^4*x^4 + 80*b^4*c*d^3*x^3 + (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 3*(40*b^4*c^2*d^2 -
17*b^2*d^4)*x^2 - (64*b^4*d^4*x^4 + 256*b^4*c*d^3*x^3 + 64*b^4*c^4 - 120*b^
2*c^2*d^2 + 51*d^4 + 24*(16*b^4*c^2*d^2 - 5*b^2*d^4)*x^2 + 16*(16*b^4*c^3*d
 - 15*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(40*b^4*c^3*d - 51*b^2*c*d^3)*x - 2*
(2*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c
^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - (40*b^3*d^4*x^3 + 120*b^3*c*d^3*x^2 + 4
0*b^3*c^3*d - 51*b*c*d^3 + 3*(40*b^3*c^2*d^2 - 17*b*d^4)*x)*cos(b*x + a))*s
in(b*x + a))/b^5
```

giac [A] time = 2.89, size = 361, normalized size = 1.39

$$\frac{\left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4\right)\cos}{1024\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*
c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3
*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/256*(8*b^3*d^4*x^3 + 24*b^
3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(4
*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

maple [B] time = 0.12, size = 1143, normalized size = 4.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+27/128*(
b*x+a)^2+3/128*sin(b*x+a)^4+45/128*sin(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2
-9/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*(b*x+a)^4)-4/b^
4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*s
in(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b
*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x
+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)+4/b^3*c*d^3*(1/4*(b*x+a)^3
*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+
3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*
sin(b*x+a)+3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b
*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b
*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*
sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8
*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)+6/b^2*c^2*d
^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x
+a)^2)-4/b^4*a^3*d^4*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)
^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-12/b^2*a*
c^2*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)-3/32*b*x-3/32*a)+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^4*a^4*d^4*sin(b*x+a)^4-
1/b^3*a^3*c*d^3*sin(b*x+a)^4+3/2/b^2*a^2*c^2*d^2*sin(b*x+a)^4-1/b*a*c^3*d*s
in(b*x+a)^4+1/4*c^4*sin(b*x+a)^4)
```

maxima [B] time = 0.42, size = 967, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^4*sin(b*x + a)^4 - 1024*a*c^3*d*sin(b*x + a)^4/b + 1536*a^2*c
^2*d^2*sin(b*x + a)^4/b^2 - 1024*a^3*c*d^3*sin(b*x + a)^4/b^3 + 256*a^4*d^4
*sin(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2
*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x +
 a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8
*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b
*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^2*c*d^3
/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - s
```

```
in(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1)
*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*s
in(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
+ a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*
(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 + 2
4*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
+ 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a^2*
d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x
+ a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
4*a) + 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos
(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2
- 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*c
os(4*b*x + 4*a) - 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) -
 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) + 128*(2*(b*x + a)^3 - 3*
b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

mupad [B] time = 1.94, size = 576, normalized size = 2.22

$$-\frac{192\,d^4\,\cos\left(2\,a+2\,b\,x\right)-3\,d^4\,\cos\left(4\,a+4\,b\,x\right)+128\,b^4\,c^4\,\cos\left(2\,a+2\,b\,x\right)-32\,b^4\,c^4\,\cos\left(4\,a+4\,b\,x\right)-256\,b^3\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(192*d^4*cos(2*a + 2*b*x) - 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a +
 2*b*x) - 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) + 32
*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) + 24*b^2*c^2
*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) + 24*b^2*d^4*x^2*c
os(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) - 32*b^4*d^4*x^4*cos(4*a
 + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) + 32*b^3*d^4*x^3*sin(4*a + 4*b
*x) + 384*b*c*d^3*sin(2*a + 2*b*x) - 12*b*c*d^3*sin(4*a + 4*b*x) + 384*b*d^
4*x*sin(2*a + 2*b*x) - 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^2*x^2*co
s(2*a + 2*b*x) - 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*d^3*x*cos
(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) + 48*b^2*c*d^3*x*cos(4*a +
 4*b*x) - 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*cos(2*a + 2*
b*x) - 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*sin(2*a + 2*b
*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 96*b^3*c^2*d^2*x*sin(4*a + 4*b*x
) + 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

sympy [A] time = 13.31, size = 935, normalized size = 3.60

$$\begin{cases} \frac{c^4\sin^4(a+bx)}{4b} + \frac{5c^3dx\sin^4(a+bx)}{8b} - \frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{3c^3dx\cos^4(a+bx)}{8b} + \frac{15c^2d^2x^2\sin^4(a+bx)}{16b} - \frac{9c^2d^2x^2\sin^2(a+bx)\cos^2(a}{8b} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**4/(4*b) + 5*c**3*d*x*sin(a + b*x)**4/(8*b) -
3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 3*c**3*d*x*cos(a + b*x)*
*4/(8*b) + 15*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) - 9*c**2*d**2*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/(8*b) - 9*c**2*d**2*x**2*cos(a + b*x)**4/(16*b)
 + 5*c*d**3*x**3*sin(a + b*x)**4/(8*b) - 3*c*d**3*x**3*sin(a + b*x)**2*cos(
a + b*x)**2/(4*b) - 3*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 5*d**4*x**4*sin(a
 + b*x)**4/(32*b) - 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*
d**4*x**4*cos(a + b*x)**4/(32*b) + 5*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(8
*b**2) + 3*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 15*c**2*d**2*x*si
n(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c**2*d**2*x*sin(a + b*x)*cos(a + b*
x)**3/(8*b**2) + 15*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 9*c
*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 5*d**4*x**3*sin(a + b*x)
**3*cos(a + b*x)/(8*b**2) + 3*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b**
2) - 15*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 9*c**2*d**2*cos(a + b*x)**4/(
32*b**3) - 51*c*d**3*x*sin(a + b*x)**4/(64*b**3) + 9*c*d**3*x*sin(a + b*x)*
*2*cos(a + b*x)**2/(32*b**3) + 45*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 51*d
**4*x**2*sin(a + b*x)**4/(128*b**3) + 9*d**4*x**2*sin(a + b*x)**2*cos(a + b
*x)**2/(64*b**3) + 45*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 51*c*d**3*sin(
a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*c*d**3*sin(a + b*x)*cos(a + b*x)**3
/(64*b**4) - 51*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 45*d**4*x*s
in(a + b*x)*cos(a + b*x)**3/(64*b**4) + 51*d**4*sin(a + b*x)**4/(256*b**5)
- 45*d**4*cos(a + b*x)**4/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 +
 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a), True))
```

3.24 $\int (c+dx)^3 \cos(a+bx) \sin^3(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{3d^3\sin^3(a+bx)\cos(a+bx)}{128b^4}-\frac{45d^3\sin(a+bx)\cos(a+bx)}{256b^4}-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}$$

```
[Out] 45/256*d^3*x/b^3-3/32*(d*x+c)^3/b-45/256*d^3*cos(b*x+a)*sin(b*x+a)/b^4+9/32
*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-9/32*d^2*(d*x+c)*sin(b*x+a)^2/b^3-3/
128*d^3*cos(b*x+a)*sin(b*x+a)^3/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)^
3/b^2-3/32*d^2*(d*x+c)*sin(b*x+a)^4/b^3+1/4*(d*x+c)^3*sin(b*x+a)^4/b
```

Rubi [A] time = 0.17, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4404, 3311, 32, 2635, 8}

$$-\frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3}-\frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin^3(a+bx)\cos(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+b}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^3*x)/(256*b^3) - (3*(c + d*x)^3)/(32*b) - (45*d^3*Cos[a + b*x]*Sin[a
+ b*x])/(256*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) -
(9*d^2*(c + d*x)*Sin[a + b*x]^2)/(32*b^3) - (3*d^3*Cos[a + b*x]*Sin[a + b*x
]^3)/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3)/(16*b^2) - (
3*d^2*(c + d*x)*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^3*Sin[a + b*x]^4)/(4*
b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^3\sin^4(a+bx)}{4b} - \frac{(3d)\int (c+dx)^2\sin^4(a+bx)\,dx}{4b}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)\sin^3(a+bx)}{16b^2} - \frac{3d^2(c+dx)\sin^4(a+bx)}{32b^3} + \frac{(c$$

$$= \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{32b^2} - \frac{9d^2(c+dx)\sin^2(a+bx)}{32b^3} - \frac{3d^3$$

$$= -\frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos(a+bx$$

$$= \frac{45d^3x}{256b^3} - \frac{3(c+dx)^3}{32b} - \frac{45d^3\cos(a+bx)\sin(a+bx)}{256b^4} + \frac{9d(c+dx)^2\cos$$

Mathematica [A] time = 0.89, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+4b(c+dx)\cos(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-6d\sin(2(a}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 6*d*(-16*(-d^2 + 2*b^2*
```

```
(c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)
])/(1024*b^4)
```

fricas [A] time = 0.49, size = 283, normalized size = 1.44

$$\frac{40\,b^3d^3x^3+120\,b^3cd^2x^2+8\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\left(8\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^4-8\left(16\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(40*b^3*d^3*x^3 + 120*b^3*c*d^2*x^2 + 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2
*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 -
8*(16*b^3*d^3*x^3 + 48*b^3*c*d^2*x^2 + 16*b^3*c^3 - 15*b*c*d^2 + 3*(16*b^3*
c^2*d - 5*b*d^3)*x)*cos(b*x + a)^2 + 3*(40*b^3*c^2*d - 17*b*d^3)*x - 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - (40*b^
2*d^3*x^2 + 80*b^2*c*d^2*x + 40*b^2*c^2*d - 17*d^3)*cos(b*x + a))*sin(b*x +
 a))/b^4
```

giac [A] time = 3.97, size = 241, normalized size = 1.23

$$\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*
d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^2
*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/
b^4 - 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*x
 + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(
2*b*x + 2*a)/b^4
```

maple [B] time = 0.02, size = 594, normalized size = 3.03

$$\frac{d^3\left(\frac{(bx+a)^3\left(\sin^4(bx+a)\right)}{4}-\frac{3(bx+a)^2\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{4}-\frac{3(bx+a)\left(\sin^4(bx+a)\right)}{32}-\frac{3\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{128}-\frac{27bx}{256}-\frac{27a}{256}+\frac{9(bx+a)(c\cdots}{3\cdots}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)*sin(b*x+a)^4-3/128
*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-27/256*b*x-27/256*a+9/32*(b*x+a)*
cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)+3/16*(b*x+a)^3)-3/b^3*a*d^3*(1/4*(b
*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*sin(b*x+a)^2)+3/b
^2*c*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*si
n(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-3/32*s
in(b*x+a)^2)+3/b^3*a^2*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2
*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x
+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+3/b*c^
2*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
-3/32*b*x-3/32*a)-1/4/b^3*a^3*d^3*sin(b*x+a)^4+3/4/b^2*a^2*c*d^2*sin(b*x+a)
^4-3/4/b*a*c^2*d*sin(b*x+a)^4+1/4*c^3*sin(b*x+a)^4)
```

maxima [B] time = 0.38, size = 549, normalized size = 2.80

$$\frac{256\,c^3\sin(bx+a)^4 - \frac{768\,ac^2d\sin(bx+a)^4}{b} + \frac{768\,a^2cd^2\sin(bx+a)^4}{b^2} - \frac{256\,a^3d^3\sin(bx+a)^4}{b^3} + \frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,b}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(256*c^3*sin(b*x + a)^4 - 768*a*c^2*d*sin(b*x + a)^4/b + 768*a^2*c*d
^2*sin(b*x + a)^4/b^2 - 256*a^3*d^3*sin(b*x + a)^4/b^3 + 24*(4*(b*x + a)*co
s(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2
*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*(
b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a
) + 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4
*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a
) + 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos(
4*b*x + 4*a) - 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*
b*x + 4*a) + 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3 -
 3*b*x - 3*a)*cos(4*b*x + 4*a) - 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x
 + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) + 96*(2*(b*x + a)^2 - 1)*s
in(2*b*x + 2*a))*d^3/b^3)/b
```

mupad [B] time = 1.71, size = 366, normalized size = 1.87

$$-\frac{24\,d^3\,\sin(2\,a+2\,b\,x) - \frac{3\,d^3\,\sin(4\,a+4\,b\,x)}{4} + 32\,b^3\,c^3\,\cos(2\,a+2\,b\,x) - 8\,b^3\,c^3\,\cos(4\,a+4\,b\,x) - 48\,b^2\,c^2\,d\,\sin(2\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -(24*d^3*sin(2*a + 2*b*x) - (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*a
 + 2*b*x) - 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) + 6*
b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) - 8*b^3*d^3*x^
3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) + 6*b^2*d^3*x^2*sin(4*
a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) + 3*b*c*d^2*cos(4*a + 4*b*x) - 48*
b*d^3*x*cos(2*a + 2*b*x) + 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*x*cos(
2*a + 2*b*x) - 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(2*a + 2
*b*x) + 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a + 2*b*x)
 - 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

sympy [A] time = 7.28, size = 602, normalized size = 3.07

$$\begin{cases} \frac{c^3\sin^4(a+bx)}{4b} + \frac{15c^2dx\sin^4(a+bx)}{32b} - \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{9c^2dx\cos^4(a+bx)}{32b} + \frac{15cd^2x^2\sin^4(a+bx)}{32b} - \frac{9cd^2x^2\sin^2(a+bx)\cos^2(a+ \cdots}{16b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**4/(4*b) + 15*c**2*d*x*sin(a + b*x)**4/(32*b)
- 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 9*c**2*d*x*cos(a + b*
x)**4/(32*b) + 15*c*d**2*x**2*sin(a + b*x)**4/(32*b) - 9*c*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**2/(16*b) - 9*c*d**2*x**2*cos(a + b*x)**4/(32*b) + 5
*d**3*x**3*sin(a + b*x)**4/(32*b) - 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*x
)**2/(16*b) - 3*d**3*x**3*cos(a + b*x)**4/(32*b) + 15*c**2*d*sin(a + b*x)**
3*cos(a + b*x)/(32*b**2) + 9*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2)
+ 15*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 9*c*d**2*x*sin(a + b
*x)*cos(a + b*x)**3/(16*b**2) + 15*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(
32*b**2) + 9*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 15*c*d**2*s
in(a + b*x)**4/(64*b**3) + 9*c*d**2*cos(a + b*x)**4/(64*b**3) - 51*d**3*x*s
in(a + b*x)**4/(256*b**3) + 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b
**3) + 45*d**3*x*cos(a + b*x)**4/(256*b**3) - 51*d**3*sin(a + b*x)**3*cos(a
 + b*x)/(256*b**4) - 45*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b,
 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3*cos
(a), True))
```

3.25 $\int (c+dx)^2 \cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=134

$$-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{(c$$

```
[Out] -3/16*c*d*x/b-3/32*d^2*x^2/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-3/32*
d^2*sin(b*x+a)^2/b^3+1/8*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)^3/b^2-1/32*d^2*sin
(b*x+a)^4/b^3+1/4*(d*x+c)^2*sin(b*x+a)^4/b
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4404, 3310}

$$\frac{d(c+dx)\sin^3(a+bx)\cos(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{d^2\sin^4(a+bx)}{32b^3}-\frac{3d^2\sin^2(a+bx)}{32b^3}+\frac{(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*c*d*x)/(16*b) - (3*d^2*x^2)/(32*b) + (3*d*(c + d*x)*Cos[a + b*x]*Sin[a
 + b*x])/(16*b^2) - (3*d^2*Sin[a + b*x]^2)/(32*b^3) + (d*(c + d*x)*Cos[a + b
*x]*Sin[a + b*x]^3)/(8*b^2) - (d^2*Sin[a + b*x]^4)/(32*b^3) + ((c + d*x)^2*
Sin[a + b*x]^4)/(4*b)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)^2\sin^4(a+bx)}{4b} - \frac{d\int (c+dx)\sin^4(a+bx)\,dx}{2b}$$
$$= \frac{d(c+dx)\cos(a+bx)\sin^3(a+bx)}{8b^2} - \frac{d^2\sin^4(a+bx)}{32b^3} + \frac{(c+dx)^2\sin^4(a}{4b}$$
$$= \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(a+bx)}{32b^3} + \frac{d(c+dx)\cos(}{}$$
$$= -\frac{3cdx}{16b} - \frac{3d^2x^2}{32b} + \frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{16b^2} - \frac{3d^2\sin^2(a+bx}{32b^3}$$

Mathematica [A] time = 0.49, size = 91, normalized size = 0.68

$$\frac{-16\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)-4bd(c+dx)(\sin(4(a+bx))-8\sin(}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 8*b^2*(c + d*x)^
2)*Cos[4*(a + b*x)] - 4*b*d*(c + d*x)*(-8*Sin[2*(a + b*x)] + Sin[4*(a + b*x
)]))/(256*b^3)
```

fricas [A] time = 0.46, size = 159, normalized size = 1.19

$$\frac{5\,b^2d^2x^2+10\,b^2cdx+\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4-\left(16\,b^2d^2x^2+32\,b^2cdx+16\,b^2c^2-5\,d^2\right)}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(5*b^2*d^2*x^2 + 10*b^2*c*d*x + (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*
c^2 - d^2)*cos(b*x + a)^4 - (16*b^2*d^2*x^2 + 32*b^2*c*d*x + 16*b^2*c^2 - 5
*d^2)*cos(b*x + a)^2 - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 5*(b*d^2*x +
 b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

giac [A] time = 0.20, size = 145, normalized size = 1.08

$$\frac{\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{16\,b^3}-\frac{\left(bd^2x+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^3
 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b^
3 - 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*sin
(2*b*x + 2*a)/b^3
```

maple [B] time = 0.02, size = 260, normalized size = 1.94

$$\frac{d^2\left(\frac{(bx+a)^2\left(\sin^4(bx+a)\right)}{4}-\frac{(bx+a)\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{2}+\frac{3(bx+a)^2}{32}-\frac{\left(\sin^4(bx+a)\right)}{32}-\frac{3\left(\sin^2(bx+a)\right)}{32}\right)}{b^2}-\frac{2a\,d^2\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4}+\frac{\left(\sin^3(bx\right.}{}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)^2-1/32*sin(b*x+a)^4-
3/32*sin(b*x+a)^2)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3
+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+2/b*c*d*(1/4*(b*x+a)*sin(b*x+a
)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-3/32*b*x-3/32*a)+1/4/b^2*
a^2*d^2*sin(b*x+a)^4-1/2/b*a*c*d*sin(b*x+a)^4+1/4*c^2*sin(b*x+a)^4)
```

maxima [B] time = 0.36, size = 263, normalized size = 1.96

$$64\,c^2\sin(bx+a)^4-\frac{128\,acd\sin(bx+a)^4}{b}+\frac{64\,a^2d^2\sin(bx+a)^4}{b^2}+\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,b}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/256*(64*c^2*sin(b*x + a)^4 - 128*a*c*d*sin(b*x + a)^4/b + 64*a^2*d^2*sin(
b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x +
 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(4
*b*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*
x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 16*(2*(b*x +
a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) + 32*(b*x + a)*si
n(2*b*x + 2*a))*d^2/b^2)/b
```

mupad [B] time = 1.25, size = 202, normalized size = 1.51

$$8\,d^2\,\cos(2\,a+2\,b\,x)-\frac{d^2\,\cos(4\,a+4\,b\,x)}{2}-16\,b^2\,c^2\,\cos(2\,a+2\,b\,x)+4\,b^2\,c^2\,\cos(4\,a+4\,b\,x)+16\,b\,c\,d\,\sin(2\,a+2\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (8*d^2*cos(2*a + 2*b*x) - (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a + 2
*b*x) + 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) - 2*b*c*d*si
n(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) + 4*b^2*d^2*x^2*cos(4*a +
4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) - 2*b*d^2*x*sin(4*a + 4*b*x) - 32*b^2*
c*d*x*cos(2*a + 2*b*x) + 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

sympy [A] time = 3.62, size = 320, normalized size = 2.39

$$\begin{cases} \frac{c^2\sin^4(a+bx)}{4b} + \frac{5cdx\sin^4(a+bx)}{16b} - \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{3cdx\cos^4(a+bx)}{16b} + \frac{5d^2x^2\sin^4(a+bx)}{32b} - \frac{3d^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b} - \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**4/(4*b) + 5*c*d*x*sin(a + b*x)**4/(16*b) - 3*
c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 3*c*d*x*cos(a + b*x)**4/(16*b
) + 5*d**2*x**2*sin(a + b*x)**4/(32*b) - 3*d**2*x**2*sin(a + b*x)**2*cos(a
+ b*x)**2/(16*b) - 3*d**2*x**2*cos(a + b*x)**4/(32*b) + 5*c*d*sin(a + b*x)*
*3*cos(a + b*x)/(16*b**2) + 3*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) +
5*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 3*d**2*x*sin(a + b*x)*cos
(a + b*x)**3/(16*b**2) - 5*d**2*sin(a + b*x)**4/(64*b**3) + 3*d**2*cos(a +
b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*
cos(a), True))
```

3.26 $\int (c + dx) \cos(a + bx) \sin^3(a + bx) \, dx$

Optimal. Leaf size=72

$$\frac{d \sin^3(a + bx) \cos(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} - \frac{3dx}{32b}$$

```
[Out] -3/32*d*x/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16*d*cos(b*x+a)*sin(b*x+a)^3
/b^2+1/4*(d*x+c)*sin(b*x+a)^4/b
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4404, 2635, 8}

$$\frac{d \sin^3(a + bx) \cos(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} + \frac{(c + dx) \sin^4(a + bx)}{4b} - \frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-3*d*x)/(32*b) + (3*d*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) + (d*Cos[a + b*x
]*Sin[a + b*x]^3)/(16*b^2) + ((c + d*x)*Sin[a + b*x]^4)/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\sin^3(a+bx)\,dx = \frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{d\int \sin^4(a+bx)\,dx}{4b}$$
$$= \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4(a+bx)}{4b} - \frac{(3d)\int \sin^2(a+bx)}{16b}$$
$$= \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+dx)\sin^4(}{4b}$$
$$= -\frac{3dx}{32b} + \frac{3d\cos(a+bx)\sin(a+bx)}{32b^2} + \frac{d\cos(a+bx)\sin^3(a+bx)}{16b^2} + \frac{(c+}{}$$

Mathematica [A] time = 0.11, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx))-2bx\cos(2(a+bx)))}{16b^2} - \frac{d(\sin(4(a+bx))-4bx\cos(4(a+bx)))}{128b^2} + \frac{c\sin^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sin[a + b*x]^4)/(4*b) + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)]))
/(16*b^2) - (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

fricas [A] time = 0.44, size = 76, normalized size = 1.06

$$\frac{8(bdx+bc)\cos(bx+a)^4 + 5bdx - 16(bdx+bc)\cos(bx+a)^2 - \left(2d\cos(bx+a)^3 - 5d\cos(bx+a)\right)\sin(bx+a}{32b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 + 5*b*d*x - 16*(b*d*x + b*c)*cos(b*x +
 a)^2 - (2*d*cos(b*x + a)^3 - 5*d*cos(b*x + a))*sin(b*x + a))/b^2
```

giac [A] time = 3.71, size = 75, normalized size = 1.04

$$\frac{(bdx+bc)\cos(4bx+4a)}{32b^2} - \frac{(bdx+bc)\cos(2bx+2a)}{8b^2} - \frac{d\sin(4bx+4a)}{128b^2} + \frac{d\sin(2bx+2a)}{16b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*a
)/b^2 - 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

maple [A] time = 0.02, size = 85, normalized size = 1.18

$$\frac{\frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4}+\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{3bx}{32}-\frac{3a}{32}\right)}{b}-\frac{da\left(\sin^4(bx+a)\right)}{4b}+\frac{c\left(\sin^4(bx+a)\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)-3/32*b*x-3/32*a)-1/4/b*d*a*sin(b*x+a)^4+1/4*c*sin(b*x+a)^4)
```

maxima [A] time = 0.34, size = 92, normalized size = 1.28

$$\frac{32\,c\sin\left(bx+a\right)^4-\frac{32\,ad\sin(bx+a)^4}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)-16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)+8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/128*(32*c*sin(b*x + a)^4 - 32*a*d*sin(b*x + a)^4/b + (4*(b*x + a)*cos(4*b
*x + 4*a) - 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) + 8*sin(2*b*x
+ 2*a))*d/b)/b
```

mupad [B] time = 0.25, size = 94, normalized size = 1.31

$$\frac{2\,d\,\sin\left(2\,a+2\,b\,x\right)-\frac{d\,\sin(4\,a+4\,b\,x)}{4}-2\,b\,c\sin\left(2\,a+2\,b\,x\right)^2+8\,b\,c\sin\left(a+b\,x\right)^2+4\,b\,d\,x\,\left(2\sin\left(a+b\,x\right)^2-1\right)-}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (2*d*sin(2*a + 2*b*x) - (d*sin(4*a + 4*b*x))/4 - 2*b*c*sin(2*a + 2*b*x)^2 +
 8*b*c*sin(a + b*x)^2 + 4*b*d*x*(2*sin(a + b*x)^2 - 1) - b*d*x*(2*sin(2*a +
 2*b*x)^2 - 1))/(32*b^2)
```

sympy [A] time = 1.83, size = 138, normalized size = 1.92

$$\begin{cases} \frac{c\sin^4(a+bx)}{4b}+\frac{5dx\sin^4(a+bx)}{32b}-\frac{3dx\sin^2(a+bx)\cos^2(a+bx)}{16b}-\frac{3dx\cos^4(a+bx)}{32b}+\frac{5d\sin^3(a+bx)\cos(a+bx)}{32b^2}+\frac{3d\sin(a+bx)\cos^3(a+bx)}{32b^2} \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**4/(4*b) + 5*d*x*sin(a + b*x)**4/(32*b) - 3*d*x*s
in(a + b*x)**2*cos(a + b*x)**2/(16*b) - 3*d*x*cos(a + b*x)**4/(32*b) + 5*d*
sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 3*d*sin(a + b*x)*cos(a + b*x)**3/(
32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)**3*cos(a), True))
```

3.27 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}-\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\right.}{8d}$$

```
[Out] 1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d-1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2
*a-2*b*c/d)/d
```

Rubi [A] time = 0.23, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral
[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*Sin
Integral[(2*b*c)/d + 2*b*x])/(4*d) - (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b
*c)/d + 4*b*x])/(8*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)}-\frac{\sin(4a+4bx)}{8(c+dx)}\right)dx\\ &= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx\right)+\frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx\right)+\frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2b\right.}{c+dx}\\ &= -\frac{\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d}+\frac{\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right.}{4}\end{aligned}$$

Mathematica [A] time = 0.39, size = 110, normalized size = 0.85

$$-\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right)-2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right)-2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b}{}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/8*(CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] - 2*CosIntegral[(
2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*SinIntegral
[(2*b*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
d
```

fricas [A] time = 0.49, size = 156, normalized size = 1.21

$$\frac{2\left(\mathrm{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-\left(\mathrm{Ci}\left(\frac{4(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{4(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-2\cos\left(-\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")

[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) - 2*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d))/d

giac [C] time = 2.11, size = 6046, normalized size = 46.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")

[Out] -1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 - 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*

```
b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(
b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
 + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8
*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^
2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(
b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/
d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*
tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan
(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x +
 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integra
l(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) - 4*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 4*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2
*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2
- 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*
c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(
b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)
^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(
2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2
*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d
))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4
*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2
*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/
d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan
```

```
(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)
^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a
)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(a)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*t
an(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2
*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b
*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2
- 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_inte
gral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x
 + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)
^2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(
2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 4*
sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 8*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 16*sin_integral(2*
(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 8*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 16*sin_integral(2*(b
*d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*ta
n(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*
c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x -
 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)
^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 +
 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d
)^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan
(b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c
/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^
2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2
- 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integra
l(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral
```

(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 - 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 16*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 4*real_

```
part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c
/d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 4*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*real_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d)) - 2*im
ag_part(cos_integral(2*b*x + 2*b*c/d)) + 2*imag_part(cos_integral(-2*b*x -
2*b*c/d)) - imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*sin_integral(4*(b
*d*x + b*c)/d) - 4*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(2*a)^2*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + d*tan(
2*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2 + d*tan(2*a)^
2*tan(2*b*c/d)^2 + d*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2*tan(b*c/d)^2 +
d*tan(a)^2*tan(b*c/d)^2 + d*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2 + d*
tan(a)^2 + d*tan(2*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

maple [A] time = 0.02, size = 178, normalized size = 1.38

$$\frac{-\frac{b\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\cdots\right.}{d}\right.}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b
*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(2*Si(2*b*x+2*a+2*(-a*d
+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b
*c)/d)/d))
```

maxima [C] time = 0.47, size = 274, normalized size = 2.12

$$\frac{b\left(-2i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_1\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(-2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/
d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*
(b*c - a*d)/d) - 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
```

```
*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d
)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*
(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a + b\,x)\,\sin (a + b\,x)^3}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a + bx)\cos(a + bx)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x), x)
```

3.28 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\sin\left(4a-\frac{4bc}{d}\right.}{2}$$

```
[Out] -1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2
*a-2*b*c/d)/d^2+1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*c
/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)+1/8*sin(4*b*x+4
*a)/d/(d*x+c)
```

Rubi [A] time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) + Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2} - \frac{\sin(4a+4bx)}{8(c+dx)^2}\right) dx \\
&= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d} - \frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d} \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)} + \frac{\sin(4a+4bx)}{8d(c+dx)} - \frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d} + \frac{\left(b\cos\right.}{} \\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2} - \frac{b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2} - \frac{\sin(2a+2b}{4d(c+dx)}
\end{aligned}$$

Mathematica [A] time = 1.23, size = 151, normalized size = 0.84

$$\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right) - 4b\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right) - 4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right) + 4b\sin\left(4a-\frac{4bc}{d}\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*a - (4
*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - (2*d*Sin[2*(a + b*x)])/(c + d*x)
+ (d*Sin[4*(a + b*x)])/(c + d*x) - 4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*
b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/
(8*d^2)
```

fricas [A] time = 0.51, size = 245, normalized size = 1.37

$$\frac{2\,(bdx+bc)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d)
- 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + (
(b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - ((b*d*x + b*c)*cos_integral(4*
(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(
b*c - a*d)/d) + 4*(d*cos(b*x + a)^3 - d*cos(b*x + a))*sin(b*x + a))/(d^3*x
+ c*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 256, normalized size = 1.43

$$\frac{-\frac{b^2\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{32}+\frac{b^2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right.}{d}}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+4*(4*Si(4*b*x+4*a+4
*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(
-a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si
(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c
)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

maxima [C] time = 0.53, size = 301, normalized size = 1.68

$$\frac{b^2\left(-2i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_2\left(\cdots\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(b^2*(-2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**2, x)
```

3.29 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=229

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}+\frac{b^2\cos\left(4a-\right.}{}$$

```
[Out] -1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)+1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2*
cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*
x)/d^3+b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2+1/16*sin(4*b*x+4*a)/d/
(d*x+c)^2
```

Rubi [A] time = 0.34, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\mathrm{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) + (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) + (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) + Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) + (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3} - \frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx \\
&= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx \\
&= -\frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d} \\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} - \frac{b^2\int \frac{\mathrm{s}}{}}{} \\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} - \frac{\sin(2a+2bx)}{8d(c+dx)^2} + \frac{\sin(4a+4bx)}{16d(c+dx)^2} + \frac{\left(b^2\,\mathrm{co}\right.}{} \\
&= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)} + \frac{b\cos(4a+4bx)}{4d^2(c+dx)} + \frac{b^2\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3} - \frac{b^2\mathrm{Ci}\left(\right.}{}
\end{aligned}$$

Mathematica [A] time = 2.76, size = 199, normalized size = 0.87

$$\frac{-2\left(4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)+4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2b(c+dx)\cos(2(a+bx))+d\sin(2(a+bx)))}{(c+dx)^2}\right)+16b^2\text{ s}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*(4*b*(c +
d*x)*Cos[4*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 - 2*(4*b^2*CosInte
gral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*(2*b*(c + d*x)*Cos[2*(a +
 b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + 4*b^2*Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*(c + d*x))/d]) + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
(c + d*x))/d])/(16*d^3)
```

fricas [A] time = 0.65, size = 423, normalized size = 1.85

$$8\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+2\,bd^2x+2\,bcd-10\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+4\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(b*d^2*x + b*c*d)*cos(b*x + a)^4 + 2*b*d^2*x + 2*b*c*d - 10*(b*d^2*x
 + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-4*(
b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) + 2*(d^2
*cos(b*x + a)^3 - d^2*cos(b*x + a))*sin(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) + 2*((b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(
b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 329, normalized size = 1.44

$$-\frac{-\frac{b^3\left(-\frac{2\sin(4bx+4a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{8\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{8\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}}{d}\right)}{32}+\frac{b^3\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2 d}+\cdots\right)}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/32*b^3*(-2*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d+2*(-4*cos(4*b*x+4
*a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c
)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*b^3*
(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+
c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2
*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

maxima [C] time = 0.68, size = 336, normalized size = 1.47

$$\frac{b^3\left(-2i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_3\left(\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(b^3*(-2*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**3, x)
```

3.30 $\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\sin\left(4\right.}{}$$

```
[Out] 4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/12*b*cos(4*b*x+4*a
)/d^2/(d*x+c)^2-4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2*
sin(2*b*x+2*a)/d^3/(d*x+c)+1/24*sin(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*sin(4*b*
x+4*a)/d^3/(d*x+c)
```

Rubi [A] time = 0.39, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2}{}\right.}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) + (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) + Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4} - \frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$= -\left(\frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx\right) + \frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d} - \frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{\sin(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\int}{}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} + \frac{\text{si}}{2}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{12d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{6d^3(c+dx)} + \frac{\text{si}}{2}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2} + \frac{b\cos(4a+4bx)}{12d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4} + \frac{4b^3}{}$$

Mathematica [A] time = 2.25, size = 316, normalized size = 1.10

$$-8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+}{d}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*Sin[2*
a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^2)*Si
n[4*a]) + 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin[2*a]
)*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x)*Sin
[4*a])*Sin[4*b*x] - 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegral[(2*
b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d]) + 32
*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] - Sin
[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(24*d^4*(c + d*x)^3)
```

fricas [B] time = 0.70, size = 588, normalized size = 2.05

$$bd^3x+4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4+bcd^2-5\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2-8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/6*(b*d^3*x + 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 + b*c*d^2 - 5*(b*d^3*x
+ b*c*d^2)*cos(b*x + a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)
*sin_integral(2*(b*d*x + b*c)/d) - ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) + 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*
c^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b
^3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/
d) - 2*((8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3
 - (5*b^2*d^3*x^2 + 10*b^2*c*d^2*x + 5*b^2*c^2*d - d^3)*cos(b*x + a))*sin(b
*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 404, normalized size = 1.41

$$\frac{-\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3((bx+a)d-da+cb)^3d}+\frac{-\frac{8\cos(4bx+4a)}{3((bx+a)d-da+cb)^2d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{32}+}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/32*b^4*(-4/3*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^3/d+4/3*(-2*cos(4*b
*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d-2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+
4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-
a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x+2*a)/((
b*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(
2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-
a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d
))
```

maxima [C] time = 0.93, size = 386, normalized size = 1.34

$$\frac{b^4\left(-2i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+2i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_4\right.}{16\left(b^3c^3d-3\,ab^2c^2d^2+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/16*(b^4*(-2*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 2*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) - 2*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
```

```
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (a+b\,x)\,\sin (a+b\,x)^3}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)*sin(a + b*x)^3)/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3 (a+bx)\cos (a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)/(c + d*x)**4, x)
```

3.31 $\int (c+dx)^m \cot(a+bx)\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\cot(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\,dx$$

Mathematica [A] time = 2.58, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x], x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cos(bx+a)\csc(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

maple [A] time = 0.16, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)\,(c+d\,x)^m}{\sin(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x),x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\csc(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x), x)
```

3.32 $\int (c+dx)^4 \cot(a+bx)\,dx$

Optimal. Leaf size=151

$$-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{2id(c+dx)^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)}{}$$

```
[Out] -1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polyl
og(2,exp(2*I*(b*x+a)))/b^2+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+
3*I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(
b*x+a)))/b^5
```

Rubi [A] time = 0.22, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4\text{I}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((-I/5)*(c + d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2
*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2
*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx)\,dx = -\frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3\log\left(1-e^{2i(a+bx)}\right)dx}{b}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{\left(6id^2\right)\int (}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+d}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+d}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+d}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2} + \frac{3d^2(c+d}{}$$

Mathematica [B] time = 6.11, size = 799, normalized size = 5.29

$$\frac{1}{5}id^4x^5+icd^3x^4+\frac{d^4\log\left(1-e^{-i(a+bx)}\right)x^4}{b}+\frac{d^4\log\left(1+e^{-i(a+bx)}\right)x^4}{b}+2ic^2d^2x^3+\frac{4cd^3\log\left(1-e^{-i(a+bx)}\right)x^3}{b}+\frac{4cd^3\log}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x],x]
```

```
[Out] ((2*I)*c^3*d*Pi*x)/b + (2*I)*c^2*d^2*x^3 + I*c*d^3*x^4 + (I/5)*d^4*x^5 - ((
4*I)*c^3*d*x*ArcTan[Tan[a]])/b + 2*c^3*d*x^2*Cot[a] + (2*c^3*d*Pi*Log[1 + E
^((-2*I)*b*x)])/b^2 + (6*c^2*d^2*x^2*Log[1 - E^((-I)*(a + b*x))])/b + (4*c*
d^3*x^3*Log[1 - E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 - E^((-I)*(a + b*x)
)])/b + (6*c^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))])/b + (4*c*d^3*x^3*Log[1
+ E^((-I)*(a + b*x))])/b + (d^4*x^4*Log[1 + E^((-I)*(a + b*x))])/b + (4*c^3
*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b + (4*c^3*d*ArcTan[Tan[a]]
*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))])/b^2 - (2*c^3*d*Pi*Log[Cos[b*x]]
)/b^2 + (c^4*Log[Sin[a + b*x]])/b - (4*c^3*d*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLog[2,
-E^((-I)*(a + b*x))])/b^2 + ((4*I)*d^2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*PolyLo
g[2, E^((-I)*(a + b*x))])/b^2 - ((2*I)*c^3*d*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))])/b^2 + (12*c^2*d^2*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (2
4*c*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*PolyLog[3, -E^
```

```
((-I)*(a + b*x))])/b^3 + (12*c^2*d^2*PolyLog[3, E^((-I)*(a + b*x))])/b^3 +
(24*c*d^3*x*PolyLog[3, E^((-I)*(a + b*x))])/b^3 + (12*d^4*x^2*PolyLog[3, E^
((-I)*(a + b*x))])/b^3 - ((24*I)*c*d^3*PolyLog[4, -E^((-I)*(a + b*x))])/b^4
 - ((24*I)*d^4*x*PolyLog[4, -E^((-I)*(a + b*x))])/b^4 - ((24*I)*c*d^3*PolyL
og[4, E^((-I)*(a + b*x))])/b^4 - ((24*I)*d^4*x*PolyLog[4, E^((-I)*(a + b*x)
)])/b^4 - (24*d^4*PolyLog[5, -E^((-I)*(a + b*x))])/b^5 - (24*d^4*PolyLog[5,
 E^((-I)*(a + b*x))])/b^5 - 2*c^3*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Se
c[a]^2]
```

fricas [C] time = 0.60, size = 1204, normalized size = 7.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) + 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4
*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^
3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (4*I*b^
3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 -
12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
+ a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b*
x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*
c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4
*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*
b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x + a
)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2
*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a)\csc(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a), x)
```

maple [B] time = 0.17, size = 1150, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+12/b^3*
c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))
-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^
2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2
*I*c^2*d^2*x^3-2*I*c^3*d*x^2-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*p
olylog(5,exp(I*(b*x+a)))/b^5+I*c^4*x-2/b*c^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(
exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-1/5*I*d^4*x^5+4/b*c^3*d*ln(e
xp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*
(b*x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,-ex
p(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp
(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*po
lylog(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+2*I/b^4*d
^4*a^4*x-4*I/b^2*c^3*d*a^2+8*I/b^3*c^2*d^2*a^3-6*I/b^4*c*d^3*a^4-4*I/b^2*d^
4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*
I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x
+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,ex
p(I*(b*x+a)))+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*
x+a))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x
+a))-1)-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a
))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-8*I
/b^3*c*d^3*a^3*x+12*I/b^2*c^2*d^2*a^2*x-8*I/b*c^3*d*a*x-12*I/b^2*c*d^3*poly
log(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12
*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(
b*x+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+a
)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3
```

maxima [B] time = 0.62, size = 1262, normalized size = 8.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/10*(10*c^4*log(sin(b*x + a)) - 40*a*c^3*d*log(sin(b*x + a))/b + 60*a^2*c^
2*d^2*log(sin(b*x + a))/b^2 - 40*a^3*c*d^3*log(sin(b*x + a))/b^3 + 10*a^4*d
^4*log(sin(b*x + a))/b^4 + (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 10*I*a*
d^4)*(b*x + a)^4 - 240*d^4*polylog(5, -e^(I*b*x + I*a)) - 240*d^4*polylog(5
, e^(I*b*x + I*a)) + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 - 20*I*a^2*d^4)*(b
*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 - 60*I*a^2*b*c*d^3 + 20*I
*a^3*d^4)*(b*x + a)^2 + (10*I*(b*x + a)^4*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)
*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(b*x + a
)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3 - 40*I*a^3*
d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-10*I*(b*x + a)^
4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^2 + 120
*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*d + 120*I*a*b^2*c
^2*d^2 - 120*I*a^2*b*c*d^3 + 40*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a),
 -cos(b*x + a) + 1) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 120*I*a^2*b*
c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4
)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^2*d^4)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 1
20*I*a^2*b*c*d^3 - 40*I*(b*x + a)^3*d^4 + 40*I*a^3*d^4 + (-120*I*b*c*d^3 +
120*I*a*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*a^
2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 5*((b*x + a)^4*d^4 + 4*(b*c*d^3
- a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2
+ 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 5*((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (240*I
*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*polylog(4, -e^(I*b*x + I*a))
+ (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*polylog(4, e^(I*b*x +
 I*a)) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 120*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*p
olylog(3, e^(I*b*x + I*a)))/b^4)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,(c+d\,x)^4}{\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^4 \cos(a + bx) \csc(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a),x)
[Out] Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x), x)
```

3.33 $\int (c+dx)^3 \cot(a+bx)\,dx$

Optimal. Leaf size=127

$$\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^4}{4d}$$

```
[Out] -1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d*(d*x+c)^2*pol
ylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^
3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.19, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((-I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((
3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*
x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^(
(2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cot(a+bx)\,dx = -\frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2\log\left(1-e^{2i(a+bx)}\right)dx}{b}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\operatorname{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{\left(3id^2\right)\int (c$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\operatorname{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\operatorname{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\operatorname{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} + \frac{3d^2(c+dx$$

Mathematica [B] time = 2.67, size = 560, normalized size = 4.41

$$6b^4c^2dx^2\cot(a) - 6b^4c^2dx^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} + 4b^3c^3\log(\sin(a+bx)) - 12ib^3c^2dx\tan^{-1}(\tan(a)) + 12b$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x],x]
```

```
[Out] ((6*I)*b^3*c^2*d*Pi*x + (4*I)*b^4*c*d^2*x^3 + I*b^4*d^3*x^4 - (12*I)*b^3*c^
2*d*x*ArcTan[Tan[a]] + 6*b^4*c^2*d*x^2*Cot[a] + 6*b^2*c^2*d*Pi*Log[1 + E^((
-2*I)*b*x)] + 12*b^3*c*d^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 4*b^3*d^3*x^3*
Log[1 - E^((-I)*(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 + E^((-I)*(a + b*x))]
+ 4*b^3*d^3*x^3*Log[1 + E^((-I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] - 6*b^2*c^2*d*Pi*Log[Cos[b*x]] + 4*b^3*c^3*Log[Sin
[a + b*x]] - 12*b^2*c^2*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + (
12*I)*b^2*d^2*x*(2*c + d*x)*PolyLog[2, -E^((-I)*(a + b*x))] + (12*I)*b^2*d^
2*x*(2*c + d*x)*PolyLog[2, E^((-I)*(a + b*x))] - (6*I)*b^2*c^2*d*PolyLog[2,
 E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 24*b*c*d^2*PolyLog[3, -E^((-I)*(a + b*
x))] + 24*b*d^3*x*PolyLog[3, -E^((-I)*(a + b*x))] + 24*b*c*d^2*PolyLog[3, E
^((-I)*(a + b*x))] + 24*b*d^3*x*PolyLog[3, E^((-I)*(a + b*x))] - (24*I)*d^3
*PolyLog[4, -E^((-I)*(a + b*x))] - (24*I)*d^3*PolyLog[4, E^((-I)*(a + b*x))
] - 6*b^4*c^2*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(4*b^4)
```

fricas [C] time = 0.58, size = 814, normalized size = 6.41

$$\frac{6i\,d^3\text{polylog}\,(4, \cos\,(bx+a)+i\,\sin\,(bx+a))-6i\,d^3\text{polylog}\,(4, \cos\,(bx+a)-i\,\sin\,(bx+a))-6i\,d^3\text{polylog}\,(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x +
a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*
dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a)
+ 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*
d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
+ a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(b*d^3*x + b*c*d^2)*
polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3
, cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*
x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos\,(bx+a)\csc\,(bx+a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a), x)
```

maple [B] time = 0.10, size = 783, normalized size = 6.17

$$\frac{6ic\,d^2a^2x}{b^2}-\frac{6ic^2dax}{b}-\frac{6ic\,d^2\,\text{polylog}\left(2, \mathrm{e}^{i(bx+a)}\right)x}{b^2}-\frac{6ic\,d^2\,\text{polylog}\left(2, -\mathrm{e}^{i(bx+a)}\right)x}{b^2}-\frac{d^3a^3\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^4}+\frac{2d^3a^3\,\text{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2
*polylog(2,-exp(I*(b*x+a)))*x-6*I/b*c^2*d*a*x+6*I*d^3*polylog(4,exp(I*(b*x+
a)))/b^4+I*c^3*x-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b^4*d^3*a^3*ln(exp(I*
(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(
I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(
I*(b*x+a)))*x-3/2*I/b^4*a^4*d^3+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-I*c*
d^2*x^3-3/2*I*c^2*d*x^2-1/4*I*d^3*x^4+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*
ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b
*x+a))-1)-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,exp(I*
(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*c^2*d*a^2-2*I/b^
3*a^3*d^3*x+4*I/b^3*c*d^2*a^3-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I
/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/
b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d
^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln
(exp(I*(b*x+a))+1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+6/b^2*c^2*d*a*ln(
exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+
a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3
```

maxima [B] time = 0.52, size = 747, normalized size = 5.88

$$\frac{4\,c^3\log\left(\sin\left(bx+a\right)\right)-\frac{12\,ac^2d\log(\sin(bx+a))}{b}+\frac{12\,a^2cd^2\log(\sin(bx+a))}{b^2}-\frac{4\,a^3d^3\log(\sin(bx+a))}{b^3}+\frac{-i\,(bx+a)^4d^3+\left(-4i\,bcd^2+4i\,ad^3\right)(bx+a}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(4*c^3*log(sin(b*x + a)) - 12*a*c^2*d*log(sin(b*x + a))/b + 12*a^2*c*d^
2*log(sin(b*x + a))/b^2 - 4*a^3*d^3*log(sin(b*x + a))/b^3 + (-I*(b*x + a)^4
*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x + a)^3 + 24*I*d^3*polylog(4, -e^(I*b
*x + I*a)) + 24*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-6*I*b^2*c^2*d + 12*I*
a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a)^2 + (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2
 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^
3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d
^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b
*c*d^2 - 12*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1)
+ (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 +
 (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-12*I*b
^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*
c*d^2 + 24*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I*a)) + 2*((b*x + a)^3*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(
b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + 2*((b
```

```
*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a
) + 1) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*a))
+ 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx) \csc(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x), x)
```

3.34 $\int (c+dx)^2 \cot(a+bx)\,dx$

Optimal. Leaf size=93

$$\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

```
[Out] -1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b-I*d*(d*x+c)*polylog(2
,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.17, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3717, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((-I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b - (I*
d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, E^((2*I)
*(a + b*x))])/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cot(a+bx)\,dx &= -\frac{i(c+dx)^3}{3d}-2i\int\frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(2d)\int(c+dx)\log\left(1-e^{2i(a+bx)}\right)dx}{b}\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{\left(id^2\right)\int\mathrm{Li}_2\,(}{b}\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\,\mathrm{Subst}\left(\int\right.}{}\\
&= -\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\mathrm{Li}_3\left(e^{2i(a+}\right.}{2b^3}\end{aligned}$$

Mathematica [B] time = 1.41, size = 356, normalized size = 3.83

$$3b^3cdx^2\cot(a)-3b^3cdx^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)}+3b^2c^2\log(\sin(a+bx))-6ib^2cdx\tan^{-1}(\tan(a))+6b^2cd$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x],x]
```

```
[Out] ((3*I)*b^2*c*d*Pi*x + I*b^3*d^2*x^3 - (6*I)*b^2*c*d*x*ArcTan[Tan[a]] + 3*b^
3*c*d*x^2*Cot[a] + 3*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 3*b^2*d^2*x^2*Log[1
```

```
 - E^((-I)*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 6*b^2*
c*d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 6*b*c*d*ArcTan[Tan[a]]*Lo
g[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 3*b*c*d*Pi*Log[Cos[b*x]] + 3*b^2*
c^2*Log[Sin[a + b*x]] - 6*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]
]] + (6*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b*d^2*x*PolyLog[
2, E^((-I)*(a + b*x))] - (3*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[
a]]))] + 6*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 6*d^2*PolyLog[3, E^((-I)*(
a + b*x))] - 3*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2])/(3*b
^3)
```

fricas [C] time = 0.50, size = 498, normalized size = 5.35

$$2\,d^2\text{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+2\,d^2\text{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)+2\,d^2\text{polylog}\left(3,-c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) + 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*
I*b*c*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*d
ilog(cos(b*x + a) - I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x
+ a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cos(bx+a)\csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a), x)
```

maple [B] time = 0.07, size = 468, normalized size = 5.03

$$\frac{d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}-\frac{2d^2a^2\ln\left(e^{i(bx+a)}\right)}{b^3}+\frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}+\frac{4ic}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b*d^2
*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp
(I*(b*x+a))+1)*x^2+4/3*I/b^3*d^2*a^3-I*c*d*x^2-4*I/b*c*d*a*x+2*d^2*polylog(
3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+I*c^2*x-2/b*c^2*
ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1
)-1/3*I*d^2*x^3+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)
))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+4/b^2*c*d*a*ln(exp(I*(b*x+a)))-2/b^2*c*
d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*a^2*x-2*I/b^2*c*d*a^2-2*I/b^2*d^2*poly
log(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*
d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))
```

maxima [B] time = 0.47, size = 404, normalized size = 4.34

$$6\,c^2 \log\left(\sin\left(bx + a\right)\right) - \frac{12\,acd\log(\sin(bx+a))}{b} + \frac{6\,a^2d^2\log(\sin(bx+a))}{b^2} + \frac{-2i\,(bx+a)^3 d^2 + \left(-6i\,bcd + 6i\,ad^2\right)(bx+a)^2 + 12\,d^2\mathrm{Li}_3(-e^{(i\,bx + i\,a)}) + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/6*(6*c^2*log(sin(b*x + a)) - 12*a*c*d*log(sin(b*x + a))/b + 6*a^2*d^2*log
(sin(b*x + a))/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x
+ a)^2 + 12*d^2*polylog(3, -e^(I*b*x + I*a)) + 12*d^2*polylog(3, e^(I*b*x +
 I*a)) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d +
12*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-12*I*b*
c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-12*I*b*c
*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 3*((b*x + a)
^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*cos(b*x + a) + 1) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a + b\,x\right)\,\left(c + d\,x\right)^2}{\sin\left(a + b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cos(a + bx) \csc(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x), x)
```

3.35 $\int (c + dx) \cot(a + bx) \, dx$

Optimal. Leaf size=65

$$-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] -1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2,exp(2
*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.10, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] ((-I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b - ((I/2
)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\,dx &= -\frac{i(c+dx)^2}{2d}-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx\\ &= -\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{d\int \log\left(1-e^{2i(a+bx)}\right)\,dx}{b}\\ &= -\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}+\frac{(id)\,\text{Subst}\left(\int \frac{\log(1-x)}{x}\,dx,x,e^{2i(a+bx)}\right)}{2b^2}\\ &= -\frac{i(c+dx)^2}{2d}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [B] time = 5.11, size = 188, normalized size = 2.89

$$-\frac{d\csc(a)\sec(a)\left(b^2x^2e^{i\tan^{-1}(\tan(a))}+\frac{\tan(a)\left(i\text{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a)\right.}\right.\right.}{\sqrt{\tan^2(a)+1}}\right.}{2b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Cot[a + b*x],x]
```

```
[Out] (d*x^2*Cot[a])/2 + (c*(Log[Cos[a + b*x]] + Log[Tan[a + b*x]]))/b - (d*Csc[a
]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) -
 Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*
x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + A
rcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/S
qrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [B] time = 0.50, size = 250, normalized size = 3.85

$$\frac{-i\,d\text{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+i\,d\text{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)+i\,d\text{Li}_2\left(-\cos\left(bx+a\right)+i\,\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*dilog(-cos
(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b*c - a*d
)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) +
1))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \cos (bx + a) \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*csc(b*x + a), x)
```

maple [B] time = 0.07, size = 215, normalized size = 3.31

$$-\frac{id\,x^2}{2}+icx+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-\frac{id\,a^2}{b^2}+\frac{d\ln\left(e^{i(bx+a)}+1\right)x}{b}-\frac{id\operatorname{polylo}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] -1/2*I*d*x^2+I*c*x+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)-2/
b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(exp(I*(b*x+a))+1)*x
-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln
(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(
b*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.46, size = 189, normalized size = 2.91

$$\frac{-i\,b^2dx^2 - 2i\,b^2cx - 2i\,bdx\arctan\left(\sin\left(bx+a\right), -\cos\left(bx+a\right)+1\right) + 2i\,bc\arctan\left(\sin\left(bx+a\right), \cos\left(bx+a\right)-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x - 2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x
+ a) + 1) + 2*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x +
```

```
2*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - 2*I*d*dilog(-e^(I*b*x +
I*a)) - 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) + (b*d*x + b*c)*log(cos(b*x + a)^2 + si
n(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cos(a+bx)\ (c+dx)}{\sin(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x),x)
```

```
[Out] int((cos(a + b*x)*(c + d*x))/sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x), x)
```

3.36 $\int \frac{\cot(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x),x]
```

```
[Out] Defer[Int][Cot[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{c+dx} \, dx = \int \frac{\cot(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 3.66, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x),x]
```

```
[Out] Integrate[Cot[a + b*x]/(c + d*x), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x), x)
```

3.37 $\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx=\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 6.98, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Cot[a + b*x]/(c + d*x)^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(cos(b*x + a)*csc(b*x + a)/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)/(c + d*x)**2, x)
```

3.38 $\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\cot(a+bx)\csc(a+bx)(c+dx)^m, x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Mathematica [A] time = 2.93, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cos(bx+a)\csc(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a)\csc(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a)\left(\csc^2(bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a)\csc(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a + b\,x)\,(c + d\,x)^m}{\sin(a + b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2,x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cos(a + bx)\csc^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)*csc(a + b*x)**2, x)
```

3.39 $\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=208

$$-\frac{24id^4\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^5}+\frac{24id^4\text{Li}_4\left(e^{i(a+bx)}\right)}{b^5}-\frac{24d^3(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2}{b}$$

```
[Out] -8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*
(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I
*(b*x+a)))/b^3-24*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)
*polylog(3,exp(I*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I
*d^4*polylog(4,exp(I*(b*x+a)))/b^5
```

Rubi [A] time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-1$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^4\csc(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\csc(a+bx)\,dx}{b}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} - \frac{\left(12d^2\right)\int (c+dx}{}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\mathrm{L}}{b^3}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\mathrm{L}}{b^3}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\mathrm{L}}{b^3}$$
$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^4\csc(a+bx)}{b} + \frac{12id^2(c+dx)^2\mathrm{L}}{b^3}$$

Mathematica [A] time = 1.36, size = 308, normalized size = 1.48

$$\frac{8id\left(\frac{3d\left(b^2(c+dx)^2\mathrm{Li}_2(-\cos(a+bx)-i\sin(a+bx))+2ibd(c+dx)\mathrm{Li}_3(-\cos(a+bx)-i\sin(a+bx))-2d^2\mathrm{Li}_4(-\cos(a+bx)-i\sin(a+bx))\right)}{b^3} - \frac{3d\left(b^2(c+dx)^2\mathrm{Li}_2(\mathrm{co}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-2*b*(c + d*x)^4*Csc[a] + (8*I)*d*((2*I)*(c + d*x)^3*ArcTanh[Cos[a + b*x]
+ I*Sin[a + b*x]] + (3*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[
a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]
- 2*d^2*PolyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]))/b^3 - (3*d*(b^2*(c + d
*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyL
og[3, Cos[a + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Si
n[a + b*x]]))/b^3) + b*(c + d*x)^4*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] -
 b*(c + d*x)^4*Sec[a/2]*Sec[(a + b*x)/2]*Sin[(b*x)/2])/(2*b^2)
```

fricas [C] time = 0.55, size = 1021, normalized size = 4.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c
^4 - 12*I*d^4*polylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I
*d^4*polylog(4, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*poly
log(4, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*
c*d^3*x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x +
a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x -
6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*
b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*s
in(b*x + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^
2*x + b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b
^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a
^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin
(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3
+ 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3
*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3
 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^
3*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x +
 b*c*d^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^
4*x + b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
 + 12*(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*
x + a))/(b^5*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \cos(bx + a) \csc(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^2, x)
```

maple [B] time = 0.12, size = 716, normalized size = 3.44

$$\frac{8d^4a^3 \operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b^5} - \frac{24d^3c \operatorname{polylog}\left(3, -e^{i(bx+a)}\right)}{b^4} + \frac{24d^3c \operatorname{polylog}\left(3, e^{i(bx+a)}\right)}{b^4} - \frac{8d\,c^3 \operatorname{arctanh}\left(e^{i(bx+a)}\right)}{b^2} - 24$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] 8/b^5*d^4*a^3*arctanh(exp(I*(b*x+a)))-24/b^4*d^3*c*polylog(3,-exp(I*(b*x+a)
))+24/b^4*d^3*c*polylog(3,exp(I*(b*x+a)))-8/b^2*d*c^3*arctanh(exp(I*(b*x+a)
))-24/b^4*d^4*polylog(3,-exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,exp(I*(b*x+
a)))*x-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I/b^3*d^3*c*polylog(2,-ex
p(I*(b*x+a)))*x-24*I/b^3*d^3*c*polylog(2,exp(I*(b*x+a)))*x+24*I*d^4*polylog
(4,exp(I*(b*x+a)))/b^5-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^4
)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+12/b^2*d^2*c^2*ln(1-exp(I*(b*x+a)))
*x+12/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a-12/b^2*d^2*c^2*ln(exp(I*(b*x+a))+1
)*x-12/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)*a+12/b^2*d^3*c*ln(1-exp(I*(b*x+a)))
*x^2-12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a^2-12/b^2*d^3*c*ln(exp(I*(b*x+a))+1
)*x^2+12/b^4*d^3*c*ln(exp(I*(b*x+a))+1)*a^2+4/b^2*d^4*ln(1-exp(I*(b*x+a)))*
x^3+4/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^3-4/b^2*d^4*ln(exp(I*(b*x+a))+1)*x^3-4
/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^3-24/b^4*d^3*c*a^2*arctanh(exp(I*(b*x+a)))+
24/b^3*d^2*c^2*a*arctanh(exp(I*(b*x+a)))+12*I/b^3*d^4*polylog(2,-exp(I*(b*x
+a)))*x^2-12*I/b^3*d^4*polylog(2,exp(I*(b*x+a)))*x^2+12*I/b^3*d^2*c^2*polyl
og(2,-exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*polylog(2,exp(I*(b*x+a)))
```

maxima [B] time = 0.84, size = 2944, normalized size = 14.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*c^
3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b)
- 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*
a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*a*
c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^2) + 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*
x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*
b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
 (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x +
a))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) - 2*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*co
s(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a)
```

```
+ 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*l
og(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(
b*x + a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x +
 2*a) + 1)*b^4) + c^4/sin(b*x + a) - 4*a*c^3*d/(b*sin(b*x + a)) + 6*a^2*c^2
*d^2/(b^2*sin(b*x + a)) - 4*a^3*c*d^3/(b^3*sin(b*x + a)) + a^4*d^4/(b^4*sin
(b*x + a)) - ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c
*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x +
a))*cos(2*b*x + 2*a) - (4*I*(b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(
b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*(b*x + a)^3*
d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (4*I*(
b*x + a)^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^
2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*cos(b*x +
a) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*
(b*c*d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d
^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b^2
*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 - 12*I*a^2*d^4 + (-24*I*b*
c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
(12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^4 + 12*a^2*d^4 + 24*(b*c*
d^3 - a*d^4)*(b*x + a) - 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*I*b^2*c^2*d
^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*a^2*d^4 + (24*I*b*c*d^3 -
 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (2*I*(b*
x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 1
2*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) + (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*
d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4
)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I
*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2*d
^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (2*I*(b*x + a)^3*d^4 + (6*I*
b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*
d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 24
*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
 + I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4 + (-
24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4)*cos(2*b*x + 2*a) + 24*(b*c*
d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a))
```

```
 - (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a*d^4 + (24*I*b*c*d^3 + 24*I*
(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*a) - 24*(b*c*d^3 + (b*x + a)*d^4
- a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*cos(2*b*x + 2*a) +
b^4*sin(2*b*x + 2*a) + I*b^4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4}{{\sin\left(a+b\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx) \csc^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*csc(a + b*x)**2, x)
```

3.40 $\int (c+dx)^3 \cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=146

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{6d(c+dx)^2\tanh^{-}}{b^2}$$

```
[Out] -6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(
d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+
a)))/b^3-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)
))/b^4
```

Rubi [A] time = 0.12, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x]
)/b + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*
x))])/b^4 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{(3d)\int (c+dx)^2\csc(a+bx)\,dx}{b}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}-\frac{\left(6d^2\right)\int (c+dx)$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\text{Li}_2}{b^3}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\text{Li}_2}{b^3}$$
$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\csc(a+bx)}{b}+\frac{6id^2(c+dx)\text{Li}_2}{b^3}$$

Mathematica [B] time = 1.17, size = 311, normalized size = 2.13

$$-\frac{b^3c^3\csc(a+bx)+3b^3c^2dx\csc(a+bx)+3b^3cd^2x^2\csc(a+bx)+b^3d^3x^3\csc(a+bx)-3b^2c^2d\log\left(1-e^{i(a+bx)}\right)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((b^3*c^3*Csc[a + b*x] + 3*b^3*c^2*d*x*Csc[a + b*x] + 3*b^3*c*d^2*x^2*Csc[
a + b*x] + b^3*d^3*x^3*Csc[a + b*x] - 3*b^2*c^2*d*Log[1 - E^(I*(a + b*x))]
- 6*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x))] - 3*b^2*d^3*x^2*Log[1 - E^(I*(a +
b*x))] + 3*b^2*c^2*d*Log[1 + E^(I*(a + b*x))] + 6*b^2*c*d^2*x*Log[1 + E^(I*
(a + b*x))] + 3*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))] - (6*I)*b*d^2*(c + d*x
)*PolyLog[2, -E^(I*(a + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a +
 b*x))] + 6*d^3*PolyLog[3, -E^(I*(a + b*x))] - 6*d^3*PolyLog[3, E^(I*(a + b
*x))])/b^4)
```

fricas [C] time = 0.55, size = 669, normalized size = 4.58

$$-\frac{2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 - 6\,d^3\mathrm{polylog}\left(3, \cos\left(bx+a\right) + i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right) - 6\,d^3\mathrm{pol}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*d^3*p
olylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, co
s(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(cos(b*x + a) + I*s
in(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(b*x + a)
- I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(-cos(b*
x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(-
cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2
*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*
(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*
d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^4*s
in(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos\left(bx+a\right)\csc\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^2, x)

maple [B] time = 0.09, size = 433, normalized size = 2.97

$$-\frac{6id^3\operatorname{polylog}\left(2,\mathrm{e}^{i(bx+a)}\right)x}{b^3}+\frac{6d^2c\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{b^2}+\frac{6d^2c\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a}{b^3}-\frac{6d^2c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x}{b^2}-\frac{6d^2c\ln\left(\mathrm{e}^{i(b}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x)

[Out] -6*I/b^3*d^2*c*polylog(2,exp(I*(b*x+a)))+6/b^2*d^2*c*ln(1-exp(I*(b*x+a)))*x +6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6/b^ 3*d^2*c*ln(exp(I*(b*x+a))+1)*a-6/b^2*d*c^2*arctanh(exp(I*(b*x+a)))-6/b^4*d^ 3*a^2*arctanh(exp(I*(b*x+a)))-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^3*po lylog(3,exp(I*(b*x+a)))/b^4+12/b^3*d^2*c*a*arctanh(exp(I*(b*x+a)))+6*I/b^3* d^2*c*polylog(2,-exp(I*(b*x+a)))-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-3/ b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^2-2*I*(d^ 3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)+3/b^ 2*d^3*ln(1-exp(I*(b*x+a)))*x^2-3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2+6*I/b^3*d ^3*polylog(2,-exp(I*(b*x+a)))*x

maxima [B] time = 0.56, size = 1770, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")

[Out] -1/2*(3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b* x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b *x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1) *b) - 6*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b* x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b *x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2* b*x + 2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(c

```
os(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x
+ a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*b^3) + 2*c^3/sin(b*x + a) - 6*a*c^2*d/(b*sin(b*x + a)) + 6*a^2*c*d^2
/(b^2*sin(b*x + a)) - 2*a^3*d^3/(b^3*sin(b*x + a)) - 2*((6*(b*x + a)^2*d^3
+ 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*
a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1)
 + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
 a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a
)^2)*cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2
 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a
)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b*c*d^2
 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2
*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x +
2*a))*dilog(e^(I*b*x + I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*
d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x
 + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b
*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-12*I*d^3*cos(2*b*x + 2*
a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, -e^(I*b*x + I*a)) - (12
*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, e^
(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x +
a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) + 2
*I*b^3))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)\,(c+dx)^3}{\sin(a+bx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**2, x)
```

3.41 $\int (c+dx)^2 \cot(a+bx) \csc(a+bx)\,dx$

Optimal. Leaf size=90

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

```
[Out] -4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*pol
ylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3
```

Rubi [A] time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4410, 4183, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/
b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((2*I)*d^2*PolyLog[2, E^
(I*(a + b*x))])/b^3
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx = -\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{(2d)\int (c+dx)\csc(a+bx)\,dx}{b}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}-\frac{\left(2d^2\right)\int\log\left(1-\right.}{b^2}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{\left(2id^2\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\csc(a+bx)}{b}+\frac{2id^2\text{Li}_2\left(-e^{i(a+bx}\right.}{b^3}$$

Mathematica [B] time = 2.04, size = 234, normalized size = 2.60

$$\frac{-2b^2\csc(a)(c+dx)^2+b^2\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)(c+dx)^2\csc\left(\frac{1}{2}(a+bx)\right)-b^2\sec\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)(c+dx)^2\sec\left(\frac{1}{2}(a+bx)\right)}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] (-8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] - 2*b^2*(c + d*x)^2*Csc[a]
+ 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*x +
ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x
+ ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*Poly
Log[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + b^2*(c + d*
x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] - b^2*(c + d*x)^2*Sec[a/2]*Sec[
(a + b*x)/2]*Sin[(b*x)/2])/(2*b^3)
```

fricas [B] time = 0.53, size = 375, normalized size = 4.17

$$-\frac{b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\text{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-i\,d^2\text{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*dilo
g(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(
b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x
 + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*c*d - a*d^2)*log(-1/2*c
os(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a))/(b^3*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cos(bx+a) \csc(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*csc(b*x + a)^2, x)
```

maple [B] time = 0.03, size = 233, normalized size = 2.59

$$-\frac{d^2x^2}{b\sin(bx+a)}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)x}{b^2}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)a}{b^3}-\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)x}{b^2}-\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)a}{b^3}-\frac{2ia}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -1/b*d^2/sin(b*x+a)*x^2+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp
(I*(b*x+a)))*a-2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x-2/b^3*d^2*ln(exp(I*(b*x+a))
+1)*a-2*I/b^3*d^2*dilog(1-exp(I*(b*x+a)))+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+
1)-2/b^3*a*d^2*ln(csc(b*x+a)-cot(b*x+a))-2/b*c*d/sin(b*x+a)*x+2/b^2*c*d*ln(
csc(b*x+a)-cot(b*x+a))-1/b*c^2/sin(b*x+a)
```

maxima [B] time = 0.52, size = 556, normalized size = 6.18

$$\frac{\left(2\,bd^2x+2\,bcd-2\left(bd^2x+bcd\right)\cos\left(2\,bx+2\,a\right)-\left(2i\,bd^2x+2i\,bcd\right)\sin\left(2\,bx+2\,a\right)\right)\arctan\left(\sin\left(bx+a\right),\cos\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] ((2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (2*I*b*d^2*x
 + 2*I*b*c*d)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (
2*b*c*d*cos(2*b*x + 2*a) + 2*I*b*c*d*sin(2*b*x + 2*a) - 2*b*c*d)*arctan2(si
n(b*x + a), cos(b*x + a) - 1) - (2*b*d^2*x*cos(2*b*x + 2*a) + 2*I*b*d^2*x*s
in(2*b*x + 2*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + 2*(d^2*cos(2*b*x + 2*a)
 + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 2*(d^2*cos(2*b*x
 + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) - (I*b*d^2*x
 + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(2*b*x + 2*a) + (b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
 (-I*b*d^2*x - I*b*c*d + (I*b*d^2*x + I*b*c*d)*cos(2*b*x + 2*a) - (b*d^2*x
+ b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
+ a) + 1) - (2*I*b^2*d^2*x^2 + 4*I*b^2*c*d*x + 2*I*b^2*c^2)*sin(b*x + a))/(
-I*b^3*cos(2*b*x + 2*a) + b^3*sin(2*b*x + 2*a) + I*b^3)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)\,\left(c+d\,x\right)^2}{\sin (a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**2, x)
```

3.42 $\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=30

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -d*arctanh(cos(b*x+a))/b^2-(d*x+c)*csc(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4410, 3770}

$$-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - ((c + d*x)*Csc[a + b*x])/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot(a+bx)\csc(a+bx)\,dx &= -\frac{(c+dx)\csc(a+bx)}{b}+\frac{d\int\csc(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.06, size = 131, normalized size = 4.37

$$\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{c\csc(a+bx)}{b}-\frac{dx\csc(a)}{b}+\frac{dx\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)}{2b}-\frac{dx\sec(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x],x]
```

```
[Out] -((d*x*Csc[a])/b) - (c*Csc[a + b*x])/b - (d*Log[Cos[a/2 + (b*x)/2]])/b^2 +
(d*Log[Sin[a/2 + (b*x)/2]])/b^2 + (d*x*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x
)/2])/(2*b) - (d*x*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b)
```

fricas [B] time = 0.47, size = 62, normalized size = 2.07

$$-\frac{2\,bdx + d\log\left(\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right) - d\log\left(-\frac{1}{2}\,\cos\left(bx+a\right)+\frac{1}{2}\right)\sin\left(bx+a\right) + 2\,bc}{2\,b^{2}\sin\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos
(b*x + a) + 1/2)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

giac [B] time = 0.75, size = 801, normalized size = 26.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 +
b*d*x*tan(1/2*b*x)^2 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b
*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + d*log(4
*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^
2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^2*tan(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(4*(tan(1/2*b
*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a
)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*t
an(1/2*b*x)*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1
/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*t
an(1/2*b*x)^2 + b*c*tan(1/2*a)^2 + b*d*x + d*log(4*(tan(1/2*b*x)^4*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*
b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x) -
d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(
1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(
1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(
```

```
1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*ta
n(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a) - d*log(4*(tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(
1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*
tan(1/2*a) + b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2
*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

maple [A] time = 0.02, size = 52, normalized size = 1.73

$$-\frac{dx}{b\sin(bx+a)}+\frac{d\ln(\csc(bx+a)-\cot(bx+a))}{b^2}-\frac{c}{b\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x)
```

```
[Out] -1/b*d/sin(b*x+a)*x+1/b^2*d*ln(csc(b*x+a)-cot(b*x+a))-1/b*c/sin(b*x+a)
```

maxima [B] time = 0.38, size = 259, normalized size = 8.63

$$-\frac{\frac{\left(4\,(bx+a)\cos(bx+a)\sin(2\,bx+2\,a)-4\,(bx+a)\cos(2\,bx+2\,a)\sin(bx+a)+\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2-2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos(bx+a)^2+\sin(bx+\cdots\right.\right.}{\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\cdots\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - 4*(b*x + a)*cos(2*b*x +
2*a)*sin(b*x + a) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (co
s(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(b*x + a))*
d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) +
2*c/sin(b*x + a) - 2*a*d/(b*sin(b*x + a)))/b
```

mupad [B] time = 2.28, size = 88, normalized size = 2.93

$$-\frac{d\,\ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{b^2}+\frac{d\,\ln\left(d\,2\mathrm{i}-d\,\mathrm{e}^{a\,1\mathrm{i}}\,\mathrm{e}^{b\,x\,1\mathrm{i}}\,2\mathrm{i}\right)}{b^2}-\frac{\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\,\left(c+d\,x\right)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

```
[Out] (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 - (d*log(exp(a*1i + b*x*1i)*
1i + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*(exp(a*2i + b*x*2i) -
1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos(a+bx)\csc^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**2, x)
```

3.43 $\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 16.76, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x),x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{2\cos(bx+a)\sin(2bx+2a) - 2\cos(2bx+2a)\sin(bx+a) + \frac{\left(bd^2x+bcd+\left(bd^2x+bcd\right)\cos(2bx+2a)^2+\left(bd^2x+bcd\right)\sin(2bx+\cdots}{\cdots}}{bdx+(bd\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin
(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*
cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*
x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) + (b*d^2*x + b*c*d + (b*d^2*x +
b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2
*x + b*c*d)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x
 + a)), x) + 2*cos(b*x + a)*sin(2*b*x + 2*a) - 2*cos(2*b*x + 2*a)*sin(b*x +
 a) + 2*sin(b*x + a))/(b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x +
b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^2\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(a + bx \right)} \csc^{2}{\left(a + bx \right)}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c),x)
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x), x)
```

3.44 $\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 20.30, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (a + bx) \csc^2 (a + bx)}{(c + dx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**2/(c + d*x)**2, x)
```

3.45 $\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\cot(a+bx)\csc^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a)^2,x)
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx = \int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Mathematica [A] time = 6.31, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot(a+bx)\csc^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]*Csc[a + b*x]^2, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cos(bx+a)\csc(bx+a)^3\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\left(\csc^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
[Out] int((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)\csc(bx+a)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*csc(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^m}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3,x)
[Out] int((cos(a + b*x)*(c + d*x)^m)/sin(a + b*x)^3, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.46 $\int (c+dx)^4 \cot(a+bx) \csc^2(a+bx) \, dx$

Optimal. Leaf size=137

$$\frac{3d^4 \text{Li}_3\left(e^{2i(a+bx)}\right)}{b^5}-\frac{6id^3(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^4}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(a+bx)}{b^2}-\frac{(c+dx)^4}{}$$

```
[Out] -2*I*d*(d*x+c)^3/b^2-2*d*(d*x+c)^3*cot(b*x+a)/b^2-1/2*(d*x+c)^4*csc(b*x+a)^
2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b^3-6*I*d^3*(d*x+c)*polylog(2,ex
p(2*I*(b*x+a)))/b^4+3*d^4*polylog(3,exp(2*I*(b*x+a)))/b^5
```

Rubi [A] time = 0.26, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4410, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{6id^3(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}+\frac{3d^4\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^5}+\frac{6d^2(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\cot(a}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (2*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x
)^4*Csc[a + b*x]^2)/(2*b) + (6*d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*
PolyLog[3, E^((2*I)*(a + b*x))])/b^5
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{(2d)\int (c+dx)^3\csc^2(a+bx)\,dx}{b}$$
$$= -\frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{\left(6d^2\right)\int (c+dx)^2 \text{c}}{b^2}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} - \frac{(12}{}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{6d^2}{}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{6d^2}{}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{6d^2}{}$$
$$= -\frac{2id(c+dx)^3}{b^2} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\csc^2(a+bx)}{2b} + \frac{6d^2}{}$$

Mathematica [B] time = 6.61, size = 504, normalized size = 3.68

$$\frac{6c^2d^2\csc(a)(\sin(a)\log(\sin(a)\cos(bx)+\cos(a)\sin(bx))-bx\cos(a))}{b^3\left(\sin^2(a)+\cos^2(a)\right)} + \frac{2\csc(a)\csc(a+bx)\left(c^3d\sin(bx)+3c^2d^2x\,\text{s}\right.}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*((c + d*x)^4*Csc[a + b*x]^2)/b - (d^4*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b
*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*
x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^5 + (6*c^2*d^2*Cs
c[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*
(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2
*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (6*c*d^3*Csc[
a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]])
- Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b
*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x +
```

```
ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/
Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.70, size = 1071, normalized size = 7.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos
(b*x + a)*sin(b*x + a) + (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12
*I*b*c*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-12*I*b
*d^4*x - 12*I*b*c*d^3 + (12*I*b*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2)*dilog
(cos(b*x + a) - I*sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*d^3 + (12*I*b*d
^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))
+ (12*I*b*d^4*x + 12*I*b*c*d^3 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a
)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x
 + b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2
)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
 b^2*c^2*d^2 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2)*
log(cos(b*x + a) - I*sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b
*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4 - (b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c
*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(
b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^
2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*
d^3 - a^2*d^4)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 12
*(d^4*cos(b*x + a)^2 - d^4)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 12*
(d^4*cos(b*x + a)^2 - d^4)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(
d^4*cos(b*x + a)^2 - d^4)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 12*(
d^4*cos(b*x + a)^2 - d^4)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*
cos(b*x + a)^2 - b^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cos(bx+a)\csc(bx+a)^3\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*csc(b*x + a)^3, x)
```

maple [B] time = 0.13, size = 716, normalized size = 5.23

$$-\frac{24id^3cax}{b^3}+\frac{12d^4\,\mathrm{polylog}\left(3,-\mathrm{e}^{i(bx+a)}\right)}{b^5}+\frac{12d^4\,\mathrm{polylog}\left(3,\mathrm{e}^{i(bx+a)}\right)}{b^5}+\frac{6d^4a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^5}-\frac{12d^4a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] -24*I/b^3*d^3*c*a*x+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,
exp(I*(b*x+a)))/b^5+6/b^5*d^4*a^2*ln(exp(I*(b*x+a))-1)-12/b^5*d^4*a^2*ln(ex
p(I*(b*x+a)))+6/b^3*d^2*c^2*ln(exp(I*(b*x+a))-1)+6/b^3*d^2*c^2*ln(exp(I*(b*
x+a))+1)-12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-6/b^5*d^4*a^2*ln(1-exp(I*(b*x+a)
))+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*d^4*ln(exp(I*(b*x+a))+1)*x^2-4*
I/b^2*d^4*x^3+8*I/b^5*d^4*a^3+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*e
xp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a
))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*
I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*
x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2+12/
b^3*d^3*c*ln(exp(I*(b*x+a))+1)*x+12/b^3*d^3*c*ln(1-exp(I*(b*x+a)))*x+12/b^4
*d^3*c*ln(1-exp(I*(b*x+a)))*a+24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))-12/b^4*d^3*
c*a*ln(exp(I*(b*x+a))-1)-12*I/b^4*d^3*c*polylog(2,-exp(I*(b*x+a)))-12*I/b^4
*d^3*c*polylog(2,exp(I*(b*x+a)))+12*I/b^4*d^4*a^2*x-12*I/b^2*d^3*c*x^2-12*I
/b^4*d^3*c*a^2-12*I/b^4*d^4*polylog(2,-exp(I*(b*x+a)))*x-12*I/b^4*d^4*polyl
og(2,exp(I*(b*x+a)))*x
```

maxima [B] time = 0.89, size = 4540, normalized size = 33.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*si
n(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*
b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) -
 cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*c^2*d^2/((2*(
```

```
2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x
 + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*si
n(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x
 + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a)
+ sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b
*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b
*x + 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*
b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*
a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) -
 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (2*(b
*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x + a)
*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*s
in(4*b*x + 4*a) + sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2
*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2
*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*
x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2
- sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x +
2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x
+ 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*s
in(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x
+ 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x +
a)*sin(2*b*x + 2*a))*c^2*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a)
- cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*
b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1
)*b^2) - 12*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2
*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*
cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1
) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*c
os(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x -
 (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x +
 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x +
4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^3) + 6*(8*
(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x
+ a)^2*cos(2*b*x + 2*a) - 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2
```

*b*x + 2*a))*cos(4*b*x + 4*a) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a
) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) -
1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
+ 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*cos(b*x + a) + 1) - 4*((b*x + a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(
2*b*x + 2*a) + a)*sin(4*b*x + 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/
((2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(
2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) -
4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^4) - c^4/sin(b*x + a)^2 +
4*a*c^3*d/(b*sin(b*x + a)^2) - 6*a^2*c^2*d^2/(b^2*sin(b*x + a)^2) + 4*a^3*
c*d^3/(b^3*sin(b*x + a)^2) - a^4*d^4/(b^4*sin(b*x + a)^2) + 2*((6*(b*x + a)
^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*
d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 1
2*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*(b*x + a)^2*d^4 + (-24*I*b*
c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(
b*x + a) + 1) - (6*(b*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b
*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x
+ a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x +
a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (12*
I*(b*x + a)^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + (-8*I*b*
c*d^3 - 4*(-2*I*a - 1)*d^4)*(b*x + a)^3 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2)
*cos(2*b*x + 2*a) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a*d^4 + 12*(b*c*d^3
+ (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 24*(b*c*d^3 + (b*x + a)*d^4 -
a*d^4)*cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4)
*sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(2*
b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (12*b*c*d^3 + 12*(b*x + a)*d^4 - 12*a
d^4 + 12(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 24*(b*c*d^3
+ (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-12*I*b*c*d^3 - 12*I*(b*x + a)
*d^4 + 12*I*a*d^4)*sin(4*b*x + 4*a) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 -
24*I*a*d^4)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-3*I*(b*x + a)^2*d^
4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*
c*d^3 + 6*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12
*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 +
2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(
b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*
a*d^4)*(b*x + a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x
+ a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*

```
x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x +
a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
+ 1) + (-12*I*d^4*cos(4*b*x + 4*a) + 24*I*d^4*cos(2*b*x + 2*a) + 12*d^4*sin
(4*b*x + 4*a) - 24*d^4*sin(2*b*x + 2*a) - 12*I*d^4)*polylog(3, -e^(I*b*x +
I*a)) + (-12*I*d^4*cos(4*b*x + 4*a) + 24*I*d^4*cos(2*b*x + 2*a) + 12*d^4*si
n(4*b*x + 4*a) - 24*d^4*sin(2*b*x + 2*a) - 12*I*d^4)*polylog(3, e^(I*b*x +
I*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*s
in(4*b*x + 4*a) + (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
 a)^3 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) + 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) - 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\left(c+d\,x\right)^4}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^4)/sin(a + b*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.47 $\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=115

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(c+dx)^2}{2b^2}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*x
+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*polylog(2,exp(2*
I*(b*x+a)))/b^4
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4410, 4184, 3717, 2190, 2279, 2391}

$$-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\cot(a+bx)}{2b^2}-\frac{(c+dx)^3\csc^2(a+bx)}{2b}-\frac{3id(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - (
(c + d*x)^3*Csc[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a +
b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{(3d)\int (c+dx)^2\csc^2(a+bx)\,dx}{2b}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{\left(3d^2\right)\int (c+dx)\,c}{b^2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} - \frac{(}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3}{}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\csc^2(a+bx)}{2b} + \frac{3}{}$$

Mathematica [B] time = 6.41, size = 277, normalized size = 2.41

$$\frac{3cd^2\csc(a)(\sin(a)\log(\sin(a)\cos(bx)+\cos(a)\sin(bx))-bx\cos(a))}{b^3\left(\sin^2(a)+\cos^2(a)\right)}+\frac{3\csc(a)\csc(a+bx)\left(c^2d\sin(bx)+2cd^2x\sin\right.}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -1/2*((c + d*x)^3*Csc[a + b*x]^2)/b + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[
Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (
3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*
x]))/(2*b^2) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x
*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Ta
n[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Arc
Tan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + A
rcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]
^2 + Sin[a]^2)])
```

fricas [B] time = 0.75, size = 587, normalized size = 5.10

$$\frac{b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3+3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d\right)\cos\left(bx+a\right)\sin\left(bx+a\right)+\left(-3i\,d^3\cos\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (-3*I*d^3*cos(b
*x + a)^2 + 3*I*d^3)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*d^3*cos(b*
x + a)^2 - 3*I*d^3)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*d^3*cos(b*x
 + a)^2 - 3*I*d^3)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*d^3*cos(b*
x + a)^2 + 3*I*d^3)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d^3*x + b*
c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x +
a) + 1) - 3*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) - I*sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3 - (b*c*d^2 - a*d^3)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*(b*c*d
^2 - a*d^3 - (b*c*d^2 - a*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*
I*sin(b*x + a) + 1/2) - 3*(b*d^3*x + a*d^3 - (b*d^3*x + a*d^3)*cos(b*x + a)
^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d^3*x + a*d^3 - (b*d^3*x
```

```
 + a*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1))/(b^4*cos
(b*x + a)^2 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos(bx+a)\csc(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*csc(b*x + a)^3, x)
```

maple [B] time = 0.10, size = 409, normalized size = 3.56

$$\frac{2b\,d^3x^3\mathrm{e}^{2i(bx+a)}-3id^3x^2\mathrm{e}^{2i(bx+a)}+6bc\,d^2x^2\mathrm{e}^{2i(bx+a)}-6ic\,d^2x\,\mathrm{e}^{2i(bx+a)}+6b\,c^2dx\,\mathrm{e}^{2i(bx+a)}-3ic^2d\,\mathrm{e}^{2i(bx+a)}+3id^3x}{b^2\left(\mathrm{e}^{2i(bx+a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3
*I*c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3/b^3*d^2*c*ln(exp(I*(b*x+a))-1)+3/b^3
*d^2*c*ln(exp(I*(b*x+a))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))-3*I/b^2*d^3*x^2-
6*I/b^3*d^3*a*x-3*I/b^4*d^3*a^2+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x-3*I/b^4*d^
3*polylog(2,-exp(I*(b*x+a)))+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(
1-exp(I*(b*x+a)))*a-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/b^4*d^3*a*ln(ex
p(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.70, size = 1044, normalized size = 9.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*cos(4*b*x + 4
*a) - 12*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)
*sin(4*b*x + 4*a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*sin(2*b*x + 2*a))*arctan
2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d^2*cos(4*b*x + 4*a) - 12*b*c*d^
2*cos(2*b*x + 2*a) + 6*I*b*c*d^2*sin(4*b*x + 4*a) - 12*I*b*c*d^2*sin(2*b*x
+ 2*a) + 6*b*c*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*b*d^3*x*co
```

```
s(4*b*x + 4*a) - 12*b*d^3*x*cos(2*b*x + 2*a) + 6*I*b*d^3*x*sin(4*b*x + 4*a)
 - 12*I*b*d^3*x*sin(2*b*x + 2*a) + 6*b*d^3*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)*cos(4*b*x + 4*a) + (-4*I*b^3*
d^3*x^3 - 4*I*b^3*c^3 - 6*b^2*c^2*d + (-12*I*b^3*c*d^2 + 6*b^2*d^3)*x^2 + (
-12*I*b^3*c^2*d + 12*b^2*c*d^2)*x)*cos(2*b*x + 2*a) - (6*d^3*cos(4*b*x + 4*
a) - 12*d^3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*
x + 2*a) + 6*d^3)*dilog(-e^(I*b*x + I*a)) - (6*d^3*cos(4*b*x + 4*a) - 12*d^
3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x + 2*a) +
 6*d^3)*dilog(e^(I*b*x + I*a)) + (-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*
x - 3*I*b*c*d^2)*cos(4*b*x + 4*a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(2*b*x +
 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (
-3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*cos(4*b*x + 4*a)
+ (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(
4*b*x + 4*a) - 6*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*
x)*sin(4*b*x + 4*a) + (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*d + 6*(2*b^3
*c*d^2 + I*b^2*d^3)*x^2 + 12*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2*b*x + 2*a))
/(-2*I*b^4*cos(4*b*x + 4*a) + 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(4*b*x +
4*a) - 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,(c+d\,x)^3}{\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3,x)
```

```
[Out] int((cos(a + b*x)*(c + d*x)^3)/sin(a + b*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx)\csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*csc(a + b*x)**3, x)
```

3.48 $\int (c+dx)^2 \cot(a+bx) \csc^2(a+bx)\, dx$

Optimal. Leaf size=54

$$\frac{d^2 \log(\sin(a+bx))}{b^3} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

```
[Out] -d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+c)^2*csc(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b
^3
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4410, 4184, 3475}

$$-\frac{d(c+dx)\cot(a+bx)}{b^2} + \frac{d^2\log(\sin(a+bx))}{b^3} - \frac{(c+dx)^2\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -((d*(c + d*x)*Cot[a + b*x])/b^2) - ((c + d*x)^2*Csc[a + b*x]^2)/(2*b) + (d
^2*Log[Sin[a + b*x]])/b^3
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2 \cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d\int (c+dx)\csc^2(a+bx)\,dx}{b}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2\int \cot(a+bx)\,dx}{b^2}$$
$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\csc^2(a+bx)}{2b} + \frac{d^2\log(\sin(a+bx))}{b^3}$$

Mathematica [C] time = 0.89, size = 94, normalized size = 1.74

$$\frac{-b^2(c+dx)^2\csc^2(a+bx) + 2bd\csc(a)\sin(bx)(c+dx)\csc(a+bx) - 2id^2\tan^{-1}(\tan(a+bx)) - 2bd^2x\cot(a) + d^2 \text{l}}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] ((2*I)*b*d^2*x - (2*I)*d^2*ArcTan[Tan[a + b*x]] - 2*b*d^2*x*Cot[a] - b^2*(c
 + d*x)^2*Csc[a + b*x]^2 + d^2*Log[Sin[a + b*x]^2] + 2*b*d*(c + d*x)*Csc[a]
*Csc[a + b*x]*Sin[b*x])/(2*b^3)
```

fricas [A] time = 0.72, size = 102, normalized size = 1.89

$$\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\left(bd^2x + bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right) + 2\left(d^2\cos\left(bx+a\right)^2 - d^2\right)\log\left(\frac{1}{2}\,\sin\left(bx+a\right)\right)}{2\left(b^3\cos\left(bx+a\right)^2 - b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + 2*(d^2*cos(b*x + a)^2 - d^2)*log(1/2*sin(b*x + a)))/(b^3*co
s(b*x + a)^2 - b^3)
```

giac [B] time = 2.88, size = 3482, normalized size = 64.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/8*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*ta
```

n(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d
*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*
b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)
^4 + b^2*d^2*x^2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2
 + 2*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a
)^3 + b^2*d^2*x^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 4*
b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 + 4*b*d^2*x*
tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*
d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)^2*tan
(1/2*a)^3 - 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)*tan
(1/2*a)^4 - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*ta
n(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*
tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x
)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1
/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*
x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4
 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c
^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 24*
b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*
b*x)*tan(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 - 4*b*d^2*x*tan(1/2*b*x)^3 -
 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) + 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*
tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b
*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2
*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t

an(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2
*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c*d*x*tan(1/2*a)^2 -
24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 + 16*d^2*log(16*(tan(1/2*b*x)^8*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*
tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b
*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2
*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*t
an(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + t
an(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2
*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 - 4*b*d^2*x*tan(1/2*a)^3 +
8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 +
tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)
^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*
a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*
tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a
) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2
*a)^3 + b^2*d^2*x^2 + 2*b^2*c^2*tan(1/2*b*x)^2 - 4*b*c*d*tan(1/2*b*x)^3 - 2
4*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*tan(1/2*a)^2 - 24*b*c*d*tan(1
/2*b*x)*tan(1/2*a)^2 - 4*b*c*d*tan(1/2*a)^3 + 2*b^2*c*d*x + 4*b*d^2*x*tan(1
/2*b*x) - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(
1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*ta
n(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^
4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b
*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2
*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*t
an(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)
*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*
x)^2 + 4*b*d^2*x*tan(1/2*a) - 8*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2
*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)
^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)
^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(
1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + ta
n(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a)
- 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x
)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a
)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) - 4*d^2*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1

```
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*a)^2 + b^2*c^2 + 4*b*c*d*tan(1/2*b*x) + 4*b*c*d*tan
(1/2*a))/(b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*tan(1/2*b*x)^3*tan(1/2*a)
^3 + b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^3*tan(1/2*b*x)^3*tan(1/2*a) - 4*
b^3*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^3*tan(1/2*b*x)*tan(1/2*a)^3 + b^3*tan
(1/2*b*x)^2 + 2*b^3*tan(1/2*b*x)*tan(1/2*a) + b^3*tan(1/2*a)^2)
```

maple [A] time = 0.03, size = 95, normalized size = 1.76

$$-\frac{d^2x^2}{2b\sin(bx+a)^2}-\frac{d^2\cot(bx+a)\,x}{b^2}+\frac{d^2\ln(\sin(bx+a))}{b^3}-\frac{cdx}{b\sin(bx+a)^2}-\frac{cd\cot(bx+a)}{b^2}-\frac{c^2}{2b\sin(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] -1/2/b*d^2/sin(b*x+a)^2*x^2-1/b^2*d^2*cot(b*x+a)*x+d^2*ln(sin(b*x+a))/b^3-1
/b*c*d/sin(b*x+a)^2*x-1/b^2*c*d*cot(b*x+a)-1/2/b*c^2/sin(b*x+a)^2
```

maxima [B] time = 0.36, size = 1130, normalized size = 20.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) - 1)*co
s(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*
a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2
*b*x + 2*a) - 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 - (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) - 2*(b*x + a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*a*d^2/((2*(2*cos(2
*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)
^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a)^
2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 - 4*(b*x + a)^2*cos(2*b*x + 2*a) - 4*(
(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a)
 + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*co
s(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a)
 - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) - 1)*log(cos(b*x + a)^2 + sin(
```

```
b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*(2*cos(2*b*x + 2*a) - 1)*cos(4*b*x +
4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)^2 + 4*s
in(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a
) - 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 4*((b*x
+ a)^2*sin(2*b*x + 2*a) + b*x - (b*x + a)*cos(2*b*x + 2*a) + a)*sin(4*b*x +
 4*a) + 4*(b*x + a)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b^2) - c^2/sin(b*x + a)^2 + 2*a*c*d/(b*sin(b*x + a)^2) - a^2
*d^2/(b^2*sin(b*x + a)^2))/b
```

mupad [B] time = 2.56, size = 147, normalized size = 2.72

$$\frac{\frac{(c+d\,x)^2}{b}+\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{1+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}-2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}}-\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}+\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x)^2)/sin(a + b*x)^3,x)
```

```
[Out] ((c + d*x)^2/b + (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(exp(a*4i + b*x*4i) -
2*exp(a*2i + b*x*2i) + 1) - (d^2*x*2i)/b^2 + (b*c^2 - c*d*2i - d^2*x*2i + b
*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) - 1)) + (d^2*log(exp(a*2i)*e
xp(b*x*2i) - 1))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx) \csc^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*csc(a + b*x)**3, x)
```

3.49 $\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx$

Optimal. Leaf size=35

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

```
[Out] -1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*csc(b*x+a)^2/b
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4410, 3767, 8}

$$-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
```

```
[Out] -(d*Cot[a + b*x])/(2*b^2) - ((c + d*x)*Csc[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\cot(a+bx)\csc^2(a+bx)\,dx = -\frac{(c+dx)\csc^2(a+bx)}{2b} + \frac{d\int \csc^2(a+bx)\,dx}{2b}$$
$$= -\frac{(c+dx)\csc^2(a+bx)}{2b} - \frac{d\,\text{Subst}(\int 1\,dx, x, \cot(a+bx))}{2b^2}$$
$$= -\frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\csc^2(a+bx)}{2b}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 1.37

$$-\frac{d\cot(a+bx)}{2b^2} - \frac{c\csc^2(a+bx)}{2b} - \frac{dx\csc^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]*Csc[a + b*x]^2,x]
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) - (d*x*Csc[a + b*x]^2)
/(2*b)
```

fricas [A] time = 0.58, size = 44, normalized size = 1.26

$$\frac{bdx + d\cos(bx+a)\sin(bx+a) + bc}{2\left(b^2\cos(bx+a)^2 - b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="fricas")
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2 - b^2)
```

giac [B] time = 0.36, size = 526, normalized size = 15.03

$$-\frac{bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="giac")
[Out] -1/8*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4
```

```
+ 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*ta
n(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b
*c*tan(1/2*b*x)^4 + 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*ta
n(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^2*tan(1/2
*a)^3 + b*c*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*
b*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 -
 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)*ta
n(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/
2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3
+ b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/
2*b*x)^2 + 2*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2*tan(1/2*a)^2)
```

maple [A] time = 0.03, size = 61, normalized size = 1.74

$$\frac{\frac{d\left(-\frac{bx+a}{2\sin(bx+a)^2}-\frac{\cot(bx+a)}{2}\right)}{b}+\frac{da}{2b\sin(bx+a)^2}-\frac{c}{2\sin(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(-1/2*(b*x+a)/sin(b*x+a)^2-1/2*cot(b*x+a))+1/2/b*d*a/sin(b*x+a)^
2-1/2*c/sin(b*x+a)^2)
```

maxima [B] time = 0.39, size = 287, normalized size = 8.20

$$\frac{\frac{2(4(bx+a)\cos(2bx+2a)^2+4(bx+a)\sin(2bx+2a)^2-(2(bx+a)\cos(2bx+2a)+\sin(2bx+2a))\cos(4bx+4a)-2(bx+a)\cos(2bx+2a)-(2(bx+a)\sin(2bx+}{(2(2\cos(2bx+2a)-1)\cos(4bx+4a)-\cos(4bx+4a)^2-4\cos(2bx+2a)^2-\sin(4bx+4a)^2+4\sin(4bx+4a)\sin(2bx+2a)-4\sin(2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 - (
2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - 2*(b*x
+ a)*cos(2*b*x + 2*a) - (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) +
1)*sin(4*b*x + 4*a) + sin(2*b*x + 2*a))*d/((2*(2*cos(2*b*x + 2*a) - 1)*cos(
4*b*x + 4*a) - cos(4*b*x + 4*a)^2 - 4*cos(2*b*x + 2*a)^2 - sin(4*b*x + 4*a)
^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) - 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b
*x + 2*a) - 1)*b) - c/sin(b*x + a)^2 + a*d/(b*sin(b*x + a)^2))/b
```

mupad [B] time = 1.72, size = 53, normalized size = 1.51

$$\frac{d\,1\mathrm{i}-\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\left(-b\,\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*(c + d*x))/sin(a + b*x)^3,x)
```

```
[Out] (d*1i - exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*2
i) - 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos(a+bx)\csc^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*csc(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*csc(a + b*x)**3, x)
```

3.50 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.33, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x), x]
```

fricas [A] time = 1.24, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\csc(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)*csc(b*x + a)^3/(d*x + c), x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)\left(\csc^3(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos(a+b\,x)}{\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\csc^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x), x)
```

3.51 $\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 10.64, size = 0, normalized size = 0.00

$$\int \frac{\cot(a+bx)\csc^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]*Csc[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos(bx+a)\csc(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*csc(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{\cos \left(bx+a\right) \left(\csc^{3}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(cos(a + b*x)/(sin(a + b*x)^3*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(a+bx \right)}\csc^{3}{\left(a+bx \right)}}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*csc(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*csc(a + b*x)**3/(c + d*x)**2, x)
```

3.52 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2
-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.45, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - \frac{\left(15d^2\right)\int$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}$$

Mathematica [A] time = 2.24, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx)) - \cos(2(a+bx))\left(16b^2(c+dx)^2-15d^2\right)\right) - 15\sqrt{\pi}\,d^2\cos\left(2a-\frac{2bc}{d}\right)C}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

fricas [A] time = 0.80, size = 222, normalized size = 1.13

$$-\frac{15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - 2\left(16\,b^3d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

giac [C] time = 3.06, size = 1198, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(64*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I
*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d
^2*((I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2)/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d -
36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
 - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sq
rt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2
 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3
)/d^3 + (I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c
)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d
 - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)
*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 48*(-I*sqrt(p
i)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqr
t(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
```

```
1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.03, size = 234, normalized size = 1.19

$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\cdots\right)}{8b\sqrt{\frac{b}{d}}}\cdots\right)}{4b}\cdots\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.55, size = 275, normalized size = 1.40

$$\frac{\sqrt{2}\left(160\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)-8\left(16\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\,\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
+ a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*
d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) + ((15*I - 15)*4^(1/4)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (-
(15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (1
```

```
5*I - 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx) \sin(a+bx) (c+dx)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
[Out] Timed out
```

3.53 $\int (c+dx)^{3/2} \cos(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=168

$$-\frac{3\sqrt{\pi}\, d^{3/2} \sin\left(2a-\frac{2bc}{d}\right) C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\, d^{3/2} \cos\left(2a-\frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}-\frac{(c+}{}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*Fre
snelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b
^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.29, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}\, d^{3/2} \sin\left(2a-\frac{2bc}{d}\right) \text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\, d^{3/2} \cos\left(2a-\frac{2bc}{d}\right) S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2b}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\right)\int \frac{\sin}{}}{32}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\cos\left(2\right.\right.}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d\cos\left(2\right.\right.}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}} -$$

Mathematica [A] time = 0.84, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}\,d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\,\sqrt{c+dx}\,(4b(c+dx)\cos(2(a}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

fricas [A] time = 0.80, size = 167, normalized size = 0.99

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) + 3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - 4\left(2\,b^2dx+3\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

giac [C] time = 0.51, size = 743, normalized size = 4.42

$$-\frac{16\left(\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\frac{i\sqrt{\pi}\left(16\,b^2c^2+8i\,bcd-3\,d^2\right)d\,\mathrm{erf}\left(-\cdots\right.}{\sqrt{bd}\left(\cdots\right.}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(16*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*
sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*
a*d)/d)/b^2)/d^2) + 8*(-I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c -
2*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.02, size = 187, normalized size = 1.11

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.56, size = 256, normalized size = 1.52

$$-\frac{\sqrt{2}\left(32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^{2}\cos\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)-24\sqrt{2}\sqrt{dx+c}\,bd\sin\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)-\left(-(3i+3)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^{2}\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c + a
*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d) -
(-(3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (3
*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(2*I*b/d)) - ((3*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)
*cos(-2*(b*c - a*d)/d) - (3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin
(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

sympy [B] time = 41.48, size = 665, normalized size = 3.96

$$-\frac{5\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{32d\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{21\sqrt{\pi}\sqrt{\frac{d}{b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

3.54 $\int \sqrt{c+dx} \cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

```
[Out] 1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^(
1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(1/2
)/b
```

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\,\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int \sqrt{c+dx}\,\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b} - \frac{(d\,\mathrm{si}}{}$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{}\right.}{4b}$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}}{}$$

Mathematica [A] time = 0.26, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - \sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

fricas [A] time = 0.75, size = 125, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right) - 2\left(2b\cos(bx+a)^2\right.}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
```

```
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

giac [C] time = 0.56, size = 402, normalized size = 2.83

$$-\frac{4\left(\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2i\,bc-2i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2i\,bc+2i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{i\sqrt{\pi}(4\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,b}{\sqrt{b}}\right.}{d}\right.}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*s
qrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2
*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - I*sqrt(pi)
*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x
 + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)/d
```

maple [A] time = 0.02, size = 142, normalized size = 1.00

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))
```

maxima [C] time = 0.45, size = 209, normalized size = 1.47

$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}\,b\cos\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d)
 + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I +
 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(
b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

sympy [B] time = 6.14, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
```

```
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

3.55 $\int \sqrt{c+dx}\,\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

```
[Out] 1/8*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*d^(
1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2)
)*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(2*b*x+2*a)*(d*x+c)^(1/2
)/b
```

Rubi [A] time = 0.22, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{8b^{3/2}}-\frac{\sqrt{\pi}\sqrt{d}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}}-\frac{\sqrt{c+dx}\cos(2a+2bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(4*b) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (2*b*
c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(8*b^(3/2)) -
 (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*S
in[2*a - (2*b*c)/d])/(8*b^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}\sqrt{c+dx}\,\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int \sqrt{c+dx}\,\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{d\int \frac{\cos(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b}$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{\left(d\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{\sqrt{c+dx}}\,dx}{8b} - \frac{(d\,\mathrm{si}}{}$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Subst}\left(\int \cos\left(\frac{2bx^2}{d}\right)dx, x, \sqrt{}\right.}{4b}$$
$$= -\frac{\sqrt{c+dx}\,\cos(2a+2bx)}{4b} + \frac{\sqrt{d}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{8b^{3/2}} - \frac{\sqrt{d}}{}$$

Mathematica [A] time = 0.02, size = 134, normalized size = 0.94

$$\frac{\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{\pi}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-2\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(2(a+bx))}{8b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-2*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + Sqrt[Pi]*Cos[2*a - (2*b*c)/d
]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[Pi]*FresnelS[(2*Sqr
t[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(8*b*Sqrt[b/d])
```

fricas [A] time = 0.47, size = 125, normalized size = 0.88

$$\frac{\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right)-2\left(2b\cos(bx+a)^2\right.}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(pi*d*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) - pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(p
```

```
i*d)))*sin(-2*(b*c - a*d)/d) - 2*(2*b*cos(b*x + a)^2 - b)*sqrt(d*x + c))/b^
2
```

giac [C] time = 0.40, size = 402, normalized size = 2.83

$$-\frac{4\left(\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c-\frac{i\sqrt{\pi}(4bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{i}{\sqrt{b}}\right.}{d}\right.}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(4*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*s
qrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2
*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - I*sqrt(pi)
*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(
pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x
 + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 142, normalized size = 1.00

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/16/b*d*Pi^(1
/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))
```

maxima [C] time = 0.60, size = 209, normalized size = 1.47

$$-\frac{\sqrt{2}\left(8\sqrt{2}\sqrt{dx+c}\,b\cos\left(\frac{2\left((dx+c)b-bc+ad\right)}{d}\right)+\left((i-1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2\,(bc-ad)}{d}\right)+(i+1)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\sin\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/64*sqrt(2)*(8*sqrt(2)*sqrt(d*x + c)*b*cos(2*((d*x + c)*b - b*c + a*d)/d)
 + ((I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (I +
 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(2*I*b/d)) + (-(I + 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*cos(-2*(
b*c - a*d)/d) - (I - 1)*4^(1/4)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*
d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(1/2), x)
```

sympy [B] time = 6.21, size = 389, normalized size = 2.74

$$-\frac{b^{\frac{3}{2}}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{5}{2}}\cos\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{3}{4}\right)\Gamma\left(\frac{5}{4}\right){}_2F_3\left(\left.\begin{matrix}\frac{3}{4},\frac{5}{4}\\ \frac{3}{2},\frac{7}{4},\frac{9}{4}\end{matrix}\right|-\frac{b^2(c+dx)^2}{d^2}\right)}{4d^{\frac{5}{2}}\Gamma\left(\frac{7}{4}\right)\Gamma\left(\frac{9}{4}\right)}-\frac{\sqrt{b}\sqrt{\frac{d}{b}}\,(c+dx)^{\frac{3}{2}}\sin\left(2a-\frac{2bc}{d}\right)\Gamma\left(\frac{1}{4}\right)\Gamma\left(\frac{3}{4}\right)\cdots}{8d^{\frac{3}{2}}\Gamma\left(\frac{5}{4}\right)\Gamma\left(\frac{7}{4}\right)\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -b**(3/2)*sqrt(d/b)*(c + d*x)**(5/2)*cos(2*a - 2*b*c/d)*gamma(3/4)*gamma(5/
4)*hyper((3/4, 5/4), (3/2, 7/4, 9/4), -b**2*(c + d*x)**2/d**2)/(4*d**(5/2)*
gamma(7/4)*gamma(9/4)) - sqrt(b)*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c
/d)*gamma(1/4)*gamma(3/4)*hyper((1/4, 3/4), (1/2, 5/4, 7/4), -b**2*(c + d*x
)**2/d**2)/(8*d**(3/2)*gamma(5/4)*gamma(7/4)) + sqrt(pi)*c*sqrt(d/b)*sin(2*
```

```
a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) + sqr
t(pi)*c*sqrt(d/b)*cos(2*a - 2*b*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d
*sqrt(b/d)))/(2*d) + sqrt(pi)*x*sqrt(d/b)*sin(2*a - 2*b*c/d)*fresnelc(2*b*s
qrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2 + sqrt(pi)*x*sqrt(d/b)*cos(2*a - 2*b
*c/d)*fresnels(2*b*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/2
```

3.56 $\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=168

$$-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2}-\frac{(c+$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-3/32*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(
2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*Fre
snelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b
^(5/2)+3/16*d*sin(2*b*x+2*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.28, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{32b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{32b^{5/2}}+\frac{3d\sqrt{c+dx}\,\sin(2a+2b}{16b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(4*b) - (3*d^(3/2)*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(32*b^(5/
2)) - (3*d^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[
Pi])]*Sin[2*a - (2*b*c)/d])/(32*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b
*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{(3d)\int \sqrt{c+dx}\,\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\right)\int \frac{\sin}{}}{32}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d^2\cos\left(2\right.\right.}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} + \frac{3d\sqrt{c+dx}\,\sin(2a+2bx)}{16b^2} - \frac{\left(3d\cos\left(2a\right.\right.}{}$$
$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{4b} - \frac{3d^{3/2}\sqrt{\pi}\,\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{32b^{5/2}} -$$

Mathematica [A] time = 0.04, size = 157, normalized size = 0.93

$$\frac{-3\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 3\sqrt{\pi}\,d\cos\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) - 2\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(4b(c+dx)\cos(2(a}{32d^2\left(\frac{b}{d}\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-3*d*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sq
rt[Pi]] - 3*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2
*a - (2*b*c)/d] - 2*Sqrt[b/d]*Sqrt[c + d*x]*(4*b*(c + d*x)*Cos[2*(a + b*x)]
 - 3*d*Sin[2*(a + b*x)]))/(32*(b/d)^(5/2)*d^2)
```

fricas [A] time = 0.50, size = 167, normalized size = 0.99

$$-\frac{3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{S}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right) + 3\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) - 4\left(2\,b^2dx + 3\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(3*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 3*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)
*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(2*b^2*d*x + 3*b*d*cos(b*x + a)*
sin(b*x + a) + 2*b^2*c - 4*(b^2*d*x + b^2*c)*cos(b*x + a)^2)*sqrt(d*x + c))
/b^3
```

giac [C] time = 1.02, size = 743, normalized size = 4.42

$$-\frac{16\left(\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{2ibc-2iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}-\frac{i\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-2ibc+2iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}\right)c^2+d^2\left(\frac{i\sqrt{\pi}\left(16b^2c^2+8ibcd-3d^2\right)d\,\mathrm{erf}\left(-\cdots\right)}{\sqrt{bd}\left(\cdots\right)}\cdots\right)}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/64*(16*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*
sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt
(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*
a*d)/d)/b^2)/d^2) + 8*(-I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*
I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c -
2*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 187, normalized size = 1.11

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/8/b*d*(1/4/b
*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^(1
/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+
sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.55, size = 256, normalized size = 1.52

$$-\frac{\sqrt{2}\left(32\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-24\sqrt{2}\,\sqrt{dx+c}\,bd\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-\left(-(3i+3)\cdot 4^{\frac{1}{4}}\sqrt{\pi}\,d^2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*sqrt(2)*(32*sqrt(2)*(d*x + c)^(3/2)*b^2*cos(2*((d*x + c)*b - b*c + a
*d)/d) - 24*sqrt(2)*sqrt(d*x + c)*b*d*sin(2*((d*x + c)*b - b*c + a*d)/d) -
(-(3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (3
*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(2*I*b/d)) - ((3*I - 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)
*cos(-2*(b*c - a*d)/d) - (3*I + 3)*4^(1/4)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin
(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(3/2), x)
```

sympy [B] time = 42.22, size = 665, normalized size = 3.96

$$-\frac{5\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)\Gamma\left(\frac{1}{4}\right)}{32d\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{\pi}\sqrt{\frac{d}{b}}\,(c+dx)^2\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2b\sqrt{c+dx}}{\sqrt{\pi}\,d\sqrt{\frac{b}{d}}}\right)}{2d}-\frac{21\sqrt{\pi}\sqrt{\frac{d}{b}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] -5*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sq
rt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1/4)/(32*d*gamma(9/4)) + sqrt(pi)*sqr
t(d/b)*(c + d*x)**2*sin(2*a - 2*b*c/d)*fresnelc(2*b*sqrt(c + d*x)/(sqrt(pi)
*d*sqrt(b/d)))/(2*d) - 21*sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d
)*fresnels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(32*d*gam
ma(11/4)) + sqrt(pi)*sqrt(d/b)*(c + d*x)**2*cos(2*a - 2*b*c/d)*fresnels(2*b
*sqrt(c + d*x)/(sqrt(pi)*d*sqrt(b/d)))/(2*d) - 15*sqrt(pi)*d*sqrt(d/b)*sin(
2*a - 2*b*c/d)*fresnelc(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(1
/4)/(512*b**2*gamma(9/4)) - 63*sqrt(pi)*d*sqrt(d/b)*cos(2*a - 2*b*c/d)*fres
nels(2*sqrt(b)*sqrt(c + d*x)/(sqrt(pi)*sqrt(d)))*gamma(3/4)/(512*b**2*gamma
(11/4)) + 5*sqrt(d/b)*(c + d*x)**(3/2)*sin(2*a - 2*b*c/d)*sin(2*b*c/d + 2*b
*x)*gamma(1/4)/(64*sqrt(b)*sqrt(d)*gamma(9/4)) - 21*sqrt(d/b)*(c + d*x)**(3
/2)*cos(2*a - 2*b*c/d)*cos(2*b*c/d + 2*b*x)*gamma(3/4)/(64*sqrt(b)*sqrt(d)*
gamma(11/4)) + 15*sqrt(d)*sqrt(d/b)*sqrt(c + d*x)*sin(2*a - 2*b*c/d)*cos(2*
b*c/d + 2*b*x)*gamma(1/4)/(256*b**(3/2)*gamma(9/4)) + 63*sqrt(d)*sqrt(d/b)*
sqrt(c + d*x)*sin(2*b*c/d + 2*b*x)*cos(2*a - 2*b*c/d)*gamma(3/4)/(256*b**(3
/2)*gamma(11/4))
```

3.57 $\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+5/16*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2
-15/128*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*Pi^(1/2)/b^(7/2)+15/128*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/64*d^2*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.33, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4406, 12, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{128b^{7/2}}+\frac{15\sqrt{\pi}\,d^{5/2}\sin\left(2a-\frac{2bc}{d}\right)S\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{128b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\cos(2a}{64b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b^3) - ((c + d*x)^(5/2)*Cos[2*a
 + 2*b*x])/(4*b) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(128*b^(7/2)) + (15*d^(5/2)*Sqrt[
Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c
)/d])/(128*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(16*b^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin(a+bx)\,dx = \int \frac{1}{2}(c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= \frac{1}{2}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{(5d)\int (c+dx)^{3/2}\cos(2a+2bx)\,dx}{8b}$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx)^{3/2}\sin(2a+2bx)}{16b^2} - \frac{\left(15d^2\right)\int$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} + \frac{5d(c+dx$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{64b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{4b} - \frac{15d^{5/2}\sqrt{\pi}$$

Mathematica [A] time = 1.06, size = 179, normalized size = 0.91

$$\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(20bd(c+dx)\sin(2(a+bx)) - \cos(2(a+bx))\left(16b^2(c+dx)^2-15d^2\right)\right) - 15\sqrt{\pi}\,d^2\cos\left(2a-\frac{2bc}{d}\right)C}{128b^3\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x],x]
```

```
[Out] (-15*d^2*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])
/Sqrt[Pi]] + 15*d^2*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
*Sin[2*a - (2*b*c)/d] + 2*Sqrt[b/d]*Sqrt[c + d*x]*(-((-15*d^2 + 16*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)]) + 20*b*d*(c + d*x)*Sin[2*(a + b*x)]))/(128*b^3*S
qrt[b/d])
```

fricas [A] time = 0.68, size = 222, normalized size = 1.13

$$-\frac{15\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{2(bc-ad)}{d}\right)C\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{2(bc-ad)}{d}\right) - 2\left(16b^3d^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/128*(15*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d
*x + c)*sqrt(b/(pi*d))) - 15*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x +
 c)*sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*
x + 16*b^3*c^2 - 15*b*d^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 -
 15*b*d^2)*cos(b*x + a)^2 + 40*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)*sin(b*x +
 a))*sqrt(d*x + c))/b^4
```

giac [C] time = 0.68, size = 1198, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(64*(I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I
*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 12*c*d
^2*((I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d
)/b^2)/d^2 + (-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d -
36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
 - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sq
rt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d^2
 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^3
)/d^3 + (I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d^3)*d
*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*x + c
)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*c^2*d
 - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)
*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 48*(-I*sqrt(p
i)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqr
t(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
```

```
1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) +
 2*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 2*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 234, normalized size = 1.19

$$\frac{-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{4b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/8/b*d*(1/4/b
*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+c)
^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos(2
*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a*d
-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.47, size = 275, normalized size = 1.40

$$\sqrt{2}\left(160\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-8\left(16\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^3-15\sqrt{2}\,\sqrt{dx+c}\,bd^2\right)\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/1024*sqrt(2)*(160*sqrt(2)*(d*x + c)^(3/2)*b^2*d*sin(2*((d*x + c)*b - b*c
+ a*d)/d) - 8*(16*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x + c)*b*
d^2)*cos(2*((d*x + c)*b - b*c + a*d)/d) + ((15*I - 15)*4^(1/4)*sqrt(pi)*d^3
*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(
b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (-
(15*I + 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (1
```

```
5*I - 15)*4^(1/4)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(-2*I*b/d)))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+bx)\sin(a+bx)(c+dx)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.58 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=406

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{16b^{7/2}}$$

```
[Out] 5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2+
1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*d^
(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*d^2*sin(
b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 1.14, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int\left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$
$$=\frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx$$
$$=\frac{(c+dx)^{5/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b}+\frac{(5d)\int (c+dx)^{3}}{}$$
$$=\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{(c+dx)^{5}}{}$$
$$=\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c+}}{}$$
$$=\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c+}}{}$$
$$=\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}-\frac{15d^2\sqrt{c+}}{}$$
$$=\frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2}-\frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2}+\frac{15d^{5/2}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [C] time = 15.07, size = 1171, normalized size = 2.88

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{2\pi}\,C\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])
/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)
/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*
b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3
```

```
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

fricas [A] time = 0.59, size = 370, normalized size = 0.91

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-405}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

giac [C] time = 3.34, size = 2453, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/1728*(72*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((sqrt(6)*sqrt(pi)*(12*b^2*
c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sq
rt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d
*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d
^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6
*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(6)*sqrt(p
i)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2
*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x
+ c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3
*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b
^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(
d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*
b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^
2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)
*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqr
t(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*
```

```
a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*
d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 3
6*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b
*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)
/b^3)/d^3) - 36*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b
*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*
I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.05, size = 474, normalized size = 1.17

$$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fresne}\cdots\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{4b}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*
```

```
x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.60, size = 543, normalized size = 1.34

$$-\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-2160\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left((5i+5)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\mathrm{c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) + ((5*I + 5)*9^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(
1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-(b*c - a*d)/d) + (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((405*I - 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (405*I + 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-I*b/d)) + (-(5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12
*(d*x + c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c +
a*d)/d) - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x
+ c)*b - b*c + a*d)/d))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.59 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}}{}\right.}{24b^{5/2}}$$

```
[Out] 1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d^
(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^
(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos(b*x+a)
*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.68, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b} + \frac{d\int \sqrt{c+dx}\,\text{si}}{8}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin}{4b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin}{4b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} + \frac{(c+dx)^{3/2}\sin}{4b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{8b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{24b^2} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos(}{}$$

Mathematica [C] time = 9.14, size = 677, normalized size = 1.92

$$\frac{d\left(\sqrt{2\pi}\sqrt{\frac{b}{d}}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\sqrt{\frac{b}{d}}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a\right.\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
+ 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sq
rt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2
*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
 + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[
```

```
b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x
]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqr
t[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x
)] + 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

fricas [A] time = 0.63, size = 298, normalized size = 0.84

$$\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 2.77, size = 1529, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/288*(12*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sqrt(2)*
sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
```

```
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c
)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I
*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - 4*(sqrt(6)*sqrt(pi)*(6*b*c
 + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*
sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.04, size = 386, normalized size = 1.09

$$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+c)\cdots}{\cdots}}{d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
```

```
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.59, size = 495, normalized size = 1.40

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{144\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\,\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)-216\,\sqrt{dx+c}\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^
2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x + c)
*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(27*I - 27)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (27*I + 27)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(I*b/d)) + ((27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*
d)/d) + (27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

3.60 $\int \sqrt{c+dx}\,\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right)}{4b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1
/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(b*x+
a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.47, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx)-\frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx-\frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}dx}{24b}-\frac{d}{}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int}{24b}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b}-\frac{\sqrt{c+dx}\sin(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}}{}$$
$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}}{}\right.}{12b^{3/2}}$$

Mathematica [C] time = 5.36, size = 264, normalized size = 0.87

$$\frac{\frac{\sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))}{\sqrt{\frac{b}{d}}}-9i\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c}{}\right)}{\sqrt{-\frac{ib(c+dx}{d}}}\right.}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (((-9*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (Sqrt[6*Pi]*Cos[3*a - (3*
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d]*Sqrt
[c + d*x]*Sin[3*(a + b*x)])/Sqrt[b/d])/(72*b)
```

fricas [A] time = 0.56, size = 245, normalized size = 0.81

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

giac [C] time = 2.85, size = 838, normalized size = 2.76

$$-\frac{\frac{\sqrt{6}\,\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c -
 I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(
6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b) - 6*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 6*I*sqrt(d*x + c)*d*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a
*d)/d)/b)/d
```

maple [A] time = 0.04, size = 294, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.57, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) + ((9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(I*b/d)) + (-(9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx}\,\sin^2(a + bx)\cos(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

3.61 $\int \sqrt{c+dx}\,\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+}}{\sqrt{d}}\right.}{4b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/72*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1
/8*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin(b*x+
a)*(d*x+c)^(1/2)/b-1/12*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.47, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\right.}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (
3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sin[
a + b*x])/(4*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\cos(a+bx) - \frac{1}{4}\sqrt{c+dx}\cos(3a+3bx)\right) dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\cos(a+bx)\,dx - \frac{1}{4}\int \sqrt{c+dx}\cos(3a+3bx)\,dx$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{d\int \frac{\sin(3a+3bx)}{\sqrt{c+dx}}dx}{24b} - \frac{d}{\cdots}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)\int \cdots}{24b}$$
$$= \frac{\sqrt{c+dx}\sin(a+bx)}{4b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{12b} + \frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Subst}\cdots}{\cdots}$$
$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\cdots}{\cdots}\right)}{12b^{3/2}}$$

Mathematica [C] time = 3.05, size = 264, normalized size = 0.87

$$\frac{\frac{\sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3(a+bx))}{\sqrt{\frac{b}{d}}} - 9i\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c\cdots}{\cdots}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}\cdots\right.}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (((-9*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt
[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/
Sqrt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (Sqrt[6*Pi]*Cos[3*a - (3*
b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[6*Pi]*FresnelC[
Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] - 6*Sqrt[b/d]*Sqrt
[c + d*x]*Sin[3*(a + b*x)])/Sqrt[b/d])/(72*b)
```

fricas [A] time = 0.63, size = 245, normalized size = 0.81

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)S\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 9\sqrt{2}\,\pi d\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) + sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*cos(b*x + a)^2 - b)*sqrt(d*x +
 c)*sin(b*x + a))/b^2
```

giac [C] time = 3.87, size = 838, normalized size = 2.76

$$\frac{\sqrt{6}\,\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9\,\sqrt{2}\,\sqrt{\pi}\,(2\,bc}{}$$

$$-\frac{}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/144*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c -
 I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(
6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(
b^2*d^2) + 1)*b) - 6*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 6*I*sqrt(d*x + c)*d*e^((3*I*
(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c
)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c
 - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a
*d)/d)/b)/d
```

maple [A] time = 0.00, size = 294, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+ \cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/16/b*d*2^(1/2)*
Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)
*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d
)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresne
lS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/
d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.58, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{72\sqrt{dx+c}\,b^2\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left(-(i+1)\cdot 9^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i-1)\cdot \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 72*sqrt
(d*x + c)*b^2*sin(((d*x + c)*b - b*c + a*d)/d)/d + (-(I + 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2
)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(
3*I*b/d)) + ((9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/
d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(I*b/d)) + (-(9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)
*cos(-(b*c - a*d)/d) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \sin^2(a + bx) \cos(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x), x)
```

3.62 $\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\ \cdots}{\cdots}\right)}{24b^{5/2}}$$

```
[Out] 1/4*(d*x+c)^(3/2)*sin(b*x+a)/b-1/12*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b+1/144*d^
(3/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^
(3/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))
*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*cos(b*x+a)
*(d*x+c)^(1/2)/b^2-1/24*d*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.57, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(8*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*x
])/(24*b^2) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt
[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a -
 (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/
2)) - (d^(3/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[
d]]*Sin[3*a - (3*b*c)/d])/(24*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + ((
c + d*x)^(3/2)*Sin[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(12
*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\cos(a+bx)-\frac{1}{4}(c+dx)^{3/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\cos(a+bx)\,dx-\frac{1}{4}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{4b}-\frac{(c+dx)^{3/2}\sin(3a+3bx)}{12b}+\frac{d\int \sqrt{c+dx}\,\sin}{8b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(}{4b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(}{4b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}+\frac{(c+dx)^{3/2}\sin(}{4b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{8b^2}-\frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{24b^2}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(a\right.}{}$$

Mathematica [C] time = 8.95, size = 677, normalized size = 1.92

$$\frac{d\left(\sqrt{2\pi}\sqrt{\frac{b}{d}}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\sqrt{\frac{b}{d}}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\right.\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/S
qrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d
])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
+ 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*Sq
rt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) - (c*(-(Sqrt[2
*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - S
qrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]
 + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqrt[3]*b*Sqrt[
```

```
b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x
]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sqr
t[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*x
)] + 2*b*x*Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

fricas [A] time = 0.82, size = 298, normalized size = 0.84

$$\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt
(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(
b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-(b*c - a*d)/d) - sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt
(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(b*d*cos(b*x + a)^3 -
3*b*d*cos(b*x + a) - 2*(b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2)
*sin(b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 6.94, size = 1529, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/288*(12*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sqrt(2)*
sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
```

```
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c
)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/
d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(-2*I
*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - 4*(sqrt(6)*sqrt(pi)*(6*b*c
 + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*
sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*
x + c)*b + I*b*c - I*a*d)/d)/b + 6*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 386, normalized size = 1.09

$$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+c)^{\frac{3}{2}}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/8/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/8/b*d*(-1/2/b*d
*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(-
1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^
```

```
(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.61, size = 495, normalized size = 1.40

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{144\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+24\,\sqrt{dx+c}\,b^2\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\,\sqrt{dx+c}\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/d - 144*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d + 24*sqrt(d*x + c)*b^
2*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*cos(((d*x + c)
*b - b*c + a*d)/d) + ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(27*I - 27)*s
qrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (27*I + 27)*sqrt(
2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt
(I*b/d)) + ((27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*
d)/d) + (27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d
))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin^2(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x), x)
```

3.63 $\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=406

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(a-\frac{bc}{d}\right)S\left(\right.}{16b^{7/2}}$$

```
[Out] 5/8*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/72*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^2+
1/4*(d*x+c)^(5/2)*sin(b*x+a)/b-1/12*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-5/864*d^
(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/864*d^(5/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d
^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*
(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/16*d^2*sin(
b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.67, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{5\sqrt{\frac{\pi}{6}}d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(8*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a +
 3*b*x])/(72*b^2) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[
b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]
*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])
/(144*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/
2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(
16*b^(7/2)) - (15*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(16*b^3) + ((c + d*x)^(5/
2)*Sin[a + b*x])/(4*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(144*b^3) -
 ((c + d*x)^(5/2)*Sin[3*a + 3*b*x])/(12*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{4}(c+dx)^{5/2}\cos(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\cos(a+bx)\,dx - \frac{1}{4}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{4b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{12b} + \frac{(5d)\int (c+dx)^{3}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{(c+dx)^{5}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{8b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{72b^2} + \frac{15d^{5/2}\sqrt{\cdots}}{}$$

Mathematica [C] time = 13.63, size = 1171, normalized size = 2.88

$$-\frac{ie^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b} - \frac{\left(-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) - \sqrt{2\pi}\,C\left(\cdots\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/8*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])
/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))
/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]*
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)
/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*
b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(8*b^3) + ((b/d)^(3
```

```
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*
c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]*F
resnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (
4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b*(
c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(32*b^5) - (c
^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c
+ d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a
- (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(24*Sqr
t[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi
]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) +
 Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*C
os[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(
Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) - ((b/d)^(3/2
)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c
^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b
*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d
*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*Sin
[3*(a + b*x)])))/(288*Sqrt[3]*b^5)
```

fricas [A] time = 0.69, size = 370, normalized size = 0.91

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405\,\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-405}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) + 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(10*(b^2*d^2*x
+ b^2*c*d)*cos(b*x + a)^3 - 30*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (12*b^3
*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 35*b*d^2 - (12*b^3*d^2*x^2 + 24*b^3*
c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/
b^4
```

giac [C] time = 2.46, size = 2453, normalized size = 6.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/1728*(72*(sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) - 3*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*
x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((sqrt(6)*sqrt(pi)*(12*b^2*
c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 9*(sq
rt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)
*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d
*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)
/b^2)/d^2 - 9*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + (sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d
^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6
*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e
^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) - d^3*((sqrt(6)*sqrt(p
i)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2
*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x
+ c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3
*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b
^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b
*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(
d*x + c)^(3/2)*b^2*c*d - 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*
b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)
*b + I*b*c - I*a*d)/d)/b^3)/d^3 - 27*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^
2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 2*(4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)
*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqr
t(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*
```

a*d)/d)/b^3)/d^3 + (sqrt(6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*
d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^3) + 6*(-12*I*(d*x + c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 3
6*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b
*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)
/b^3)/d^3) - 36*(sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*sqrt(2)*sqrt(pi)
*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b
) + sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) - 6*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b
*c + 3*I*a*d)/d)/b + 18*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d
)/d)/b - 18*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*
I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d

maple [A] time = 0.00, size = 474, normalized size = 1.17

$$\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fresne}\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x)

[Out] 2/d*(1/8/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/8/b*d*(-1/2/b*d
*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1/2
)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin((
a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/2
4/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(-1/6/b*d*(d*

```
x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2)*
sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.60, size = 543, normalized size = 1.34

$$-\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-2160\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)+\left((5i+5)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,bd^2\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\,c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456*(240*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d) - 2160*
(d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d) + ((5*I + 5)*9^(1/4)*s
qrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(
1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt
(d*x + c)*sqrt(3*I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-(b*c - a*d)/d) + (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((405*I - 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (405*I + 405)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(-I*b/d)) + (-(5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1
/4)*cos(-3*(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d
^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + 24*(12
*(d*x + c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*sin(3*((d*x + c)*b - b*c +
a*d)/d) - 216*(4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*sin(((d*x
+ c)*b - b*c + a*d)/d))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.64 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S}{4096b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 1.05, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} - \frac{(5d)\int (c+}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{5d(c+dx}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c-}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c-}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c-}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c-}}{}$$

Mathematica [A] time = 14.25, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\,\cos(2(a+bx)) + 256b^3c^2\sqrt{c+dx}\,\cos(4(a+bx)) - 1024b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx)) + 25}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

fricas [A] time = 0.88, size = 406, normalized size = 1.00

$$15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 3.28, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16384*(512*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 + 24*c*d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*
I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(
d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4
```

I(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b
^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*
d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d
^2 + 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*
d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b
*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2
*I*b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 19
2*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2
 + 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b
+ 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*
I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
 c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^
2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b
- 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c
^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d +
48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*
b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)
/d)/b^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 1
5*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*
I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)
*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192
*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf
(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +

```
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*d*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(
d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c
)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*
I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.04, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\cdots\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.54, size = 547, normalized size = 1.34

$$-\frac{\left(1280\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-10240\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-15\sqrt{dx+c}\,\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 102
40*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
+ 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*
b - b*c + a*d)/d) + (-(480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^
2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
 ((30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d)
+ (30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))
*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((
480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.65 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{}}{}\right.}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.67, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,d$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int \sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{co}}{}$$

Mathematica [A] time = 3.08, size = 393, normalized size = 1.12

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

fricas [A] time = 0.58, size = 316, normalized size = 0.90

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 4.75, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(64*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(
-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c -
 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c
)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
- 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16
*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
```

```
sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*
b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c
- I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*
sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) - 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b +
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.04, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}+\frac{d(dx+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

maxima [C] time = 0.51, size = 503, normalized size = 1.43

$$\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{1024\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+768\sqrt{dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 1024
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x + c
)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 768*sqrt(d*x + c)*b^2*sin(2*((d*
x + c)*b - b*c + a*d)/d) - ((48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (
-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (6*
I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - (-(48*I - 48)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I + 48)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.66 $\int \sqrt{c+dx}\,\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\,\cdots}{\cdots}\right.}{64b^{3/2}}$$

```
[Out] -1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x
+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.50, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)-\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx\right)+\frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right.}{6}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Su}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}+\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}-\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a\right.}{}$$

Mathematica [A] time = 0.81, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{128}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

fricas [A] time = 0.81, size = 244, normalized size = 0.82

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-8\,\pi d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

giac [C] time = 2.83, size = 818, normalized size = 2.74

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}(8\,bc+i\,d)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4i\,bc-4i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{2}\sqrt{\pi}(8\,bc-i\,d)d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4i\,bc+4i\,ad}{d}\right)}}{\sqrt{bd}\left(-\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+8\left(-\frac{i\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(-I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

maple [A] time = 0.03, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.49, size = 425, normalized size = 1.42

$$\left(\frac{32\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{128\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left(-(8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(8i+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 128*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + (-(8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*I +
8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I
- 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3(a+bx)\cos(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

3.67 $\int \sqrt{c+dx}\,\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\ \cdots}{\cdots}\right.}{64b^{3/2}}$$

```
[Out] -1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelS(2*b^(1/2)*2^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^
(3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/
2))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x
+c)^(1/2)/b+1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.46, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) + (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) - (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) + (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$

$$= -\left(\frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right.}{6}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Su}}{}$$

$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b} + \frac{\sqrt{c+dx}\cos(4a+4bx)}{32b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a\right.}{}$$

Mathematica [A] time = 0.29, size = 264, normalized size = 0.88

$$\frac{-\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)+\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{128}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] - Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

fricas [A] time = 0.85, size = 244, normalized size = 0.82

$$-\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{C}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\,\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)-8\,\pi d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_si
n(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 8*pi*d*sq
rt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d
))) + 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin
(-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 16*b*cos(b*x + a)^2 + 5*b)*sqr
t(d*x + c))/b^2
```

giac [C] time = 1.07, size = 818, normalized size = 2.74

$$-\frac{\frac{i\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+8\left(-\frac{i\sqrt{\cdots}}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/256*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(-I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))+1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.48, size = 425, normalized size = 1.42

$$\left(\frac{32\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{128\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left(-(8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)-(8i+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 128*sqr
t(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + (-(8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((8*I +
8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I
- 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3{(a+bx)}\cos{(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x), x)
```

3.68 $\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{}}{}\right.}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b+3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2-3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.57, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) + ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) + (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) - (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,d$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{(3d)\int\sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{co}}{}$$

Mathematica [A] time = 2.37, size = 393, normalized size = 1.12

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + 3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] + 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
+ 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

fricas [A] time = 0.68, size = 316, normalized size = 0.90

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2
*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fr
esnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) - 4
8*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d))) - 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(
b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
+ 10*b^2*d*x + 10*b^2*c - 32*(b^2*d*x + b^2*c)*cos(b*x + a)^2 - 3*(2*b*d*co
s(b*x + a)^3 - 5*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 6.77, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(64*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(
-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c -
 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c
)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b^2*c^2
- 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16
*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
```

```
sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*
b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*
I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + 16*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a
*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c
- I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*
sqrt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) +
 8*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b) - 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b +
16*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(
d*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)
*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)S\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}+\frac{d(dx+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

maxima [C] time = 0.50, size = 503, normalized size = 1.43

$$\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{1024\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\,\sqrt{dx+c}\,b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+768\,\sqrt{dx+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d - 1024
*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x + c
)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) + 768*sqrt(d*x + c)*b^2*sin(2*((d*
x + c)*b - b*c + a*d)/d) - ((48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d
^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (
-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (6*
I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sq
rt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*
cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - (-(48*I - 48)*4^(1/4
)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I + 48)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.69 $\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{\frac{2}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\right.}{4096b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2+15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.70, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) - (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) + ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) + (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) - (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right) dx$$

$$= -\left(\frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx\right) + \frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,d$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} - \frac{(5d)\int (c+d}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} + \frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b} + \frac{5d(c+dx)^3}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

$$= \frac{15d^2\sqrt{c+dx}\cos(2a+2bx)}{128b^3} - \frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b} - \frac{15d^2\sqrt{c+}}{}$$

Mathematica [A] time = 9.97, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\cos(2(a+bx)) + 256b^3c^2\sqrt{c+dx}\cos(4(a+bx)) - 1024b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx)) + 256}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]*Sin[a + b*x]^3,x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] - 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] + 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] + 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] - 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] -
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

fricas [A] time = 0.91, size = 406, normalized size = 1.00

$$\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) -
 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 4*(320*b^3*d^2*x^2 + 640*b^3*c*d*x +
 320*b^3*c^2 + 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*x + 64*b^3*c^2 - 15*b*d^2)*c
os(b*x + a)^4 - 255*b*d^2 - 8*(128*b^3*d^2*x^2 + 256*b^3*c*d*x + 128*b^3*c^
2 - 75*b*d^2)*cos(b*x + a)^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 5*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 4.04, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/16384*(512*(-I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2
*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)))*c^3 + 24*c*d^2*((-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)
*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*
I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(-8*I*(
d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-4
```

```
*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (I*sqrt(2)*sqrt(pi)*(64*b
^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 4*I*(-8*I*(d*x + c)^(3/2)*b*d + 16*I*sqrt(d*x + c)*b*c*
d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d
^2 + 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*
x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*
d)/d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b
*d - 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2
*I*b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 19
2*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x +
c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d
/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2
 + 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b
+ 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*
I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)*b^3) - 4*I*(-64*I*(d*x + c)^(5/2)*b^2*d + 192*I*(d*x +
 c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^
2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b
- 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c
^2*d - 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) +
1)*b^3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d +
48*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*
b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)
/d)/b^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 1
5*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*
I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)
*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192
*(I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)*b) - I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf
(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c +
```

```
 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 4*sqrt(d*x + c)*d*e
^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(
d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c
)*b + 2*I*b*c - 2*I*a*d)/d)/b - 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*
I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.63, size = 547, normalized size = 1.34

$$-\frac{\left(1280\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-10240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) - 102
40*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c
)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d)
+ 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*
b - b*c + a*d)/d) + (-(480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2
)^(1/4)*cos(-2*(b*c - a*d)/d) - (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^
2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) +
 ((30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d)
+ (30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))
*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b
^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + ((
480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) + (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.70 $\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=267

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{8b}-$$

```
[Out] -1/8*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/
d)^m)-1/8*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+
c)/d)^m)-1/8*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x+c)
/d)/b/((-I*b*(d*x+c)/d)^m)-1/8*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+c)/d
)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.28, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+\right.}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(8*b*
(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(
8*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a -
(b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(8*b*(((-I)*b*(
c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/
d])/(8*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)]])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^m \sin(a+bx) + \frac{1}{4}(c+dx)^m \sin(3a+3bx)\right) dx$$
$$= \frac{1}{4}\int (c+dx)^m \sin(a+bx)\,dx + \frac{1}{4}\int (c+dx)^m \sin(3a+3bx)\,dx$$
$$= \frac{1}{8}i\int e^{-i(a+bx)}(c+dx)^m\,dx - \frac{1}{8}i\int e^{i(a+bx)}(c+dx)^m\,dx + \frac{1}{8}i\int e^{-i(3a+3bx}$$
$$= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{8b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)}{}$$

Mathematica [A] time = 0.46, size = 250, normalized size = 0.94

$$\frac{e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(3e^{\frac{2i(ad+bc)}{d}}\left(-e^{2ia}\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)-e^{\frac{2ibc}{d}}\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)\right)-3^{-m}}{24b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((c + d*x)^m*(3*E^(((2*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-I
)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1 +
 m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (E^((6*I)*a)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d] + E^(((6*I)*b*c)/d)*(((-I)*b*
(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*x))/d])/(3^m*((b^2*(c + d*x)^2
)/d^2)^m)))/(24*b*E^(((3*I)*(b*c + a*d))/d))
```

fricas [A] time = 0.68, size = 184, normalized size = 0.69

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right)+3e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}\Gamma\left(m+1,\frac{ibdx+ibc}{d}\right)+3e^{\left(-\frac{dm\log\left(-\frac{ib}{d}\right)+ibc-iad}{d}\right)}\Gamma\left(m+\right.}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/24*(e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m + 1, (3*I*b*d*
x + 3*I*b*c)/d) + 3*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)*gamma(m + 1, (I
*b*d*x + I*b*c)/d) + 3*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*d)/d)*gamma(m + 1
, (-I*b*d*x - I*b*c)/d) + e^(-(d*m*log(-3*I*b/d) + 3*I*b*c - 3*I*a*d)/d)*ga
mma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right) \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^m, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*sin(a + b*x)*cos(a + b*x)**2, x)
```

3.71 $\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=205

$$-\frac{8d^4\cos^3(a+bx)}{81b^5}-\frac{160d^4\cos(a+bx)}{27b^5}-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}-\frac{8d^3(c+dx)\sin(a+bx)\cos^2(a+bx)}{27b^4}+\frac{4d^2(c$$

[Out] -160/27*d^4*cos(b*x+a)/b^5+8/3*d^2*(d*x+c)^2*cos(b*x+a)/b^3-8/81*d^4*cos(b*x+a)^3/b^5+4/9*d^2*(d*x+c)^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^4*cos(b*x+a)^3/b-160/27*d^3*(d*x+c)*sin(b*x+a)/b^4+8/9*d*(d*x+c)^3*sin(b*x+a)/b^2-8/27*d^3*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^4+4/9*d*(d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)/b^2

Rubi [A] time = 0.20, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 3296, 2638, 3310}

$$-\frac{160d^3(c+dx)\sin(a+bx)}{27b^4}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^3(c+dx)\sin(a+bx)\cos^2}{27b^4}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]

[Out] (-160*d^4*Cos[a + b*x])/(27*b^5) + (8*d^2*(c + d*x)^2*Cos[a + b*x])/(3*b^3) - (8*d^4*Cos[a + b*x]^3)/(81*b^5) + (4*d^2*(c + d*x)^2*Cos[a + b*x]^3)/(9*b^3) - ((c + d*x)^4*Cos[a + b*x]^3)/(3*b) - (160*d^3*(c + d*x)*Sin[a + b*x])/(27*b^4) + (8*d*(c + d*x)^3*Sin[a + b*x])/(9*b^2) - (8*d^3*(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(27*b^4) + (4*d*(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
```

```
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^4\cos^3(a+bx)}{3b}+\frac{(4d)\int (c+dx)^3\cos^3(a+bx)\,dx}{3b}$$

$$= \frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^4\cos^3(a+bx)}{3b}+\frac{4d(c+dx)^3\cos^2(}{9}$$

$$= -\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d^2(c+dx)^2\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^4\cos^3(a+bx}{3b}$$

$$= \frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d^2(c+dx)^2\cos^3(a+b}{9b^3}$$

$$= -\frac{16d^4\cos(a+bx)}{27b^5}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+\frac{4d}{}$$

$$= -\frac{160d^4\cos(a+bx)}{27b^5}+\frac{8d^2(c+dx)^2\cos(a+bx)}{3b^3}-\frac{8d^4\cos^3(a+bx)}{81b^5}+$$

Mathematica [A] time = 1.55, size = 150, normalized size = 0.73

$$-\frac{-24bd(c+dx)\sin(a+bx)\left(\cos(2(a+bx))\left(3b^2(c+dx)^2-2d^2\right)+15b^2(c+dx)^2-82d^2\right)+81\cos(a+bx)\left(b^4(c\right.}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/324*(81*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x]
 + (8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)] -
 24*b*d*(c + d*x)*(-82*d^2 + 15*b^2*(c + d*x)^2 + (-2*d^2 + 3*b^2*(c + d*x)
^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/b^5
```

fricas [A] time = 0.49, size = 294, normalized size = 1.43

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+27\,b^4c^4-36\,b^2c^2d^2+8\,d^4+18\left(9\,b^4c^2d^2-2\,b^2d^4\right)x^2+36\left(3\,b^4c^3d-2\,b^2cd^3\right)x\right)\mathrm{cc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/81*((27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 - 36*b^2*c^2*d^2 +
8*d^4 + 18*(9*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d - 2*b^2*c*d^3)
*x)*cos(b*x + a)^3 - 24*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*d^2 - 2
0*d^4)*cos(b*x + a) - 12*(6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d -
40*b*c*d^3 + (3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d - 2*b*c*d^3 + (
9*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 + 2*(9*b^3*c^2*d^2 - 20*b*d^4)*x
)*sin(b*x + a))/b^5
```

giac [A] time = 0.25, size = 350, normalized size = 1.71

$$-\frac{\left(27\,b^4d^4x^4+108\,b^4cd^3x^3+162\,b^4c^2d^2x^2+108\,b^4c^3dx+27\,b^4c^4-36\,b^2d^4x^2-72\,b^2cd^3x-36\,b^2c^2d^2+8\,d^4\right)\mathrm{c}}{324\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/324*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*
c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8
*d^4)*cos(3*b*x + 3*a)/b^5 - 1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b
^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^5 + 1/27*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x
^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*sin(3*b*x + 3*a
)/b^5 + (b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d - 6*b*
d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

maple [B] time = 0.06, size = 835, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*cos(b*x+a)^3+4/9*(b*x+a)^3*(2+cos(b*x+a)^2)*
sin(b*x+a)+8/3*(b*x+a)^2*cos(b*x+a)-160/27*cos(b*x+a)-16/3*(b*x+a)*sin(b*x+
a)+4/9*(b*x+a)^2*cos(b*x+a)^3-8/27*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)-8/81
*cos(b*x+a)^3)-4/b^4*a*d^4*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+co
s(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*co
s(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b^3*c*d^3*(-1/3*(b*x+a)^3*co
s(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*
x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2/27*(2+cos(b*x+a)^2)*sin(b*x+a))+
6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin
(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*co
s(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*co
s(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x
+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*
x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+12/b^3*a^2*c*d^3*(-1/3*(
b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-12/b^2*a*c^2*d^2*(-1/3
*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+4/b*c^3*d*(-1/3*(b*x
+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^4*a^4*d^4*cos(b*x+a
)^3+4/3/b^3*a^3*c*d^3*cos(b*x+a)^3-2/b^2*a^2*c^2*d^2*cos(b*x+a)^3+4/3/b*a*c
^3*d*cos(b*x+a)^3-1/3*c^4*cos(b*x+a)^3)
```

maxima [B] time = 0.47, size = 889, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/324*(108*c^4*cos(b*x + a)^3 - 432*a*c^3*d*cos(b*x + a)^3/b + 648*a^2*c^2
*d^2*cos(b*x + a)^3/b^2 - 432*a^3*c*d^3*cos(b*x + a)^3/b^3 + 108*a^4*d^4*co
s(b*x + a)^3/b^4 + 36*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x +
 a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c^3*d/b - 108*(3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a
*c^2*d^2/b^2 + 108*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*c*d^3/b^3 - 36*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*a^3*d^4/b^4 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 -
 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a)
)*c^2*d^2/b^2 - 36*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
- 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a
))*a*c*d^3/b^3 + 18*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2
 - 2)*cos(b*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x +
```

```
a))*a^2*d^4/b^4 + 12*(3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27
*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x +
 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 12*(3*(3*(b*x + a)^3
 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x + a)^2 - 2)*sin(b*x +
 a))*a*d^4/b^4 + ((27*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) +
81*((b*x + a)^4 - 12*(b*x + a)^2 + 24)*cos(b*x + a) - 12*(3*(b*x + a)^3 - 2
*b*x - 2*a)*sin(3*b*x + 3*a) - 324*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a)
)*d^4/b^4)/b
```

mupad [B] time = 1.90, size = 448, normalized size = 2.19

$$\frac{4\,x\cos\left(a+b\,x\right)^3\,\left(14\,c\,d^3-3\,b^2\,c^3\,d\right)}{9\,b^3}-\frac{\cos\left(a+b\,x\right)^3\,\left(27\,b^4\,c^4-252\,b^2\,c^2\,d^2+488\,d^4\right)}{81\,b^5}-\frac{8\,\cos\left(a+b\,x\right)\,\sin\left(a+\right.}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (4*x*cos(a + b*x)^3*(14*c*d^3 - 3*b^2*c^3*d))/(9*b^3) - (cos(a + b*x)^3*(48
8*d^4 + 27*b^4*c^4 - 252*b^2*c^2*d^2))/(81*b^5) - (8*cos(a + b*x)*sin(a + b
*x)^2*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^5) - (4*cos(a + b*x)^2*sin(a + b*x)*(
14*c*d^3 - 3*b^2*c^3*d))/(9*b^4) - (d^4*x^4*cos(a + b*x)^3)/(3*b) - (8*sin(
a + b*x)^3*(20*c*d^3 - 3*b^2*c^3*d))/(27*b^4) + (8*d^4*x^3*sin(a + b*x)^3)/
(9*b^2) - (8*x*sin(a + b*x)^3*(20*d^4 - 9*b^2*c^2*d^2))/(27*b^4) + (2*x^2*c
os(a + b*x)^3*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^3) - (4*c*d^3*x^3*cos(a + b*x)
^3)/(3*b) + (4*d^4*x^3*cos(a + b*x)^2*sin(a + b*x))/(3*b^2) + (8*d^4*x^2*co
s(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (8*c*d^3*x^2*sin(a + b*x)^3)/(3*b^2) -
 (4*x*cos(a + b*x)^2*sin(a + b*x)*(14*d^4 - 9*b^2*c^2*d^2))/(9*b^4) + (4*c*
d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (16*c*d^3*x*cos(a + b*x)*sin(a +
 b*x)^2)/(3*b^3)
```

sympy [A] time = 7.19, size = 646, normalized size = 3.15

$$\begin{cases} -\frac{c^4\cos^3(a+bx)}{3b}-\frac{4c^3dx\cos^3(a+bx)}{3b}-\frac{2c^2d^2x^2\cos^3(a+bx)}{b}-\frac{4cd^3x^3\cos^3(a+bx)}{3b}-\frac{d^4x^4\cos^3(a+bx)}{3b}+\frac{8c^3d\sin^3(a+bx)}{9b^2}+\frac{4c^3d\sin(a+bx)}{3b^2} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**3/(3*b) - 4*c**3*d*x*cos(a + b*x)**3/(3*b) -
 2*c**2*d**2*x**2*cos(a + b*x)**3/b - 4*c*d**3*x**3*cos(a + b*x)**3/(3*b) -
 d**4*x**4*cos(a + b*x)**3/(3*b) + 8*c**3*d*sin(a + b*x)**3/(9*b**2) + 4*c*
```

```
*3*d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 8*c**2*d**2*x*sin(a + b*x)**3/
(3*b**2) + 4*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 + 8*c*d**3*x**2*
sin(a + b*x)**3/(3*b**2) + 4*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**2/b**2
+ 8*d**4*x**3*sin(a + b*x)**3/(9*b**2) + 4*d**4*x**3*sin(a + b*x)*cos(a + b
*x)**2/(3*b**2) + 8*c**2*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 28*c*
*2*d**2*cos(a + b*x)**3/(9*b**3) + 16*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)
/(3*b**3) + 56*c*d**3*x*cos(a + b*x)**3/(9*b**3) + 8*d**4*x**2*sin(a + b*x)
**2*cos(a + b*x)/(3*b**3) + 28*d**4*x**2*cos(a + b*x)**3/(9*b**3) - 160*c*d
**3*sin(a + b*x)**3/(27*b**4) - 56*c*d**3*sin(a + b*x)*cos(a + b*x)**2/(9*b
**4) - 160*d**4*x*sin(a + b*x)**3/(27*b**4) - 56*d**4*x*sin(a + b*x)*cos(a
+ b*x)**2/(9*b**4) - 160*d**4*sin(a + b*x)**2*cos(a + b*x)/(27*b**5) - 488*
d**4*cos(a + b*x)**3/(81*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**
2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**2, True))
```

3.72 $\int (c+dx)^3 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=151

$$\frac{2d^3\sin^3(a+bx)}{27b^4}-\frac{14d^3\sin(a+bx)}{9b^4}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}$$

```
[Out] 4/3*d^2*(d*x+c)*cos(b*x+a)/b^3+2/9*d^2*(d*x+c)*cos(b*x+a)^3/b^3-1/3*(d*x+c)
^3*cos(b*x+a)^3/b-14/9*d^3*sin(b*x+a)/b^4+2/3*d*(d*x+c)^2*sin(b*x+a)/b^2+1/
3*d*(d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)/b^2+2/27*d^3*sin(b*x+a)^3/b^4
```

Rubi [A] time = 0.13, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 3296, 2637, 2633}

$$\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}+\frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d(c+dx)^2\sin(a+bx)}{3b^2}+\frac{d(c+dx)^2\sin(a+bx)\cos^2(a+bx)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*(c + d*x)*Cos[a + b*x])/(3*b^3) + (2*d^2*(c + d*x)*Cos[a + b*x]^3)/(
9*b^3) - ((c + d*x)^3*Cos[a + b*x]^3)/(3*b) - (14*d^3*Sin[a + b*x])/(9*b^4)
 + (2*d*(c + d*x)^2*Sin[a + b*x])/(3*b^2) + (d*(c + d*x)^2*Cos[a + b*x]^2*S
in[a + b*x])/(3*b^2) + (2*d^3*Sin[a + b*x]^3)/(27*b^4)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d\int (c+dx)^2\cos^3(a+bx)\,dx}{b}$$
$$= \frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{d(c+dx)^2\cos^2(a}{3b}$$
$$= \frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a+bx)}{3b}+\frac{2d(c+dx)^2\sin(a}{3b^2}$$
$$= \frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a}{3b}$$
$$= \frac{4d^2(c+dx)\cos(a+bx)}{3b^3}+\frac{2d^2(c+dx)\cos^3(a+bx)}{9b^3}-\frac{(c+dx)^3\cos^3(a}{3b}$$

Mathematica [A] time = 0.89, size = 127, normalized size = 0.84

$$\frac{-27b(c+dx)\cos(a+bx)\left(b^2(c+dx)^2-6d^2\right)-3b(c+dx)\cos(3(a+bx))\left(3b^2(c+dx)^2-2d^2\right)+2d\sin(a+bx)\left(c\right.}{108b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (-27*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 3*b*(c + d*x)*(-
2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 2*d*(-82*d^2 + 45*b^2*(c + d*
x)^2 + (-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[a + b*x])/(108*b^
4)
```

fricas [A] time = 0.70, size = 183, normalized size = 1.21

$$-\frac{3\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+3\,b^3c^3-2\,bcd^2+\left(9\,b^3c^2d-2\,bd^3\right)x\right)\cos\left(bx+a\right)^3-36\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)-}{27}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*(3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 - 2*b*c*d^2 + (9*b^3*
c^2*d - 2*b*d^3)*x)*cos(b*x + a)^3 - 36*(b*d^3*x + b*c*d^2)*cos(b*x + a) -
(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - 40*d^3 + (9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

giac [A] time = 1.91, size = 231, normalized size = 1.53

$$-\frac{\left(3\,b^3d^3x^3+9\,b^3cd^2x^2+9\,b^3c^2dx+3\,b^3c^3-2\,bd^3x-2\,bcd^2\right)\cos\left(3\,bx+3\,a\right)}{36\,b^4}-\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/36*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^
3*x - 2*b*c*d^2)*cos(3*b*x + 3*a)/b^4 - 1/4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
+ 3*b^3*c^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 + 1/108
*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^
4 + 3/4*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

maple [B] time = 0.01, size = 447, normalized size = 2.96

$$\frac{\frac{d^3\left(-\frac{(bx+a)^3\left(\cos^3(bx+a)\right)}{3}+\frac{(bx+a)^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}-\frac{4\sin(bx+a)}{3}+\frac{4(bx+a)\cos(bx+a)}{3}+\frac{2(bx+a)\left(\cos^3(bx+a)\right)}{9}-\frac{2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{27}\right)}{b^3}-\frac{3a\,d^3\left(-\frac{(bx+a)^2(}{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*cos(b*x+a)^3+1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*
sin(b*x+a)-4/3*sin(b*x+a)+4/3*(b*x+a)*cos(b*x+a)+2/9*(b*x+a)*cos(b*x+a)^3-2
/27*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b^3*a*d^3*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2
/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/
b^2*c*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x
+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*cos(b*x+a
)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-6/b^2*a*c*d^2*(-1/3*(b*x+a)*cos(b*x+a)
```

```
^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b*c^2*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/
9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/b^3*a^3*d^3*cos(b*x+a)^3-1/b^2*a^2*c*d^2
*cos(b*x+a)^3+1/b*a*c^2*d*cos(b*x+a)^3-1/3*c^3*cos(b*x+a)^3)
```

maxima [B] time = 0.37, size = 505, normalized size = 3.34

$$-\frac{36\,c^3\cos\left(bx+a\right)^3-\frac{108\,ac^2d\cos(bx+a)^3}{b}+\frac{108\,a^2cd^2\cos(bx+a)^3}{b^2}-\frac{36\,a^3d^3\cos(bx+a)^3}{b^3}+\frac{9\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^3*cos(b*x + a)^3 - 108*a*c^2*d*cos(b*x + a)^3/b + 108*a^2*c*d^
2*cos(b*x + a)^3/b^2 - 36*a^3*d^3*cos(b*x + a)^3/b^3 + 9*(3*(b*x + a)*cos(3
*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))
*c^2*d/b - 18*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - si
n(3*b*x + 3*a) - 9*sin(b*x + a))*a*c*d^2/b^2 + 9*(3*(b*x + a)*cos(3*b*x + 3
*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a^2*d^3
/b^3 + 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b
*x + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*c*d^2/b
^2 - 3*((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x
 + a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*a*d^3/b^3
 + (3*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) + 27*((b*x + a)^3 - 6*
b*x - 6*a)*cos(b*x + a) - (9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 81*((b*x +
 a)^2 - 2)*sin(b*x + a))*d^3/b^3)/b
```

mupad [B] time = 1.34, size = 290, normalized size = 1.92

$$\frac{\cos\left(a+b\,x\right)^3\,\left(14\,c\,d^2-3\,b^2\,c^3\right)}{9\,b^3}-\frac{2\sin\left(a+b\,x\right)^3\,\left(20\,d^3-9\,b^2\,c^2\,d\right)}{27\,b^4}-\frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(14\,d^3-9\,b^2\,c^2\,d\right.}{9\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (cos(a + b*x)^3*(14*c*d^2 - 3*b^2*c^3))/(9*b^3) - (2*sin(a + b*x)^3*(20*d^3
 - 9*b^2*c^2*d))/(27*b^4) - (cos(a + b*x)^2*sin(a + b*x)*(14*d^3 - 9*b^2*c^
2*d))/(9*b^4) + (x*cos(a + b*x)^3*(14*d^3 - 9*b^2*c^2*d))/(9*b^3) - (d^3*x^
3*cos(a + b*x)^3)/(3*b) + (2*d^3*x^2*sin(a + b*x)^3)/(3*b^2) + (4*c*d^2*cos
(a + b*x)*sin(a + b*x)^2)/(3*b^3) + (4*d^3*x*cos(a + b*x)*sin(a + b*x)^2)/(
3*b^3) + (4*c*d^2*x*sin(a + b*x)^3)/(3*b^2) - (c*d^2*x^2*cos(a + b*x)^3)/b
+ (d^3*x^2*cos(a + b*x)^2*sin(a + b*x))/b^2 + (2*c*d^2*x*cos(a + b*x)^2*sin
(a + b*x))/b^2
```

sympy [A] time = 3.95, size = 391, normalized size = 2.59

$$\begin{cases} -\frac{c^3\cos^3(a+bx)}{3b} - \frac{c^2dx\cos^3(a+bx)}{b} - \frac{cd^2x^2\cos^3(a+bx)}{b} - \frac{d^3x^3\cos^3(a+bx)}{3b} + \frac{2c^2d\sin^3(a+bx)}{3b^2} + \frac{c^2d\sin(a+bx)\cos^2(a+bx)}{b^2} + \frac{4cd^2x\sin^3(}{3b^2} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**3/(3*b) - c**2*d*x*cos(a + b*x)**3/b - c*d**
2*x**2*cos(a + b*x)**3/b - d**3*x**3*cos(a + b*x)**3/(3*b) + 2*c**2*d*sin(a
 + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**2/b**2 + 4*c*d**2*x
*sin(a + b*x)**3/(3*b**2) + 2*c*d**2*x*sin(a + b*x)*cos(a + b*x)**2/b**2 +
2*d**3*x**2*sin(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)*
*2/b**2 + 4*c*d**2*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*c*d**2*cos(a
+ b*x)**3/(9*b**3) + 4*d**3*x*sin(a + b*x)**2*cos(a + b*x)/(3*b**3) + 14*d*
*3*x*cos(a + b*x)**3/(9*b**3) - 40*d**3*sin(a + b*x)**3/(27*b**4) - 14*d**3
*sin(a + b*x)*cos(a + b*x)**2/(9*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2
/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a)**2, True))
```

3.73 $\int (c+dx)^2 \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}+\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}-\frac{(c+dx)^2\cos^3(}{3b}$$

```
[Out] 4/9*d^2*cos(b*x+a)/b^3+2/27*d^2*cos(b*x+a)^3/b^3-1/3*(d*x+c)^2*cos(b*x+a)^3
/b+4/9*d*(d*x+c)*sin(b*x+a)/b^2+2/9*d*(d*x+c)*cos(b*x+a)^2*sin(b*x+a)/b^2
```

Rubi [A] time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4405, 3310, 3296, 2638}

$$\frac{4d(c+dx)\sin(a+bx)}{9b^2}+\frac{2d(c+dx)\sin(a+bx)\cos^2(a+bx)}{9b^2}+\frac{2d^2\cos^3(a+bx)}{27b^3}+\frac{4d^2\cos(a+bx)}{9b^3}-\frac{(c+dx)^2\cos^3(}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (4*d^2*Cos[a + b*x])/(9*b^3) + (2*d^2*Cos[a + b*x]^3)/(27*b^3) - ((c + d*x)
^2*Cos[a + b*x]^3)/(3*b) + (4*d*(c + d*x)*Sin[a + b*x])/(9*b^2) + (2*d*(c +
 d*x)*Cos[a + b*x]^2*Sin[a + b*x])/(9*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{(2d)\int (c+dx)\cos^3(a+bx)\,dx}{3b}$$
$$=\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{2d(c+dx)\cos^2(a+bx)\,\text{s}}{9b^2}$$
$$=\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{4d(c+dx)\sin(a+bx)}{9b^2}+$$
$$=\frac{4d^2\cos(a+bx)}{9b^3}+\frac{2d^2\cos^3(a+bx)}{27b^3}-\frac{(c+dx)^2\cos^3(a+bx)}{3b}+\frac{4d(c\,}{}$$

Mathematica [A] time = 0.49, size = 86, normalized size = 0.83

$$-\frac{27\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)+\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)-6bd(c+dx)(9\sin(a+bx)+\sin(3(a}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/108*(27*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + (-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] - 6*b*d*(c + d*x)*(9*Sin[a + b*x] + Sin[3*(a + b*x)
]))/b^3
```

fricas [A] time = 0.72, size = 100, normalized size = 0.97

$$-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^3-12\,d^2\cos\left(bx+a\right)-6\left(2\,bd^2x+2\,bcd+\left(bd^2x+bcd\right)\cos}{27\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/27*((9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(b*x + a)^3 -
12*d^2*cos(b*x + a) - 6*(2*b*d^2*x + 2*b*c*d + (b*d^2*x + b*c*d)*cos(b*x +
a)^2)*sin(b*x + a))/b^3
```

giac [A] time = 4.95, size = 137, normalized size = 1.33

$$-\frac{\left(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-2\,d^2\right)\cos\left(3\,bx+3\,a\right)}{108\,b^3}-\frac{\left(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,d^2\right)\cos\left(bx+a\right)}{4\,b^3}+\frac{\left(bd^2x+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/108*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x + 3*a)/
b^3 - 1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)/b^3 +
1/18*(b*d^2*x + b*c*d)*sin(3*b*x + 3*a)/b^3 + 1/2*(b*d^2*x + b*c*d)*sin(b*x
 + a)/b^3
```

maple [B] time = 0.01, size = 204, normalized size = 1.98

$$\frac{\frac{d^2\left(-\frac{(bx+a)^2\left(\cos^3(bx+a)\right)}{3}+\frac{2(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}+\frac{2\left(\cos^3(bx+a)\right)}{27}+\frac{4\cos(bx+a)}{9}\right)}{b^2}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^3(bx+a)\right)}{3}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}\right)}{b^2}+\frac{2cd\left(-\frac{(bx+}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*cos(b*x+a)^3+2/9*(b*x+a)*(2+cos(b*x+a)^2)*si
n(b*x+a)+2/27*cos(b*x+a)^3+4/9*cos(b*x+a))-2/b^2*a*d^2*(-1/3*(b*x+a)*cos(b*
x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+2/b*c*d*(-1/3*(b*x+a)*cos(b*x+a)^3+
1/9*(2+cos(b*x+a)^2)*sin(b*x+a))-1/3/b^2*a^2*d^2*cos(b*x+a)^3+2/3/b*a*c*d*c
os(b*x+a)^3-1/3*c^2*cos(b*x+a)^3)
```

maxima [B] time = 0.37, size = 243, normalized size = 2.36

$$-\frac{36\,c^2\cos\left(bx+a\right)^3-\frac{72\,acd\cos(bx+a)^3}{b}+\frac{36\,a^2d^2\cos(bx+a)^3}{b^2}+\frac{6\,(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/108*(36*c^2*cos(b*x + a)^3 - 72*a*c*d*cos(b*x + a)^3/b + 36*a^2*d^2*cos(
b*x + a)^3/b^2 + 6*(3*(b*x + a)*cos(3*b*x + 3*a) + 9*(b*x + a)*cos(b*x + a)
 - sin(3*b*x + 3*a) - 9*sin(b*x + a))*c*d/b - 6*(3*(b*x + a)*cos(3*b*x + 3*
a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*a*d^2/b^
2 + ((9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) + 27*((b*x + a)^2 - 2)*cos(b*x +
a) - 6*(b*x + a)*sin(3*b*x + 3*a) - 54*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

mupad [B] time = 1.13, size = 145, normalized size = 1.41

$$\frac{12\,d^2\,\cos\left(a+b\,x\right)+2\,d^2\cos\left(a+b\,x\right)^3-9\,b^2\,c^2\cos\left(a+b\,x\right)^3+12\,b\,d^2\,x\,\sin\left(a+b\,x\right)-9\,b^2\,d^2\,x^2\cos\left(a+b\,x\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (12*d^2*cos(a + b*x) + 2*d^2*cos(a + b*x)^3 - 9*b^2*c^2*cos(a + b*x)^3 + 12
*b*d^2*x*sin(a + b*x) - 9*b^2*d^2*x^2*cos(a + b*x)^3 + 12*b*c*d*sin(a + b*x
) - 18*b^2*c*d*x*cos(a + b*x)^3 + 6*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) + 6
*b*c*d*cos(a + b*x)^2*sin(a + b*x))/(27*b^3)
```

sympy [A] time = 2.03, size = 216, normalized size = 2.10

$$\begin{cases} -\frac{c^2\cos^3(a+bx)}{3b}-\frac{2cdx\cos^3(a+bx)}{3b}-\frac{d^2x^2\cos^3(a+bx)}{3b}+\frac{4cd\sin^3(a+bx)}{9b^2}+\frac{2cd\sin(a+bx)\cos^2(a+bx)}{3b^2}+\frac{4d^2x\sin^3(a+bx)}{9b^2}+\frac{2d^2x\sin(a+b}{3} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**3/(3*b) - 2*c*d*x*cos(a + b*x)**3/(3*b) - d*
*2*x**2*cos(a + b*x)**3/(3*b) + 4*c*d*sin(a + b*x)**3/(9*b**2) + 2*c*d*sin(
a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*x*sin(a + b*x)**3/(9*b**2) + 2*d
**2*x*sin(a + b*x)*cos(a + b*x)**2/(3*b**2) + 4*d**2*sin(a + b*x)**2*cos(a
+ b*x)/(9*b**3) + 14*d**2*cos(a + b*x)**3/(27*b**3), Ne(b, 0)), ((c**2*x +
c*d*x**2 + d**2*x**3/3)*sin(a)*cos(a)**2, True))
```

3.74 $\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=51

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

```
[Out] -1/3*(d*x+c)*cos(b*x+a)^3/b+1/3*d*sin(b*x+a)/b^2-1/9*d*sin(b*x+a)^3/b^2
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4405, 2633}

$$-\frac{d\sin^3(a+bx)}{9b^2}+\frac{d\sin(a+bx)}{3b^2}-\frac{(c+dx)\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)*Cos[a + b*x]^3)/(3*b) + (d*Sin[a + b*x])/(3*b^2) - (d*Sin[a + b
*x]^3)/(9*b^2)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cos^2(a+bx)\sin(a+bx)\,dx &= -\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\int \cos^3(a+bx)\,dx}{3b}\\ &= -\frac{(c+dx)\cos^3(a+bx)}{3b}-\frac{d\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(a+bx)\right)}{3b^2}\\ &= -\frac{(c+dx)\cos^3(a+bx)}{3b}+\frac{d\sin(a+bx)}{3b^2}-\frac{d\sin^3(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.15, size = 71, normalized size = 1.39

$$\frac{d(\sin(a+bx)-bx\cos(a+bx))}{4b^2}+\frac{d(\sin(3(a+bx))-3bx\cos(3(a+bx)))}{36b^2}-\frac{c\cos^3(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -1/3*(c*Cos[a + b*x]^3)/b + (d*(-(b*x*Cos[a + b*x]) + Sin[a + b*x]))/(4*b^2
) + (d*(-3*b*x*Cos[3*(a + b*x)] + Sin[3*(a + b*x)]))/(36*b^2)
```

fricas [A] time = 0.68, size = 46, normalized size = 0.90

$$-\frac{3\,(bdx+bc)\cos\,(bx+a)^3-\left(d\cos\,(bx+a)^2+2\,d\right)\sin\,(bx+a)}{9\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/9*(3*(b*d*x + b*c)*cos(b*x + a)^3 - (d*cos(b*x + a)^2 + 2*d)*sin(b*x + a
))/b^2
```

giac [A] time = 0.17, size = 69, normalized size = 1.35

$$-\frac{(bdx+bc)\cos\,(3\,bx+3\,a)}{12\,b^2}-\frac{(bdx+bc)\cos\,(bx+a)}{4\,b^2}+\frac{d\sin\,(3\,bx+3\,a)}{36\,b^2}+\frac{d\sin\,(bx+a)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/12*(b*d*x + b*c)*cos(3*b*x + 3*a)/b^2 - 1/4*(b*d*x + b*c)*cos(b*x + a)/b
^2 + 1/36*d*sin(3*b*x + 3*a)/b^2 + 1/4*d*sin(b*x + a)/b^2
```

maple [A] time = 0.01, size = 71, normalized size = 1.39

$$\frac{\frac{d\left(-\frac{(bx+a)\left(\cos^3(bx+a)\right)}{3}+\frac{\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{9}\right)}{b}+\frac{da\left(\cos^3(bx+a)\right)}{3b}-\frac{c\left(\cos^3(bx+a)\right)}{3}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(-1/3*(b*x+a)*cos(b*x+a)^3+1/9*(2+cos(b*x+a)^2)*sin(b*x+a))+1/3/
b*d*a*cos(b*x+a)^3-1/3*c*cos(b*x+a)^3)
```

maxima [A] time = 0.39, size = 86, normalized size = 1.69

$$-\frac{12\,c\cos\left(bx+a\right)^{3}-\frac{12\,ad\cos(bx+a)^{3}}{b}+\frac{(3\,(bx+a)\cos(3\,bx+3\,a)+9\,(bx+a)\cos(bx+a)-\sin(3\,bx+3\,a)-9\,\sin(bx+a))d}{b}}{36\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/36*(12*c*cos(b*x + a)^3 - 12*a*d*cos(b*x + a)^3/b + (3*(b*x + a)*cos(3*b
*x + 3*a) + 9*(b*x + a)*cos(b*x + a) - sin(3*b*x + 3*a) - 9*sin(b*x + a))*d
/b)/b
```

mupad [B] time = 0.95, size = 58, normalized size = 1.14

$$\frac{\frac{2\,d\,\sin(a+b\,x)}{9}-b\left(\frac{c\cos(a+b\,x)^{3}}{3}+\frac{d\,x\cos(a+b\,x)^{3}}{3}\right)+\frac{d\cos(a+b\,x)^{2}\,\sin(a+b\,x)}{9}}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x),x)
```

```
[Out] ((2*d*sin(a + b*x))/9 - b*((c*cos(a + b*x)^3)/3 + (d*x*cos(a + b*x)^3)/3) +
 (d*cos(a + b*x)^2*sin(a + b*x))/9)/b^2
```

sympy [A] time = 0.87, size = 85, normalized size = 1.67

$$\begin{cases} -\frac{c\cos^{3}\left(a+bx\right)}{3b}-\frac{dx\cos^{3}\left(a+bx\right)}{3b}+\frac{2d\sin^{3}\left(a+bx\right)}{9b^{2}}+\frac{d\sin\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b^{2}} & \text{for}\ b\neq 0 \\ \left(cx+\frac{dx^{2}}{2}\right)\sin\left(a\right)\cos^{2}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**3/(3*b) - d*x*cos(a + b*x)**3/(3*b) + 2*d*sin(a
 + b*x)**3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**2/(3*b**2), Ne(b, 0)), (
(c*x + d*x**2/2)*sin(a)*cos(a)**2, True))
```

3.75 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

```
[Out] 1/4*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/4*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d+1
/4*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cos\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(4*d) + (CosIntegral[
(b*c)/d + b*x]*Sin[a - (b*c)/d])/(4*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c
)/d + b*x])/(4*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(
4*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)}+\frac{\sin(3a+3bx)}{4(c+dx)}\right)dx\\ &= \frac{1}{4}\int \frac{\sin(a+bx)}{c+dx}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{c+dx}\,dx\\ &= \frac{1}{4}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx+\frac{1}{4}\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\\ &= \frac{\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{4d}+\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{4d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}\right.}{4d}\end{aligned}$$

Mathematica [A] time = 0.30, size = 100, normalized size = 0.83

$$\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + CosIntegral[b*(c/d +
 x)]*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/(4*d)
```

fricas [A] time = 0.62, size = 152, normalized size = 1.26

$$\frac{\left(\text{Ci}\left(\frac{bdx+bc}{d}\right)+\text{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\text{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)+2\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*((cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*sin(-
(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d*x
+ b*c)/d))*sin(-3*(b*c - a*d)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral(3*(
b*d*x + b*c)/d) + 2*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/d
```

giac [C] time = 3.83, size = 6279, normalized size = 51.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan
(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_inte
gral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3
/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 +
 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b
*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(
3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2
 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_int
egral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*ta
n(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - i
mag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3
/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/
2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*ta
n(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*
tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(
3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 4*imag_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 8*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(
1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*ta
n(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral(-3*b*x -
```

```
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*
d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((
b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 4*imag_part(co
s_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2
*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x + b*c)/d)*ta
n(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + imag_part(cos_integ
ral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/
d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*
tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*t
an(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/
2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*
tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(c
os_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 -
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2
*b*c/d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*
tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2
*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x +
3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_part(cos_integr
al(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d) + 2*real_par
t(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2*r
eal_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(3/2*b*c/d)
^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan
(3/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(
1/2*a)^2*tan(3/2*b*c/d)^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*
a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(-b*x - b*c/d))*
tan(3/2*a)^2*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b
*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integ
ral(-b*x - b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_pa
rt(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)
+ 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) - 2*real_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a
)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*t
an(1/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan
(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x - 3
*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)*tan(1/2*b*c
/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c
/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*
a)^2*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b
*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*real_part(cos_integ
```

```
ral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*rea
l_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan
(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2
+ imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 + imag_pa
rt(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*a)^2 - 2*sin_integr
al(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 - 2*sin_integral((b*d*x + b
*c)/d)*tan(3/2*a)^2*tan(1/2*a)^2 + 4*imag_part(cos_integral(3*b*x + 3*b*c/d
))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) - 4*imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + 8*sin_integral(3*(b*d
*x + b*c)/d)*tan(3/2*a)*tan(1/2*a)^2*tan(3/2*b*c/d) + imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + imag_part(cos_integral(
b*x + b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-b*x -
 b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x - 3*
b*c/d))*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)*t
an(3/2*a)^2*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2
*tan(3/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*t
an(3/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)^2*tan(3/2
*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/
d)^2 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b*c/d
)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 - 2*s
in_integral((b*d*x + b*c)/d)*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + 4*imag_part(co
s_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) - 4*imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 8*s
in_integral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*a)*tan(1/2*b*c/d) + 4*ima
g_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d
) - 4*imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d) + 8*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)*tan(3/2*b*c/d)^2*t
an(1/2*b*c/d) - imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(1
/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c
/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
 + imag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d)^2
- 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 - 2*sin_i
ntegral((b*d*x + b*c)/d)*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_inte
gral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integr
al(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-b*
x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral(3*(b*d*x + b*c)/d
)*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a
)^2*tan(1/2*b*c/d)^2 + 4*imag_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a
)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 4*imag_part(cos_integral(-3*b*x - 3*b*c
/d))*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 + 8*sin_integral(3*(b*d*x +
 b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integra
l(3*b*x + 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - imag_part(cos_integ
```

ral(b*x + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integra
l(-b*x - b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + imag_part(cos_integral
(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x +
b*c)/d)*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(b*x +
b*c/d))*tan(3/2*a)^2*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*t
an(3/2*a)^2*tan(1/2*a) + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2
*a)*tan(1/2*a)^2 + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*t
an(1/2*a)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)^2*tan(3
/2*b*c/d) + 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)^2*tan(3/
2*b*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(1/2*a)^2*tan(3/2*
b*c/d) - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2*tan(3/2*b
*c/d) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d
)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d)
^2 + 2*real_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 + 2
*real_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(3/2*b*c/d)^2 - 2*real
_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) - 2*real_part(
cos_integral(-b*x - b*c/d))*tan(3/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_i
ntegral(b*x + b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) + 2*real_part(cos_integra
l(-b*x - b*c/d))*tan(1/2*a)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(b*x
 + b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x
- b*c/d))*tan(3/2*b*c/d)^2*tan(1/2*b*c/d) + 2*real_part(cos_integral(3*b*x
+ 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 + 2*real_part(cos_integral(-3*b*x -
 3*b*c/d))*tan(3/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(b*x + b*c
/d))*tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-b*x - b*c/d))*
tan(1/2*a)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*ta
n(3/2*b*c/d)*tan(1/2*b*c/d)^2 - 2*real_part(cos_integral(-3*b*x - 3*b*c/d))
*tan(3/2*b*c/d)*tan(1/2*b*c/d)^2 - imag_part(cos_integral(3*b*x + 3*b*c/d))
*tan(3/2*a)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*a)^2 - imag_pa
rt(cos_integral(-b*x - b*c/d))*tan(3/2*a)^2 + imag_part(cos_integral(-3*b*x
 - 3*b*c/d))*tan(3/2*a)^2 - 2*sin_integral(3*(b*d*x + b*c)/d)*tan(3/2*a)^2
+ 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*a)^2 + imag_part(cos_integral(3*b
*x + 3*b*c/d))*tan(1/2*a)^2 - imag_part(cos_integral(b*x + b*c/d))*tan(1/2*
a)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)^2 - imag_part(cos_i
ntegral(-3*b*x - 3*b*c/d))*tan(1/2*a)^2 + 2*sin_integral(3*(b*d*x + b*c)/d)
*tan(1/2*a)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1/2*a)^2 + 4*imag_part(
cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) - 4*imag_part(cos_
integral(-3*b*x - 3*b*c/d))*tan(3/2*a)*tan(3/2*b*c/d) + 8*sin_integral(3*(b
*d*x + b*c)/d)*tan(3/2*a)*tan(3/2*b*c/d) - imag_part(cos_integral(3*b*x + 3
*b*c/d))*tan(3/2*b*c/d)^2 + imag_part(cos_integral(b*x + b*c/d))*tan(3/2*b*
c/d)^2 - imag_part(cos_integral(-b*x - b*c/d))*tan(3/2*b*c/d)^2 + imag_part
(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d)^2 - 2*sin_integral(3*(b*d*x
 + b*c)/d)*tan(3/2*b*c/d)^2 + 2*sin_integral((b*d*x + b*c)/d)*tan(3/2*b*c/d
)^2 + 4*imag_part(cos_integral(b*x + b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) - 4*
imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*a)*tan(1/2*b*c/d) + 8*sin_int

```
egral((b*d*x + b*c)/d)*tan(1/2*a)*tan(1/2*b*c/d) + imag_part(cos_integral(3
*b*x + 3*b*c/d))*tan(1/2*b*c/d)^2 - imag_part(cos_integral(b*x + b*c/d))*ta
n(1/2*b*c/d)^2 + imag_part(cos_integral(-b*x - b*c/d))*tan(1/2*b*c/d)^2 - i
mag_part(cos_integral(-3*b*x - 3*b*c/d))*tan(1/2*b*c/d)^2 + 2*sin_integral(
3*(b*d*x + b*c)/d)*tan(1/2*b*c/d)^2 - 2*sin_integral((b*d*x + b*c)/d)*tan(1
/2*b*c/d)^2 + 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*a) + 2*rea
l_part(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*a) + 2*real_part(cos_integra
l(b*x + b*c/d))*tan(1/2*a) + 2*real_part(cos_integral(-b*x - b*c/d))*tan(1/
2*a) - 2*real_part(cos_integral(3*b*x + 3*b*c/d))*tan(3/2*b*c/d) - 2*real_p
art(cos_integral(-3*b*x - 3*b*c/d))*tan(3/2*b*c/d) - 2*real_part(cos_integr
al(b*x + b*c/d))*tan(1/2*b*c/d) - 2*real_part(cos_integral(-b*x - b*c/d))*t
an(1/2*b*c/d) + imag_part(cos_integral(3*b*x + 3*b*c/d)) + imag_part(cos_in
tegral(b*x + b*c/d)) - imag_part(cos_integral(-b*x - b*c/d)) - imag_part(co
s_integral(-3*b*x - 3*b*c/d)) + 2*sin_integral(3*(b*d*x + b*c)/d) + 2*sin_i
ntegral((b*d*x + b*c)/d))/(d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan
(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(3/2*a)
^2*tan(1/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*
b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^
2*tan(1/2*a)^2 + d*tan(3/2*a)^2*tan(3/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(3/2*b
*c/d)^2 + d*tan(3/2*a)^2*tan(1/2*b*c/d)^2 + d*tan(1/2*a)^2*tan(1/2*b*c/d)^2
 + d*tan(3/2*b*c/d)^2*tan(1/2*b*c/d)^2 + d*tan(3/2*a)^2 + d*tan(1/2*a)^2 +
d*tan(3/2*b*c/d)^2 + d*tan(1/2*b*c/d)^2 + d)
```

maple [A] time = 0.01, size = 167, normalized size = 1.38

$$\frac{\frac{b\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{12}+\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/b*(1/12*b*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*
x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)+1/4*b*(Si(b*x+a+(-a*d+b*c)/d)*
cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d))
```

maxima [C] time = 0.43, size = 273, normalized size = 2.26

$$-\frac{b\left(i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(i\,E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_1\left(-\frac{3i\,bc+3i\,(bx+}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*(b*(I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_int
egral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b*(I*
exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral
_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) + b*
(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_e(1, -
(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(exp_integral_e
(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1, -(3*I*b*c
+ 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(a+b\,x\right)^2\,\sin \left(a+b\,x\right)}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(a + bx \right)} \cos^{2}{\left(a + bx \right)}}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x), x)
```

3.76 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=168

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

```
[Out] 3/4*b*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^2+1/4*b*Ci(b*c/d+b*x)*cos(a-b*c/
d)/d^2-3/4*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/4*b*Si(b*c/d+b*x)*sin
(a-b*c/d)/d^2-1/4*sin(b*x+a)/d/(d*x+c)-1/4*sin(3*b*x+3*a)/d/(d*x+c)
```

Rubi [A] time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{4d^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(4*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(4*d^2) - Sin[a + b*x]/(4*d*(c + d
*x)) - Sin[3*a + 3*b*x]/(4*d*(c + d*x)) - (b*Sin[a - (b*c)/d]*SinIntegral[(
b*c)/d + b*x])/(4*d^2) - (3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d +
3*b*x])/(4*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(a+bx)}{4(c+dx)^2}+\frac{\sin(3a+3bx)}{4(c+dx)^2}\right)dx \\
&= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int \frac{\cos(3a+3bx)}{c+dx}\,dx}{4d} \\
&= -\frac{\sin(a+bx)}{4d(c+dx)}-\frac{\sin(3a+3bx)}{4d(c+dx)}+\frac{\left(3b\cos\left(3a-\frac{3bc}{d}\right)\right)\int \frac{\cos\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}+\frac{\left(b\cos\left(\right.\right.}{} \\
&= \frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\sin(a+bx)}{4d(c+dx)}-
\end{aligned}
$$

Mathematica [A] time = 1.09, size = 139, normalized size = 0.83

$$
-\frac{-b\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)-3b\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right)+b\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+3b\sin\left(3a-\frac{3bc}{d}\right)}{4d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -1/4*(-(b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)]) - 3*b*Cos[3*a - (3*b*c
)/d]*CosIntegral[(3*b*(c + d*x))/d] + (d*Sin[a + b*x])/(c + d*x) + (d*Sin[3
*(a + b*x)])/(c + d*x) + b*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 3*b*
Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^2
```

fricas [A] time = 0.83, size = 233, normalized size = 1.39

$$-\frac{8\,d\cos\left(bx+a\right)^{2}\sin\left(bx+a\right)+6\left(bdx+bc\right)\sin\left(-\frac{3\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{3\left(bdx+bc\right)}{d}\right)+2\left(bdx+bc\right)\sin\left(-\frac{bc-ad}{d}\right)\mathrm{Si}\left(\frac{bdx+bc}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(8*d*cos(b*x + a)^2*sin(b*x + a) + 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/
d)*sin_integral(3*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*si
n_integral((b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral((b*d*x + b*c)/d)
+ (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) - 3*((b
*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-3
*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 240, normalized size = 1.43

$$\frac{\frac{b^{2}\left(-\frac{3\sin(3bx+3a)}{((bx+a)d-da+cb)d}+\frac{\frac{9\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}}{d}\right)}{12}+\frac{b^{2}\left(-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+\cdots}{d}\right)}{d}\cdots}{}\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/12*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*
(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-
a*d+b*c)/d)/d)/d)+1/4*b^2*(-sin(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*
```

```
d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d
))
```

maxima [C] time = 0.52, size = 300, normalized size = 1.79

$$-\frac{b^2\left(iE_2\left(\frac{ibc+i(bx+a)d-iad}{d}\right)-iE_2\left(-\frac{ibc+i(bx+a)d-iad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(iE_2\left(\frac{3ibc+3i(bx+a)d-3iad}{d}\right)-iE_2\left(-\frac{3ibc+3i(bx-}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(b^2*(I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^2
*(I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(exp_in
tegral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(2, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b*c*d + (
b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**2, x)
```

3.77 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=221

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right.}{8d^3}$$

```
[Out] -1/8*b*cos(b*x+a)/d^2/(d*x+c)-3/8*b*cos(3*b*x+3*a)/d^2/(d*x+c)-1/8*b^2*cos(
a-b*c/d)*Si(b*c/d+b*x)/d^3-9/8*b^2*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3-9
/8*b^2*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/8*b^2*Ci(b*c/d+b*x)*sin(a-b
*c/d)/d^3-1/8*sin(b*x+a)/d/(d*x+c)^2-1/8*sin(3*b*x+3*a)/d/(d*x+c)^2
```

Rubi [A] time = 0.32, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^3}-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[a + b*x])/(8*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(8*d^2*(c + d*
x)) - (9*b^2*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(8*d^3) -
 (b^2*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^3) - Sin[a + b*x]/(
8*d*(c + d*x)^2) - Sin[3*a + 3*b*x]/(8*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/
d]*SinIntegral[(b*c)/d + b*x])/(8*d^3) - (9*b^2*Cos[3*a - (3*b*c)/d]*SinInt
egral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(a+bx)}{4(c+dx)^3} + \frac{\sin(3a+3bx)}{4(c+dx)^3}\right)dx$$

$$= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx + \frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{8d} + \frac{(3b)\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx}{8d}$$

$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{b^2\int \frac{\sin(a+}{c+d}}{8d^2}$$

$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{\sin(a+bx)}{8d(c+dx)^2} - \frac{\sin(3a+3bx)}{8d(c+dx)^2} - \frac{\left(9b^2\cos\left(\right.\right.}{}$$

$$= -\frac{b\cos(a+bx)}{8d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{8d^2(c+dx)} - \frac{9b^2\text{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{8d^3} - \frac{b^2\text{Ci}\left(\right.}{}$$

Mathematica [A] time = 2.56, size = 181, normalized size = 0.82

$$-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right) + b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right) + b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) + 9b^2\cos\left(3a-\frac{3b}{}\right.}{8d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/8*(9*b^2*CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + b^2*CosIn
tegral[b*(c/d + x)]*Sin[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin
[a + b*x]))/(c + d*x)^2 + (d*(3*b*(c + d*x)*Cos[3*(a + b*x)] + d*Sin[3*(a +
 b*x)]))/(c + d*x)^2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + 9*b^
2*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d])/d^3
```

fricas [A] time = 0.53, size = 393, normalized size = 1.78

$$-\frac{8\,d^2\cos\left(bx+a\right)^2\sin\left(bx+a\right)+24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3+18\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(8*d^2*cos(b*x + a)^2*sin(b*x + a) + 24*(b*d^2*x + b*c*d)*cos(b*x + a
)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-3*(b*c - a*d)/d)*sin_in
tegral(3*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-(b
*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^2*x + b*c*d)*cos(b*x +
 a) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x + b*c)/d))*sin
(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(3*
(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-3*(b
*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 313, normalized size = 1.42

$$\frac{\frac{b^3\left(-\frac{3\sin(3bx+3a)}{2((bx+a)d-da+cb)^2d}+\frac{-\frac{9\cos(3bx+3a)}{2((bx+a)d-da+cb)d}-\frac{9\left(\frac{3\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}-\frac{3\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{2d}}{d}\right)}{12}+\frac{b^3\left(-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2d}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/12*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2*(-3*cos(3*b*
x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d)/d)/d)+1/4*b
^3*(-1/2*sin(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a
+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*
sin((-a*d+b*c)/d)/d)/d)/d))
```

maxima [C] time = 0.65, size = 335, normalized size = 1.52

$$-\frac{b^3\left(i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(i\,E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_3\left(-\frac{3i\,bc+3i\,(bx-}{d}\right.\right.}{8\left(b^2c^2d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(b^3*(I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^3
*(I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(exp_in
tegral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(3, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**3, x)
```

3.78 $\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=270

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}+\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)}{8d^4}$$

```
[Out] -9/8*b^3*Ci(3*b*c/d+3*b*x)*cos(3*a-3*b*c/d)/d^4-1/24*b^3*Ci(b*c/d+b*x)*cos(
a-b*c/d)/d^4-1/24*b*cos(b*x+a)/d^2/(d*x+c)^2-1/8*b*cos(3*b*x+3*a)/d^2/(d*x+
c)^2+9/8*b^3*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/24*b^3*Si(b*c/d+b*x)*
sin(a-b*c/d)/d^4-1/12*sin(b*x+a)/d/(d*x+c)^3+1/24*b^2*sin(b*x+a)/d^3/(d*x+c
)-1/12*sin(3*b*x+3*a)/d/(d*x+c)^3+3/8*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)
```

Rubi [A] time = 0.38, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{24d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{8d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[a + b*x])/(24*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(8*d^2*(c + d
*x)^2) - (b^3*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(24*d^4) - (9*b^
3*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(8*d^4) - Sin[a + b*
x]/(12*d*(c + d*x)^3) + (b^2*Sin[a + b*x])/(24*d^3*(c + d*x)) - Sin[3*a + 3
*b*x]/(12*d*(c + d*x)^3) + (3*b^2*Sin[3*a + 3*b*x])/(8*d^3*(c + d*x)) + (b^
3*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(24*d^4) + (9*b^3*Sin[3*a -
(3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(8*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(a+bx)}{4(c+dx)^4} + \frac{\sin(3a+3bx)}{4(c+dx)^4}\right) dx$$

$$= \frac{1}{4}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx + \frac{1}{4}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(a+bx)}{12d(c+dx)^3} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{12d} + \frac{b\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx}{4d}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} - \frac{\sin(3a+3bx)}{12d(c+dx)^3} - \frac{b^2\int \frac{\sin(}{(c+}}{24}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} + \frac{b^2\sin(a+bx)}{24d^3(c+dx)} - \frac{\sin(3a}{12d(c+}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{\sin(a+bx)}{12d(c+dx)^3} + \frac{b^2\sin(a+bx)}{24d^3(c+dx)} - \frac{\sin(3a}{12d(c+}$$

$$= -\frac{b\cos(a+bx)}{24d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{8d^2(c+dx)^2} - \frac{b^3\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{24d^4} - \frac{9b^3\cos\left(3\right.}{}$$

Mathematica [A] time = 1.82, size = 300, normalized size = 1.11

$$-\frac{b^3(c+dx)^3\left(\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)-\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right)\right)+27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]
) + d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*S
in[3*a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*d*(c + d*x)*Sin[a])*Sin
[b*x] - d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x)*Sin[3*a]
)*Sin[3*b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] -
 Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) + 27*b^3*(c + d*x)^3*(Cos[3*a -
 (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*b*c)/d]*SinIntegr
al[(3*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

fricas [B] time = 0.67, size = 558, normalized size = 2.07

$$-\frac{24\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3-54\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\text{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-2\left(b^3d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/48*(24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - 54*(b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*
x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*s
in(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) - 16*(b*d^3*x + b*c*d^2)*c
os(b*x + a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*co
s_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 27*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 8*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - (9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d -
 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x
+ c^3*d^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.01, size = 381, normalized size = 1.41

$$\frac{\frac{b^4\left(-\frac{\sin(3bx+3a)}{((bx+a)d-da+cb)^3 d}+\frac{-\frac{3\cos(3bx+3a)}{2((bx+a)d-da+cb)^2 d}-\frac{3\left(-\frac{3\sin(3bx+3a)}{((bx+a)d-da+cb)d}+\frac{3\left(\frac{9\,\mathrm{Si}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\sin\left(\frac{-3da+3cb}{d}\right)}{d}+\frac{9\,\mathrm{Ci}\left(3bx+3a+\frac{-3da+3cb}{d}\right)\cos\left(\frac{-3da+3cb}{d}\right)}{d}\right)}{d}\right)}{2d}}{d}\right)}{12}+\frac{b^4(\cdots}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/12*b^4*(-sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^3/d+(-3/2*cos(3*b*x+3*a)
/((b*x+a)*d-d*a+c*b)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*
Si(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b
*c)/d)*cos(3*(-a*d+b*c)/d)/d)/d)/d)/d)+1/4*b^4*(-1/3*sin(b*x+a)/((b*x+a)*d-
d*a+c*b)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/
((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+
(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d))
```

maxima [C] time = 0.88, size = 385, normalized size = 1.43

$$-\frac{b^4\left(i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)-i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(i\,E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)-i\,E_4\left(-\frac{3i\,bc+3i\,(b\cdots}{}\right.\right.}{8\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/8*(b^4*(I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) - I*exp_i
ntegral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) + b^4
*(I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_inte
gral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d)
+ b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_integral_
e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(exp_in
```

```
tegral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(4, -(
3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d))/((b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*
b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x))/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**2/(c + d*x)**4, x)
```

3.79 $\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\right.}{b}$$

```
[Out] 1/8*(d*x+c)^(1+m)/d/(1+m)+I*exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(
d*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-I*(d*x+c)^m*GAMMA(1+m,4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3307, 2181}

$$\frac{i2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{4ib(c+dx)}{d}\right)}{b}-\frac{i2^{-2(m+3)}e^{-4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\text{G}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(8*d*(1 + m)) + (I*E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Ga
mma[1 + m, ((-4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)
^m) - (I*(c + d*x)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*
E^((4*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m - \frac{1}{8}(c+dx)^m\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} - \frac{1}{8}\int (c+dx)^m\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} - \frac{1}{16}\int e^{-i(4a+4bx)}(c+dx)^m\,dx - \frac{1}{16}\int e^{i(4a+4bx)}(c+dx)^m$$
$$= \frac{(c+dx)^{1+m}}{8d(1+m)} + \frac{i4^{-3-m}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{4ib(c+}{d}\right.}{b}$$

Mathematica [A] time = 1.04, size = 213, normalized size = 1.31

$$\frac{4^{-m-3}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(-id(m+1)\left(-\frac{ib(c+dx)}{d}\right)^m\left(\cos\left(4a-\frac{4bc}{d}\right)-i\sin\left(4a-\frac{4bc}{d}\right)\right)\Gamma\left(m+1,\frac{4ib(c+dx)}{d}\right)+id(}{bd(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (4^(-3 - m)*(c + d*x)^m*(2^(3 + 2*m)*b*(c + d*x)*((b^2*(c + d*x)^2)/d^2)^m
- I*d*(1 + m)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d]*
(Cos[4*a - (4*b*c)/d] - I*Sin[4*a - (4*b*c)/d]) + I*d*(1 + m)*((I*b*(c + d*
x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d]*(Cos[4*a - (4*b*c)/d] + I*Sin
[4*a - (4*b*c)/d])))/(b*d*(1 + m)*((b^2*(c + d*x)^2)/d^2)^m)
```

fricas [A] time = 0.59, size = 134, normalized size = 0.83

$$\frac{(-i\,dm-i\,d)e^{\left(-\frac{dm\log\left(\frac{4i\,b}{d}\right)-4i\,bc+4i\,ad}{d}\right)}\Gamma\left(m+1,\frac{4i\,bdx+4i\,bc}{d}\right)+(i\,dm+i\,d)e^{\left(-\frac{dm\log\left(-\frac{4i\,b}{d}\right)+4i\,bc-4i\,ad}{d}\right)}\Gamma\left(m+1,\frac{-4i\,bdx-4i\,bc}{d}\right)+8}{64\,(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/64*((-I*d*m - I*d)*e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m
+ 1, (4*I*b*d*x + 4*I*b*c)/d) + (I*d*m + I*d)*e^(-(d*m*log(-4*I*b/d) + 4*I*
b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d) + 8*(b*d*x + b*c)*
(d*x + c)^m)/(b*d*m + b*d)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^2 \sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^2, x)
```

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^2(bx+a)\right)\left(\sin^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(dm+d)\int (dx+c)^m \cos(4\,bx+4\,a)\,dx - e^{\left(m\log(dx+c)+\log(dx+c)\right)}}{8\,(dm+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/8*((d*m + d)*integrate((d*x + c)^m*cos(4*b*x + 4*a), x) - e^(m*log(d*x +
 c) + log(d*x + c)))/(d*m + d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2 \sin(a+b\,x)^2 (c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^m, x)
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.80 $\int (c+dx)^4 \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=131

$$-\frac{3d^4\sin(4a+4bx)}{1024b^5}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}+\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{(c+d}{}$$

```
[Out] 1/40*(d*x+c)^5/d+3/256*d^3*(d*x+c)*cos(4*b*x+4*a)/b^4-1/32*d*(d*x+c)^3*cos(
4*b*x+4*a)/b^2-3/1024*d^4*sin(4*b*x+4*a)/b^5+3/128*d^2*(d*x+c)^2*sin(4*b*x+
4*a)/b^3-1/32*(d*x+c)^4*sin(4*b*x+4*a)/b
```

Rubi [A] time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3}+\frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4}-\frac{d(c+dx)^3\cos(4a+4bx)}{32b^2}-\frac{3d^4\sin(4a+4bx)}{1024b^5}-\frac{(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^5/(40*d) + (3*d^3*(c + d*x)*Cos[4*a + 4*b*x])/(256*b^4) - (d*(c +
 d*x)^3*Cos[4*a + 4*b*x])/(32*b^2) - (3*d^4*Sin[4*a + 4*b*x])/(1024*b^5) +
(3*d^2*(c + d*x)^2*Sin[4*a + 4*b*x])/(128*b^3) - ((c + d*x)^4*Sin[4*a + 4*b
*x])/(32*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4 - \frac{1}{8}(c+dx)^4\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^5}{40d} - \frac{1}{8}\int (c+dx)^4\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^5}{40d} - \frac{(c+dx)^4\sin(4a+4bx)}{32b} + \frac{d\int (c+dx)^3\sin(4a+4bx)\,dx}{8b}$$
$$= \frac{(c+dx)^5}{40d} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} - \frac{(c+dx)^4\sin(4a+4bx)}{32b} + \frac{(3d}{}$$
$$= \frac{(c+dx)^5}{40d} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} + \frac{3d^2(c+dx)^2\sin(4a+4bx)}{128b^3} -$$
$$= \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} + \frac{3}{}$$
$$= \frac{(c+dx)^5}{40d} + \frac{3d^3(c+dx)\cos(4a+4bx)}{256b^4} - \frac{d(c+dx)^3\cos(4a+4bx)}{32b^2} - \frac{3}{}$$

Mathematica [A] time = 1.27, size = 132, normalized size = 1.01

$$\frac{20bd(c+dx)\cos(4(a+bx))\left(3d^2-8b^2(c+dx)^2\right) - 5\sin(4(a+bx))\left(32b^4(c+dx)^4 - 24b^2d^2(c+dx)^2 + 3d^4\right) + 12}{5120b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (128*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) +
20*b*d*(c + d*x)*(3*d^2 - 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] - 5*(3*d^4 -
24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Sin[4*(a + b*x)])/(5120*b^5)
```

fricas [B] time = 0.62, size = 466, normalized size = 3.56

$$\frac{32\,b^5d^4x^5 + 160\,b^5cd^3x^4 - 40\left(8\,b^3d^4x^3 + 24\,b^3cd^3x^2 + 8\,b^3c^3d - 3\,bcd^3 + 3\left(8\,b^3c^2d^2 - bd^4\right)x\right)\cos\left(bx+a\right)^4 + 40}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/1280*(32*b^5*d^4*x^5 + 160*b^5*c*d^3*x^4 - 40*(8*b^3*d^4*x^3 + 24*b^3*c*d
^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a
)^4 + 40*(8*b^5*c^2*d^2 - b^3*d^4)*x^3 + 40*(8*b^5*c^3*d - 3*b^3*c*d^3)*x^2
 + 40*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^
```

```
3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(32*b^5*c^4 - 24*b^3*c^2*d^2 + 3*b
*d^4)*x - 5*(2*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4 - 24*b^2*c^
2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^3*d - 3*b^2*
c*d^3)*x)*cos(b*x + a)^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 32*b^4*c^4
 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^2 + 16*(8*b^4*c^
3*d - 3*b^2*c*d^3)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

giac [A] time = 1.11, size = 224, normalized size = 1.71

$$\frac{1}{40}\,d^4x^5+\frac{1}{8}\,cd^3x^4+\frac{1}{4}\,c^2d^2x^3+\frac{1}{4}\,c^3dx^2+\frac{1}{8}\,c^4x-\frac{\left(8\,b^3d^4x^3+24\,b^3cd^3x^2+24\,b^3c^2d^2x+8\,b^3c^3d-3\,bd^4x-3\,bcd^3\right)c}{256\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/40*d^4*x^5 + 1/8*c*d^3*x^4 + 1/4*c^2*d^2*x^3 + 1/4*c^3*d*x^2 + 1/8*c^4*x
- 1/256*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d
- 3*b*d^4*x - 3*b*c*d^3)*cos(4*b*x + 4*a)/b^5 - 1/1024*(32*b^4*d^4*x^4 + 12
8*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4*c^3*d*x + 32*b^4*c^4 - 24*b
^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 + 3*d^4)*sin(4*b*x + 4*a)/b^5
```

maple [B] time = 0.09, size = 1915, normalized size = 14.62

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*((b*x+a)^4*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b
*x+a)^3*cos(b*x+a)^2+3/4*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a
)+3/32*(b*x+a)*cos(b*x+a)^2-3/64*cos(b*x+a)*sin(b*x+a)-21/256*b*x-21/256*a-
7/16*(b*x+a)^3-1/10*(b*x+a)^5-(b*x+a)^4*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/4*(b*x+a)^3*sin(b*x+a)^4+3/4*(b*x+a)^2*(-1/4*(
sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*sin(b*x
+a)^4+3/128*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-4/b^4*a*d^4*((b*x+a)^
3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)^2*cos(b*x+a)^2+3/
8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/128*(b*x+a)^2-3/128*
sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*c
os(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*(b*x+a)*(-1/4*(sin
(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128*sin(b*x+a)^4)+4/b
^3*c*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(b*x+a)
^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-21/12
8*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3
+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^4+3/8*
```

```
(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+3/128
*sin(b*x+a)^4)+6/b^4*a^2*d^4*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x
+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a
-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*co
s(b*x+a))-12/b^3*a*c*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2
*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/1
2*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a))+6/b^2*c^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*
x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8
*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))
-4/b^4*a^3*d^4*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*
x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*
x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+12/b^3*a^2*c*d^3*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(
-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)-12/b^2*a*c^2*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1
/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))
*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+4/b*c^3*d*((b*x+a)*(-1/2*cos(
b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(
-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a
)^4)+1/b^4*a^4*d^4*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+
1/8*b*x+1/8*a)-4/b^3*a^3*c*d^3*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)
*sin(b*x+a)+1/8*b*x+1/8*a)+6/b^2*a^2*c^2*d^2*(-1/4*cos(b*x+a)^3*sin(b*x+a)+
1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-4/b*a*c^3*d*(-1/4*cos(b*x+a)^3*sin
(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^4*(-1/4*cos(b*x+a)^3*sin
(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

maxima [B] time = 0.37, size = 735, normalized size = 5.61

$$\frac{160\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^4 - \frac{640\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^3d}{b} + \frac{960\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2c^2d^2}{b^2} - \frac{640\,(4\,bx+4\,a-\sin(4\,bx}{b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/5120*(160*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^4 - 640*(4*b*x + 4*a - sin(4
*b*x + 4*a))*a*c^3*d/b + 960*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c^2*d^2/b
^2 - 640*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*c*d^3/b^3 + 160*(4*b*x + 4*a
- sin(4*b*x + 4*a))*a^4*d^4/b^4 + 160*(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*
x + 4*a) - cos(4*b*x + 4*a))*c^3*d/b - 480*(8*(b*x + a)^2 - 4*(b*x + a)*sin
(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c^2*d^2/b^2 + 480*(8*(b*x + a)^2 - 4*(b
```

```
*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*c*d^3/b^3 - 160*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^3*d^4/b^4 + 40*(3
2*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4
*b*x + 4*a))*c^2*d^2/b^2 - 80*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*
a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*c*d^3/b^3 + 40*(32*(b*x + a)
^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a)
)*a^2*d^4/b^4 + 20*(32*(b*x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a)
 - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a))*c*d^3/b^3 - 20*(32*(b*
x + a)^4 - 3*(8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*
x - 3*a)*sin(4*b*x + 4*a))*a*d^4/b^4 + (128*(b*x + a)^5 - 20*(8*(b*x + a)^3
 - 3*b*x - 3*a)*cos(4*b*x + 4*a) - 5*(32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
sin(4*b*x + 4*a))*d^4/b^4)/b
```

mupad [B] time = 1.68, size = 349, normalized size = 2.66

$$-\frac{15\,d^4\,\sin\left(4\,a+4\,b\,x\right)-640\,b^5\,c^4\,x+160\,b^4\,c^4\,\sin\left(4\,a+4\,b\,x\right)-128\,b^5\,d^4\,x^5+160\,b^3\,c^3\,d\,\cos\left(4\,a+4\,b\,x\right)-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -(15*d^4*sin(4*a + 4*b*x) - 640*b^5*c^4*x + 160*b^4*c^4*sin(4*a + 4*b*x) -
128*b^5*d^4*x^5 + 160*b^3*c^3*d*cos(4*a + 4*b*x) - 1280*b^5*c^3*d*x^2 - 640
*b^5*c*d^3*x^4 - 120*b^2*c^2*d^2*sin(4*a + 4*b*x) + 160*b^3*d^4*x^3*cos(4*a
 + 4*b*x) - 1280*b^5*c^2*d^2*x^3 - 120*b^2*d^4*x^2*sin(4*a + 4*b*x) + 160*b
^4*d^4*x^4*sin(4*a + 4*b*x) - 60*b*c*d^3*cos(4*a + 4*b*x) - 60*b*d^4*x*cos(
4*a + 4*b*x) + 960*b^4*c^2*d^2*x^2*sin(4*a + 4*b*x) - 240*b^2*c*d^3*x*sin(4
*a + 4*b*x) + 640*b^4*c^3*d*x*sin(4*a + 4*b*x) + 480*b^3*c^2*d^2*x*cos(4*a
+ 4*b*x) + 480*b^3*c*d^3*x^2*cos(4*a + 4*b*x) + 640*b^4*c*d^3*x^3*sin(4*a +
 4*b*x))/(5120*b^5)
```

sympy [A] time = 13.62, size = 1231, normalized size = 9.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**4*x*sin(a + b*x)**4/8 + c**4*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**4*x*cos(a + b*x)**4/8 + c**3*d*x**2*sin(a + b*x)**4/4 + c**3*d*x**
2*sin(a + b*x)**2*cos(a + b*x)**2/2 + c**3*d*x**2*cos(a + b*x)**4/4 + c**2*
d**2*x**3*sin(a + b*x)**4/4 + c**2*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**
2/2 + c**2*d**2*x**3*cos(a + b*x)**4/4 + c*d**3*x**4*sin(a + b*x)**4/8 + c*
d**3*x**4*sin(a + b*x)**2*cos(a + b*x)**2/4 + c*d**3*x**4*cos(a + b*x)**4/8
```

```
 + d**4*x**5*sin(a + b*x)**4/40 + d**4*x**5*sin(a + b*x)**2*cos(a + b*x)**2
/20 + d**4*x**5*cos(a + b*x)**4/40 + c**4*sin(a + b*x)**3*cos(a + b*x)/(8*b
) - c**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) + c**3*d*x*sin(a + b*x)**3*cos(
a + b*x)/(2*b) - c**3*d*x*sin(a + b*x)*cos(a + b*x)**3/(2*b) + 3*c**2*d**2*
x**2*sin(a + b*x)**3*cos(a + b*x)/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)*cos
(a + b*x)**3/(4*b) + c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)/(2*b) - c*d**
3*x**3*sin(a + b*x)*cos(a + b*x)**3/(2*b) + d**4*x**4*sin(a + b*x)**3*cos(a
 + b*x)/(8*b) - d**4*x**4*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c**3*d*sin(a
 + b*x)**4/(8*b**2) - c**3*d*cos(a + b*x)**4/(8*b**2) - 3*c**2*d**2*x*sin(a
 + b*x)**4/(32*b**2) + 9*c**2*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b*
*2) - 3*c**2*d**2*x*cos(a + b*x)**4/(32*b**2) - 3*c*d**3*x**2*sin(a + b*x)*
*4/(32*b**2) + 9*c*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - 3*
c*d**3*x**2*cos(a + b*x)**4/(32*b**2) - d**4*x**3*sin(a + b*x)**4/(32*b**2)
 + 3*d**4*x**3*sin(a + b*x)**2*cos(a + b*x)**2/(16*b**2) - d**4*x**3*cos(a
+ b*x)**4/(32*b**2) - 3*c**2*d**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) +
3*c**2*d**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**3) - 3*c*d**3*x*sin(a + b*x
)**3*cos(a + b*x)/(16*b**3) + 3*c*d**3*x*sin(a + b*x)*cos(a + b*x)**3/(16*b
**3) - 3*d**4*x**2*sin(a + b*x)**3*cos(a + b*x)/(32*b**3) + 3*d**4*x**2*sin
(a + b*x)*cos(a + b*x)**3/(32*b**3) + 3*c*d**3*sin(a + b*x)**4/(64*b**4) +
3*c*d**3*cos(a + b*x)**4/(64*b**4) + 3*d**4*x*sin(a + b*x)**4/(256*b**4) -
9*d**4*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*b**4) + 3*d**4*x*cos(a + b*x)
**4/(256*b**4) + 3*d**4*sin(a + b*x)**3*cos(a + b*x)/(256*b**5) - 3*d**4*si
n(a + b*x)*cos(a + b*x)**3/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2
+ 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**2*cos(a)**2, True))
```

3.81 $\int (c+dx)^3 \cos^2(a+bx) \sin^2(a+bx) \, dx$

Optimal. Leaf size=105

$$\frac{3d^3\cos(4a+4bx)}{1024b^4}+\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

```
[Out] 1/32*(d*x+c)^4/d+3/1024*d^3*cos(4*b*x+4*a)/b^4-3/128*d*(d*x+c)^2*cos(4*b*x+
4*a)/b^2+3/256*d^2*(d*x+c)*sin(4*b*x+4*a)/b^3-1/32*(d*x+c)^3*sin(4*b*x+4*a)
/b
```

Rubi [A] time = 0.13, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3}-\frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2}+\frac{3d^3\cos(4a+4bx)}{1024b^4}-\frac{(c+dx)^3\sin(4a+4bx)}{32b}+\frac{(c+dx)^4}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^4/(32*d) + (3*d^3*Cos[4*a + 4*b*x])/(1024*b^4) - (3*d*(c + d*x)^2
*Cos[4*a + 4*b*x])/(128*b^2) + (3*d^2*(c + d*x)*Sin[4*a + 4*b*x])/(256*b^3)
 - ((c + d*x)^3*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^3 - \frac{1}{8}(c+dx)^3\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^4}{32d} - \frac{1}{8}\int (c+dx)^3\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^4}{32d} - \frac{(c+dx)^3\sin(4a+4bx)}{32b} + \frac{(3d)\int (c+dx)^2\sin(4a+4bx)\,d}{32b}$$
$$= \frac{(c+dx)^4}{32d} - \frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2} - \frac{(c+dx)^3\sin(4a+4bx)}{32b} + \frac{(3}{}$$
$$= \frac{(c+dx)^4}{32d} - \frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2} + \frac{3d^2(c+dx)\sin(4a+4bx)}{256b^3} -$$
$$= \frac{(c+dx)^4}{32d} + \frac{3d^3\cos(4a+4bx)}{1024b^4} - \frac{3d(c+dx)^2\cos(4a+4bx)}{128b^2} + \frac{3d^2(c+}{}$$

Mathematica [A] time = 0.66, size = 106, normalized size = 1.01

$$\frac{-4b(c+dx)\sin(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)-3d\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)+32b^4x\left(4c^3+6c^2dx+4\right.}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(-d^2 + 8*b^2*(
c + d*x)^2)*Cos[4*(a + b*x)] - 4*b*(c + d*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*S
in[4*(a + b*x)])/(1024*b^4)
```

fricas [B] time = 0.50, size = 308, normalized size = 2.93

$$\frac{4\,b^4d^3x^4+16\,b^4cd^2x^3-3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^4+3\left(8\,b^4c^2d-b^2d^3\right)x^2+3\left(8\,b^2d^3x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/128*(4*b^4*d^3*x^4 + 16*b^4*c*d^2*x^3 - 3*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
 + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 + 3*(8*b^4*c^2*d - b^2*d^3)*x^2 + 3*(8
*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 2*(8*b^
4*c^3 - 3*b^2*c*d^2)*x - 2*(2*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 8*b^3*c^3
 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^3 - (8*b^3*d^3*x^3 +
 24*b^3*c*d^2*x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(
b*x + a))*sin(b*x + a))/b^4
```

giac [A] time = 0.23, size = 153, normalized size = 1.46

$$\frac{1}{32}\,d^3x^4+\frac{1}{8}\,cd^2x^3+\frac{3}{16}\,c^2dx^2+\frac{1}{8}\,c^3x-\frac{3\left(8\,b^2d^3x^2+16\,b^2cd^2x+8\,b^2c^2d-d^3\right)\cos\left(4\,bx+4\,a\right)}{1024\,b^4}-\frac{\left(8\,b^3d^3x^3+24\,b^3c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/32*d^3*x^4 + 1/8*c*d^2*x^3 + 3/16*c^2*d*x^2 + 1/8*c^3*x - 3/1024*(8*b^2*d
^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(4*b*x + 4*a)/b^4 - 1/256*(
8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b*d^3*x -
 3*b*c*d^2)*sin(4*b*x + 4*a)/b^4
```

maple [B] time = 0.02, size = 1074, normalized size = 10.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*((b*x+a)^3*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/16*(
b*x+a)^2*cos(b*x+a)^2+3/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)
-21/128*(b*x+a)^2-3/128*sin(b*x+a)^2-3/32*(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*
x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-3/16*(b*x+a)^2*sin(b*x+a)^
4+3/8*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)
+3/128*sin(b*x+a)^4)-3/b^3*a*d^3*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/
64*a-1/12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a
)+3/8*b*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a)
)*cos(b*x+a))+3/b^2*c*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)-1/8*(b*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/
12*(b*x+a)^3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b
*x+3/8*a)-1/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b
*x+a))+3/b^3*a^2*d^3*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/
16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*
cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-6/b^2*a*c*d^2*((b*x+a)*(-1/2*c
os(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a
)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*
x+a)^4)+3/b*c^2*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*
(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-1/b^3*a^3*d^3*(-1/4*cos(b*x+a)^3*
sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+3/b^2*a^2*c*d^2*(-1/4*c
os(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-3/b*a*c^2*d
```

```
*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c^3
*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

maxima [B] time = 0.36, size = 442, normalized size = 4.21

$$\frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^3 - \frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ac^2d}{b} + \frac{96\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2cd^2}{b^2} - \frac{32\,(4\,bx+4\,a-\sin(4\,bx+4\,a))}{b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(32*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^3 - 96*(4*b*x + 4*a - sin(4*b
*x + 4*a))*a*c^2*d/b + 96*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*c*d^2/b^2 -
32*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^3*d^3/b^3 + 24*(8*(b*x + a)^2 - 4*(b*
x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c^2*d/b - 48*(8*(b*x + a)^2 - 4
*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*c*d^2/b^2 + 24*(8*(b*x +
a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a^2*d^3/b^3 + 4*(32
*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*
b*x + 4*a))*c*d^2/b^2 - 4*(32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) -
 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a))*a*d^3/b^3 + (32*(b*x + a)^4 - 3*(8
*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) - 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*
b*x + 4*a))*d^3/b^3)/b
```

mupad [B] time = 1.69, size = 329, normalized size = 3.13

$$x^2\left(\frac{3\,c^2\,d}{64}+\frac{9\,d^3}{512\,b^2}\right)+x^2\left(\frac{9\,c^2\,d}{64}-\frac{9\,d^3}{512\,b^2}\right)+x\left(\frac{c^3}{32}+\frac{9\,c\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^3}{32}-\frac{9\,c\,d^2}{256\,b^2}\right)+\frac{d^3\,x^4}{32}-\frac{x\,\cos(4\,a+4\,b\,x)\,\left(\frac{c^3}{4}+\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] x^2*((3*c^2*d)/64 + (9*d^3)/(512*b^2)) + x^2*((9*c^2*d)/64 - (9*d^3)/(512*b
^2)) + x*(c^3/32 + (9*c*d^2)/(256*b^2)) + x*((3*c^3)/32 - (9*c*d^2)/(256*b^
2)) + (d^3*x^4)/32 - (x*cos(4*a + 4*b*x)*(c^3/4 + (9*c*d^2)/(32*b^2)))/8 +
(x*cos(4*a + 4*b*x)*(c^3/8 - (3*c*d^2)/(64*b^2)))/4 + (c*d^2*x^3)/8 + (cos(
4*a + 4*b*x)*((3*d^3)/128 - (3*b^2*c^2*d)/16))/(8*b^4) + (sin(4*a + 4*b*x)*
(3*c*d^2 - 8*b^2*c^3))/(256*b^3) - (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/8 + (9*
d^3)/(64*b^2)))/8 + (x^2*cos(4*a + 4*b*x)*((3*c^2*d)/16 - (3*d^3)/(128*b^2)
))/4 - (d^3*x^3*sin(4*a + 4*b*x))/(32*b) + (3*x*sin(4*a + 4*b*x)*(d^3 - 8*b
^2*c^2*d))/(256*b^3) - (3*c*d^2*x^2*sin(4*a + 4*b*x))/(32*b)
```

sympy [A] time = 7.94, size = 835, normalized size = 7.95

$$\begin{cases} \frac{c^3x\sin^4(a+bx)}{8} + \frac{c^3x\sin^2(a+bx)\cos^2(a+bx)}{4} + \frac{c^3x\cos^4(a+bx)}{8} + \frac{3c^2dx^2\sin^4(a+bx)}{16} + \frac{3c^2dx^2\sin^2(a+bx)\cos^2(a+bx)}{8} + \frac{3c^2dx^2\cos^4(a+bx)}{16} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^2(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**3*x*sin(a + b*x)**4/8 + c**3*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**3*x*cos(a + b*x)**4/8 + 3*c**2*d*x**2*sin(a + b*x)**4/16 + 3*c**2*
d*x**2*sin(a + b*x)**2*cos(a + b*x)**2/8 + 3*c**2*d*x**2*cos(a + b*x)**4/16
 + c*d**2*x**3*sin(a + b*x)**4/8 + c*d**2*x**3*sin(a + b*x)**2*cos(a + b*x)
**2/4 + c*d**2*x**3*cos(a + b*x)**4/8 + d**3*x**4*sin(a + b*x)**4/32 + d**3
*x**4*sin(a + b*x)**2*cos(a + b*x)**2/16 + d**3*x**4*cos(a + b*x)**4/32 + c
**3*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**3*sin(a + b*x)*cos(a + b*x)**3/
(8*b) + 3*c**2*d*x*sin(a + b*x)**3*cos(a + b*x)/(8*b) - 3*c**2*d*x*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + 3*c*d**2*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*
b) - 3*c*d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d**3*x**3*sin(a + b
*x)**3*cos(a + b*x)/(8*b) - d**3*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b) -
3*c**2*d*sin(a + b*x)**4/(32*b**2) - 3*c**2*d*cos(a + b*x)**4/(32*b**2) - 3
*c*d**2*x*sin(a + b*x)**4/(64*b**2) + 9*c*d**2*x*sin(a + b*x)**2*cos(a + b*
x)**2/(32*b**2) - 3*c*d**2*x*cos(a + b*x)**4/(64*b**2) - 3*d**3*x**2*sin(a
+ b*x)**4/(128*b**2) + 9*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**2/(64*b**2
) - 3*d**3*x**2*cos(a + b*x)**4/(128*b**2) - 3*c*d**2*sin(a + b*x)**3*cos(a
 + b*x)/(64*b**3) + 3*c*d**2*sin(a + b*x)*cos(a + b*x)**3/(64*b**3) - 3*d**
3*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + 3*d**3*x*sin(a + b*x)*cos(a +
b*x)**3/(64*b**3) + 3*d**3*sin(a + b*x)**4/(256*b**4) + 3*d**3*cos(a + b*x)
**4/(256*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*
x**4/4)*sin(a)**2*cos(a)**2, True))
```

3.82 $\int (c+dx)^2 \cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=79

$$\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

```
[Out] 1/24*(d*x+c)^3/d-1/64*d*(d*x+c)*cos(4*b*x+4*a)/b^2+1/256*d^2*sin(4*b*x+4*a)
/b^3-1/32*(d*x+c)^2*sin(4*b*x+4*a)/b
```

Rubi [A] time = 0.12, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$-\frac{d(c+dx)\cos(4a+4bx)}{64b^2}+\frac{d^2\sin(4a+4bx)}{256b^3}-\frac{(c+dx)^2\sin(4a+4bx)}{32b}+\frac{(c+dx)^3}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^3/(24*d) - (d*(c + d*x)*Cos[4*a + 4*b*x])/(64*b^2) + (d^2*Sin[4*a
 + 4*b*x])/(256*b^3) - ((c + d*x)^2*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^2 - \frac{1}{8}(c+dx)^2\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^3}{24d} - \frac{1}{8}\int (c+dx)^2\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^3}{24d} - \frac{(c+dx)^2\sin(4a+4bx)}{32b} + \frac{d\int (c+dx)\sin(4a+4bx)\,dx}{16b}$$
$$= \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} - \frac{(c+dx)^2\sin(4a+4bx)}{32b} + \frac{d^2}{}$$
$$= \frac{(c+dx)^3}{24d} - \frac{d(c+dx)\cos(4a+4bx)}{64b^2} + \frac{d^2\sin(4a+4bx)}{256b^3} - \frac{(c+dx)^2}{}$$

Mathematica [A] time = 0.42, size = 77, normalized size = 0.97

$$\frac{-3\sin(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)-12bd(c+dx)\cos(4(a+bx))+32b^3x\left(3c^2+3cdx+d^2x^2\right)}{768b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (32*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 12*b*d*(c + d*x)*Cos[4*(a + b*x)] -
 3*(-d^2 + 8*b^2*(c + d*x)^2)*Sin[4*(a + b*x)])/(768*b^3)
```

fricas [B] time = 0.44, size = 180, normalized size = 2.28

$$\frac{8\,b^3d^2x^3+24\,b^3cdx^2-24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+24\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+3\left(8\,b^3c^2-bd^2\right)x-3\left(2\right.}{192}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(8*b^3*d^2*x^3 + 24*b^3*c*d*x^2 - 24*(b*d^2*x + b*c*d)*cos(b*x + a)^4
 + 24*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(8*b^3*c^2 - b*d^2)*x - 3*(2*(8*
b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a)^3 - (8*b^2*d^2*x
^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(b*x + a))*sin(b*x + a))/b^3
```

giac [A] time = 0.20, size = 94, normalized size = 1.19

$$\frac{1}{24}\,d^2x^3+\frac{1}{8}\,cdx^2+\frac{1}{8}\,c^2x-\frac{\left(bd^2x+bcd\right)\cos\left(4\,bx+4\,a\right)}{64\,b^3}-\frac{\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\sin\left(4\,bx+4\,a\right)}{256\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/24*d^2*x^3 + 1/8*c*d*x^2 + 1/8*c^2*x - 1/64*(b*d^2*x + b*c*d)*cos(4*b*x +
 4*a)/b^3 - 1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*sin(4*b*
x + 4*a)/b^3
```

maple [B] time = 0.02, size = 519, normalized size = 6.57

$$\frac{d^2\left((bx+a)^2\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)\left(\cos^2(bx+a)\right)}{8}+\frac{\cos(bx+a)\sin(bx+a)}{16}+\frac{7bx}{64}+\frac{7a}{64}-\frac{(bx+a)^3}{12}-(bx+a)^2\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*((b*x+a)^2*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/8*(b
*x+a)*cos(b*x+a)^2+1/16*cos(b*x+a)*sin(b*x+a)+7/64*b*x+7/64*a-1/12*(b*x+a)^
3-(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1
/8*(b*x+a)*sin(b*x+a)^4-1/32*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a))-2/b^
2*a*d^2*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+
1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/
8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+2/b*c*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a
)+1/2*b*x+1/2*a)-1/16*(b*x+a)^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)
^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)+1/b^2*a^2*d
^2*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)-2
/b*a*c*d*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/
8*a)+c^2*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/
8*a))
```

maxima [B] time = 0.34, size = 232, normalized size = 2.94

$$24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c^2-\frac{48\,(4\,bx+4\,a-\sin(4\,bx+4\,a))acd}{b}+\frac{24\,(4\,bx+4\,a-\sin(4\,bx+4\,a))a^2d^2}{b^2}+\frac{12\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,b\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/768*(24*(4*b*x + 4*a - sin(4*b*x + 4*a))*c^2 - 48*(4*b*x + 4*a - sin(4*b*
x + 4*a))*a*c*d/b + 24*(4*b*x + 4*a - sin(4*b*x + 4*a))*a^2*d^2/b^2 + 12*(8
*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*c*d/b - 12*
(8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4*a))*a*d^2/b^2
```

```
 + (32*(b*x + a)^3 - 12*(b*x + a)*cos(4*b*x + 4*a) - 3*(8*(b*x + a)^2 - 1)*
sin(4*b*x + 4*a))*d^2/b^2)/b
```

mupad [B] time = 1.31, size = 179, normalized size = 2.27

$$x\left(\frac{c^2}{32}+\frac{3\,d^2}{256\,b^2}\right)+x\left(\frac{3\,c^2}{32}-\frac{3\,d^2}{256\,b^2}\right)+\frac{d^2\,x^3}{24}+\frac{\sin\left(4\,a+4\,b\,x\right)\,\left(d^2-8\,b^2\,c^2\right)}{256\,b^3}-\frac{x\,\cos\left(4\,a+4\,b\,x\right)\,\left(\frac{c^2}{4}+\frac{3\,d^2}{32\,b^2}\right)}{8}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] x*(c^2/32 + (3*d^2)/(256*b^2)) + x*((3*c^2)/32 - (3*d^2)/(256*b^2)) + (d^2*
x^3)/24 + (sin(4*a + 4*b*x)*(d^2 - 8*b^2*c^2))/(256*b^3) - (x*cos(4*a + 4*b
*x)*(c^2/4 + (3*d^2)/(32*b^2)))/8 + (x*cos(4*a + 4*b*x)*(c^2/8 - d^2/(64*b^
2)))/4 + (c*d*x^2)/8 - (d^2*x^2*sin(4*a + 4*b*x))/(32*b) - (c*d*cos(4*a + 4
*b*x))/(64*b^2) - (c*d*x*sin(4*a + 4*b*x))/(16*b)
```

sympy [A] time = 3.98, size = 493, normalized size = 6.24

$$\begin{cases} \frac{c^2x\sin^4(a+bx)}{8}+\frac{c^2x\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{c^2x\cos^4(a+bx)}{8}+\frac{cdx^2\sin^4(a+bx)}{8}+\frac{cdx^2\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{cdx^2\cos^4(a+bx)}{8}+ \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^2(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c**2*x*sin(a + b*x)**4/8 + c**2*x*sin(a + b*x)**2*cos(a + b*x)**
2/4 + c**2*x*cos(a + b*x)**4/8 + c*d*x**2*sin(a + b*x)**4/8 + c*d*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/4 + c*d*x**2*cos(a + b*x)**4/8 + d**2*x**3*sin(
a + b*x)**4/24 + d**2*x**3*sin(a + b*x)**2*cos(a + b*x)**2/12 + d**2*x**3*c
os(a + b*x)**4/24 + c**2*sin(a + b*x)**3*cos(a + b*x)/(8*b) - c**2*sin(a +
b*x)*cos(a + b*x)**3/(8*b) + c*d*x*sin(a + b*x)**3*cos(a + b*x)/(4*b) - c*d
*x*sin(a + b*x)*cos(a + b*x)**3/(4*b) + d**2*x**2*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - d**2*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b) - c*d*sin(a + b*x)
**4/(16*b**2) - c*d*cos(a + b*x)**4/(16*b**2) - d**2*x*sin(a + b*x)**4/(64*
b**2) + 3*d**2*x*sin(a + b*x)**2*cos(a + b*x)**2/(32*b**2) - d**2*x*cos(a +
 b*x)**4/(64*b**2) - d**2*sin(a + b*x)**3*cos(a + b*x)/(64*b**3) + d**2*sin
(a + b*x)*cos(a + b*x)**3/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*
x**3/3)*sin(a)**2*cos(a)**2, True))
```

3.83 $\int (c + dx)\cos^2(a + bx)\sin^2(a + bx)\,dx$

Optimal. Leaf size=53

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

```
[Out] 1/16*(d*x+c)^2/d-1/128*d*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)*sin(4*b*x+4*a)/b
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2638}

$$-\frac{d\cos(4a+4bx)}{128b^2}-\frac{(c+dx)\sin(4a+4bx)}{32b}+\frac{(c+dx)^2}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^2/(16*d) - (d*Cos[4*a + 4*b*x])/(128*b^2) - ((c + d*x)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx) - \frac{1}{8}(c+dx)\cos(4a+4bx)\right) dx$$
$$= \frac{(c+dx)^2}{16d} - \frac{1}{8}\int (c+dx)\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^2}{16d} - \frac{(c+dx)\sin(4a+4bx)}{32b} + \frac{d\int \sin(4a+4bx)\,dx}{32b}$$
$$= \frac{(c+dx)^2}{16d} - \frac{d\cos(4a+4bx)}{128b^2} - \frac{(c+dx)\sin(4a+4bx)}{32b}$$

Mathematica [A] time = 0.29, size = 54, normalized size = 1.02

$$-\frac{8(a+bx)(ad-2bc-bdx)+4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] -1/128*(8*(a + b*x)*(-2*b*c + a*d - b*d*x) + d*Cos[4*(a + b*x)] + 4*b*(c + 
d*x)*Sin[4*(a + b*x)])/b^2
```

fricas [A] time = 0.59, size = 85, normalized size = 1.60

$$\frac{b^2dx^2 - d\cos(bx+a)^4 + 2\,b^2cx + d\cos(bx+a)^2 - 2\left(2\,(bdx+bc)\cos(bx+a)^3 - (bdx+bc)\cos(bx+a)\right)\sin(}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(b^2*d*x^2 - d*cos(b*x + a)^4 + 2*b^2*c*x + d*cos(b*x + a)^2 - 2*(2*(b
*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*sin(b*x + a))/b^2
```

giac [A] time = 1.36, size = 48, normalized size = 0.91

$$\frac{1}{16}\,dx^2 + \frac{1}{8}\,cx - \frac{d\cos(4\,bx+4\,a)}{128\,b^2} - \frac{(bdx+bc)\sin(4\,bx+4\,a)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/16*d*x^2 + 1/8*c*x - 1/128*d*cos(4*b*x + 4*a)/b^2 - 1/32*(b*d*x + b*c)*si
n(4*b*x + 4*a)/b^2
```

maple [B] time = 0.02, size = 194, normalized size = 3.66

$$\frac{\frac{d\left((bx+a)\left(-\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{16}+\frac{\left(\sin^2(bx+a)\right)}{16}-(bx+a)\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)-\frac{\left(\sin^4(bx+a)\right)}{16}\right)}{b}-\frac{da\left(-\frac{\left(\cos^3(bx+a)\right)\sin}{4}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b*d*((b*x+a)*(-1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/16*(b*x+a)
^2+1/16*sin(b*x+a)^2-(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
+3/8*b*x+3/8*a)-1/16*sin(b*x+a)^4)-1/b*d*a*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/
8*cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a)+c*(-1/4*cos(b*x+a)^3*sin(b*x+a)+1/8*
cos(b*x+a)*sin(b*x+a)+1/8*b*x+1/8*a))
```

maxima [B] time = 0.33, size = 96, normalized size = 1.81

$$\frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))c-\frac{4\,(4\,bx+4\,a-\sin(4\,bx+4\,a))ad}{b}+\frac{\left(8\,(bx+a)^2-4\,(bx+a)\sin(4\,bx+4\,a)-\cos(4\,bx+4\,a)\right)d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/128*(4*(4*b*x + 4*a - sin(4*b*x + 4*a))*c - 4*(4*b*x + 4*a - sin(4*b*x +
4*a))*a*d/b + (8*(b*x + a)^2 - 4*(b*x + a)*sin(4*b*x + 4*a) - cos(4*b*x + 4
*a))*d/b)/b
```

mupad [B] time = 1.04, size = 57, normalized size = 1.08

$$\frac{c\,x}{8}+\frac{d\,x^2}{16}-\frac{d\,\cos(4\,a+4\,b\,x)}{128\,b^2}-\frac{c\,\sin(4\,a+4\,b\,x)}{32\,b}-\frac{d\,x\,\sin(4\,a+4\,b\,x)}{32\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x),x)
```

```
[Out] (c*x)/8 + (d*x^2)/16 - (d*cos(4*a + 4*b*x))/(128*b^2) - (c*sin(4*a + 4*b*x)
)/(32*b) - (d*x*sin(4*a + 4*b*x))/(32*b)
```

sympy [A] time = 2.00, size = 238, normalized size = 4.49

$$\begin{cases} \frac{cx\sin^4(a+bx)}{8}+\frac{cx\sin^2(a+bx)\cos^2(a+bx)}{4}+\frac{cx\cos^4(a+bx)}{8}+\frac{dx^2\sin^4(a+bx)}{16}+\frac{dx^2\sin^2(a+bx)\cos^2(a+bx)}{8}+\frac{dx^2\cos^4(a+bx)}{16}+\frac{c\sin^3(a+}{} \\ \left(cx+\frac{dx^2}{2}\right)\sin^2(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((c*x*sin(a + b*x)**4/8 + c*x*sin(a + b*x)**2*cos(a + b*x)**2/4 +
c*x*cos(a + b*x)**4/8 + d*x**2*sin(a + b*x)**4/16 + d*x**2*sin(a + b*x)**2*
cos(a + b*x)**2/8 + d*x**2*cos(a + b*x)**4/16 + c*sin(a + b*x)**3*cos(a + b
*x)/(8*b) - c*sin(a + b*x)*cos(a + b*x)**3/(8*b) + d*x*sin(a + b*x)**3*cos(
a + b*x)/(8*b) - d*x*sin(a + b*x)*cos(a + b*x)**3/(8*b) - d*sin(a + b*x)**4
/(32*b**2) - d*cos(a + b*x)**4/(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(
a)**2*cos(a)**2, True))
```

3.84 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

```
[Out] -1/8*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d+1/8*ln(d*x+c)/d+1/8*Si(4*b*c/d+4*
b*x)*sin(4*a-4*b*c/d)/d
```

Rubi [A] time = 0.14, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(8*d) + Log[c + d*x]
/(8*d) + (Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(8*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{8(c+dx)}-\frac{\cos(4a+4bx)}{8(c+dx)}\right)dx\\ &= \frac{\log(c+dx)}{8d}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{c+dx}\,dx\\ &= \frac{\log(c+dx)}{8d}-\frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx+\frac{1}{8}\sin\left(4a-\frac{4bc}{d}\right)\int\\ &= -\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\log(c+dx)}{8d}+\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4bx\right)}{8d}\end{aligned}$$

Mathematica [A] time = 0.16, size = 65, normalized size = 0.83

$$\frac{-\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right)+\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b(c+dx)}{d}\right)+\log(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (-(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]) + Log[c + d*x] + Si
n[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

fricas [A] time = 0.49, size = 88, normalized size = 1.13

$$-\frac{\left(\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)-2\,\log\left(dx+c\right)}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/16*((cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-4*(b*d*x + b*c)/d))
*cos(-4*(b*c - a*d)/d) - 2*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*
c)/d) - 2*log(d*x + c))/d
```

giac [C] time = 0.22, size = 669, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(d*x + c))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*a)^2*tan(2*b*c/d) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^
2*tan(2*b*c/d) - 2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*
b*c/d)^2 + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
)^2 - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 2*log(abs
(d*x + c))*tan(2*a)^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2
 + real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*real_part(cos_i
ntegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*real_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 2*log(abs(d*x + c))*tan(2*b*c/d)
^2 + real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + real_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a) - 2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a) + 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) - 2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*c/d) + 2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*c/d) - 4*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 2*log(ab
s(d*x + c)) - real_part(cos_integral(4*b*x + 4*b*c/d)) - real_part(cos_inte
gral(-4*b*x - 4*b*c/d)))/(d*tan(2*a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2 + d*ta
n(2*b*c/d)^2 + d)
```

maple [A] time = 0.03, size = 105, normalized size = 1.35

$$-\frac{\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{8d}-\frac{\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{8d}+\frac{\ln\left((bx+a)\,d-da+cb\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] -1/8*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d-1/8*Ci(4*b*x+4*a+4*
(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d+1/8*ln((b*x+a)*d-d*a+c*b)/d
```

maxima [C] time = 0.41, size = 160, normalized size = 2.05

$$\frac{b\left(E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_1\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)+b\left(-i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+i\,E_1\left(-\frac{4i\,bc+4}{}\right.\right.}{16\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/16*(b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_i
ntegral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/
d) + b*(-I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + I*e
xp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a
*d)/d) + 2*b*log(b*c + (b*x + a)*d - a*d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x), x)
```

3.85 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=104

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

```
[Out] -1/8/d/(d*x+c)+1/8*cos(4*b*x+4*a)/d/(d*x+c)+1/2*b*cos(4*a-4*b*c/d)*Si(4*b*c
/d+4*b*x)/d^2+1/2*b*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2
```

Rubi [A] time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{2d^2}+\frac{\cos(4a+4bx)}{8d(c+dx)}-\frac{1}{8d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -1/(8*d*(c + d*x)) + Cos[4*a + 4*b*x]/(8*d*(c + d*x)) + (b*CosIntegral[(4*b
*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(2*d^2) + (b*Cos[4*a - (4*b*c)/d]*SinI
ntegral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{1}{8(c+dx)^2}-\frac{\cos(4a+4bx)}{8(c+dx)^2}\right)dx\\ &= -\frac{1}{8d(c+dx)}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx\\ &= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\sin(4a+4bx)}{c+dx}\,dx}{2d}\\ &= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{\left(b\sin\right.}{}\\ &= -\frac{1}{8d(c+dx)}+\frac{\cos(4a+4bx)}{8d(c+dx)}+\frac{b\text{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{2d^2}+\frac{b\cos\left(4a-\right.}{}\end{aligned}$$

Mathematica [A] time = 0.43, size = 81, normalized size = 0.78

$$\frac{4b\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right)+4b\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(\cos(4(a+bx))-1)}{c+dx}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*(-1 + Cos[4*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(4*b*(c + d*x))/d]
*Sin[4*a - (4*b*c)/d] + 4*b*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/(8*d^2)
```

fricas [A] time = 0.55, size = 138, normalized size = 1.33

$$\frac{4\,d\cos\left(bx+a\right)^{4}-4\,d\cos\left(bx+a\right)^{2}+2\,(bdx+bc)\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right)+\left((bdx+bc)\,\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right)+(bd\right.}{4\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*cos(b*x + a)^4 - 4*d*cos(b*x + a)^2 + 2*(b*d*x + b*c)*cos(-4*(b*c
- a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b*d*x + b*c)*cos_integral(4*(
b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b
*c - a*d)/d))/(d^3*x + c*d^2)
```

giac [C] time = 0.97, size = 3218, normalized size = 30.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2
*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 2*b*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*
tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*d*x + b*c)/
d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + b*d*x*imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*
c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 4*b*d*x*imag_part(cos_integral(4*b*x + 4*b*
c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 8*b*d*x*sin_integral
(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 2*b*c*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 2*b*c*r
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/
d) - b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b
*c/d)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)
```

```
^2 - 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 - 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^
2*tan(2*a)*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*
tan(2*a)^2*tan(2*b*c/d)^2 - b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)^2*tan(2*b*c/d)^2 + 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
a)^2*tan(2*b*c/d)^2 + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*a) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*b*x)^2*tan(2*a) - b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^
2*tan(2*a)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 -
 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)
 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c
/d) + 4*b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*
tan(2*b*c/d) - 4*b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d) + 8*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d) + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2*tan(2*b*c/d) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*a)^2*tan(2*b*c/d) - b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*b*x)^2*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2*tan(2*b*c/d)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 - 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*a)*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^
2*tan(2*b*c/d)^2 - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
*tan(2*b*c/d)^2 + 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*
c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - b*d*
x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 2*b*d*x*sin_inte
gral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 2*b*c*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 2*b*c*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a) - b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(2*a)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 2*b*c*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 2*b*c*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 4*b*d*x*imag_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*b*d*x*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 8*b*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) + 2*b*c*real_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*b*c*real_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - b*d*x*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*c/d)^2 - 2*b*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 -
 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2
*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + b*
c*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - b*c*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 2*b*c*sin_integral(4*(b*d*x + b
```

```
*c)/d)*tan(2*b*x)^2 + 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a) + 2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - b*c*ima
g_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + b*c*imag_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*ta
n(2*a)^2 - 2*b*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) -
2*b*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 4*b*c*imag
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*b*c*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 8*b*c*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) - b*c*imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*b*c/d)^2 + b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*c/d)^2 - 2*b*c*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 -
d*tan(2*b*x)^2*tan(2*b*c/d)^2 - 2*d*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - d*
tan(2*a)^2*tan(2*b*c/d)^2 + b*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))
- b*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*b*d*x*sin_integral(4*
(b*d*x + b*c)/d) + 2*b*c*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)
+ 2*b*c*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 2*b*c*real_par
t(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 2*b*c*real_part(cos_integra
l(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + b*c*imag_part(cos_integral(4*b*x + 4*b*
c/d)) - b*c*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*b*c*sin_integral(
4*(b*d*x + b*c)/d) - d*tan(2*b*x)^2 - 2*d*tan(2*b*x)*tan(2*a) - d*tan(2*a)^
2)/(d^3*x*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + c*d^2*tan(2*b*x)^2*tan(2
*a)^2*tan(2*b*c/d)^2 + d^3*x*tan(2*b*x)^2*tan(2*a)^2 + d^3*x*tan(2*b*x)^2*t
an(2*b*c/d)^2 + d^3*x*tan(2*a)^2*tan(2*b*c/d)^2 + c*d^2*tan(2*b*x)^2*tan(2*
a)^2 + c*d^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + c*d^2*tan(2*a)^2*tan(2*b*c/d)^2
+ d^3*x*tan(2*b*x)^2 + d^3*x*tan(2*a)^2 + d^3*x*tan(2*b*c/d)^2 + c*d^2*tan(
2*b*x)^2 + c*d^2*tan(2*a)^2 + c*d^2*tan(2*b*c/d)^2 + d^3*x + c*d^2)
```

maple [A] time = 0.03, size = 156, normalized size = 1.50

$$\frac{-\frac{b^2\left(-\frac{4\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{4\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}\right)}{32}-\frac{b^2}{8((bx+a)d-da+cb)d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/32*b^2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4
*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(
-a*d+b*c)/d)/d)/d)-1/8*b^2/((b*x+a)*d-d*a+c*b)/d)
```

maxima [C] time = 0.44, size = 171, normalized size = 1.64

$$\frac{64\,b^2\left(E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_2\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^2\left(64i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-64i\,E}{1024\left(bcd+(bx+a)d^2-ad^2\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d)
+ exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c
- a*d)/d) - b^2*(64*I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*
d)/d) - 64*I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*s
in(-4*(b*c - a*d)/d) - 128*b^2)/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**2, x)
```

3.86 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=127

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{1}{16d(c+dx)^2}$$

```
[Out] -1/16/d/(d*x+c)^2+b^2*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^3+1/16*cos(4*b*x
+4*a)/d/(d*x+c)^2-b^2*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/4*b*sin(4*b*
x+4*a)/d^2/(d*x+c)
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{}{16}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -1/(16*d*(c + d*x)^2) + Cos[4*a + 4*b*x]/(16*d*(c + d*x)^2) + (b^2*Cos[4*a
- (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/d^3 - (b*Sin[4*a + 4*b*x])/(4*
d^2*(c + d*x)) - (b^2*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/
d^3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx &= \int \left(\frac{1}{8(c+dx)^3}-\frac{\cos(4a+4bx)}{8(c+dx)^3}\right)dx\\
&= -\frac{1}{16d(c+dx)^2}-\frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx}{4d}\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{b^2\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{d^2}\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}-\frac{b\sin(4a+4bx)}{4d^2(c+dx)}+\frac{\left(b^2\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\text{co}}{}}{d^2}\\
&= -\frac{1}{16d(c+dx)^2}+\frac{\cos(4a+4bx)}{16d(c+dx)^2}+\frac{b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b\sin(4}{4d^2(c}
\end{aligned}$$

Mathematica [A] time = 0.85, size = 105, normalized size = 0.83

$$\frac{16b^2\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right)-16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right)+\frac{d(-4b(c+dx)\sin(4(a+bx))+d\cos(4(a+bx))-d)}{(c+dx)^2}}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] (16*b^2*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (d*(-d + d*Co
s[4*(a + b*x)] - 4*b*(c + d*x)*Sin[4*(a + b*x)]))/(c + d*x)^2 - 16*b^2*Sin[
4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(16*d^3)
```

fricas [B] time = 0.47, size = 255, normalized size = 2.01

$$d^2\cos\left(bx+a\right)^4 - d^2\cos\left(bx+a\right)^2 - 2\left(b^2d^2x^2 + 2\,b^2cdx + b^2c^2\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{4\,(bdx+bc)}{d}\right) + \left(\left(b^2d^2x^2 + 2\,b^2cd\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + ((b^2*d
^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(4*(b*d*x + b*c)/d) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c
- a*d)/d) - 2*(2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - (b*d^2*x + b*c*d)*cos(b
*x + a))*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [C] time = 0.55, size = 5600, normalized size = 44.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*d^
2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan
(2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^
2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*
b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*
tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*d^2*x^2*real_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*d^2*x^2*real_part(
cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^2*
```

```
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan
(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(
4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*rea
l_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*d
^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^
2 + 16*b^2*c*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(
2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_integral(4*(b*d*x
 + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^2*x^2*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^
2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*
c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*t
an(2*a)^2*tan(2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x
- 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(co
s_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^2*x^2*si
n_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*c*d*x*real
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 3
2*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)
*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 8*b^2*c^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^2*c^2*sin_in
tegral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^2*c*d*
x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/
d)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(
2*b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
a)*tan(2*b*c/d)^2 + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)
*tan(2*b*c/d)^2 + 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 8*b^2*c^2*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 16*b^2*c^2*sin_integral(4*
(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 8*b^2*c*d*x*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d*x*r
eal_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*
d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*d^2*x
```

```
^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^2*c*d*x
*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 32*b^2*c
*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a) - 4*b^2*d^2*x^2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 - 4*b^2*d^2*x^2*real_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 16*b^2*c*d*x*imag_part
(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^2*c*d*x*im
ag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 32*b^2*
c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 16*b^2*d^
2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 16*b
^2*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)
+ 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)
*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 32*b^2*c*d*x*sin_integral(4*(b*d*
x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d) - 4*b^2*d^2*x^2*real_part(cos_integral(
4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 16*b^2*c*d*x*imag_part(cos_in
tegral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^2*c*d*x*imag_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 32*b^2*c*d*x*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 4*b*d^2*x*tan(2*b*x)^2
*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2*tan(2*b
*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2
 + 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 8*b
^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*d^2*x^
2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) - 16*b^2*d^2*x^2*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a) - 8*b^2*c^2*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b
*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*a)^2 - 8*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)^2 + 8*b^2*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/
d) + 16*b^2*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d) + 8*b^2*c^
2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 8*b^
2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) +
 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d) + 32*
b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d) +
```

```
32*b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d
) - 8*b^2*c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c
/d) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*
b*c/d) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)
 - 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 - 8*
b^2*c*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2 + 8*b^2*
c^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 8*b^
2*c^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 1
6*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)^2 + 4*b*c*d
*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 4*b*c*d*tan(2*b*x)*tan(2*a)^2*tan(2
*b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*
d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d)) + 4*b^2*c^2*real_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 + 4*b^2*c^2*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 - 16*b^2*c*d*x*imag_part(cos_integral(4*b*
x + 4*b*c/d))*tan(2*a) + 16*b^2*c*d*x*imag_part(cos_integral(-4*b*x - 4*b*c
/d))*tan(2*a) - 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b
*d^2*x*tan(2*b*x)^2*tan(2*a) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2 + 4*b*d^2*x*tan(2*b*x)*tan(2*a)^2 + 16*b^2*c*d*x*imag_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(-
4*b*x - 4*b*c/d))*tan(2*b*c/d) + 32*b^2*c*d*x*sin_integral(4*(b*d*x + b*c)/
d)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*a)*tan(2*b*c/d) + 16*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)*tan(2*b*c/d) - 4*b^2*c^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c
/d)^2 - 4*b*d^2*x*tan(2*b*x)*tan(2*b*c/d)^2 - 4*b*d^2*x*tan(2*a)*tan(2*b*c/
d)^2 + 8*b^2*c*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)) + 8*b^2*c*d*x*r
eal_part(cos_integral(-4*b*x - 4*b*c/d)) - 8*b^2*c^2*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*a) + 8*b^2*c^2*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a) - 16*b^2*c^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a) + 4*b
*c*d*tan(2*b*x)^2*tan(2*a) + 4*b*c*d*tan(2*b*x)*tan(2*a)^2 + 8*b^2*c^2*imag
_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d) - 8*b^2*c^2*imag_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) + 16*b^2*c^2*sin_integral(4*(b*d*
x + b*c)/d)*tan(2*b*c/d) - 4*b*c*d*tan(2*b*x)*tan(2*b*c/d)^2 - d^2*tan(2*b*
x)^2*tan(2*b*c/d)^2 - 4*b*c*d*tan(2*a)*tan(2*b*c/d)^2 - 2*d^2*tan(2*b*x)*ta
n(2*a)*tan(2*b*c/d)^2 - d^2*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*real_part
(cos_integral(4*b*x + 4*b*c/d)) + 4*b^2*c^2*real_part(cos_integral(-4*b*x -
 4*b*c/d)) - 4*b*d^2*x*tan(2*b*x) - 4*b*d^2*x*tan(2*a) - 4*b*c*d*tan(2*b*x)
 - d^2*tan(2*b*x)^2 - 4*b*c*d*tan(2*a) - 2*d^2*tan(2*b*x)*tan(2*a) - d^2*ta
n(2*a)^2)/(d^5*x^2*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2
*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*b*x)^2*tan(2*a)^2 + d^5*x
^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + d^5*x^2*tan(2*a)^2*tan(2*b*c/d)^2 + c^2*d^
3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*a)^
2 + 2*c*d^4*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*a)^2*tan(2*b*c/
d)^2 + d^5*x^2*tan(2*b*x)^2 + d^5*x^2*tan(2*a)^2 + c^2*d^3*tan(2*b*x)^2*tan
```

```
(2*a)^2 + d^5*x^2*tan(2*b*c/d)^2 + c^2*d^3*tan(2*b*x)^2*tan(2*b*c/d)^2 + c^
2*d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 2*c*d^4*x*tan(2*b*x)^2 + 2*c*d^4*x*tan(2*
a)^2 + 2*c*d^4*x*tan(2*b*c/d)^2 + d^5*x^2 + c^2*d^3*tan(2*b*x)^2 + c^2*d^3*
tan(2*a)^2 + c^2*d^3*tan(2*b*c/d)^2 + 2*c*d^4*x + c^2*d^3)
```

maple [A] time = 0.03, size = 193, normalized size = 1.52

$$\frac{-\frac{b^3\left(-\frac{2\cos(4bx+4a)}{((bx+a)d-da+cb)^2 d}-\frac{2\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{d}\right)}{32}-\frac{b^3}{16((bx+a)d-da+cb)^2 d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/32*b^3*(-2*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d-2*(-4*sin(4*b*x+4
*a)/((b*x+a)*d-d*a+c*b)/d+4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c
)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)-1/16*b^3
/((b*x+a)*d-d*a+c*b)^2/d)
```

maxima [C] time = 0.49, size = 206, normalized size = 1.62

$$\frac{64\,b^3\left(E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_3\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^3\left(64i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-64i\,E_3 \cdots\right.}{1024\left(b^2c^2d-2\,abcd^2+(bx+a)^2d^3+a^2d^3+2\left(bcd^2-ad^3\right)(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/1024*(64*b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d)
+ exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c
- a*d)/d) - b^3*(64*I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*
d)/d) - 64*I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*s
in(-4*(b*c - a*d)/d) - 64*b^3)/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3
+ a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^3, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**3, x)
```

3.87 $\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=158

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4bx)}{12d^2(c+dx)^2}+\frac{\cos(4a}{24d(c}$$

```
[Out] -1/24/d/(d*x+c)^3+1/24*cos(4*b*x+4*a)/d/(d*x+c)^3-1/3*b^2*cos(4*b*x+4*a)/d^
3/(d*x+c)-4/3*b^3*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*x)/d^4-4/3*b^3*Ci(4*b*c/d
+4*b*x)*sin(4*a-4*b*c/d)/d^4-1/12*b*sin(4*b*x+4*a)/d^2/(d*x+c)^2
```

Rubi [A] time = 0.23, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{4b^3\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4bc}{d}+4bx\right)}{3d^4}-\frac{b^2\cos(4a+4bx)}{3d^3(c+dx)}-\frac{b\sin(4a+4b}{12d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/(24*d*(c + d*x)^3) + Cos[4*a + 4*b*x]/(24*d*(c + d*x)^3) - (b^2*Cos[4*a
+ 4*b*x])/(3*d^3*(c + d*x)) - (4*b^3*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a
 - (4*b*c)/d])/(3*d^4) - (b*Sin[4*a + 4*b*x])/(12*d^2*(c + d*x)^2) - (4*b^3
*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{1}{8(c+dx)^4} - \frac{\cos(4a+4bx)}{8(c+dx)^4}\right) dx$$

$$= -\frac{1}{24d(c+dx)^3} - \frac{1}{8}\int \frac{\cos(4a+4bx)}{(c+dx)^4}\,dx$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} + \frac{b\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} + \frac{b^2\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{3d^2}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{\left(4b\right.}{}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{b\sin(4a+4bx)}{12d^2(c+dx)^2} - \frac{\left(4b\right.}{}$$

$$= -\frac{1}{24d(c+dx)^3} + \frac{\cos(4a+4bx)}{24d(c+dx)^3} - \frac{b^2\cos(4a+4bx)}{3d^3(c+dx)} - \frac{4b^3\text{Ci}\left(\frac{4bc}{d}+4bx\right)\sin}{3d^4}$$

Mathematica [A] time = 1.65, size = 123, normalized size = 0.78

$$-\frac{32b^3\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right) + 32b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b(c+dx)}{d}\right) + \frac{d\left(\cos(4(a+bx))\left(8b^2(c+dx)^2-d^2\right)+d(2b(c+dx)\sin(4(a+b\right.}{(c+dx)^3}}{24d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/24*(32*b^3*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + (d*((-d
^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + d*(d + 2*b*(c + d*x)*Sin[4*(a +
b*x)])))/(c + d*x)^3 + 32*b^3*Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*
x))/d])/d^4
```

fricas [B] time = 0.54, size = 406, normalized size = 2.57

$$-\frac{b^2d^3x^2 + 2\,b^2cd^2x + b^2c^2d + \left(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c^2d - d^3\right)\cos\left(bx + a\right)^4 - \left(8\,b^2d^3x^2 + 16\,b^2cd^2x + 8\,b^2c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + (8*b^2*d^3*x^2 + 16*b^2*c*d
^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^4 - (8*b^2*d^3*x^2 + 16*b^2*c*d^2*x
+ 8*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + b^3*c^3)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d
) + (2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a
))*sin(b*x + a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c
^3)*cos_integral(4*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d
))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

giac [C] time = 0.64, size = 8508, normalized size = 53.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/12*(8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integr
al(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x
^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b
*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)
^2*tan(2*a)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 + 24*b^
3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^
```

2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*d^3*x^3*im
ag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 8*b^3*d^3*
x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 16*
b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 + 32*b^
3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*ta
n(2*b*c/d) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2
*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_part(cos_int
egral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2
*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(
2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x
)^2*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)
/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4
*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*re
al_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^
2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2 - 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a
)^2*tan(2*b*c/d)^2 + 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a
)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^
2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^
2 + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a) + 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2*tan(2*a)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x +
b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_int
egral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 96
*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2
*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*
b*x)^2*tan(2*a)*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*b^3*d^3*x^3*real_part(cos_integral
(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^2*d*x*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan
(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4
*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d
*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(cos_int

egral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*a)*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c*d^2*x^2*sin_integra
l(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2*tan(2*b*x)^2
*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 8*
b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - 8*b^3*d
^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 16*b^3*d^3*
x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 48*b^3*c*d^2*x^2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 48*b^3*c*d^2*x^2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 8*b^3*d^3
*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 8*b^3*d^3*x^3*im
ag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 16*b^3*d^3*x^3*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*a)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 + 24*b^3*c^2*d*x*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c^2*d*x*sin_int
egral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)^2 - 48*b^3*c*d^2*x^2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) - 48*b^3*c*d^2*
x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d) + 3
2*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d
) - 32*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2
*b*c/d) + 64*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c
/d) + 96*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*
tan(2*a)*tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*
c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 192*b^3*c^2*d*x*sin_integral(4*(
b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 48*b^3*c*d^2*x^2*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 48*b^3*c*d^2*
x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 16*
b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x
)^2*tan(2*a)^2*tan(2*b*c/d) - 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x +
4*b*c/d))*tan(2*b*c/d)^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*c/d)^2 - 16*b^3*d^3*x^3*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*x)^2*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*sin_integral(4*(b*d*x +
 b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(cos_integ
ral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*real_part(
cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*c^3*real_p

```
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2 - 1
6*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*a)^2*tan(2*b*c/d)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b
*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*a)^2*ta
n(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*x)^2 - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 +
 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 16*b^3*
d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 48*b^3*c^2*d*x
*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 48*b^3*c^
2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*
b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 24*b^3*
c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 48*b^3*c*d
^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*b^2*d^3*x^2*tan(2*b*x
)^2*tan(2*a)^2 - 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b
*x)^2*tan(2*a)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*b*x)^2*tan(2*a)^2 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)
^2*tan(2*a)^2 - 16*b^3*d^3*x^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan
(2*b*c/d) - 16*b^3*d^3*x^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*
b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)
^2*tan(2*b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*x)^2*tan(2*b*c/d) + 96*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(2*b*c/d) - 96*b^3*c*d^2*x^2*imag_part(cos_integral(
-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d) + 192*b^3*c*d^2*x^2*sin_integral(4
*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d) + 32*b^3*c^3*imag_part(cos_integral
(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) - 32*b^3*c^3*imag_par
t(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) + 64*b
^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d) +
 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b
*c/d) + 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
*tan(2*b*c/d) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*b*c/d)^2 - 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c
/d)^2 - 4*b^2*d^3*x^2*tan(2*b*x)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos
_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_pa
rt(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*c^3
*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2*tan(2*b*c/d)^2 - 48*b^3*c^2*d
*x*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 48*b^
3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2
 - 16*b^2*d^3*x^2*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - 4*b^2*d^3*x^2*tan(2*
a)^2*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*a)^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d
))*tan(2*a)^2*tan(2*b*c/d)^2 + 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*t
```

an(2*a)^2*tan(2*b*c/d)^2 + 4*b^2*c^2*d*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)
^2 + 8*b^3*d^3*x^3*imag_part(cos_integral(4*b*x + 4*b*c/d)) - 8*b^3*d^3*x^3
*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 16*b^3*d^3*x^3*sin_integral(4*
(b*d*x + b*c)/d) + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*
tan(2*b*x)^2 - 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*b*x)^2 + 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 4
8*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 48*b^3*
c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 16*b^3*c^3*r
eal_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*a) + 16*b^3*c^3*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*tan(2*a) - 24*b^3*c^
2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 24*b^3*c^2*d*x*
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2 - 48*b^3*c^2*d*x*sin_i
ntegral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*a)
^2 - 48*b^3*c*d^2*x^2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)
 - 48*b^3*c*d^2*x^2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)
- 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2*tan(2*b*
c/d) - 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2*ta
n(2*b*c/d) + 96*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*
a)*tan(2*b*c/d) - 96*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d))*
tan(2*a)*tan(2*b*c/d) + 192*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*a)*tan(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*a)^2*tan(2*b*c/d) + 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d)
)*tan(2*a)^2*tan(2*b*c/d) - 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4
*b*c/d))*tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*
b*c/d))*tan(2*b*c/d)^2 - 48*b^3*c^2*d*x*sin_integral(4*(b*d*x + b*c)/d)*tan
(2*b*c/d)^2 - 8*b^2*c*d^2*x*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^3*c^3*real_p
art(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 16*b^3*c^3*rea
l_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 32*b^2*c*d
^2*x*tan(2*b*x)*tan(2*a)*tan(2*b*c/d)^2 - 2*b*d^3*x*tan(2*b*x)^2*tan(2*a)*t
an(2*b*c/d)^2 - 8*b^2*c*d^2*x*tan(2*a)^2*tan(2*b*c/d)^2 - 2*b*d^3*x*tan(2*b
*x)*tan(2*a)^2*tan(2*b*c/d)^2 + 24*b^3*c*d^2*x^2*imag_part(cos_integral(4*b
*x + 4*b*c/d)) - 24*b^3*c*d^2*x^2*imag_part(cos_integral(-4*b*x - 4*b*c/d))
 + 48*b^3*c*d^2*x^2*sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*d^3*x^2*tan(2*b
*x)^2 + 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*x)^2 - 8
*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*x)^2 + 16*b^3*c^
3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*x)^2 + 48*b^3*c^2*d*x*real_part(c
os_integral(4*b*x + 4*b*c/d))*tan(2*a) + 48*b^3*c^2*d*x*real_part(cos_integ
ral(-4*b*x - 4*b*c/d))*tan(2*a) - 16*b^2*d^3*x^2*tan(2*b*x)*tan(2*a) - 4*b^
2*d^3*x^2*tan(2*a)^2 - 8*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*t
an(2*a)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2
- 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*b^2*c^2*d*tan(2
*b*x)^2*tan(2*a)^2 - 48*b^3*c^2*d*x*real_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(2*b*c/d) - 48*b^3*c^2*d*x*real_part(cos_integral(-4*b*x - 4*b*c/d))*t
an(2*b*c/d) + 32*b^3*c^3*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*
tan(2*b*c/d) - 32*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a

```
)*tan(2*b*c/d) + 64*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*
b*c/d) + 4*b^2*d^3*x^2*tan(2*b*c/d)^2 - 8*b^3*c^3*imag_part(cos_integral(4*
b*x + 4*b*c/d))*tan(2*b*c/d)^2 + 8*b^3*c^3*imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(2*b*c/d)^2 - 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d)*tan(2
*b*c/d)^2 - 4*b^2*c^2*d*tan(2*b*x)^2*tan(2*b*c/d)^2 - 16*b^2*c^2*d*tan(2*b*
x)*tan(2*a)*tan(2*b*c/d)^2 - 2*b*c*d^2*tan(2*b*x)^2*tan(2*a)*tan(2*b*c/d)^2
 - 4*b^2*c^2*d*tan(2*a)^2*tan(2*b*c/d)^2 - 2*b*c*d^2*tan(2*b*x)*tan(2*a)^2*
tan(2*b*c/d)^2 + 24*b^3*c^2*d*x*imag_part(cos_integral(4*b*x + 4*b*c/d)) -
24*b^3*c^2*d*x*imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 48*b^3*c^2*d*x*s
in_integral(4*(b*d*x + b*c)/d) - 8*b^2*c*d^2*x*tan(2*b*x)^2 + 16*b^3*c^3*re
al_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a) + 16*b^3*c^3*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a) - 32*b^2*c*d^2*x*tan(2*b*x)*tan(2*a) -
 2*b*d^3*x*tan(2*b*x)^2*tan(2*a) - 8*b^2*c*d^2*x*tan(2*a)^2 - 2*b*d^3*x*tan
(2*b*x)*tan(2*a)^2 - 16*b^3*c^3*real_part(cos_integral(4*b*x + 4*b*c/d))*ta
n(2*b*c/d) - 16*b^3*c^3*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c
/d) + 8*b^2*c*d^2*x*tan(2*b*c/d)^2 + 2*b*d^3*x*tan(2*b*x)*tan(2*b*c/d)^2 +
2*b*d^3*x*tan(2*a)*tan(2*b*c/d)^2 + 4*b^2*d^3*x^2 + 8*b^3*c^3*imag_part(cos
_integral(4*b*x + 4*b*c/d)) - 8*b^3*c^3*imag_part(cos_integral(-4*b*x - 4*b
*c/d)) + 16*b^3*c^3*sin_integral(4*(b*d*x + b*c)/d) - 4*b^2*c^2*d*tan(2*b*x
)^2 - 16*b^2*c^2*d*tan(2*b*x)*tan(2*a) - 2*b*c*d^2*tan(2*b*x)^2*tan(2*a) -
4*b^2*c^2*d*tan(2*a)^2 - 2*b*c*d^2*tan(2*b*x)*tan(2*a)^2 + 4*b^2*c^2*d*tan(
2*b*c/d)^2 + 2*b*c*d^2*tan(2*b*x)*tan(2*b*c/d)^2 + d^3*tan(2*b*x)^2*tan(2*b
*c/d)^2 + 2*b*c*d^2*tan(2*a)*tan(2*b*c/d)^2 + 2*d^3*tan(2*b*x)*tan(2*a)*tan
(2*b*c/d)^2 + d^3*tan(2*a)^2*tan(2*b*c/d)^2 + 8*b^2*c*d^2*x + 2*b*d^3*x*tan
(2*b*x) + 2*b*d^3*x*tan(2*a) + 4*b^2*c^2*d + 2*b*c*d^2*tan(2*b*x) + d^3*tan
(2*b*x)^2 + 2*b*c*d^2*tan(2*a) + 2*d^3*tan(2*b*x)*tan(2*a) + d^3*tan(2*a)^2
)/(d^7*x^3*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^
2*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*b*x)^2*tan(2*a)^2 + d^7*x^3*tan
(2*b*x)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c^2*d^5*x*
tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^2*tan(2*a)^
2 + 3*c*d^6*x^2*tan(2*b*x)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*a)^2*tan(2*
b*c/d)^2 + c^3*d^4*tan(2*b*x)^2*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3*tan(2*b
*x)^2 + d^7*x^3*tan(2*a)^2 + 3*c^2*d^5*x*tan(2*b*x)^2*tan(2*a)^2 + d^7*x^3*
tan(2*b*c/d)^2 + 3*c^2*d^5*x*tan(2*b*x)^2*tan(2*b*c/d)^2 + 3*c^2*d^5*x*tan(
2*a)^2*tan(2*b*c/d)^2 + 3*c*d^6*x^2*tan(2*b*x)^2 + 3*c*d^6*x^2*tan(2*a)^2 +
 c^3*d^4*tan(2*b*x)^2*tan(2*a)^2 + 3*c*d^6*x^2*tan(2*b*c/d)^2 + c^3*d^4*tan
(2*b*x)^2*tan(2*b*c/d)^2 + c^3*d^4*tan(2*a)^2*tan(2*b*c/d)^2 + d^7*x^3 + 3*
c^2*d^5*x*tan(2*b*x)^2 + 3*c^2*d^5*x*tan(2*a)^2 + 3*c^2*d^5*x*tan(2*b*c/d)^
2 + 3*c*d^6*x^2 + c^3*d^4*tan(2*b*x)^2 + c^3*d^4*tan(2*a)^2 + c^3*d^4*tan(2
*b*c/d)^2 + 3*c^2*d^5*x + c^3*d^4)
```

maple [A] time = 0.03, size = 230, normalized size = 1.46

$$-\frac{-\frac{b^4\left(-\frac{4\cos(4bx+4a)}{3((bx+a)d-da+cb)^3 d}-\frac{4\left(-\frac{2\sin(4bx+4a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{8\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{8\left(\frac{4\operatorname{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\operatorname{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}}{d}\right)}{3d}\right)}{32}-\cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x)

[Out] 1/b*(-1/32*b^4*(-4/3*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^3/d-4/3*(-2*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d+2*(-4*cos(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)/d)-1/24*b^4/((b*x+a)*d-d*a+c*b)^3/d)

maxima [C] time = 0.61, size = 256, normalized size = 1.62

$$\frac{3\,b^4\left(E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)+E_4\left(-\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)\right)\cos\left(-\frac{4\,(bc-ad)}{d}\right)-b^4\left(3i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-3i\,E_4\left(-\cdots\right.\right.}{48\left(b^3c^3d-3\,ab^2c^2d^2+3\,a^2bcd^3+(bx+a)^3d^4-a^3d^4+3\left(bcd^3-ad^4\right)(bx+a)^2+3\left(b^2c^2d^2-2\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)^2*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")

[Out] 1/48*(3*b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*(b*c - a*d)/d) - b^4*(3*I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d) - 3*I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*(b*c - a*d)/d) - 2*b^4)/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^2)/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^2(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**2/(c + d*x)**4, x)
```

3.88 $\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}+\cdots$$

```
[Out] -1/16*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/16*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*
x+c)/d)^m)-1/32*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*b*(d*x
+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I*b*(d*x+
c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*5^(-1-m)*exp(5*I*(a-b*c
/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/32*5^(-
1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/((I*b*(d*x+c
)/d)^m)
```

Rubi [A] time = 0.40, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3308, 2181}

$$-\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}-\frac{3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+\cdots\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(16*b
*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/
(16*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) - (3^(-1 - m)*E^((3*I)*(a
- (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d])/(32*b*(((-I)*
b*(c + d*x))/d)^m) - (3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((3*I)*b*(c + d*x
))/d])/(32*b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + (5^(-1 - m)*E
^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d])/(3
2*b*(((-I)*b*(c + d*x))/d)^m) + (5^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((5*I)
*b*(c + d*x))/d])/(32*b*E^((5*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m \sin(a+bx) + \frac{1}{16}(c+dx)^m \sin(3a+3bx) - \frac{1}{16}(c+d\right.$$
$$= \frac{1}{16}\int (c+dx)^m \sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^m \sin(5a+5bx)\,dx$$
$$= \frac{1}{32}i\int e^{-i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32}i\int e^{i(3a+3bx)}(c+dx)^m\,dx - \frac{1}{32}i\int$$
$$= -\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b} - \frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+a}{}$$

Mathematica [A] time = 0.63, size = 376, normalized size = 0.92

$$\frac{e^{-\frac{5i(ad+bc)}{d}}(c+dx)^m\left(-5\,3^{-m}e^{\frac{2i(ad+bc)}{d}}\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)+e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^m*(30*E^(((4*I)*(b*c + a*d))/d)*(-((E^((2*I)*a)*Gamma[1 + m, ((-
I)*b*(c + d*x))/d])/(((-I)*b*(c + d*x))/d)^m) - (E^(((2*I)*b*c)/d)*Gamma[1
+ m, (I*b*(c + d*x))/d])/((I*b*(c + d*x))/d)^m) - (5*E^(((2*I)*(b*c + a*d))
/d)*(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
 + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*b*(c + d*
x))/d]))/(3^m*((b^2*(c + d*x)^2)/d^2)^m) + (3*(E^((10*I)*a)*((I*b*(c + d*x)
)/d)^m*Gamma[1 + m, ((-5*I)*b*(c + d*x))/d] + E^(((10*I)*b*c)/d)*(((-I)*b*(
```

```
c + d*x))/d)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(5^m*((b^2*(c + d*x)^2
)/d^2)^m)))/(480*b*E^(((5*I)*(b*c + a*d))/d))
```

fricas [A] time = 0.72, size = 276, normalized size = 0.68

$$\frac{3\,e^{\left(-\frac{dm\log\left(\frac{5i\,b}{d}\right)-5i\,bc+5i\,ad}{d}\right)}\Gamma\left(m+1,\frac{5i\,bdx+5i\,bc}{d}\right)-5\,e^{\left(-\frac{dm\log\left(\frac{3i\,b}{d}\right)-3i\,bc+3i\,ad}{d}\right)}\Gamma\left(m+1,\frac{3i\,bdx+3i\,bc}{d}\right)-30\,e^{\left(-\frac{dm\log\left(\frac{i\,b}{d}\right)-i\,bc+i\,ad}{d}\right)}\Gamma\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, (5*I*b*
d*x + 5*I*b*c)/d) - 5*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*gamma(m
 + 1, (3*I*b*d*x + 3*I*b*c)/d) - 30*e^(-(d*m*log(I*b/d) - I*b*c + I*a*d)/d)
*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 30*e^(-(d*m*log(-I*b/d) + I*b*c - I*a*
d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) - 5*e^(-(d*m*log(-3*I*b/d) + 3*I*b
*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3*e^(-(d*m*log(-5
*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*gamma(m + 1, (-5*I*b*d*x - 5*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos (bx + a)^2 \sin (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

maple [F] time = 0.21, size = 0, normalized size = 0.00

$$\int (dx + c)^m \left(\cos^2 (bx + a)\right) \left(\sin^3 (bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos (bx + a)^2 \sin (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.89 $\int (c+dx)^4 \cos^2(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=330

$$-\frac{3d^4\cos(a+bx)}{b^5}-\frac{d^4\cos(3a+3bx)}{162b^5}+\frac{3d^4\cos(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3}{}$$

```
[Out] -3*d^4*cos(b*x+a)/b^5+3/2*d^2*(d*x+c)^2*cos(b*x+a)/b^3-1/8*(d*x+c)^4*cos(b*
x+a)/b-1/162*d^4*cos(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*cos(3*b*x+3*a)/b^3-1
/48*(d*x+c)^4*cos(3*b*x+3*a)/b+3/6250*d^4*cos(5*b*x+5*a)/b^5-3/500*d^2*(d*x
+c)^2*cos(5*b*x+5*a)/b^3+1/80*(d*x+c)^4*cos(5*b*x+5*a)/b-3*d^3*(d*x+c)*sin(
b*x+a)/b^4+1/2*d*(d*x+c)^3*sin(b*x+a)/b^2-1/54*d^3*(d*x+c)*sin(3*b*x+3*a)/b
^4+1/36*d*(d*x+c)^3*sin(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c)*sin(5*b*x+5*a)/b^
4-1/100*d*(d*x+c)^3*sin(5*b*x+5*a)/b^2
```

Rubi [A] time = 0.39, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{3d^3(c+dx)\sin(a+bx)}{b^4}-\frac{d^3(c+dx)\sin(3a+3bx)}{54b^4}+\frac{3d^3(c+dx)\sin(5a+5bx)}{1250b^4}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}+\frac{d^2(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-3*d^4*Cos[a + b*x])/b^5 + (3*d^2*(c + d*x)^2*Cos[a + b*x])/(2*b^3) - ((c
+ d*x)^4*Cos[a + b*x])/(8*b) - (d^4*Cos[3*a + 3*b*x])/(162*b^5) + (d^2*(c +
 d*x)^2*Cos[3*a + 3*b*x])/(36*b^3) - ((c + d*x)^4*Cos[3*a + 3*b*x])/(48*b)
+ (3*d^4*Cos[5*a + 5*b*x])/(6250*b^5) - (3*d^2*(c + d*x)^2*Cos[5*a + 5*b*x]
)/(500*b^3) + ((c + d*x)^4*Cos[5*a + 5*b*x])/(80*b) - (3*d^3*(c + d*x)*Sin[
a + b*x])/b^4 + (d*(c + d*x)^3*Sin[a + b*x])/(2*b^2) - (d^3*(c + d*x)*Sin[3
*a + 3*b*x])/(54*b^4) + (d*(c + d*x)^3*Sin[3*a + 3*b*x])/(36*b^2) + (3*d^3*
(c + d*x)*Sin[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)^3*Sin[5*a + 5*b*x])/(
100*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\sin(a+bx)+\frac{1}{16}(c+dx)^4\sin(3a+3bx)-\frac{1}{16}(c+dx)\right.$$

$$= \frac{1}{16}\int (c+dx)^4\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^4\sin(5a+5bx)\,dx+$$

$$= -\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a}{80b}$$

$$= -\frac{(c+dx)^4\cos(a+bx)}{8b}-\frac{(c+dx)^4\cos(3a+3bx)}{48b}+\frac{(c+dx)^4\cos(5a}{80b}$$

$$= \frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2\cos(}{36b^3}$$

$$= \frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx)}{8b}+\frac{d^2(c+dx)^2\cos(}{36b^3}$$

$$= -\frac{3d^4\cos(a+bx)}{b^5}+\frac{3d^2(c+dx)^2\cos(a+bx)}{2b^3}-\frac{(c+dx)^4\cos(a+bx}{8b}$$

Mathematica [A] time = 3.17, size = 238, normalized size = 0.72

$$\frac{120bd(c+dx)\sin(a+bx)\left(16\cos(2(a+bx))\left(75b^2(c+dx)^2-68d^2\right)-27\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-506250*(24*d^4 - 12*b^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Cos[a + b*x] -
 3125*(8*d^4 - 36*b^2*d^2*(c + d*x)^2 + 27*b^4*(c + d*x)^4)*Cos[3*(a + b*x)
] + 81*(24*d^4 - 300*b^2*d^2*(c + d*x)^2 + 625*b^4*(c + d*x)^4)*Cos[5*(a +
b*x)] + 120*b*d*(c + d*x)*(17475*b^2*c^2 - 101794*d^2 + 34950*b^2*c*d*x + 1
7475*b^2*d^2*x^2 + 16*(-68*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 27*
(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/(4050000*b^5)
```

fricas [A] time = 0.81, size = 471, normalized size = 1.43

$$\frac{81\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+625\,b^4c^4-300\,b^2c^2d^2+24\,d^4+150\left(25\,b^4c^2d^2-2\,b^2d^4\right)x^2+100\left(25\,b^4c^3d-6\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/253125*(81*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*
c^2*d^2 + 24*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d
 - 6*b^2*c*d^3)*x)*cos(b*x + a)^5 - 5*(16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*
x^3 + 16875*b^4*c^4 - 11700*b^2*c^2*d^2 + 1736*d^4 + 450*(225*b^4*c^2*d^2 -
 26*b^2*d^4)*x^2 + 900*(75*b^4*c^3*d - 26*b^2*c*d^3)*x)*cos(b*x + a)^3 + 12
0*(2925*b^2*d^4*x^2 + 5850*b^2*c*d^3*x + 2925*b^2*c^2*d^2 - 6284*d^4)*cos(b
*x + a) + 60*(1950*b^3*d^4*x^3 + 5850*b^3*c*d^3*x^2 + 1950*b^3*c^3*d - 1256
8*b*c*d^3 - 27*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d^
3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^4 + (975*b^3*d^4*x^3 + 292
5*b^3*c*d^3*x^2 + 975*b^3*c^3*d - 434*b*c*d^3 + (2925*b^3*c^2*d^2 - 434*b*d
^4)*x)*cos(b*x + a)^2 + 2*(2925*b^3*c^2*d^2 - 6284*b*d^4)*x)*sin(b*x + a))/
b^5
```

giac [A] time = 0.87, size = 531, normalized size = 1.61

$$\frac{\left(625\,b^4d^4x^4+2500\,b^4cd^3x^3+3750\,b^4c^2d^2x^2+2500\,b^4c^3dx+625\,b^4c^4-300\,b^2d^4x^2-600\,b^2cd^3x-300\,b^2c^2d^2+\right.}{50000\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/50000*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500
*b^4*c^3*d*x + 625*b^4*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^
2*d^2 + 24*d^4)*cos(5*b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d
^3*x^3 + 162*b^4*c^2*d^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^
2 - 72*b^2*c*d^3*x - 36*b^2*c^2*d^2 + 8*d^4)*cos(3*b*x + 3*a)/b^5 - 1/8*(b^
4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 -
 12*b^2*d^4*x^2 - 24*b^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*cos(b*x + a)/b^
5 - 1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c
^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(5*b*x + 5*a)/b^5 + 1/108*(3*b^3*d^4*x^3 +
 9*b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*s
in(3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
+ b^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*sin(b*x + a)/b^5
```

maple [B] time = 0.12, size = 1812, normalized size = 5.49

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x)

[Out] 1/b*(1/b^4*d^4*(-1/3*(b*x+a)^4*(2+sin(b*x+a)^2)*cos(b*x+a)+8/15*(b*x+a)^3*sin(b*x+a)+8/5*(b*x+a)^2*cos(b*x+a)-3424/1125*cos(b*x+a)-3424/1125*(b*x+a)*sin(b*x+a)+4/45*(b*x+a)^3*sin(b*x+a)^3+4/45*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+88/3375*(b*x+a)*sin(b*x+a)^3+88/10125*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^4*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-4/25*(b*x+a)^3*sin(b*x+a)^5-12/125*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+24/625*(b*x+a)*sin(b*x+a)^5+24/3125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-4/b^4*a*d^4*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*sin(b*x+a)^5)+4/b^3*c*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*sin(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*sin(b*x+a)^5)+6/b^4*a^2*d^4*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-12/b^3*a*c*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))+6/b^2*c^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-4/b^4*a^3*d^4*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+12/b^3*a^2*c*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-12/b^2*a*c^2*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+4/b*c^3*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-4/b^3*a^3*c*d^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+6/b^2*a^2*c^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-

```
4/b*a*c^3*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c^4*(-1/5*si
n(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

maxima [B] time = 0.42, size = 1339, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/4050000*(270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^4 - 1080000*(3*co
s(b*x + a)^5 - 5*cos(b*x + a)^3)*a*c^3*d/b + 1620000*(3*cos(b*x + a)^5 - 5*
cos(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*cos(b*x + a)^5 - 5*cos(b*x + a
)^3)*a^3*c*d^3/b^3 + 270000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^4*d^4/b
^4 + 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) -
450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450
*sin(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a
)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*s
in(3*b*x + 3*a) + 450*sin(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*cos
(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a)
- 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*c*d^3/b^
3 - 4500*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 4
50*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*
sin(b*x + a))*a^3*d^4/b^4 + 450*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) -
 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x
+ a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13
500*(b*x + a)*sin(b*x + a))*c^2*d^2/b^2 - 900*(27*(25*(b*x + a)^2 - 2)*cos(
5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2
 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b
*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a*c*d^3/b^3 + 450*(27*(25*(b*x +
a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 675
0*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*
x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*a^2*d^4/b^4 + 60*(1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x +
 a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*si
n(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*c*d^3/b^3 - 60*(135
*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) - 1875*(3*(b*x + a)^3 - 2*
b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*sin(
3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x + a))*a*d^4/b^4 + (81*(625*
(b*x + a)^4 - 300*(b*x + a)^2 + 24)*cos(5*b*x + 5*a) - 3125*(27*(b*x + a)^4
 - 36*(b*x + a)^2 + 8)*cos(3*b*x + 3*a) - 506250*((b*x + a)^4 - 12*(b*x + a
)^2 + 24)*cos(b*x + a) - 1620*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*
```

```
a) + 37500*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x + 3*a) + 2025000*((b*x +
 a)^3 - 6*b*x - 6*a)*sin(b*x + a))*d^4/b^4)/b
```

mupad [B] time = 4.61, size = 816, normalized size = 2.47

$$-\frac{3\,d^4\,\cos\left(a+b\,x\right)+\frac{d^4\,\cos(3\,a+3\,b\,x)}{162}-\frac{3\,d^4\,\cos(5\,a+5\,b\,x)}{6250}+\frac{b^4\,c^4\,\cos(a+b\,x)}{8}+\frac{b^4\,c^4\,\cos(3\,a+3\,b\,x)}{48}-\frac{b^4\,c^4\,\cos(5\,a+5\,b\,x)}{80}-\frac{3\,b^2\,c^2\,d}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(3*d^4*cos(a + b*x) + (d^4*cos(3*a + 3*b*x))/162 - (3*d^4*cos(5*a + 5*b*x)
)/6250 + (b^4*c^4*cos(a + b*x))/8 + (b^4*c^4*cos(3*a + 3*b*x))/48 - (b^4*c^
4*cos(5*a + 5*b*x))/80 - (3*b^2*c^2*d^2*cos(a + b*x))/2 - (b^3*c^3*d*sin(3*
a + 3*b*x))/36 + (b^3*c^3*d*sin(5*a + 5*b*x))/100 - (3*b^2*d^4*x^2*cos(a +
b*x))/2 + (b^4*d^4*x^4*cos(a + b*x))/8 - (b^3*d^4*x^3*sin(a + b*x))/2 + 3*b
*c*d^3*sin(a + b*x) - (b^2*c^2*d^2*cos(3*a + 3*b*x))/36 + (3*b^2*c^2*d^2*co
s(5*a + 5*b*x))/500 + 3*b*d^4*x*sin(a + b*x) - (b^2*d^4*x^2*cos(3*a + 3*b*x
))/36 + (3*b^2*d^4*x^2*cos(5*a + 5*b*x))/500 + (b^4*d^4*x^4*cos(3*a + 3*b*x
))/48 - (b^4*d^4*x^4*cos(5*a + 5*b*x))/80 - (b^3*d^4*x^3*sin(3*a + 3*b*x))/
36 + (b^3*d^4*x^3*sin(5*a + 5*b*x))/100 + (b*c*d^3*sin(3*a + 3*b*x))/54 - (
3*b*c*d^3*sin(5*a + 5*b*x))/1250 - (b^3*c^3*d*sin(a + b*x))/2 + (b*d^4*x*si
n(3*a + 3*b*x))/54 - (3*b*d^4*x*sin(5*a + 5*b*x))/1250 - 3*b^2*c*d^3*x*cos(
a + b*x) + (b^4*c^3*d*x*cos(a + b*x))/2 + (b^4*c^2*d^2*x^2*cos(3*a + 3*b*x)
)/8 - (3*b^4*c^2*d^2*x^2*cos(5*a + 5*b*x))/40 - (b^2*c*d^3*x*cos(3*a + 3*b*
x))/18 + (b^4*c^3*d*x*cos(3*a + 3*b*x))/12 + (3*b^2*c*d^3*x*cos(5*a + 5*b*x
))/250 - (b^4*c^3*d*x*cos(5*a + 5*b*x))/20 + (b^4*c*d^3*x^3*cos(a + b*x))/2
 - (3*b^3*c^2*d^2*x*sin(a + b*x))/2 - (3*b^3*c*d^3*x^2*sin(a + b*x))/2 + (b
^4*c*d^3*x^3*cos(3*a + 3*b*x))/12 - (b^4*c*d^3*x^3*cos(5*a + 5*b*x))/20 + (
3*b^4*c^2*d^2*x^2*cos(a + b*x))/4 - (b^3*c^2*d^2*x*sin(3*a + 3*b*x))/12 - (
b^3*c*d^3*x^2*sin(3*a + 3*b*x))/12 + (3*b^3*c^2*d^2*x*sin(5*a + 5*b*x))/100
 + (3*b^3*c*d^3*x^2*sin(5*a + 5*b*x))/100)/b^5
```

sympy [A] time = 20.16, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**4*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**4*cos(a + b*x
)**5/(15*b) - 4*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 8*c**3*d*x
*cos(a + b*x)**5/(15*b) - 2*c**2*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/
b - 4*c**2*d**2*x**2*cos(a + b*x)**5/(5*b) - 4*c*d**3*x**3*sin(a + b*x)**2*
```

```
cos(a + b*x)**3/(3*b) - 8*c*d**3*x**3*cos(a + b*x)**5/(15*b) - d**4*x**4*si
n(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**4*x**4*cos(a + b*x)**5/(15*b) +
104*c**3*d*sin(a + b*x)**5/(225*b**2) + 52*c**3*d*sin(a + b*x)**3*cos(a + b
*x)**2/(45*b**2) + 8*c**3*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2) + 104*c*
*2*d**2*x*sin(a + b*x)**5/(75*b**2) + 52*c**2*d**2*x*sin(a + b*x)**3*cos(a
+ b*x)**2/(15*b**2) + 8*c**2*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(5*b**2) +
 104*c*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 52*c*d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**2/(15*b**2) + 8*c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**4/(5*
b**2) + 104*d**4*x**3*sin(a + b*x)**5/(225*b**2) + 52*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)**2/(45*b**2) + 8*d**4*x**3*sin(a + b*x)*cos(a + b*x)**4/(
15*b**2) + 104*c**2*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 676*c**2*
d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*c**2*d**2*cos(a + b*
x)**5/(1125*b**3) + 208*c*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3) + 1
352*c*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 3424*c*d**3*x*cos
(a + b*x)**5/(1125*b**3) + 104*d**4*x**2*sin(a + b*x)**4*cos(a + b*x)/(75*b
**3) + 676*d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 1712*d**4
*x**2*cos(a + b*x)**5/(1125*b**3) - 50272*c*d**3*sin(a + b*x)**5/(16875*b**
4) - 20456*c*d**3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 3424*c*d**3
*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*x*sin(a + b*x)**5/(1
6875*b**4) - 20456*d**4*x*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 342
4*d**4*x*sin(a + b*x)*cos(a + b*x)**4/(1125*b**4) - 50272*d**4*sin(a + b*x)
**4*cos(a + b*x)/(16875*b**5) - 303368*d**4*sin(a + b*x)**2*cos(a + b*x)**3
/(50625*b**5) - 760816*d**4*cos(a + b*x)**5/(253125*b**5), Ne(b, 0)), ((c**
4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*
*3*cos(a)**2, True))
```

3.90 $\int (c+dx)^3 \cos^2(a+bx) \sin^3(a+bx)\,dx$

Optimal. Leaf size=259

$$-\frac{3d^3\sin(a+bx)}{4b^4}-\frac{d^3\sin(3a+3bx)}{216b^4}+\frac{3d^3\sin(5a+5bx)}{5000b^4}+\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3a}{}$$

```
[Out] 3/4*d^2*(d*x+c)*cos(b*x+a)/b^3-1/8*(d*x+c)^3*cos(b*x+a)/b+1/72*d^2*(d*x+c)*
cos(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*cos(3*b*x+3*a)/b-3/1000*d^2*(d*x+c)*cos(5
*b*x+5*a)/b^3+1/80*(d*x+c)^3*cos(5*b*x+5*a)/b-3/4*d^3*sin(b*x+a)/b^4+3/8*d*
(d*x+c)^2*sin(b*x+a)/b^2-1/216*d^3*sin(3*b*x+3*a)/b^4+1/48*d*(d*x+c)^2*sin(
3*b*x+3*a)/b^2+3/5000*d^3*sin(5*b*x+5*a)/b^4-3/400*d*(d*x+c)^2*sin(5*b*x+5*
a)/b^2
```

Rubi [A] time = 0.28, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}+\frac{d^2(c+dx)\cos(3a+3bx)}{72b^3}-\frac{3d^2(c+dx)\cos(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\sin(a+bx)}{8b^2}+\frac{d(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*(c + d*x)*Cos[a + b*x])/(4*b^3) - ((c + d*x)^3*Cos[a + b*x])/(8*b) +
 (d^2*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Cos[3*a + 3*b*x])
/(48*b) - (3*d^2*(c + d*x)*Cos[5*a + 5*b*x])/(1000*b^3) + ((c + d*x)^3*Cos[
5*a + 5*b*x])/(80*b) - (3*d^3*Sin[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Sin[
a + b*x])/(8*b^2) - (d^3*Sin[3*a + 3*b*x])/(216*b^4) + (d*(c + d*x)^2*Sin[3
*a + 3*b*x])/(48*b^2) + (3*d^3*Sin[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x
)^2*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^3\sin(a+bx)+\frac{1}{16}(c+dx)^3\sin(3a+3bx)-\frac{1}{16}(c+dx)^3\right.$$
$$=\frac{1}{16}\int (c+dx)^3\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^3\sin(5a+5bx)\,dx+\frac{1}{8}$$
$$=-\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a}{80b}$$
$$=-\frac{(c+dx)^3\cos(a+bx)}{8b}-\frac{(c+dx)^3\cos(3a+3bx)}{48b}+\frac{(c+dx)^3\cos(5a}{80b}$$
$$=\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos(3a}{72b^3}$$
$$=\frac{3d^2(c+dx)\cos(a+bx)}{4b^3}-\frac{(c+dx)^3\cos(a+bx)}{8b}+\frac{d^2(c+dx)\cos(3a}{72b^3}$$

Mathematica [A] time = 1.45, size = 369, normalized size = 1.42

$$\frac{3375b^3c^3\cos(5(a+bx))+10125b^3c^2dx\cos(5(a+bx))+10125b^3cd^2x^2\cos(5(a+bx))+3375b^3d^3x^3\cos(5(a+bx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-33750*b*(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 1875*b*(c + d
*x)*(-2*d^2 + 3*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 3375*b^3*c^3*Cos[5*(a +
 b*x)] - 810*b*c*d^2*Cos[5*(a + b*x)] + 10125*b^3*c^2*d*x*Cos[5*(a + b*x)]
- 810*b*d^3*x*Cos[5*(a + b*x)] + 10125*b^3*c*d^2*x^2*Cos[5*(a + b*x)] + 337
5*b^3*d^3*x^3*Cos[5*(a + b*x)] + 101250*b^2*c^2*d*Sin[a + b*x] - 202500*d^3
*Sin[a + b*x] + 202500*b^2*c*d^2*x*Sin[a + b*x] + 101250*b^2*d^3*x^2*Sin[a
+ b*x] + 5625*b^2*c^2*d*Sin[3*(a + b*x)] - 1250*d^3*Sin[3*(a + b*x)] + 1125
0*b^2*c*d^2*x*Sin[3*(a + b*x)] + 5625*b^2*d^3*x^2*Sin[3*(a + b*x)] - 2025*b
^2*c^2*d*Sin[5*(a + b*x)] + 162*d^3*Sin[5*(a + b*x)] - 4050*b^2*c*d^2*x*Sin
[5*(a + b*x)] - 2025*b^2*d^3*x^2*Sin[5*(a + b*x)])/(270000*b^4)
```

fricas [A] time = 0.53, size = 296, normalized size = 1.14

$$\frac{135 \left(25 \, b^{3} d^{3} x^{3} + 75 \, b^{3} c d^{2} x^{2} + 25 \, b^{3} c^{3} - 6 \, b c d^{2} + 3 \left(25 \, b^{3} c^{2} d - 2 \, b d^{3}\right) x\right) \cos \left(b x + a\right)^{5} - 75 \left(75 \, b^{3} d^{3} x^{3} + 225 \, b^{3} c\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16875*(135*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 +
3*(25*b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^5 - 75*(75*b^3*d^3*x^3 + 225*b^3
*c*d^2*x^2 + 75*b^3*c^3 - 26*b*c*d^2 + (225*b^3*c^2*d - 26*b*d^3)*x)*cos(b*
x + a)^3 + 11700*(b*d^3*x + b*c*d^2)*cos(b*x + a) + (5850*b^2*d^3*x^2 + 117
00*b^2*c*d^2*x + 5850*b^2*c^2*d - 81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*
b^2*c^2*d - 2*d^3)*cos(b*x + a)^4 - 12568*d^3 + (2925*b^2*d^3*x^2 + 5850*b^
2*c*d^2*x + 2925*b^2*c^2*d - 434*d^3)*cos(b*x + a)^2)*sin(b*x + a))/b^4
```

giac [A] time = 0.28, size = 351, normalized size = 1.36

$$\frac{\left(25 \, b^{3} d^{3} x^{3} + 75 \, b^{3} c d^{2} x^{2} + 75 \, b^{3} c^{2} d x + 25 \, b^{3} c^{3} - 6 \, b d^{3} x - 6 \, b c d^{2}\right) \cos \left(5 \, b x + 5 \, a\right)}{2000 \, b^{4}} - \frac{\left(3 \, b^{3} d^{3} x^{3} + 9 \, b^{3} c d^{2} x^{2} + 9 \, b^{3} c\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2*d*x + 25*b^3*c^3 - 6
*b*d^3*x - 6*b*c*d^2)*cos(5*b*x + 5*a)/b^4 - 1/144*(3*b^3*d^3*x^3 + 9*b^3*c
*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b*c*d^2)*cos(3*b*x + 3
*a)/b^4 - 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 6*
b*d^3*x - 6*b*c*d^2)*cos(b*x + a)/b^4 - 3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*
d^2*x + 25*b^2*c^2*d - 2*d^3)*sin(5*b*x + 5*a)/b^4 + 1/432*(9*b^2*d^3*x^2 +
 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*sin(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*
x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a)/b^4
```

maple [B] time = 0.02, size = 992, normalized size = 3.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/3*(b*x+a)^3*(2+sin(b*x+a)^2)*cos(b*x+a)+2/5*(b*x+a)^2*si
n(b*x+a)-856/1125*sin(b*x+a)+4/5*(b*x+a)*cos(b*x+a)+1/15*(b*x+a)^2*sin(b*x+
a)^3+2/45*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+22/3375*sin(b*x+a)^3+1/5*(b*x
```

```
+a)^3*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-3/25*(b*x+a)^2*sin(b*x
+a)^5-6/125*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)+6/625*si
n(b*x+a)^5)-3/b^3*a*d^3*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*co
s(b*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x
+a)^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x
+a)-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos
(b*x+a))+3/b^2*c*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b
*x+a)+4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)
^2)*cos(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)
-2/25*(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a))+3/b^3*a^2*d^3*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+
a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*
x+a)-1/25*sin(b*x+a)^5)-6/b^2*a*c*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*
x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*si
n(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)+3/b*c^2*d*(-1/3*(b*x+a)*(2+sin(b*
x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b
*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25*sin(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5
*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+3/b^2*a^2*c*d^2*(-1/5*sin(b*x
+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)-3/b*a*c^2*d*(-1/5*sin(b*x+a)^2*cos(b*
x+a)^3-2/15*cos(b*x+a)^3)+c^3*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+
a)^3))
```

maxima [B] time = 0.38, size = 766, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/270000*(18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^3 - 54000*(3*cos(b*
x + a)^5 - 5*cos(b*x + a)^3)*a*c^2*d/b + 54000*(3*cos(b*x + a)^5 - 5*cos(b*
x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*a^3*d
^3/b^3 + 225*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a)
 - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) +
450*sin(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x +
a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*
sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*cos(5*
b*x + 5*a) - 75*(b*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9
*sin(5*b*x + 5*a) + 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a^2*d^3/b^3 + 1
5*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x
 + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*c*
d^2/b^2 - 15*(27*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2
 - 2)*cos(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a
)*sin(5*b*x + 5*a) + 750*(b*x + a)*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b
```

```
*x + a))*a*d^3/b^3 + (135*(25*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) -
 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*cos(3*b*x + 3*a) - 33750*((b*x + a)^3 -
 6*b*x - 6*a)*cos(b*x + a) - 81*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 625
*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) + 101250*((b*x + a)^2 - 2)*sin(b*x +
a))*d^3/b^3)/b
```

mupad [B] time = 2.57, size = 516, normalized size = 1.99

$$-\frac{\frac{3\,d^3\,\sin(a+b\,x)}{4}+\frac{d^3\,\sin(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\,\sin(5\,a+5\,b\,x)}{5000}+\frac{b^3\,c^3\,\cos(a+b\,x)}{8}+\frac{b^3\,c^3\,\cos(3\,a+3\,b\,x)}{48}-\frac{b^3\,c^3\,\cos(5\,a+5\,b\,x)}{80}-\frac{b^2\,c^2\,d\,\sin(3\,a+}{48}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*sin(a + b*x))/4 + (d^3*sin(3*a + 3*b*x))/216 - (3*d^3*sin(5*a + 5*
b*x))/5000 + (b^3*c^3*cos(a + b*x))/8 + (b^3*c^3*cos(3*a + 3*b*x))/48 - (b^
3*c^3*cos(5*a + 5*b*x))/80 - (b^2*c^2*d*sin(3*a + 3*b*x))/48 + (3*b^2*c^2*d
*sin(5*a + 5*b*x))/400 + (b^3*d^3*x^3*cos(a + b*x))/8 - (3*b^2*d^3*x^2*sin(
a + b*x))/8 - (3*b*c*d^2*cos(a + b*x))/4 - (3*b*d^3*x*cos(a + b*x))/4 + (b^
3*d^3*x^3*cos(3*a + 3*b*x))/48 - (b^3*d^3*x^3*cos(5*a + 5*b*x))/80 - (b^2*d
^3*x^2*sin(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*sin(5*a + 5*b*x))/400 - (b*c*d
^2*cos(3*a + 3*b*x))/72 + (3*b*c*d^2*cos(5*a + 5*b*x))/1000 - (3*b^2*c^2*d*
sin(a + b*x))/8 - (b*d^3*x*cos(3*a + 3*b*x))/72 + (3*b*d^3*x*cos(5*a + 5*b*
x))/1000 + (3*b^3*c^2*d*x*cos(a + b*x))/8 - (3*b^2*c*d^2*x*sin(a + b*x))/4
+ (b^3*c^2*d*x*cos(3*a + 3*b*x))/16 - (3*b^3*c^2*d*x*cos(5*a + 5*b*x))/80 +
 (3*b^3*c*d^2*x^2*cos(a + b*x))/8 - (b^2*c*d^2*x*sin(3*a + 3*b*x))/24 + (3*
b^2*c*d^2*x*sin(5*a + 5*b*x))/200 + (b^3*c*d^2*x^2*cos(3*a + 3*b*x))/16 - (
3*b^3*c*d^2*x^2*cos(5*a + 5*b*x))/80)/b^4
```

sympy [A] time = 11.14, size = 690, normalized size = 2.66

$$\begin{cases} -\frac{c^3\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c^3\cos^5(a+bx)}{15b}-\frac{c^2dx\sin^2(a+bx)\cos^3(a+bx)}{b}-\frac{2c^2dx\cos^5(a+bx)}{5b}-\frac{cd^2x^2\sin^2(a+bx)\cos^3(a+bx)}{b}-\frac{2cd^2x^2}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**3*cos(a + b*x
)**5/(15*b) - c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c**2*d*x*cos(a
 + b*x)**5/(5*b) - c*d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/b - 2*c*d**2
*x**2*cos(a + b*x)**5/(5*b) - d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**3/(3*
```

```
b) - 2*d**3*x**3*cos(a + b*x)**5/(15*b) + 26*c**2*d*sin(a + b*x)**5/(75*b**
2) + 13*c**2*d*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*c**2*d*sin(a +
 b*x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*x*sin(a + b*x)**5/(75*b**2) + 26
*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 4*c*d**2*x*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 26*d**3*x**2*sin(a + b*x)**5/(75*b**2) + 13*d
**3*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(15*b**2) + 2*d**3*x**2*sin(a + b*
x)*cos(a + b*x)**4/(5*b**2) + 52*c*d**2*sin(a + b*x)**4*cos(a + b*x)/(75*b*
*3) + 338*c*d**2*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*c*d**2*co
s(a + b*x)**5/(1125*b**3) + 52*d**3*x*sin(a + b*x)**4*cos(a + b*x)/(75*b**3
) + 338*d**3*x*sin(a + b*x)**2*cos(a + b*x)**3/(225*b**3) + 856*d**3*x*cos(
a + b*x)**5/(1125*b**3) - 12568*d**3*sin(a + b*x)**5/(16875*b**4) - 5114*d*
*3*sin(a + b*x)**3*cos(a + b*x)**2/(3375*b**4) - 856*d**3*sin(a + b*x)*cos(
a + b*x)**4/(1125*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**
3 + d**3*x**4/4)*sin(a)**3*cos(a)**2, True))
```

3.91 $\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\, dx$

Optimal. Leaf size=184

$$\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2\cos(5a+5bx)}{1000b^3}+\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)}{}$$

```
[Out] 1/4*d^2*cos(b*x+a)/b^3-1/8*(d*x+c)^2*cos(b*x+a)/b+1/216*d^2*cos(3*b*x+3*a)/
b^3-1/48*(d*x+c)^2*cos(3*b*x+3*a)/b-1/1000*d^2*cos(5*b*x+5*a)/b^3+1/80*(d*x
+c)^2*cos(5*b*x+5*a)/b+1/4*d*(d*x+c)*sin(b*x+a)/b^2+1/72*d*(d*x+c)*sin(3*b*
x+3*a)/b^2-1/200*d*(d*x+c)*sin(5*b*x+5*a)/b^2
```

Rubi [A] time = 0.20, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{d(c+dx)\sin(a+bx)}{4b^2}+\frac{d(c+dx)\sin(3a+3bx)}{72b^2}-\frac{d(c+dx)\sin(5a+5bx)}{200b^2}+\frac{d^2\cos(a+bx)}{4b^3}+\frac{d^2\cos(3a+3bx)}{216b^3}-\frac{d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (d^2*Cos[a + b*x])/(4*b^3) - ((c + d*x)^2*Cos[a + b*x])/(8*b) + (d^2*Cos[3*
a + 3*b*x])/(216*b^3) - ((c + d*x)^2*Cos[3*a + 3*b*x])/(48*b) - (d^2*Cos[5*
a + 5*b*x])/(1000*b^3) + ((c + d*x)^2*Cos[5*a + 5*b*x])/(80*b) + (d*(c + d*
x)*Sin[a + b*x])/(4*b^2) + (d*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^2) - (d*(c
+ d*x)*Sin[5*a + 5*b*x])/(200*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2 \cos^2(a+bx) \sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^2 \sin(a+bx) + \frac{1}{16}(c+dx)^2 \sin(3a+3bx) - \frac{1}{16}(c+dx)^2\right.$$
$$= \frac{1}{16}\int (c+dx)^2 \sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^2 \sin(5a+5bx)\,dx +$$
$$= -\frac{(c+dx)^2\cos(a+bx)}{8b} - \frac{(c+dx)^2\cos(3a+3bx)}{48b} + \frac{(c+dx)^2\cos(5a}{80b}$$
$$= -\frac{(c+dx)^2\cos(a+bx)}{8b} - \frac{(c+dx)^2\cos(3a+3bx)}{48b} + \frac{(c+dx)^2\cos(5a}{80b}$$
$$= \frac{d^2\cos(a+bx)}{4b^3} - \frac{(c+dx)^2\cos(a+bx)}{8b} + \frac{d^2\cos(3a+3bx)}{216b^3} - \frac{(c+dx)}{}$$

Mathematica [A] time = 0.88, size = 127, normalized size = 0.69

$$\frac{-6750\cos(a+bx)\left(b^2(c+dx)^2-2d^2\right)-125\cos(3(a+bx))\left(9b^2(c+dx)^2-2d^2\right)+27\cos(5(a+bx))\left(25b^2(c+d\right.}{54000b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-6750*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] - 125*(-2*d^2 + 9*b^2*(c + d
*x)^2)*Cos[3*(a + b*x)] + 27*(-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*(a + b*x)]
 + 30*b*d*(c + d*x)*(450*Sin[a + b*x] + 25*Sin[3*(a + b*x)] - 9*Sin[5*(a +
b*x)]))/(54000*b^3)
```

fricas [A] time = 0.47, size = 166, normalized size = 0.90

$$\frac{27\left(25\,b^2d^2x^2+50\,b^2cdx+25\,b^2c^2-2\,d^2\right)\cos\left(bx+a\right)^5-5\left(225\,b^2d^2x^2+450\,b^2cdx+225\,b^2c^2-26\,d^2\right)\cos\left(bx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/3375*(27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x + a
)^5 - 5*(225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 - 26*d^2)*cos(b*x +
a)^3 + 780*d^2*cos(b*x + a) - 30*(9*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 26*b
*d^2*x - 26*b*c*d - 13*(b*d^2*x + b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/b^3
```

giac [A] time = 0.25, size = 209, normalized size = 1.14

$$\frac{\left(25\,b^{2}d^{2}x^{2}+50\,b^{2}cdx+25\,b^{2}c^{2}-2\,d^{2}\right)\cos\left(5\,bx+5\,a\right)}{2000\,b^{3}}-\frac{\left(9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx+9\,b^{2}c^{2}-2\,d^{2}\right)\cos\left(3\,bx+3\,a\right)}{432\,b^{3}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/2000*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(5*b*x + 5*a
)/b^3 - 1/432*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*cos(3*b*x
+ 3*a)/b^3 - 1/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)
/b^3 - 1/200*(b*d^2*x + b*c*d)*sin(5*b*x + 5*a)/b^3 + 1/72*(b*d^2*x + b*c*d
)*sin(3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*sin(b*x + a)/b^3
```

maple [B] time = 0.02, size = 466, normalized size = 2.53

$$\frac{d^{2}\left(-\frac{(bx+a)^{2}\left(2+\sin^{2}(bx+a)\right)\cos(bx+a)}{3}+\frac{4\cos(bx+a)}{15}+\frac{4(bx+a)\sin(bx+a)}{15}+\frac{2(bx+a)\left(\sin^{3}(bx+a)\right)}{45}+\frac{2\left(2+\sin^{2}(bx+a)\right)\cos(bx+a)}{135}+\frac{(bx+a)^{2}\left(\frac{8}{3}+\sin^{4}(bx+a)+\frac{4\left(\sin^{2}(bx+a)\right)}{3}\right)\cos}{5}\right.}{b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/3*(b*x+a)^2*(2+sin(b*x+a)^2)*cos(b*x+a)+4/15*cos(b*x+a)+
4/15*(b*x+a)*sin(b*x+a)+2/45*(b*x+a)*sin(b*x+a)^3+2/135*(2+sin(b*x+a)^2)*co
s(b*x+a)+1/5*(b*x+a)^2*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-2/25*
(b*x+a)*sin(b*x+a)^5-2/125*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a))-
2/b^2*a*d^2*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/1
5*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/2
5*sin(b*x+a)^5)+2/b*c*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(
b*x+a)^3+2/15*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*co
s(b*x+a)-1/25*sin(b*x+a)^5)+1/b^2*a^2*d^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2
/15*cos(b*x+a)^3)-2/b*a*c*d*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)
^3)+c^2*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

maxima [B] time = 0.35, size = 375, normalized size = 2.04

$$3600\left(3\cos\left(bx+a\right)^{5}-5\cos\left(bx+a\right)^{3}\right)c^{2}-\frac{7200\left(3\cos(bx+a)^{5}-5\cos(bx+a)^{3}\right)acd}{b}+\frac{3600\left(3\cos(bx+a)^{5}-5\cos(bx+a)^{3}\right)a^{2}d^{2}}{b^{2}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/54000*(3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c^2 - 7200*(3*cos(b*x +
 a)^5 - 5*cos(b*x + a)^3)*a*c*d/b + 3600*(3*cos(b*x + a)^5 - 5*cos(b*x + a)
^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*cos(3*b*
x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin(3*b*x +
 3*a) + 450*sin(b*x + a))*c*d/b - 30*(45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b
*x + a)*cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a)
+ 25*sin(3*b*x + 3*a) + 450*sin(b*x + a))*a*d^2/b^2 + (27*(25*(b*x + a)^2 -
 2)*cos(5*b*x + 5*a) - 125*(9*(b*x + a)^2 - 2)*cos(3*b*x + 3*a) - 6750*((b*
x + a)^2 - 2)*cos(b*x + a) - 270*(b*x + a)*sin(5*b*x + 5*a) + 750*(b*x + a)
*sin(3*b*x + 3*a) + 13500*(b*x + a)*sin(b*x + a))*d^2/b^2)/b
```

mupad [B] time = 0.81, size = 249, normalized size = 1.35

$$\frac{780\,d^2\,\cos\left(a+b\,x\right)+130\,d^2\cos\left(a+b\,x\right)^3-54\,d^2\cos\left(a+b\,x\right)^5-1125\,b^2\,c^2\cos\left(a+b\,x\right)^3+675\,b^2\,c^2\cos\left(a+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (780*d^2*cos(a + b*x) + 130*d^2*cos(a + b*x)^3 - 54*d^2*cos(a + b*x)^5 - 11
25*b^2*c^2*cos(a + b*x)^3 + 675*b^2*c^2*cos(a + b*x)^5 + 780*b*d^2*x*sin(a
+ b*x) - 1125*b^2*d^2*x^2*cos(a + b*x)^3 + 675*b^2*d^2*x^2*cos(a + b*x)^5 +
 780*b*c*d*sin(a + b*x) - 2250*b^2*c*d*x*cos(a + b*x)^3 + 1350*b^2*c*d*x*co
s(a + b*x)^5 + 390*b*d^2*x*cos(a + b*x)^2*sin(a + b*x) - 270*b*d^2*x*cos(a
+ b*x)^4*sin(a + b*x) + 390*b*c*d*cos(a + b*x)^2*sin(a + b*x) - 270*b*c*d*c
os(a + b*x)^4*sin(a + b*x))/(3375*b^3)
```

sympy [A] time = 5.96, size = 382, normalized size = 2.08

$$\begin{cases} -\frac{c^2\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2c^2\cos^5(a+bx)}{15b}-\frac{2cdx\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{4cdx\cos^5(a+bx)}{15b}-\frac{d^2x^2\sin^2(a+bx)\cos^3(a+bx)}{3b}-\frac{2d^2x^2\cos^5}{15b} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c**2*cos(a + b*x
)**5/(15*b) - 2*c*d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 4*c*d*x*cos(a
 + b*x)**5/(15*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d**
2*x**2*cos(a + b*x)**5/(15*b) + 52*c*d*sin(a + b*x)**5/(225*b**2) + 26*c*d*
sin(a + b*x)**3*cos(a + b*x)**2/(45*b**2) + 4*c*d*sin(a + b*x)*cos(a + b*x)
```

```
**4/(15*b**2) + 52*d**2*x*sin(a + b*x)**5/(225*b**2) + 26*d**2*x*sin(a + b*
x)**3*cos(a + b*x)**2/(45*b**2) + 4*d**2*x*sin(a + b*x)*cos(a + b*x)**4/(15
*b**2) + 52*d**2*sin(a + b*x)**4*cos(a + b*x)/(225*b**3) + 338*d**2*sin(a +
 b*x)**2*cos(a + b*x)**3/(675*b**3) + 856*d**2*cos(a + b*x)**5/(3375*b**3),
 Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**2, True))
```

3.92 $\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+}{80b}$$

```
[Out] -1/8*(d*x+c)*cos(b*x+a)/b-1/48*(d*x+c)*cos(3*b*x+3*a)/b+1/80*(d*x+c)*cos(5*
b*x+5*a)/b+1/8*d*sin(b*x+a)/b^2+1/144*d*sin(3*b*x+3*a)/b^2-1/400*d*sin(5*b*
x+5*a)/b^2
```

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2637}

$$\frac{d\sin(a+bx)}{8b^2}+\frac{d\sin(3a+3bx)}{144b^2}-\frac{d\sin(5a+5bx)}{400b^2}-\frac{(c+dx)\cos(a+bx)}{8b}-\frac{(c+dx)\cos(3a+3bx)}{48b}+\frac{(c+dx)\cos(5a+}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)*Cos[a + b*x])/(8*b) - ((c + d*x)*Cos[3*a + 3*b*x])/(48*b) + ((c
 + d*x)*Cos[5*a + 5*b*x])/(80*b) + (d*Sin[a + b*x])/(8*b^2) + (d*Sin[3*a +
3*b*x])/(144*b^2) - (d*Sin[5*a + 5*b*x])/(400*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)\sin(a+bx) + \frac{1}{16}(c+dx)\sin(3a+3bx) - \frac{1}{16}(c+dx)\sin$$

$$= \frac{1}{16}\int (c+dx)\sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)\sin(5a+5bx)\,dx + \frac{1}{8}$$

$$= -\frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a+5}{80b}$$

$$= -\frac{(c+dx)\cos(a+bx)}{8b} - \frac{(c+dx)\cos(3a+3bx)}{48b} + \frac{(c+dx)\cos(5a+5}{80b}$$

Mathematica [A] time = 0.29, size = 94, normalized size = 0.86

$$\frac{-450b(c+dx)\cos(a+bx) - 75b(c+dx)\cos(3(a+bx)) + 45bc\cos(5(a+bx)) + 450d\sin(a+bx) + 25d\sin(3(a}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (-450*b*(c + d*x)*Cos[a + b*x] - 75*b*(c + d*x)*Cos[3*(a + b*x)] + 45*b*c*C
os[5*(a + b*x)] + 45*b*d*x*Cos[5*(a + b*x)] + 450*d*Sin[a + b*x] + 25*d*Sin
[3*(a + b*x)] - 9*d*Sin[5*(a + b*x)])/(3600*b^2)
```

fricas [A] time = 0.93, size = 76, normalized size = 0.70

$$\frac{45\,(bdx+bc)\cos(bx+a)^5 - 75\,(bdx+bc)\cos(bx+a)^3 - \left(9\,d\cos(bx+a)^4 - 13\,d\cos(bx+a)^2 - 26\,d\right)\sin(bx}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/225*(45*(b*d*x + b*c)*cos(b*x + a)^5 - 75*(b*d*x + b*c)*cos(b*x + a)^3 -
(9*d*cos(b*x + a)^4 - 13*d*cos(b*x + a)^2 - 26*d)*sin(b*x + a))/b^2
```

giac [A] time = 1.17, size = 106, normalized size = 0.97

$$\frac{(bdx+bc)\cos(5\,bx+5\,a)}{80\,b^2} - \frac{(bdx+bc)\cos(3\,bx+3\,a)}{48\,b^2} - \frac{(bdx+bc)\cos(bx+a)}{8\,b^2} - \frac{d\sin(5\,bx+5\,a)}{400\,b^2} + \frac{d\sin(3\,bx+}{144\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/80*(b*d*x + b*c)*cos(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*cos(3*b*x + 3*
a)/b^2 - 1/8*(b*d*x + b*c)*cos(b*x + a)/b^2 - 1/400*d*sin(5*b*x + 5*a)/b^2
+ 1/144*d*sin(3*b*x + 3*a)/b^2 + 1/8*d*sin(b*x + a)/b^2
```

maple [A] time = 0.02, size = 163, normalized size = 1.50

$$\frac{\frac{d\left(-\frac{(bx+a)\left(2+\sin^2(bx+a)\right)\cos(bx+a)}{3}+\frac{\left(\sin^3(bx+a)\right)}{45}+\frac{2\sin(bx+a)}{15}+\frac{(bx+a)\left(\frac{8}{3}+\sin^4(bx+a)+\frac{4\left(\sin^2(bx+a)\right)}{3}\right)\cos(bx+a)}{5}-\frac{\left(\sin^5(bx+a)\right)}{25}\right)}{b}-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)\left(\cos^3(bx+a)\right)}{5}-\frac{2\cdots}{}\right.}{b}\cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(-1/3*(b*x+a)*(2+sin(b*x+a)^2)*cos(b*x+a)+1/45*sin(b*x+a)^3+2/15
*sin(b*x+a)+1/5*(b*x+a)*(8/3+sin(b*x+a)^4+4/3*sin(b*x+a)^2)*cos(b*x+a)-1/25
*sin(b*x+a)^5)-1/b*d*a*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3)+c
*(-1/5*sin(b*x+a)^2*cos(b*x+a)^3-2/15*cos(b*x+a)^3))
```

maxima [A] time = 0.34, size = 139, normalized size = 1.28

$$\frac{240\left(3\,\cos\left(bx+a\right)^5-5\,\cos\left(bx+a\right)^3\right)c-\frac{240\left(3\,\cos(bx+a)^5-5\,\cos(bx+a)^3\right)ad}{b}+\frac{(45\,(bx+a)\cos(5\,bx+5\,a)-75\,(bx+a)\cos(3\,bx+3\,a)-45\cdots}{}}{3600\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/3600*(240*(3*cos(b*x + a)^5 - 5*cos(b*x + a)^3)*c - 240*(3*cos(b*x + a)^5
 - 5*cos(b*x + a)^3)*a*d/b + (45*(b*x + a)*cos(5*b*x + 5*a) - 75*(b*x + a)*
cos(3*b*x + 3*a) - 450*(b*x + a)*cos(b*x + a) - 9*sin(5*b*x + 5*a) + 25*sin
(3*b*x + 3*a) + 450*sin(b*x + a))*d/b)/b
```

mupad [B] time = 1.24, size = 99, normalized size = 0.91

$$\frac{26\,d\,\sin\left(a+b\,x\right)-75\,b\,c\,\cos\left(a+b\,x\right)^3+45\,b\,c\,\cos\left(a+b\,x\right)^5+13\,d\,\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)-9\,d\,\cos\left(a+b\,x\right)\cdots}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] (26*d*sin(a + b*x) - 75*b*c*cos(a + b*x)^3 + 45*b*c*cos(a + b*x)^5 + 13*d*c
os(a + b*x)^2*sin(a + b*x) - 9*d*cos(a + b*x)^4*sin(a + b*x) - 75*b*d*x*cos
(a + b*x)^3 + 45*b*d*x*cos(a + b*x)^5)/(225*b^2)
```

sympy [A] time = 3.05, size = 163, normalized size = 1.50

$$\begin{cases} -\frac{c\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2c\cos^5(a+bx)}{15b} - \frac{dx\sin^2(a+bx)\cos^3(a+bx)}{3b} - \frac{2dx\cos^5(a+bx)}{15b} + \frac{26d\sin^5(a+bx)}{225b^2} + \frac{13d\sin^3(a+bx)\cos^2(a+bx)}{45b^2} \\ \left(cx+\frac{dx^2}{2}\right)\sin^3(a)\cos^2(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((-c*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*c*cos(a + b*x)**5/(
15*b) - d*x*sin(a + b*x)**2*cos(a + b*x)**3/(3*b) - 2*d*x*cos(a + b*x)**5/(
15*b) + 26*d*sin(a + b*x)**5/(225*b**2) + 13*d*sin(a + b*x)**3*cos(a + b*x)
**2/(45*b**2) + 2*d*sin(a + b*x)*cos(a + b*x)**4/(15*b**2), Ne(b, 0)), ((c*
x + d*x**2/2)*sin(a)**3*cos(a)**2, True))
```

3.93 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=185

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d}+\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right.}{8d}$$

```
[Out] 1/8*cos(a-b*c/d)*Si(b*c/d+b*x)/d+1/16*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d-
1/16*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d-1/16*Ci(5*b*c/d+5*b*x)*sin(5*a-5*
b*c/d)/d+1/16*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d+1/8*Ci(b*c/d+b*x)*sin(a-
b*c/d)/d
```

Rubi [A] time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d}+\frac{\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(16*d) + (CosIntegra
l[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(16*d) + (CosIntegral[(b*c)/d +
b*x]*Sin[a - (b*c)/d])/(8*d) + (Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x]
)/(8*d) + (Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (C
os[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{\sin(a+bx)}{8(c+dx)}+\frac{\sin(3a+3bx)}{16(c+dx)}-\frac{\sin(5a+5bx)}{16(c+dx)}\right)dx\\ &= \frac{1}{16}\int \frac{\sin(3a+3bx)}{c+dx}\,dx-\frac{1}{16}\int \frac{\sin(5a+5bx)}{c+dx}\,dx+\frac{1}{8}\int \frac{\sin(a+bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int \frac{\sin\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right)+\frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\sin\left(\frac{3bc}{d}\right.}{c+}\\ &= -\frac{\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a-\frac{5bc}{d}\right)}{16d}+\frac{\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)\sin\left(3a-\frac{3bc}{d}\right)}{16d}+\frac{\mathrm{Ci}\left(\frac{bc}{d}+bx\right.}{}\end{aligned}$$

Mathematica [A] time = 0.49, size = 154, normalized size = 0.83

$$\frac{\sin\left(5a-\frac{5bc}{d}\right)\left(-\mathrm{Ci}\left(\frac{5b(c+dx)}{d}\right)\right)+\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right)+2\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+2\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] (-(CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d]) + CosIntegral[(3*b*
(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + 2*CosIntegral[b*(c/d + x)]*Sin[a - (b*
c)/d] + 2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Cos[3*a - (3*b*c)/d]*
SinIntegral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c +
 d*x))/d])/(16*d)
```

fricas [A] time = 0.65, size = 228, normalized size = 1.23

$$\frac{2\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\sin\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{3\,(bc-ad)}{d}\right)-\left(\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*si
n(-(b*c - a*d)/d) + (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*sin(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d) - 2*cos(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) + 4*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 253, normalized size = 1.37

$$\frac{-\frac{b\left(\frac{5\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}\right)}{80}+\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}\right)}{8}+\frac{b\left(\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b
*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)+1/8*b*(Si(b*x+a+(-a*d+b*c)/d)
*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)+1/48*b*(3*
Si(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b
*c)/d)*sin(3*(-a*d+b*c)/d)/d))
```

maxima [C] time = 0.47, size = 407, normalized size = 2.20

$$\frac{b\left(-2i\,E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b\left(-i\,E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_1\left(-\frac{3i\,bc+3i\,(\cdots}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/32*(b*(-2*I*exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*ex
p_integral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) +
b*(-I*exp_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*exp_in
tegral_e(1, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d
) + b*(I*exp_integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp
_integral_e(1, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d
)/d) - 2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) - b*(exp
_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
 -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(exp_
integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x), x)
```

3.94 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=257

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\right.}{8d^2}$$

```
[Out] -5/16*b*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*cos
(3*a-3*b*c/d)/d^2+1/8*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+5/16*b*Si(5*b*c/d+5*
b*x)*sin(5*a-5*b*c/d)/d^2-3/16*b*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^2-1/8
*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-1/8*sin(b*x+a)/d/(d*x+c)-1/16*sin(3*b*x+3
*a)/d/(d*x+c)+1/16*sin(5*b*x+5*a)/d/(d*x+c)
```

Rubi [A] time = 0.42, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d^2}+\frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{5b\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) - (5*b*Cos[5*a - (5*b*c)/
d]*CosIntegral[(5*b*c)/d + 5*b*x])/(16*d^2) - Sin[a + b*x]/(8*d*(c + d*x))
- Sin[3*a + 3*b*x]/(16*d*(c + d*x)) + Sin[5*a + 5*b*x]/(16*d*(c + d*x)) - (
b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) - (3*b*Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^2}+\frac{\sin(3a+3bx)}{16(c+dx)^2}-\frac{\sin(5a+5bx)}{16(c+dx)^2}\right)dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^2}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} + \frac{\sin(5a+5bx)}{16d(c+dx)} + \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)\int \frac{\text{co}}{}}{1}$$

$$= -\frac{\sin(a+bx)}{8d(c+dx)} - \frac{\sin(3a+3bx)}{16d(c+dx)} + \frac{\sin(5a+5bx)}{16d(c+dx)} - \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\cos\left(\frac{5b}{d}\right.}{c+}}{16d}$$

$$= \frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2} + \frac{3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2} - \frac{5b\cos\left(5a-\right.}{}$$

Mathematica [A] time = 1.47, size = 213, normalized size = 0.83

$$\frac{2b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+3b\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)-5b\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5b(c+dx)}{d}\right)-2b\sin\left(a-\frac{bc}{d}\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (2*b*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 3*b*Cos[3*a - (3*b*c)/d]*C
osIntegral[(3*b*(c + d*x))/d] - 5*b*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(
c + d*x))/d] - (2*d*Sin[a + b*x])/(c + d*x) - (d*Sin[3*(a + b*x)])/(c + d*x
) + (d*Sin[5*(a + b*x)])/(c + d*x) - 2*b*Sin[a - (b*c)/d]*SinIntegral[b*(c/
d + x)] - 3*b*Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Sin
[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

fricas [A] time = 0.91, size = 347, normalized size = 1.35

$$\frac{10\,(bdx+bc)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{3\,(bdx+bc)}{d}\right)-4\,(bdx+bc)\sin\left(-\frac{bc-ad}{d}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(10*(b*d*x + b*c)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d
) - 6*(b*d*x + b*c)*sin(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) -
 4*(b*d*x + b*c)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 2*((b*
d*x + b*c)*cos_integral((b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d
*x + b*c)/d))*cos(-(b*c - a*d)/d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a
*d)/d) - 5*((b*d*x + b*c)*cos_integral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*c
os_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 32*(d*cos(b*x + a)
^4 - d*cos(b*x + a)^2)*sin(b*x + a))/(d^3*x + c*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 365, normalized size = 1.42

$$\frac{-\frac{b^2\left(-\frac{5\sin(5bx+5a)}{((bx+a)d-da+cb)d}+\frac{\frac{25\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}+\frac{25\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}}{d}\right)}{80}+\frac{b^2\left(-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\cdots\right)}{d}\cdots}{\cdots}\right)}{8}\cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/80*b^2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)/d+5*(5*Si(5*b*x+5*a+5
*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(
-a*d+b*c)/d)/d)/d)+1/8*b^2*(-sin(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a
*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/
d)+1/48*b^2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-
a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*
d+b*c)/d)/d)/d))
```

maxima [C] time = 0.58, size = 438, normalized size = 1.70

$$b^2\left(-2i\,E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^2\left(-i\,E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_2\left(-\frac{3i\,bc}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(b^2*(-2*I*exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^2*(-I*exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^2*(I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**2, x)
```

3.95 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=338

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{16d^3}-\frac{b^2\cos\left(a-\right.}{1}$$

```
[Out] -1/16*b*cos(b*x+a)/d^2/(d*x+c)-3/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)+5/32*b*cos
(5*b*x+5*a)/d^2/(d*x+c)-1/16*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-9/32*b^2*co
s(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^3+25/32*b^2*cos(5*a-5*b*c/d)*Si(5*b*c/d+
5*b*x)/d^3+25/32*b^2*Ci(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Ci(3*b
*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^3-1/
16*sin(b*x+a)/d/(d*x+c)^2-1/32*sin(3*b*x+3*a)/d/(d*x+c)^2+1/32*sin(5*b*x+5*
a)/d/(d*x+c)^2
```

Rubi [A] time = 0.50, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{25b^2\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{32d^3}-\frac{9b^2\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}-\frac{b^2\sin\left(a-\frac{bc}{d}\right)\text{CosI}}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)) + (5*b*Cos[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (25*b^2*CosIntegral[(5*
b*c)/d + 5*b*x]*Sin[5*a - (5*b*c)/d])/(32*d^3) - (9*b^2*CosIntegral[(3*b*c)
/d + 3*b*x]*Sin[3*a - (3*b*c)/d])/(32*d^3) - (b^2*CosIntegral[(b*c)/d + b*x
]*Sin[a - (b*c)/d])/(16*d^3) - Sin[a + b*x]/(16*d*(c + d*x)^2) - Sin[3*a +
3*b*x]/(32*d*(c + d*x)^2) + Sin[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^3} + \frac{\sin(3a+3bx)}{16(c+dx)^3} - \frac{\sin(5a+5bx)}{16(c+dx)^3}\right)dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^3}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^3}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin(3a+3bx)}{32d(c+dx)^2} + \frac{\sin(5a+5bx)}{32d(c+dx)^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{16d} + \frac{(3b)\int}{}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} - \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin}{32}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} - \frac{\sin(a+bx)}{16d(c+dx)^2} - \frac{\sin}{32}$$

$$= -\frac{b\cos(a+bx)}{16d^2(c+dx)} - \frac{3b\cos(3a+3bx)}{32d^2(c+dx)} + \frac{5b\cos(5a+5bx)}{32d^2(c+dx)} + \frac{25b^2\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{32d^3}$$

Mathematica [A] time = 3.90, size = 279, normalized size = 0.83

$$\frac{-2\left(b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right) + b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(b\left(\frac{c}{d}+x\right)\right) + \frac{d(b(c+dx)\cos(a+bx)+d\sin(a+bx))}{(c+dx)^2}\right) + 25b^2\sin\left(5a-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (25*b^2*CosIntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] - 9*b^2*CosInte
gral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] - (d*(3*b*(c + d*x)*Cos[3*(a +
 b*x)] + d*Sin[3*(a + b*x)]))/(c + d*x)^2 + (d*(5*b*(c + d*x)*Cos[5*(a + b*
x)] + d*Sin[5*(a + b*x)]))/(c + d*x)^2 - 2*(b^2*CosIntegral[b*(c/d + x)]*Si
n[a - (b*c)/d] + (d*(b*(c + d*x)*Cos[a + b*x] + d*Sin[a + b*x]))/(c + d*x)^
2 + b^2*Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)]) - 9*b^2*Cos[3*a - (3*b*c
)/d]*SinIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d])/(32*d^3)
```

fricas [A] time = 0.83, size = 585, normalized size = 1.73

$$\frac{160\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5 - 224\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3 + 50\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(160*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 224*(b*d^2*x + b*c*d)*cos(b*x
+ a)^3 + 50*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-5*(b*c - a*d)/d)*sin
_integral(5*(b*d*x + b*c)/d) - 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(-3*(b*c - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b
*d^2*x + b*c*d)*cos(b*x + a) + 32*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)
*sin(b*x + a) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral((b*d*
x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-(b*d*x +
b*c)/d))*sin(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_in
tegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 25*((b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*sin(-5*(b*c - a*d)/d))/(d
^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 475, normalized size = 1.41

$$-\frac{b^3\left(-\frac{5\sin(5bx+5a)}{2((bx+a)d-da+cb)^2 d}+\frac{-\frac{25\cos(5bx+5a)}{2((bx+a)d-da+cb)d}-\frac{25\left(\frac{5\operatorname{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}-\frac{5\operatorname{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}\right)}{2d}}{d}\right)}{80}+\frac{b^3\left(-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^2 d}\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/80*b^3*(-5/2*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^2/d+5/2*(-5*cos(5*b
*x+5*a)/((b*x+a)*d-d*a+c*b)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d
+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)+1/8*
b^3*(-1/2*sin(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*
a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)
*sin((-a*d+b*c)/d)/d)/d)/d)+1/48*b^3*(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*
```

```
b)^2/d+3/2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d
+b*c)/d)/d)/d)/d))
```

maxima [C] time = 0.78, size = 473, normalized size = 1.40

$$b^3\left(-2i\,E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^3\left(-i\,E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_3\left(-\frac{3i\,bc}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/32*(b^3*(-2*I*exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^3*(-I*exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^3*(I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d
^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^2\,\sin(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**3,x)
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**3, x)
```

3.96 $\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=413

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)}{96d^4}+\frac{b^3\sin\left(a\right.}{}$$

```
[Out] 125/96*b^3*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x
)*cos(3*a-3*b*c/d)/d^4-1/48*b^3*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^4-1/48*b*cos(b
*x+a)/d^2/(d*x+c)^2-1/32*b*cos(3*b*x+3*a)/d^2/(d*x+c)^2+5/96*b*cos(5*b*x+5*
a)/d^2/(d*x+c)^2-125/96*b^3*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^4+9/32*b^3
*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Si(b*c/d+b*x)*sin(a-b*c/d)
/d^4-1/24*sin(b*x+a)/d/(d*x+c)^3+1/48*b^2*sin(b*x+a)/d^3/(d*x+c)-1/48*sin(3
*b*x+3*a)/d/(d*x+c)^3+3/32*b^2*sin(3*b*x+3*a)/d^3/(d*x+c)+1/48*sin(5*b*x+5*
a)/d/(d*x+c)^3-25/96*b^2*sin(5*b*x+5*a)/d^3/(d*x+c)
```

Rubi [A] time = 0.59, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{48d^4}-\frac{9b^3\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{125b^3\cos\left(5a-\frac{5bc}{d}\right)\mathrm{Cos}}{96d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Cos[3*a + 3*b*x])/(32*d^2*(c +
d*x)^2) + (5*b*Cos[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) - (b^3*Cos[a - (b*c)/
d]*CosIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*CosIn
tegral[(3*b*c)/d + 3*b*x])/(32*d^4) + (125*b^3*Cos[5*a - (5*b*c)/d]*CosInte
gral[(5*b*c)/d + 5*b*x])/(96*d^4) - Sin[a + b*x]/(24*d*(c + d*x)^3) + (b^2*
Sin[a + b*x])/(48*d^3*(c + d*x)) - Sin[3*a + 3*b*x]/(48*d*(c + d*x)^3) + (3
*b^2*Sin[3*a + 3*b*x])/(32*d^3*(c + d*x)) + Sin[5*a + 5*b*x]/(48*d*(c + d*x
)^3) - (25*b^2*Sin[5*a + 5*b*x])/(96*d^3*(c + d*x)) + (b^3*Sin[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) + (9*b^3*Sin[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Sin[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^2(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(a+bx)}{8(c+dx)^4} + \frac{\sin(3a+3bx)}{16(c+dx)^4} - \frac{\sin(5a+5bx)}{16(c+dx)^4}\right) dx$$

$$= \frac{1}{16}\int \frac{\sin(3a+3bx)}{(c+dx)^4}\,dx - \frac{1}{16}\int \frac{\sin(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\sin(a+bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin(3a+3bx)}{48d(c+dx)^3} + \frac{\sin(5a+5bx)}{48d(c+dx)^3} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int \frac{\cos(}{(c}}{1}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} - \frac{\sin}{48}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2 s}{48}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{\sin(a+bx)}{24d(c+dx)^3} + \frac{b^2 s}{48}$$

$$= -\frac{b\cos(a+bx)}{48d^2(c+dx)^2} - \frac{b\cos(3a+3bx)}{32d^2(c+dx)^2} + \frac{5b\cos(5a+5bx)}{96d^2(c+dx)^2} - \frac{b^3\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\right.}{48d^4}$$

Mathematica [A] time = 2.90, size = 457, normalized size = 1.11

$$\frac{-27b^3(c+dx)^3\left(\cos\left(3a-\frac{3bc}{d}\right)\operatorname{Ci}\left(\frac{3b(c+dx)}{d}\right)-\sin\left(3a-\frac{3bc}{d}\right)\operatorname{Si}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\cos\left(5a-\frac{5bc}{d}\right)\operatorname{Ci}\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^2*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-(d*Cos[3*b*x]*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*x)^2)*Si
n[3*a])) + d*Cos[5*b*x]*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2*(c + d
*x)^2)*Sin[5*a]) + d*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*
x)*Sin[3*a])*Sin[3*b*x] - d*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d
*(c + d*x)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*(b*d*(c + d*x)*Cos[a] - (-2
*d^2 + b^2*(c + d*x)^2)*Sin[a]) - d*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*
d*(c + d*x)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(Cos[a - (b*c)/d]*CosIntegra
l[b*(c/d + x)] - Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) - 27*b^3*(c +
d*x)^3*(Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*(c + d*x))/d] - Sin[3*a - (3*
b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(Cos[5*a - (5
*b*c)/d]*CosIntegral[(5*b*(c + d*x))/d] - Sin[5*a - (5*b*c)/d]*SinIntegral[
(5*b*(c + d*x))/d]))/(96*d^4*(c + d*x)^3)
```

fricas [B] time = 0.62, size = 824, normalized size = 2.00

$$\frac{160\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^5-224\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^3-250\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/192*(160*(b*d^3*x + b*c*d^2)*cos(b*x + a)^5 - 224*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^3 - 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)
*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-3*(b*c - a*d)/d)*sin_integr
al(3*(b*d*x + b*c)/d) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 64*(b*d^3*x +
b*c*d^2)*cos(b*x + a) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
 b^3*c^3)*cos_integral((b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d
) - 27*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integ
ral(3*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b
^3*c^3)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) + 125*((b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x
 + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_
integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) - 32*(2*b^2*d^3*x^2 + 4
*b^2*c*d^2*x + 2*b^2*c^2*d + (25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*
d - 2*d^3)*cos(b*x + a)^4 - (21*b^2*d^3*x^2 + 42*b^2*c*d^2*x + 21*b^2*c^2*d
 - 2*d^3)*cos(b*x + a)^2)*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*
x + c^3*d^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.03, size = 580, normalized size = 1.40

$$-\frac{b^4\left(-\frac{5\sin(5bx+5a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{25\cos(5bx+5a)}{6((bx+a)d-da+cb)^2 d}-\frac{25\left(-\frac{5\sin(5bx+5a)}{((bx+a)d-da+cb)d}+\frac{\frac{25\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}+\frac{25\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}}{d}\right)}{6d}}{d}\right)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(-1/80*b^4*(-5/3*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^3/d+5/3*(-5/2*cos(5
*b*x+5*a)/((b*x+a)*d-d*a+c*b)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b
)/d+5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+
5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-1/3*sin(b*x+a)/((
b*x+a)*d-d*a+c*b)^3/d+1/3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-si
n(b*x+a)/((b*x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+
Ci(b*x+a+(-a*d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)/d)+1/48*b^4*(-sin(3*b*x+3
*a)/((b*x+a)*d-d*a+c*b)^3/d+(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d-3/
2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d
)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/
d)/d)/d)/d))
```

maxima [C] time = 1.10, size = 523, normalized size = 1.27

$$\frac{b^4\left(-2i\,E_4\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+2i\,E_4\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+b^4\left(-i\,E_4\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+i\,E_4\left(-\frac{3i\,bc}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*(b^4*(-2*I*exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d)
+ b^4*(-I*exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + I*ex
p_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*
d)/d) + b^4*(I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) -
 I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c
 - a*d)/d) - 2*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d - I*a*d)/d) +
```

```
exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d)
- b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_int
egral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d)
 + b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_in
tegral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d
))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^
4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^2*sin(a + b*x)^3)/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{\left(c+dx\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**2/(c + d*x)**4, x)
```

3.97 $\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=144

$$\text{Int}\left(\csc(a+bx)(c+dx)^m,x\right)+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}}{2b}$$

```
[Out] 1/2*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d
)^m)+1/2*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c
)/d)^m)+Unintegrable((d*x+c)^m*csc(b*x+a),x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Csc[
a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \csc(a+bx)\,dx-\int (c+dx)^m \sin(a+bx)\,dx\\ &= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx\\ &= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^n}{}\end{aligned}$$

Mathematica [A] time = 6.38, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x], x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos\left(bx+a\right)\cot\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos\left(bx+a\right)\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos\left(bx+a\right)\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos\left(bx+a\right)\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\cot\left(a+b\,x\right)\,{\left(c+d\,x\right)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x), x)
```

3.98 $\int (c+dx)^4 \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=333

$$\frac{24d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{24d^4\cos(a+bx)}{b^5}-\frac{24id^3(c+dx)\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}$$

```
[Out] -2*(d*x+c)^4*arctanh(exp(I*(b*x+a)))/b+24*d^4*cos(b*x+a)/b^5-12*d^2*(d*x+c)
^2*cos(b*x+a)/b^3+(d*x+c)^4*cos(b*x+a)/b+4*I*d*(d*x+c)^3*polylog(2,-exp(I*(
b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-12*d^2*(d*x+c)^2
*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/
b^3-24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4+24*I*d^3*(d*x+c)*polylo
g(4,exp(I*(b*x+a)))/b^4+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylo
g(5,exp(I*(b*x+a)))/b^5+24*d^3*(d*x+c)*sin(b*x+a)/b^4-4*d*(d*x+c)^3*sin(b*x
+a)/b^2
```

Rubi [A] time = 0.28, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4408, 3296, 2638, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\frac{24id^3(c+dx)\text{PolyLog}\left(4,e^{i(a+bx)}\right)}{b^4}-\frac{12d^2(c+dx)^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^(I*(a + b*x))])/b + (24*d^4*Cos[a + b*x])/b^5 - (
12*d^2*(c + d*x)^2*Cos[a + b*x])/b^3 + ((c + d*x)^4*Cos[a + b*x])/b + ((4*I
)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((4*I)*d*(c + d*x)^3*Po
lyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a +
b*x))])/b^3 + (12*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I
)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((24*I)*d^3*(c + d*x)*P
olyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5
 - (24*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5 + (24*d^3*(c + d*x)*Sin[a + b*x
])/b^4 - (4*d*(c + d*x)^3*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
```

```
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4\csc(a+bx)\,dx - \int (c+dx)^4\sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b} - \frac{(4d)\int (c+dx)^3}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^4\cos(a+bx)}{b} + \frac{4id(c+dx)^3\mathrm{Li}_2}{b^2}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4\mathrm{co}}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12d^2(c+dx)^2\cos(a+bx)}{b^3} + \frac{(c+dx)^4\mathrm{co}}{b}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos(a}{b^3}$$

$$= -\frac{2(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{24d^4\cos(a+bx)}{b^5} - \frac{12d^2(c+dx)^2\cos(a}{b^3}$$

Mathematica [B] time = 1.32, size = 837, normalized size = 2.51

$$\frac{c^4\cos(a+bx)b^4 + d^4x^4\cos(a+bx)b^4 + 4cd^3x^3\cos(a+bx)b^4 + 6c^2d^2x^2\cos(a+bx)b^4 + 4c^3dx\cos(a+bx)b^4 + c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (b^4*c^4*Cos[a + b*x] - 12*b^2*c^2*d^2*Cos[a + b*x] + 24*d^4*Cos[a + b*x] +
 4*b^4*c^3*d*x*Cos[a + b*x] - 24*b^2*c*d^3*x*Cos[a + b*x] + 6*b^4*c^2*d^2*x
^2*Cos[a + b*x] - 12*b^2*d^4*x^2*Cos[a + b*x] + 4*b^4*c*d^3*x^3*Cos[a + b*x
] + b^4*d^4*x^4*Cos[a + b*x] + b^4*c^4*Log[1 - E^(I*(a + b*x))] + 4*b^4*c^3
*d*x*Log[1 - E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))]
+ 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 - E^(I*(a +
b*x))] - b^4*c^4*Log[1 + E^(I*(a + b*x))] - 4*b^4*c^3*d*x*Log[1 + E^(I*(a +
 b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] - b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] + (4*I)*b^3*d*(
c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
```

```
, E^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*b^2*c
*d^3*x*PolyLog[3, -E^(I*(a + b*x))] - 12*b^2*d^4*x^2*PolyLog[3, -E^(I*(a +
b*x))] + 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*b^2*c*d^3*x*PolyLo
g[3, E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] - (24*I)
*b*c*d^3*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a
+ b*x))] + (24*I)*b*c*d^3*PolyLog[4, E^(I*(a + b*x))] + (24*I)*b*d^4*x*Poly
Log[4, E^(I*(a + b*x))] + 24*d^4*PolyLog[5, -E^(I*(a + b*x))] - 24*d^4*Poly
Log[5, E^(I*(a + b*x))] - 4*b^3*c^3*d*Sin[a + b*x] + 24*b*c*d^3*Sin[a + b*x
] - 12*b^3*c^2*d^2*x*Sin[a + b*x] + 24*b*d^4*x*Sin[a + b*x] - 12*b^3*c*d^3*
x^2*Sin[a + b*x] - 4*b^3*d^4*x^3*Sin[a + b*x])/b^5
```

fricas [C] time = 0.75, size = 1367, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*
x + a)) - 24*d^4*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 2*(b^4*d^4*x^
4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 -
2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a) - (-4*I*b^3*d^
4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(
b*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b
^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-4*I*
b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilo
g(-cos(b*x + a) + I*sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 +
 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a))
+ (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*
c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*sin(b
*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3
 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b
*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 +
4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2
*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a)
+ 1) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x +
a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x +
a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x +
a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x +
 a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, cos(b*x +
```

```
a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polyl
og(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*
(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 - 2
*b*d^4)*x)*sin(b*x + a))/b^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4\cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a), x)
```

maple [B] time = 0.23, size = 1295, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] -12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(
b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x
+a)))*x^2-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+24*d^4*polylog(5,-exp(I
*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/2*(d^4*x^4*b^4+4*b^4*
c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^
4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2-4
*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))+1/2*(d^4
*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^
4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-1
2*c^2*d^2*b^2+4*I*b^3*c^3*d-24*I*b*d^4*x+24*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*
x+a))-2/b*c^4*arctanh(exp(I*(b*x+a)))-4/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-24
*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*
x+a)))+4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(
I*(b*x+a)))*x^3+8/b^4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*
arctanh(exp(I*(b*x+a)))+8/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))+6/b^3*c^2*d^2
*ln(exp(I*(b*x+a))+1)*a^2+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))+1)-2/b^5*d^4*a^4*
arctanh(exp(I*(b*x+a)))-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp
(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-6/b*c^2*d^2*ln(exp(I*(b*x
+a))+1)*x^2-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(
1-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog
(3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*p
```

```
olylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b
^2*c^3*d*polylog(2,exp(I*(b*x+a)))+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b*d^4
*ln(exp(I*(b*x+a))+1)*x^4-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I
/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3
+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/
b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a))
)*x^2+12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x
```

maxima [B] time = 0.64, size = 1538, normalized size = 4.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + (48*d^4*polylog(5, -e^(I*b*x + I*a)) - 48*d^4*po
lylog(5, e^(I*b*x + I*a)) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4
)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a
)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 - 8*I*a^3*d^4)
*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4
+ (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^
3 + 12*I*a^2*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*
a^2*b*c*d^3 - 8*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
 1) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*(a^2 - 2)*d^4
 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 -
2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3
- (a^3 - 6*a)*d^4)*(b*x + a))*cos(b*x + a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c
^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*
d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*
a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) - (8*I*b^3*c^3*d - 24*I*a*b^2*c
^2*d^2 + 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d
^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^
2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) - ((b*x + a)^4*d^4 + 4*(b*c*d^3 -
a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(co
s(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((b*x + a)^4*d^4 + 4*
(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*
x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x +
 a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (48*I*b*c*
d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*polylog(4, -e^(I*b*x + I*a)) - (-48*
```

```
I*b*c*d^3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) -
24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*
d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e
^(I*b*x + I*a)) - 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2
 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*sin(b*x + a))/b^4)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\cot\left(a+b\,x\right)\,{\left(c+d\,x\right)}^4 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^{4} \cos{\left(a + bx \right)} \cot{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x), x)
```

3.99 $\int (c+dx)^3 \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=254

$$-\frac{6id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*x+c)
^3*cos(b*x+a)/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c
)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/
b^3+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-exp(I*(b
*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*d^3*sin(b*x+a)/b^4-3*d*
(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A] time = 0.20, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (6*d^2*(c + d*x)*Cos[a + b*x]
)/b^3 + ((c + d*x)^3*Cos[a + b*x])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(
I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^2
- (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*(c + d*x)*Pol
yLog[3, E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^
4 + ((6*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (6*d^3*Sin[a + b*x])/b^4
- (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\csc(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b} - \frac{(3d)\int (c+dx)}{}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{3id(c+dx)^2\mathrm{Li}}{b^2}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\mathrm{co}}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\mathrm{co}}{b}$$
$$= -\frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\mathrm{co}}{b}$$

Mathematica [A] time = 0.94, size = 330, normalized size = 1.30

$$\frac{-2b^3(c+dx)^3\tanh^{-1}(\cos(a+bx)+i\sin(a+bx)) + 3id\left(b^2(c+dx)^2\mathrm{Li}_2(-\cos(a+bx)-i\sin(a+bx)) + 2ibd(c\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^2*(
c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)
*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a + b*x
] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I
*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*
x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]) + Cos[b*x]*(b*(c + d
*x)*(-6*d^2 + b^2*(c + d*x)^2)*Cos[a] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*Sin[
a]) - (3*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a] + b*(c + d*x)*(-6*d^2 + b^2*(c
 + d*x)^2)*Sin[a])*Sin[b*x])/b^4
```

fricas [C] time = 0.79, size = 921, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(
b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*c
os(b*x + a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(co
s(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2
*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*
c*d^2*x - 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d
^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*
x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*
sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
+ 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^
3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c
*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*po
lylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x +
 b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos(bx+a) \cot(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a), x)
```

maple [B] time = 0.15, size = 847, normalized size = 3.33

$$-\frac{6icd^2 \operatorname{polylog}\left(2, e^{i(bx+a)}\right) x}{b^2}+\frac{6icd^2 \operatorname{polylog}\left(2, -e^{i(bx+a)}\right) x}{b^2}+\frac{d^3 \ln\left(1-e^{i(bx+a)}\right) x^3}{b}+\frac{d^3 \ln\left(1-e^{i(bx+a)}\right) a^3}{b^4}-\frac{d^3 \ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a))
)/b^4-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b
*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-exp(I*(b
```

```
*x+a)))*x-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+6*I/b^2*c*d^2*polylog(2,-e
xp(I*(b*x+a)))*x+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I
*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4
*exp(I*(b*x+a))+1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*
b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*
exp(-I*(b*x+a))-2/b*c^3*arctanh(exp(I*(b*x+a)))+2/b^4*d^3*a^3*arctanh(exp(I
*(b*x+a)))-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+3/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a))+1)-1/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-6/b^3*c*d^2*a^2*arctanh(exp(I*(b
*x+a)))+6/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(I*
(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylo
g(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3/b*c^2*d*ln(
exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I
*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x
+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+1/b*d^3*ln(1-exp(I*(b*x+a)))*x
^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3
```

maxima [B] time = 0.52, size = 919, normalized size = 3.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 - (12*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 12*I*d^3*polylog(
4, e^(I*b*x + I*a)) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*arctan2
(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*
I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x
+ a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*
d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*dilog(-e^(I*b*x + I*a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x
 + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(e^
(I*b*x + I*a)) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*polylog(3, -e^(I*b*x + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*poly
```

```
log(3, e^(I*b*x + I*a)) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a
^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))/b^3)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)\cot(a+bx)(c+dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x), x)
```

3.100 $\int (c+dx)^2 \cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=171

$$-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\cos(a+bx)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d}{}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos(b
*x+a)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(
2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,
exp(I*(b*x+a)))/b^3-2*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (2*d^2*Cos[a + b*x])/b^3 + ((
c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))
])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (2*d^2*PolyL
og[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 - (2
*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2\csc(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\,dx$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} - \frac{(2d)\int (c+dx)}{b}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{2id(c+dx)\mathrm{Li}_2}{b^2}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx}{b}$$
$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx}{b}$$

Mathematica [A] time = 0.85, size = 221, normalized size = 1.29

$$\frac{\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)-\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - b^2*(c + d*x)^2*Log[1 + E^(I*(a
 + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (2*I)*b*d*(c
 + d*x)*PolyLog[2, E^(I*(a + b*x))] - 2*d^2*PolyLog[3, -E^(I*(a + b*x))] +
2*d^2*PolyLog[3, E^(I*(a + b*x))] + Cos[b*x]*((-2*d^2 + b^2*(c + d*x)^2)*Co
s[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c
 + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

fricas [C] time = 1.54, size = 558, normalized size = 3.26

$$\frac{2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+2\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-2\,d^2\mathrm{polylog}\left(3,-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 2*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-2*I*b*d^2*x - 2*I*b*c*d)*dil
```

```
og(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I*sin
(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a) + 1/
2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(b*d
^2*x + b*c*d)*sin(b*x + a))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cos(bx+a)\cot(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a), x)
```

maple [B] time = 0.13, size = 479, normalized size = 2.80

$$\frac{\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{2b^3}+\frac{\left(d^2x^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}^{-i(b}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*ln(exp(I*(b*x
+a))+1)*a+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-2
/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2
*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b*c^2*arctanh(exp(I*(b*x+a)))-1/b*d^2*
ln(exp(I*(b*x+a))+1)*x^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+2*I/b^2*d^2*pol
ylog(2,-exp(I*(b*x+a)))*x+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-e
xp(I*(b*x+a)))*a^2-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x+4/b^2*c*d*a*arct
anh(exp(I*(b*x+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+2*I/b^2*c*d*polyl
og(2,-exp(I*(b*x+a)))
```

maxima [B] time = 0.47, size = 507, normalized size = 2.96

$$\frac{c^2\left(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\right)-\frac{2\,acd\left(2\cos(bx+a)-\log(\cos(bx+a)+1)+\log(\cos(bx+a)-1)\right)}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2*cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 - (4*d^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*
a)) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^
2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(b*x + a) + (-4*I*b*c*d - 4*I*(b
*x + a)*d^2 + 4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x +
a)*d^2 - 4*I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin
(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(b
*x + a))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)\,\left(c+d\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x), x)
```

3.101 $\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=94

$$\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] -2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+(d*x+c)*cos(b*x+a)/b+I*d*polylog(2,-ex
p(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^2-d*sin(b*x+a)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\sin(a+bx)}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}-\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + ((c + d*x)*Cos[a + b*x])/b + (I
*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^
2 - (d*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx = \int (c+dx)\csc(a+bx)\,dx - \int (c+dx)\sin(a+bx)\,dx$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\int \cos(a+bx)\,dx}{b}$$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} - \frac{d\sin(a+bx)}{b^2} + $$

$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{id\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}$$

Mathematica [A] time = 0.17, size = 176, normalized size = 1.87

$$\frac{d\left(i\left(\mathrm{Li}_2\left(-e^{i(a+bx)}\right)-\mathrm{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-a\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\right)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x],x]
```

```
[Out] (c*Cos[a + b*x])/b - (c*Log[Cos[(a + b*x)/2]])/b + (c*Log[Sin[(a + b*x)/2]]
)/b + (d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) -
 a*Log[Tan[(a + b*x)/2]] + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(
```

```
I*(a + b*x))]])))/b^2 + (d*Cos[b*x]*(b*x*Cos[a] - Sin[a]))/b^2 - (d*(Cos[a]
+ b*x*Sin[a])*Sin[b*x])/b^2
```

fricas [B] time = 0.65, size = 277, normalized size = 2.95

$$\frac{2\,(bdx+bc)\cos(bx+a) - i\,d\mathrm{Li}_2(\cos(bx+a)+i\,\sin(bx+a)) + i\,d\mathrm{Li}_2(\cos(bx+a)-i\,\sin(bx+a)) - i\,d\mathrm{Li}_2(-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + b*c)*cos(b*x + a) - I*d*dilog(cos(b*x + a) + I*sin(b*x + a)
) + I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*dilog(-cos(b*x + a) + I*
sin(b*x + a)) + I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x + b*c)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a) - I*
sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
+ 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b
*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b*d*x + a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) - 2*d*sin(b*x + a))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cos(bx+a)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)*cot(b*x + a), x)
```

maple [B] time = 0.08, size = 199, normalized size = 2.12

$$\frac{d\cos(bx+a)\,x}{b} - \frac{d\sin(bx+a)}{b^2} + \frac{c\cos(bx+a)}{b} + \frac{d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{b} - \frac{d\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x}{b} + \frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] 1/b*d*cos(b*x+a)*x-d*sin(b*x+a)/b^2+1/b*c*cos(b*x+a)+1/b*d*ln(1-exp(I*(b*x+
a)))*x-1/b*d*ln(exp(I*(b*x+a))+1)*x+I/b^2*d*dilog(exp(I*(b*x+a))+1)-I/b^2*d
*dilog(1-exp(I*(b*x+a)))+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-1/b^2*d*ln(exp(I*(b
*x+a))+1)*a-1/b^2*d*a*ln(csc(b*x+a)-cot(b*x+a))+1/b*c*ln(csc(b*x+a)-cot(b*x
+a))
```

maxima [B] time = 0.45, size = 199, normalized size = 2.12

$$-\frac{2i\,bdx\arctan\left(\sin\left(bx+a\right),-\cos\left(bx+a\right)+1\right)-2i\,bc\arctan\left(\sin\left(bx+a\right),\cos\left(bx+a\right)-1\right)+\left(2i\,bdx+2i\,bc\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(b*x + a
), cos(b*x + a) + 1) - 2*(b*d*x + b*c)*cos(b*x + a) - 2*I*d*dilog(-e^(I*b*x
 + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 2*d*sin(b*x + a))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)\,\cot\left(a+b\,x\right)\,\left(c+d\,x\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)*(c + d*x),x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos\left(a+bx\right)\cot\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x), x)
```

3.102 $\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=70

$$\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -cos(a-b*c/d)*Si(b*c/d+b*x)/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(csc
(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\csc(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\csc\cdots}{\cdots}\\ &= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\csc(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A] time = 8.35, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x),x]
```

[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x), x]

fricas [A] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="fricas")

[Out] integral(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="giac")

[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c), x)

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)

[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(iE_1\left(\frac{ibdx+ibc}{d}\right)-iE_1\left(-\frac{ibdx+ibc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+2d\int \frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx+2d\int \frac{}{(dx+c)(\mathrm{cc}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x, algorithm="maxima")

[Out] 1/2*((I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a) + c), x) + 2*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +

```
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-
(b*c - a*d)/d))/d
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x))/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x), x)
```

3.103 $\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=88

$$\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}$$

```
[Out] -b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+
a)/d/(d*x+c)+Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\csc(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\text{c}}{}
\end{aligned}$$

Mathematica [A] time = 4.02, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*cot(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(i\,E_2\left(\frac{i\,bdx+i\,bc}{d}\right)-i\,E_2\left(-\frac{i\,bdx+i\,bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+2\left(d^2x+cd\right)\int\frac{\sin(bx+a)}{(dx+c)^2\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+1\right)}\,dx+2\left(d^2\right.}{2\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin(b*x +
a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*
x + a)), x) + 2*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (
d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x
 + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (exp_integr
al_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (a+b\,x)\,\cot (a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x))/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos (a+bx)\cot (a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)/(c + d*x)**2, x)
```

3.104 $\int (c+dx)^m \cot^2(a+bx)\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\cot^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*cot(b*x+a)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \cot^2(a+bx)\,dx = \int (c+dx)^m \cot^2(a+bx)\,dx$$

Mathematica [A] time = 1.20, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2, x]
```

fricas [A] time = 0.71, size = 0, normalized size = 0.00

$$\operatorname{integral}\left((dx+c)^m \cot(bx+a)^2\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*cot(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cot^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2,x)
[Out] int((d*x+c)^m*cot(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \cot(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cot(a + b*x)^2*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*cot(a + b*x)**2, x)
```

3.105 $\int (c+dx)^4 \cot^2(a+bx)\,dx$

Optimal. Leaf size=155

$$\frac{3id^4\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^5}+\frac{6d^3(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx}{}$$

```
[Out] -I*(d*x+c)^4/b-1/5*(d*x+c)^5/d-(d*x+c)^4*cot(b*x+a)/b+4*d*(d*x+c)^3*ln(1-ex
p(2*I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^3+6*d^3
*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^4+3*I*d^4*polylog(4,exp(2*I*(b*x+a))
)/b^5
```

Rubi [A] time = 0.23, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3717, 2190, 2531, 6609, 2282, 6589, 32}

$$-\frac{6id^2(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{6d^3(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^4}+\frac{3id^4\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^5}+\frac{4d(c+dx)^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^4)/b - (c + d*x)^5/(5*d) - ((c + d*x)^4*Cot[a + b*x])/b + (
4*d*(c + d*x)^3*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)^2*
PolyLog[2, E^((2*I)*(a + b*x))])/b^3 + (6*d^3*(c + d*x)*PolyLog[3, E^((2*I)
*(a + b*x))])/b^4 + ((3*I)*d^4*PolyLog[4, E^((2*I)*(a + b*x))])/b^5
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\,dx = -\frac{(c+dx)^4\cot(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\cot(a+bx)\,dx}{b} - \int (c+dx)^4\,dx$$
$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} - \frac{(8id)\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} + \frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$
$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} + \frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$
$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} + \frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$
$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} + \frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$
$$= -\frac{i(c+dx)^4}{b} - \frac{(c+dx)^5}{5d} - \frac{(c+dx)^4\cot(a+bx)}{b} + \frac{4d(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$

Mathematica [B] time = 6.72, size = 795, normalized size = 5.13

$$-\frac{e^{ia}\csc(a)\left(b^4e^{-2ia}x^4 + 2ib^3\left(1-e^{-2ia}\right)\log\left(1-e^{-i(a+bx)}\right)x^3 + 2ib^3\left(1-e^{-2ia}\right)\log\left(1+e^{-i(a+bx)}\right)x^3 - 6e^{-2ia}\left(-1+e\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2,x]
```

```
[Out] -1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)) - (2
*c*d^3*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a
))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1
 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a
+ b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x
))]))/E^((2*I)*a)))/b^4 - (d^4*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I
)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E
^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x
^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x
))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a
))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(
```

```
a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^5 + (4*c^3*
d*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(
b^2*(Cos[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c^4*Sin[b*x] + 4*c^3*d*x
*Sin[b*x] + 6*c^2*d^2*x^2*Sin[b*x] + 4*c*d^3*x^3*Sin[b*x] + d^4*x^4*Sin[b*x
]))/b - (6*c^2*d^2*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-
Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a
]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan
[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcT
an[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^3*Sqrt[Sec[a]^2*(Cos[a]^2 +
Sin[a]^2)])
```

fricas [C] time = 0.95, size = 856, normalized size = 5.52

$$-\frac{10\,b^4d^4x^4 + 40\,b^4cd^3x^3 + 60\,b^4c^2d^2x^2 + 40\,b^4c^3dx + 10\,b^4c^4 - 15i\,d^4\text{polylog}\left(4, \cos\left(2\,bx + 2\,a\right) + i\,\sin\left(2\,bx + \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/10*(10*b^4*d^4*x^4 + 40*b^4*c*d^3*x^3 + 60*b^4*c^2*d^2*x^2 + 40*b^4*c^3*
d*x + 10*b^4*c^4 - 15*I*d^4*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a
))*sin(2*b*x + 2*a) + 15*I*d^4*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x +
2*a))*sin(2*b*x + 2*a) - (-30*I*b^2*d^4*x^2 - 60*I*b^2*c*d^3*x - 30*I*b^2*c
^2*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - (30
*I*b^2*d^4*x^2 + 60*I*b^2*c*d^3*x + 30*I*b^2*c^2*d^2)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*
a) + 1/2)*sin(2*b*x + 2*a) - 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 - a^3*d^4)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(
2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^
2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x +
2*a) + 1)*sin(2*b*x + 2*a) - 20*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*
d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*log(-cos(2*b*x + 2*a) -
I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 30*(b*d^4*x + b*c*d^3)*polylog(3
, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 30*(b*d^4*x + b
*c*d^3)*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a)
+ 10*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b
^4*c^4)*cos(2*b*x + 2*a) + 2*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*b^5*c^2*d^
2*x^3 + 10*b^5*c^3*d*x^2 + 5*b^5*c^4*x)*sin(2*b*x + 2*a))/(b^5*sin(2*b*x +
2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \cot (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2, x)
```

maple [B] time = 0.13, size = 913, normalized size = 5.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2,x)
```

```
[Out] 24/b^4*d^3*c*polylog(3,-exp(I*(b*x+a)))+24/b^4*d^3*c*polylog(3,exp(I*(b*x+a
)))+24/b^4*d^4*polylog(3,-exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,exp(I*(b*x
+a)))*x-24*I/b^3*d^3*c*polylog(2,exp(I*(b*x+a)))*x-c*d^3*x^4+24*I*d^4*polyl
og(4,exp(I*(b*x+a)))/b^5-1/5*d^4*x^5-c^4*x-2*c^2*d^2*x^3-2*c^3*d*x^2+8/b^5*
d^4*a^3*ln(exp(I*(b*x+a)))-4/b^5*d^4*a^3*ln(exp(I*(b*x+a))-1)+4/b^2*d*c^3*l
n(exp(I*(b*x+a))-1)+4/b^2*d*c^3*ln(exp(I*(b*x+a))+1)-8/b^2*d*c^3*ln(exp(I*(
b*x+a)))+24*I/b^5*d^4*polylog(4,-exp(I*(b*x+a)))-6*I/b^5*d^4*a^4-2*I/b*d^4*
x^4-24*I/b^2*d^2*c^2*a*x+24*I/b^3*d^3*c*a^2*x-24*I/b^3*d^3*c*polylog(2,-exp
(I*(b*x+a)))*x-12*I/b^3*d^2*c^2*polylog(2,-exp(I*(b*x+a)))+16*I/b^4*d^3*c*a
^3-8*I/b^4*d^4*a^3*x-8*I/b*d^3*c*x^3-12*I/b*d^2*c^2*x^2-12*I/b^3*d^2*c^2*a^
2-12*I/b^3*d^4*polylog(2,-exp(I*(b*x+a)))*x^2+24/b^3*d^2*c^2*a*ln(exp(I*(b*
x+a)))-24/b^4*d^3*c*a^2*ln(exp(I*(b*x+a)))-12/b^3*d^2*c^2*a*ln(exp(I*(b*x+a
))-1)+12/b^4*d^3*c*a^2*ln(exp(I*(b*x+a))-1)-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*
d^2*x^2+4*c^3*d*x+c^4)/b/(exp(2*I*(b*x+a))-1)+12/b^2*d^2*c^2*ln(1-exp(I*(b*
x+a)))*x+12/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a+12/b^2*d^2*c^2*ln(exp(I*(b*x
+a))+1)*x+12/b^2*d^3*c*ln(1-exp(I*(b*x+a)))*x^2-12/b^4*d^3*c*ln(1-exp(I*(b*
x+a)))*a^2+12/b^2*d^3*c*ln(exp(I*(b*x+a))+1)*x^2+4/b^2*d^4*ln(1-exp(I*(b*x+
a)))*x^3+4/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^3+4/b^2*d^4*ln(exp(I*(b*x+a))+1)*
x^3-12*I/b^3*d^4*polylog(2,exp(I*(b*x+a)))*x^2-12*I/b^3*d^2*c^2*polylog(2,e
xp(I*(b*x+a)))
```

maxima [B] time = 1.21, size = 3229, normalized size = 20.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((b*x + a + 1/tan(b*x + a))*c^4 - 4*(b*x + a + 1/tan(b*x + a))*a*c^3*d/b +
 6*(b*x + a + 1/tan(b*x + a))*a^2*c^2*d^2/b^2 - 4*(b*x + a + 1/tan(b*x + a)
)*a^3*c*d^3/b^3 + (b*x + a + 1/tan(b*x + a))*a^4*d^4/b^4 + 2*((b*x + a)^2*c
os(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*
```

```
x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(co
s(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x
 + 2*a))*c^3*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*
a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2
*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a)
 + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a*c^2*d^2/((cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^2) + 6*((b*x + a)^2*cos(2*b*x +
2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
(b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
 + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a^
2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
*b^3) - 2*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2
 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 + 4*(b*x + a)*sin(2*b*x + 2*a))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^4) - (-I*(b*x + a)^5*d^4 + (-5*I*b*c*d^
3 + 5*I*a*d^4)*(b*x + a)^4 + (-10*I*b^2*c^2*d^2 + 20*I*a*b*c*d^3 - 10*I*a^2
*d^4)*(b*x + a)^3 - (20*(b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4
+ 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a))*cos(2*b*x + 2*a) - (20*I*(b*x + a)^3*d^4 + (60*I*b*c*d^3 - 60*I
*a*d^4)*(b*x + a)^2 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + 60*I*a^2*d^4)*(
b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (20*(
b*x + a)^3*d^4 + 60*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 60*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + a^2*d^4)*(b*x + a) - 20*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*
x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2
*a) + (-20*I*(b*x + a)^3*d^4 + (-60*I*b*c*d^3 + 60*I*a*d^4)*(b*x + a)^2 + (
-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a))*sin(2*b*x +
2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*(b*x + a)^5*d^4 + (5*I*
b*c*d^3 - 5*(I*a + 2)*d^4)*(b*x + a)^4 + (10*I*b^2*c^2*d^2 - 20*(I*a + 2)*b
*c*d^3 + (10*I*a^2 + 40*a)*d^4)*(b*x + a)^3 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3
 + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*
(b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))*
```

```
sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (60*b^2*c^2*d^2 - 120*a*b*c*d^3
 + 60*(b*x + a)^2*d^4 + 60*a^2*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a) - 60*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (-60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*
(b*x + a)^2*d^4 - 60*I*a^2*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a))*
sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (10*I*(b*x + a)^3*d^4 + (30*I*b*
c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*b*c*d^3 + 30*I
*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3 + 30*I*a*d^4)
*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)*(b*x + a
))*cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (10*I*(b*x + a)^3*
d^4 + (30*I*b*c*d^3 - 30*I*a*d^4)*(b*x + a)^2 + (30*I*b^2*c^2*d^2 - 60*I*a*
b*c*d^3 + 30*I*a^2*d^4)*(b*x + a) + (-10*I*(b*x + a)^3*d^4 + (-30*I*b*c*d^3
 + 30*I*a*d^4)*(b*x + a)^2 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 10*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^
4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 120*
(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog(4, -e^(I*b*x
+ I*a)) + 120*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) - d^4)*polylog
(4, e^(I*b*x + I*a)) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 +
 (-120*I*b*c*d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*cos(2*b*x + 2*a) + 12
0*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x
+ I*a)) + (120*I*b*c*d^3 + 120*I*(b*x + a)*d^4 - 120*I*a*d^4 + (-120*I*b*c*
d^3 - 120*I*(b*x + a)*d^4 + 120*I*a*d^4)*cos(2*b*x + 2*a) + 120*(b*c*d^3 +
(b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - ((b*
x + a)^5*d^4 + (5*b*c*d^3 - (5*a - 10*I)*d^4)*(b*x + a)^4 + (10*b^2*c^2*d^2
 - (20*a - 40*I)*b*c*d^3 + 10*(a^2 - 4*I*a)*d^4)*(b*x + a)^3 - (-60*I*b^2*c
^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-5
*I*b^4*cos(2*b*x + 2*a) + 5*b^4*sin(2*b*x + 2*a) + 5*I*b^4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^2\,(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2, x)
```

3.106 $\int (c+dx)^3 \cot^2(a+bx)\,dx$

Optimal. Leaf size=127

$$\frac{3d^3\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+bx)}{b}-\frac{i(c+dx)^3}{b}-\frac{(}{}$$

```
[Out] -I*(d*x+c)^3/b-1/4*(d*x+c)^4/d-(d*x+c)^3*cot(b*x+a)/b+3*d*(d*x+c)^2*ln(1-ex
p(2*I*(b*x+a)))/b^2-3*I*d^2*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^3+3/2*d^3
*polylog(3,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^3\cot(a+}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) - ((c + d*x)^3*Cot[a + b*x])/b + (
3*d*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*Po
lyLog[2, E^((2*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^4)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \cot^2(a+bx)\,dx = -\frac{(c+dx)^3\cot(a+bx)}{b} + \frac{(3d)\int (c+dx)^2\cot(a+bx)\,dx}{b} - \int (c+dx)^3\,dx$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} - \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$

$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} - \frac{(c+dx)^3\cot(a+bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b^2} -$$

Mathematica [B] time = 6.15, size = 374, normalized size = 2.94

$$-\frac{3c^2d(bx\cot(a)-\log(\sin(a+bx)))}{b^2} + \frac{3cd^2\left(-b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} - i\operatorname{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right) + ibx\left(\pi\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) - (3*c^2*d*(b*x*Cot[a]
 - Log[Sin[a + b*x]]))/b^2 + (3*c*d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*L
og[1 + E^((-2*I)*b*x)] + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + A
rcTan[Tan[a]]))] - Pi*Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan
[Tan[a]]]] - I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcT
an[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^2]))/b^3 + (d^3*(I + Cot[a])*(I*b^3*x^3
- b^3*x^3*Cot[a] + 3*b^2*x^2*Log[1 - E^((-I)*(a + b*x))] + 3*b^2*x^2*Log[1
+ E^((-I)*(a + b*x))] + (6*I)*b*x*PolyLog[2, -E^((-I)*(a + b*x))] + (6*I)*b
*x*PolyLog[2, E^((-I)*(a + b*x))] + 6*PolyLog[3, -E^((-I)*(a + b*x))] + 6*P
olyLog[3, E^((-I)*(a + b*x))])*Sin[a])/(b^4*E^(I*a)) + ((c + d*x)^3*Csc[a]*
Csc[a + b*x]*Sin[b*x])/b
```

fricas [C] time = 0.78, size = 599, normalized size = 4.72

$$-\frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx + 4\,b^3c^3 - 3\,d^3\mathrm{polylog}\left(3, \cos\left(2\,bx+2\,a\right) + i\,\sin\left(2\,bx+2\,a\right)\right)\sin\left(2\,bx+2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="fricas")

[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 3*d^3 *polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*d^3 *polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - (-6*I *b*d^3*x - 6*I*b*c*d^2)*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))*sin(2* b*x + 2*a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(cos(2*b*x + 2*a) - I*sin(2*b *x + 2*a))*sin(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/ 2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^ 2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b* x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c *d^2 - a^2*d^3)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(2* b*x + 2*a) - I*sin(2*b*x + 2*a) + 1)*sin(2*b*x + 2*a) + 4*(b^3*d^3*x^3 + 3* b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(2*b*x + 2*a) + (b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x)*sin(2*b*x + 2*a))/(b^4*sin (2*b*x + 2*a))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cot(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^3*cot(b*x + a)^2, x)

maple [B] time = 0.11, size = 573, normalized size = 4.51

$$\frac{3d^3a^2\ln\left(e^{i(bx+a)}-1\right)}{b^4}-\frac{6d^3a^2\ln\left(e^{i(bx+a)}\right)}{b^4}+\frac{3d\,c^2\ln\left(e^{i(bx+a)}-1\right)}{b^2}+\frac{3d\,c^2\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{6d\,c^2\ln\left(e^{i(bx+a)}\right)}{b^2}-\frac{2id^3}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*cot(b*x+a)^2,x)

[Out] 3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-6/b^4*d^3*a^2*ln(exp(I*(b*x+a)))+3/b^2*d *c^2*ln(exp(I*(b*x+a))-1)+3/b^2*d*c^2*ln(exp(I*(b*x+a))+1)-6/b^2*d*c^2*ln(e xp(I*(b*x+a)))-2*I/b*d^3*x^3+4*I/b^4*d^3*a^3-12*I/b^2*d^2*c*a*x+6*d^3*polyl og(3,-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-3/2*c^2*d*x^2 -c*d^2*x^3-1/4*d^3*x^4-c^3*x+3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^2*d^3*l n(1-exp(I*(b*x+a)))*x^2-3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2-2*I*(d^3*x^3+3*c

```
*d^2*x^2+3*c^2*d*x+c^3)/b/(exp(2*I*(b*x+a))-1)-6*I/b^3*d^3*polylog(2,exp(I*
(b*x+a)))*x+6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6*I/b^3*d^2*c*polylog(2,exp(
I*(b*x+a)))+12/b^3*d^2*c*a*ln(exp(I*(b*x+a)))-6/b^3*d^2*c*a*ln(exp(I*(b*x+a
))-1)-6*I/b^3*d^2*c*polylog(2,-exp(I*(b*x+a)))+6*I/b^3*d^3*a^2*x-6*I/b*d^2*
c*x^2-6*I/b^3*d^2*c*a^2-6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+6/b^2*d^2*
c*ln(1-exp(I*(b*x+a)))*x+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a
```

maxima [B] time = 0.74, size = 1945, normalized size = 15.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c^3 - 6*(b*x + a + 1/tan(b*x + a))*a*c^2
*d/b + 6*(b*x + a + 1/tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a + 1/tan(b*x
+ a))*a^3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b
*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b
*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*c^2*d/((cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b) - 6*((b*x + a)^2*cos(2*b*x +
 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x + a)^2*cos(2*b*x + 2*a) +
 (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a
) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (cos(2*b
*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x + a)*sin(2*b*x + 2*a))*a
*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*
b^2) + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 -
 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*cos(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
+ 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(-I*(b*x + a)^4*d^3 + (-4*I*b*c*d
^2 + 4*I*a*d^3)*(b*x + a)^3 - (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b
*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x +
2*a) - (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*(b*x + a)^2*d^3
+ 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*(b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 2
4*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 -
24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) + (24*b*c*d^2 + 24*(b*x
```

```
+ a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(I*b*x + I*a)) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d
^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-24*I*b*c*d^2 - 24*I*(b*x +
 a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (6*I*(b*x
+ a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d
^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*
a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos
(b*x + a) + 1) + (-24*I*d^3*cos(2*b*x + 2*a) + 24*d^3*sin(2*b*x + 2*a) + 24
*I*d^3)*polylog(3, -e^(I*b*x + I*a)) + (-24*I*d^3*cos(2*b*x + 2*a) + 24*d^3
*sin(2*b*x + 2*a) + 24*I*d^3)*polylog(3, e^(I*b*x + I*a)) - ((b*x + a)^4*d^
3 + (4*b*c*d^2 - (4*a - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3
)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b
*x + 2*a) + 4*I*b^3))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+b\,x)^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**2, x)
```

3.107 $\int (c+dx)^2 \cot^2(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{id^2\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

```
[Out] -I*(d*x+c)^2/b-1/3*(d*x+c)^3/d-(d*x+c)^2*cot(b*x+a)/b+2*d*(d*x+c)*ln(1-exp(
2*I*(b*x+a)))/b^2-I*d^2*polylog(2,exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.13, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3720, 3717, 2190, 2279, 2391, 32}

$$-\frac{id^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) - ((c + d*x)^2*Cot[a + b*x])/b + (
2*d*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^2 - (I*d^2*PolyLog[2, E^((2*I
)*(a + b*x))])/b^3
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cot^2(a+bx)\,dx = -\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{(2d)\int (c+dx)\cot(a+bx)\,dx}{b}-\int (c+dx)^2\,dx$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}-\frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}+$$

$$= -\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}-\frac{(c+dx)^2\cot(a+bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^2}-$$

Mathematica [B] time = 6.03, size = 198, normalized size = 2.04

$$-\frac{2cd(bx\cot(a)-\log(\sin(a+bx)))}{b^2}+\frac{d^2\left(-b^2x^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)}-i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(\pi-\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) - (2*c*d*(b*x*Cot[a] - Log[Sin[a + b*x
]]))/b^2 + (d^2*(I*b*x*(Pi - 2*ArcTan[Tan[a]]) + Pi*Log[1 + E^((-2*I)*b*x)]
 + 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - Pi*
Log[Cos[b*x]] - 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] - I*PolyLog
[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))] - b^2*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]
*Sqrt[Sec[a]^2]))/b^3 + ((c + d*x)^2*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

fricas [B] time = 0.72, size = 384, normalized size = 3.96

$$-\frac{6\,b^2d^2x^2+12\,b^2cdx+6\,b^2c^2+3i\,d^2\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\left(2\,bx+2\,a\right)\right)\sin\left(2\,bx+2\,a\right)-3i\,d^2\mathrm{Li}_2\left(\cos\left(2\,bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*b^2*d^2*x^2 + 12*b^2*c*d*x + 6*b^2*c^2 + 3*I*d^2*dilog(cos(2*b*x +
2*a) + I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 3*I*d^2*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a))*sin(2*b*x + 2*a) - 6*(b*c*d - a*d^2)*log(-1/2*cos(2
*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) - 6*(b*c*d - a
*d^2)*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2)*sin(2*b*x +
 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1)
*sin(2*b*x + 2*a) - 6*(b*d^2*x + a*d^2)*log(-cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a) + 1)*sin(2*b*x + 2*a) + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(2*b*x + 2*a) + 2*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 3*b^3*c^2*x)*sin(2*b*x + 2
*a))/(b^3*sin(2*b*x + 2*a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\cot(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2, x)
```

maple [B] time = 0.09, size = 297, normalized size = 3.06

$$-\frac{d^2x^3}{3}-cd\,x^2-c^2x-\frac{4id^2ax}{b^2}+\frac{2dc\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^2}+\frac{2dc\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b^2}-\frac{4dc\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}-\frac{2id^2\,\mathrm{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x-4*I/b^2*d^2*a*x+2/b^2*d*c*ln(exp(I*(b*x+a))-1)+2
/b^2*d*c*ln(exp(I*(b*x+a))+1)-4/b^2*d*c*ln(exp(I*(b*x+a)))-2*I/b^3*d^2*poly
log(2,-exp(I*(b*x+a)))-2*I/b^3*d^2*a^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^
3+2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x-2*I*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*
x+a))-1)+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a-
2*I/b*d^2*x^2-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)+4/b^3*d^2*a*ln(exp(I*(b*x+a)
))
```

maxima [B] time = 0.74, size = 646, normalized size = 6.66

$$\frac{-i b^3d^2x^3 - 3i b^3cdx^2 - 3i b^3c^2x - 6 b^2c^2 - \left(6 bd^2x + 6 bcd - 6 \left(bd^2x + bcd\right) \cos\left(2 bx + 2 a\right) - \left(6i bd^2x + 6i bcd\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (-I*b^3*d^2*x^3 - 3*I*b^3*c*d*x^2 - 3*I*b^3*c^2*x - 6*b^2*c^2 - (6*b*d^2*x
+ 6*b*c*d - 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) - (6*I*b*d^2*x + 6*I*b*c*d
)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*b*c*d*cos(
2*b*x + 2*a) + 6*I*b*c*d*sin(2*b*x + 2*a) - 6*b*c*d)*arctan2(sin(b*x + a),
cos(b*x + a) - 1) - (6*b*d^2*x*cos(2*b*x + 2*a) + 6*I*b*d^2*x*sin(2*b*x + 2
*a) - 6*b*d^2*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (I*b^3*d^2*x^3
+ (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*cos(2*b*x +
 2*a) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(-e^(I
*b*x + I*a)) - 6*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) - d^2)*dilo
g(e^(I*b*x + I*a)) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*b*d^2*x - 3*I*b*c*d)*
cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*b*d^2*x + 3*I*b*c*d + (-3*I*
b*d^2*x - 3*I*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a
))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (b^3*d^2*x^3
 + 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x
+ 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) + 3*I*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^{2} \cot^{2}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2, x)
```

3.108 $\int (c+dx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=41

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

```
[Out] -c*x-1/2*d*x^2-(d*x+c)*cot(b*x+a)/b+d*ln(sin(b*x+a))/b^2
```

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3720, 3475}

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{(c+dx)\cot(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 - ((c + d*x)*Cot[a + b*x])/b + (d*Log[Sin[a + b*x]])/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cot^2(a+bx)\,dx &= -\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\int \cot(a+bx)\,dx}{b}-\int (c+dx)\,dx\\ &= -cx-\frac{dx^2}{2}-\frac{(c+dx)\cot(a+bx)}{b}+\frac{d\log(\sin(a+bx))}{b^2}\end{aligned}$$

Mathematica [C] time = 0.48, size = 82, normalized size = 2.00

$$\frac{d\log(\sin(a+bx))}{b^2}-\frac{c\cot(a+bx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\tan^2(a+bx)\right)}{b}+\frac{dx\csc(a)\sin(bx)\csc(a+bx)}{b}-\frac{dx\csc(a)(bx\sin(a}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2,x]
```

```
[Out] -((c*Cot[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Tan[a + b*x]^2])/b) + (d
*Log[Sin[a + b*x]])/b^2 - (d*x*Csc[a]*(2*Cos[a] + b*x*Sin[a]))/(2*b) + (d*x
*Csc[a]*Csc[a + b*x]*Sin[b*x])/b
```

fricas [B] time = 0.59, size = 97, normalized size = 2.37

$$-\frac{2\,bdx-d\log\left(-\frac{1}{2}\,\cos\left(2\,bx+2\,a\right)+\frac{1}{2}\right)\sin\left(2\,bx+2\,a\right)+2\,bc+2\left(bdx+bc\right)\cos\left(2\,bx+2\,a\right)+\left(b^2dx^2+2\,b^2cx\right)}{2\,b^2\sin\left(2\,bx+2\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x - d*log(-1/2*cos(2*b*x + 2*a) + 1/2)*sin(2*b*x + 2*a) + 2*b*c
 + 2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d*x^2 + 2*b^2*c*x)*sin(2*b*x + 2
*a))/(b^2*sin(2*b*x + 2*a))
```

giac [B] time = 2.53, size = 1375, normalized size = 33.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*d*x^2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c*x*tan(1/2*b*x)*tan(1/2
*a)^2 - b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*d*x^2*tan(1/2*b*x) - b^2*d*
x^2*tan(1/2*a) - b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*b^2*c*x*tan(1/2*b*x) +
 b*d*x*tan(1/2*b*x)^2 - 2*b^2*c*x*tan(1/2*a) + 4*b*d*x*tan(1/2*b*x)*tan(1/2
*a) - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3
 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*
x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/
2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2*a) - 6*tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1/2*b*x)^2*tan(1/2*a)^4 +
2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a
```

```
)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2
*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan
(1/2*a) + b*d*x*tan(1/2*a)^2 - d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*
tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3
+ 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2
*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1
/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2
 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
 + 1))*tan(1/2*b*x)*tan(1/2*a)^2 + b*c*tan(1/2*b*x)^2 + 4*b*c*tan(1/2*b*x)*
tan(1/2*a) + b*c*tan(1/2*a)^2 - b*d*x + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/
2*b*x)^7*tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(
1/2*a)^3 + 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^
5*tan(1/2*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^
3 + tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1
/2*a) - 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1
/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan
(1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(16*(tan(1/2*b*x)^8*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^6*tan(1/2*a)^4 - 2*tan(1/2*b*x)^7*
tan(1/2*a) - 2*tan(1/2*b*x)^6*tan(1/2*a)^2 + 2*tan(1/2*b*x)^5*tan(1/2*a)^3
+ 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + tan(1/2*b*x)^6 - 2*tan(1/2*b*x)^5*tan(1/2
*a) - 6*tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)^3 + tan(1
/2*b*x)^2*tan(1/2*a)^4 + 2*tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) - 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*b*x)^2
 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2
 + 1))*tan(1/2*a) - b*c)/(b^2*tan(1/2*b*x)^2*tan(1/2*a) + b^2*tan(1/2*b*x)*
tan(1/2*a)^2 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

maple [A] time = 0.05, size = 49, normalized size = 1.20

$$-\frac{d\,x^{2}}{2}-cx-\frac{d\cot\left(bx+a\right)x}{b}+\frac{d\ln\left(\sin\left(bx+a\right)\right)}{b^{2}}-\frac{c\cot\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2,x)
[Out] -1/2*d*x^2-c*x-1/b*d*cot(b*x+a)*x+d*ln(sin(b*x+a))/b^2-1/b*c*cot(b*x+a)
```

maxima [B] time = 0.58, size = 292, normalized size = 7.12

$$-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)c-\frac{2\left(bx+a+\frac{1}{\tan(bx+a)}\right)ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2-2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\left(\mathrm{co}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a + 1/tan(b*x + a))*c - 2*(b*x + a + 1/tan(b*x + a))*a*d/b +
 ((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 - 2*(b*x
+ a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 - 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos
(b*x + a) + 1) - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 4*(b*x
 + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos
(2*b*x + 2*a) + 1)*b))/b
```

mupad [B] time = 1.57, size = 67, normalized size = 1.63

$$-\frac{d\,x^2}{2}+\frac{d\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}-1\right)}{b^2}-\frac{(c+d\,x)\,2\mathrm{i}}{b\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}-1\right)}-\frac{x\,(b\,c+d\,2\mathrm{i})}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2*(c + d*x),x)
```

```
[Out] (d*log(exp(a*2i)*exp(b*x*2i) - 1))/b^2 - (d*x^2)/2 - ((c + d*x)*2i)/(b*(exp
(a*2i + b*x*2i) - 1)) - (x*(d*2i + b*c))/b
```

sympy [A] time = 0.48, size = 104, normalized size = 2.54

$$\begin{cases} \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=0\wedge b=0 \\ \left(cx+\frac{dx^2}{2}\right)\cot^2(a) & \text{for } b=0 \\ \tilde{\infty}\left(cx+\frac{dx^2}{2}\right) & \text{for } a=-bx \\ -cx-\frac{dx^2}{2}-\frac{c}{b\tan(a+bx)}-\frac{dx}{b\tan(a+bx)}-\frac{d\log\left(\tan^2(a+bx)+1\right)}{2b^2}+\frac{d\log(\tan(a+bx))}{b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2,x)
```

```
[Out] Piecewise((zoo*(c*x + d*x**2/2), Eq(a, 0) & Eq(b, 0)), ((c*x + d*x**2/2)*co
t(a)**2, Eq(b, 0)), (zoo*(c*x + d*x**2/2), Eq(a, -b*x)), (-c*x - d*x**2/2 -
 c/(b*tan(a + b*x)) - d*x/(b*tan(a + b*x)) - d*log(tan(a + b*x)**2 + 1)/(2*
b**2) + d*log(tan(a + b*x))/b**2, True))
```

3.109 $\int \frac{\cot^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x),x]
[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx = \int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 4.52, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x),x]
[Out] Integrate[Cot[a + b*x]^2/(c + d*x), x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cot(b*x+a)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\left(bdx+(bdx+bc)\cos(2\,bx+2\,a)^2+(bdx+bc)\sin(2\,bx+2\,a)^2+bc-2\,(bdx+bc)\cos(2\,bx+2\,a)\right)\log(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b
*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integr
ate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)), x) - (b*d^3*x + b*c*d^2 + (b
*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)
^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*
x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 +
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x +
 b*c^2)*cos(b*x + a)), x) - (b*d*x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*
d*x + b*c)*sin(2*b*x + 2*a)^2 + b*c - 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log
```

```
(d*x + c) - 2*d*sin(2*b*x + 2*a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(
2*b*x + 2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)
*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(c + d*x),x)
[Out] int(cot(a + b*x)^2/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c),x)
[Out] Integral(cot(a + b*x)**2/(c + d*x), x)
```

3.110 $\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx = \int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 2.35, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^2/(c + d*x)^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cot(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\cot^2 (bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2/(d*x+c)^2,x)
[Out] int(cot(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot (a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(c + d*x)^2,x)
[Out] int(cot(a + b*x)^2/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**2/(c + d*x)**2, x)
```

3.111 $\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\csc^3(a+bx)(c+dx)^m,x\right)-\text{Int}\left(\csc(a+bx)(c+dx)^m,x\right)$$

```
[Out] -Unintegrable((d*x+c)^m*csc(b*x+a),x)+Unintegrable((d*x+c)^m*csc(b*x+a)^3,x
)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Csc[a + b*x], x] + Defer[Int][(c + d*x)^m*Csc[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx=-\int (c+dx)^m \csc(a+bx)\,dx+\int (c+dx)^m \csc^3(a+bx)\,dx$$

Mathematica [A] time = 38.57, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^2*Csc[a + b*x], x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cot(bx+a)^2 \csc(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cot^2(bx+a)\right) \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
[Out] int((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^2 \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^2*csc(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2\,(c+d\,x)^m}{\sin(a+b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x),x)
[Out] int((cot(a + b*x)^2*(c + d*x)^m)/sin(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^2(a+bx) \csc(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*cot(a + b*x)**2*csc(a + b*x), x)
```

3.112 $\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=416

$$-\frac{12d^4\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^5}+\frac{12d^4\text{Li}_3\left(e^{i(a+bx)}\right)}{b^5}-\frac{12d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}+\frac{12d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{12id^3(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-12i$$

```
[Out] -12*d^2*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^4*arctanh(exp(I*(b*x+
a)))/b-2*d*(d*x+c)^3*csc(b*x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)*csc(b*x+a)/b+1
2*I*d^3*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^4-12*I*d^3*(d*x+c)*polylog(2,e
xp(I*(b*x+a)))/b^4+12*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-12*I*d^3
*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4-12*d^4*polylog(3,-exp(I*(b*x+a)))/b^
5+6*d^2*(d*x+c)^2*polylog(3,-exp(I*(b*x+a)))/b^3+12*d^4*polylog(3,exp(I*(b*
x+a)))/b^5-6*d^2*(d*x+c)^2*polylog(3,exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)^3*po
lylog(2,-exp(I*(b*x+a)))/b^2+2*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b^2-
12*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+a)))/b^5
```

Rubi [A] time = 0.50, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186}

$$\frac{12id^3(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^4}-\frac{12id^3(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^4}+\frac{12id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}-12$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-12*d^2*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^4*ArcTanh[E
^(I*(a + b*x))])/b - (2*d*(c + d*x)^3*Csc[a + b*x])/b^2 - ((c + d*x)^4*Cot[
a + b*x]*Csc[a + b*x])/(2*b) + ((12*I)*d^3*(c + d*x)*PolyLog[2, -E^(I*(a +
b*x))])/b^4 - ((2*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((12
*I)*d^3*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^4 + ((2*I)*d*(c + d*x)^3*P
olyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^4*PolyLog[3, -E^(I*(a + b*x))])/b^5
 + (6*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (12*d^4*PolyLog[3
, E^(I*(a + b*x))])/b^5 - (6*d^2*(c + d*x)^2*PolyLog[3, E^(I*(a + b*x))])/b
^3 + ((12*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a + b*x))])/b^4 - ((12*I)*d^3*
(c + d*x)*PolyLog[4, E^(I*(a + b*x))])/b^4 - (12*d^4*PolyLog[5, -E^(I*(a +
b*x))])/b^5 + (12*d^4*PolyLog[5, E^(I*(a + b*x))])/b^5
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
```

```
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^4 \csc(a+bx)\,dx + \int (c+dx)^4 \csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)^3\csc(a+bx)}{b^2} - \frac{(c+dx)^4\cot(a$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+$$

$$= -\frac{12d^2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+$$

Mathematica [B] time = 8.30, size = 966, normalized size = 2.32

$$\frac{-c^4\log\left(1-e^{i(a+bx)}\right)b^4 - d^4x^4\log\left(1-e^{i(a+bx)}\right)b^4 - 4cd^3x^3\log\left(1-e^{i(a+bx)}\right)b^4 - 6c^2d^2x^2\log\left(1-e^{i(a+bx)}\right)b^4 - 4c$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-(b^4*c^4*Log[1 - E^(I*(a + b*x))]) + 12*b^2*c^2*d^2*Log[1 - E^(I*(a + b*x
))] - 4*b^4*c^3*d*x*Log[1 - E^(I*(a + b*x))] + 24*b^2*c*d^3*x*Log[1 - E^(I*
(a + b*x))] - 6*b^4*c^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] + 12*b^2*d^4*x^2*L
og[1 - E^(I*(a + b*x))] - 4*b^4*c*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^4*d^
4*x^4*Log[1 - E^(I*(a + b*x))] + b^4*c^4*Log[1 + E^(I*(a + b*x))] - 12*b^2*
c^2*d^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c^3*d*x*Log[1 + E^(I*(a + b*x))] -
```

```
 24*b^2*c*d^3*x*Log[1 + E^(I*(a + b*x))] + 6*b^4*c^2*d^2*x^2*Log[1 + E^(I*(
a + b*x))] - 12*b^2*d^4*x^2*Log[1 + E^(I*(a + b*x))] + 4*b^4*c*d^3*x^3*Log[
1 + E^(I*(a + b*x))] + b^4*d^4*x^4*Log[1 + E^(I*(a + b*x))] - (4*I)*b*d*(c
+ d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] + (4*I)*b*d*
(c + d*x)*(-6*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] + 12*b^2*c
^2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 24*d^4*PolyLog[3, -E^(I*(a + b*x))] +
 24*b^2*c*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 12*b^2*d^4*x^2*PolyLog[3, -E
^(I*(a + b*x))] - 12*b^2*c^2*d^2*PolyLog[3, E^(I*(a + b*x))] + 24*d^4*PolyL
og[3, E^(I*(a + b*x))] - 24*b^2*c*d^3*x*PolyLog[3, E^(I*(a + b*x))] - 12*b^
2*d^4*x^2*PolyLog[3, E^(I*(a + b*x))] + (24*I)*b*c*d^3*PolyLog[4, -E^(I*(a
+ b*x))] + (24*I)*b*d^4*x*PolyLog[4, -E^(I*(a + b*x))] - (24*I)*b*c*d^3*Pol
yLog[4, E^(I*(a + b*x))] - (24*I)*b*d^4*x*PolyLog[4, E^(I*(a + b*x))] - 24*
d^4*PolyLog[5, -E^(I*(a + b*x))] + 24*d^4*PolyLog[5, E^(I*(a + b*x))])/(2*b
^5) - (Csc[a + b*x]^2*(b*c^4*Cos[a + b*x] + 4*b*c^3*d*x*Cos[a + b*x] + 6*b*
c^2*d^2*x^2*Cos[a + b*x] + 4*b*c*d^3*x^3*Cos[a + b*x] + b*d^4*x^4*Cos[a + b
*x] + 4*c^3*d*Sin[a + b*x] + 12*c^2*d^2*x*Sin[a + b*x] + 12*c*d^3*x^2*Sin[a
 + b*x] + 4*d^4*x^3*Sin[a + b*x]))/(2*b^2)
```

fricas [C] time = 2.06, size = 2762, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 b^4*c^4)*cos(b*x + a) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c
^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d
 - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 12*I*(b^
3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*d^4
*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*x^
3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 -
 2*b*d^4)*x)*cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*
c^3*d + 24*I*b*c*d^3 + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*
d - 24*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^2 - 12*I*(b
^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*d
^4*x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 24*I*b*c*d^3 + (-4*I*b^3*d^4*
x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 24*I*b*c*d^3 - 12*I*(b^3*c^2*d^2
 - 2*b*d^4)*x)*cos(b*x + a)^2 + 12*I*(b^3*c^2*d^2 - 2*b*d^4)*x)*dilog(-cos(
b*x + a) - I*sin(b*x + a)) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*
b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*
x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c
^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(
cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c
```

```
^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^
4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 +
4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)
*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(
a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^
4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4
- 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) +
 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c^2*d^2 - 4*(a^3 - 6*a)*
b*c*d^3 + (a^4 - 12*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 2)*b^2*c
^2*d^2 - 4*(a^3 - 6*a)*b*c*d^3 + (a^4 - 12*a^2)*d^4)*cos(b*x + a)^2)*log(-1
/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^
3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a
^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
+ 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2
)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*co
s(b*x + a)^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x
 + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2
*d^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*
d^4)*x^2 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d
^2 + 4*(a^3 - 6*a)*b*c*d^3 - (a^4 - 12*a^2)*d^4 + 6*(b^4*c^2*d^2 - 2*b^2*d^
4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(b^4*c^3*d - 6*b
^2*c*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 24*(d^4*cos(b*x + a)
^2 - d^4)*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*(d^4*cos(b*x + a)^
2 - d^4)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) - 24*(d^4*cos(b*x + a)^2
 - d^4)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) - 24*(d^4*cos(b*x + a)^2
 - d^4)*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b
*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x
+ a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3 + (24*I*b*d^4*x + 24
*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (24
*I*b*d^4*x + 24*I*b*c*d^3 + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2)*
polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3
+ (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (
b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c
^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*
x + a)^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12
*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - 2*d^4 - (b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2 - 2*d^4)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) - I
*sin(b*x + a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c
^3*d)*sin(b*x + a))/(b^5*cos(b*x + a)^2 - b^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cot(bx+a)^2 \csc(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^2*csc(b*x + a), x)
```

maple [B] time = 0.19, size = 1673, normalized size = 4.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 6/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*c^2*d^2*polylog(3,exp(I*(b*x
+a)))+1/2/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))-6/b^3*d^4*polylog(3,exp(I*(b*x+a
)))*x^2+6/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-12*d^4*polylog(3,-exp(I*(b
*x+a)))/b^5+12*d^4*polylog(3,exp(I*(b*x+a)))/b^5-12*d^4*polylog(5,-exp(I*(b
*x+a)))/b^5+12*d^4*polylog(5,exp(I*(b*x+a)))/b^5+1/b*c^4*arctanh(exp(I*(b*x
+a)))-6/b^5*d^4*a^2*ln(1-exp(I*(b*x+a)))+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2
-6/b^3*d^4*ln(exp(I*(b*x+a))+1)*x^2+2/b^2*c^3*d*ln(exp(I*(b*x+a))+1)*a-4/b^
4*c*d^3*a^3*arctanh(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*arctanh(exp(I*(b*x+a)
))-4/b^2*c^3*d*a*arctanh(exp(I*(b*x+a)))-3/b^3*c^2*d^2*ln(exp(I*(b*x+a))+1)
*a^2+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^4*x^4*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b
*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(3*I*(b*x+a))+d^4*x^4*b*exp(I*(b*x+a))
+4*c^3*d*x*b*exp(3*I*(b*x+a))+4*c*d^3*x^3*b*exp(I*(b*x+a))-4*I*d^4*x^3*exp(
3*I*(b*x+a))+c^4*b*exp(3*I*(b*x+a))+6*c^2*d^2*x^2*b*exp(I*(b*x+a))-12*I*c^2
*d^2*x*exp(3*I*(b*x+a))+4*c^3*d*x*b*exp(I*(b*x+a))+4*I*d^4*x^3*exp(I*(b*x+a
))+4*I*c^3*d*exp(I*(b*x+a))+c^4*b*exp(I*(b*x+a))-12*I*c*d^3*x^2*exp(3*I*(b*
x+a))+12*I*c*d^3*x^2*exp(I*(b*x+a))-4*I*c^3*d*exp(3*I*(b*x+a))+12*I*c^2*d^2
*x*exp(I*(b*x+a)))+24/b^4*c*d^3*a*arctanh(exp(I*(b*x+a)))-1/2/b^5*d^4*a^4*l
n(exp(I*(b*x+a))+1)+1/b^5*d^4*a^4*arctanh(exp(I*(b*x+a)))+2/b*c^3*d*ln(exp(
I*(b*x+a))+1)*x-2/b*c^3*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c^3*d*ln(1-exp(I*(b*
x+a)))*a+3/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+12/b^3*c*d^3*polylog(3,-exp(I
*(b*x+a)))*x+3/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))-3/b*c^2*d^2*ln(1-exp(I*
(b*x+a)))*x^2-12/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-1/2/b*d^4*ln(1-exp(I
*(b*x+a)))*x^4+1/2/b*d^4*ln(exp(I*(b*x+a))+1)*x^4+2/b*c*d^3*ln(exp(I*(b*x+a
))+1)*x^3-2/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3-2/b^4*c*d^3*ln(1-exp(I*(b*x+a)
))*a^3+2/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a^3-12/b^3*d^3*c*ln(exp(I*(b*x+a))+
1)*x+12/b^3*d^3*c*ln(1-exp(I*(b*x+a)))*x+12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*
a-12/b^4*c*d^3*ln(exp(I*(b*x+a))+1)*a-2*I/b^2*d^4*polylog(2,-exp(I*(b*x+a))
```

```
)*x^3+12*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,exp(I
*(b*x+a)))*x^3-12*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x+12*I/b^4*d^4*polylo
g(2,-exp(I*(b*x+a)))*x+12*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))-12*I/b^4*c
*d^3*polylog(4,exp(I*(b*x+a)))-2*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))+2*I
/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))+12*I/b^4*c*d^3*polylog(2,-exp(I*(b*x+a
)))+6/b^5*d^4*a^2*ln(exp(I*(b*x+a))+1)-12/b^3*c^2*d^2*arctanh(exp(I*(b*x+a)
))-12/b^5*d^4*a^2*arctanh(exp(I*(b*x+a)))-12*I/b^4*d^3*c*polylog(2,exp(I*(b
*x+a)))-12*I/b^4*d^4*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^3*polylog(2,-e
xp(I*(b*x+a)))*x^2+6*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-6*I/b^2*poly
log(2,-exp(I*(b*x+a)))*c^2*d^2*x+6*I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d^2*
x
```

maxima [B] time = 5.49, size = 6952, normalized size = 16.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 4*a*c^3*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 6*a^2*c^2*d^2*(2*cos(b*x + a
)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2
 - 4*a^3*c*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1)
 - log(cos(b*x + a) - 1))/b^3 + a^4*d^4*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1
) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^4 + 4*((2*(b*x + a)^4*
d^4 - 24*b^2*c^2*d^2 + 48*a*b*c*d^3 - 24*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x
 + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x +
a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b
*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3
- 6*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^4*d^4 - 12*b^2*c^2*d
^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c
^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a
) + (2*I*(b*x + a)^4*d^4 - 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4
 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d
^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d
^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*sin(4*
b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + 48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + 48
*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2
+ 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d +
 48*I*a*b^2*c^2*d^2 + (-48*I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (24
```

*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*a^2*d^4 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*sin(4*b*x + 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 - 48*I*a^2*d^4)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(4*b *x + 4*a) - 4*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c ^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2* c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^ 2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^4)*(b*x + a)^2 + (8*I*b^3* c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^4*d^4 + (-16*I*b*c*d^3 + 16*I*a*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a ^2 + 48*I)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 + (-48* I*a^2 + 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-4*I*(b*x + a)^4*d^4 - 16*b^ 3*c^3*d + 48*a*b^2*c^2*d^2 - 48*a^2*b*c*d^3 + 16*a^3*d^4 - 16*(I*b*c*d^3 + (-I*a + 1)*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a + 1)*b*c*d^3 + (-24*I*a^2 + 48*a)*d^4)*(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a + 1)*b^2* c^2*d^2 + (-48*I*a^2 + 96*a)*b*c*d^3 + (16*I*a^3 - 48*a^2)*d^4)*(b*x + a))* cos(3*b*x + 3*a) + (-4*I*(b*x + a)^4*d^4 + 16*b^3*c^3*d - 48*a*b^2*c^2*d^2 + 48*a^2*b*c*d^3 - 16*a^3*d^4 + (-16*I*b*c*d^3 - 16*(-I*a - 1)*d^4)*(b*x + a)^3 + (-24*I*b^2*c^2*d^2 - 48*(-I*a - 1)*b*c*d^3 + (-24*I*a^2 - 48*a)*d^4) *(b*x + a)^2 + (-16*I*b^3*c^3*d - 48*(-I*a - 1)*b^2*c^2*d^2 + (-48*I*a^2 - 96*a)*b*c*d^3 + (16*I*a^3 + 48*a^2)*d^4)*(b*x + a))*cos(b*x + a) - (8*b^3*c ^3*d - 24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*(a^3 - 6*a)*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c* d^3 + (a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a) ^3*d^4 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b* c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-8*I*b^3*c^3* d + 24*I*a*b^2*c^2*d^2 - 8*I*(b*x + a)^3*d^4 + (-24*I*a^2 + 48*I)*b*c*d^3 + (8*I*a^3 - 48*I*a)*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I *b^2*c^2*d^2 + 48*I*a*b*c*d^3 + (-24*I*a^2 + 48*I)*d^4)*(b*x + a))*sin(4*b* x + 4*a) - (16*I*b^3*c^3*d - 48*I*a*b^2*c^2*d^2 + 16*I*(b*x + a)^3*d^4 + (4 8*I*a^2 - 96*I)*b*c*d^3 + (-16*I*a^3 + 96*I*a)*d^4 + (48*I*b*c*d^3 - 48*I*a *d^4)*(b*x + a)^2 + (48*I*b^2*c^2*d^2 - 96*I*a*b*c*d^3 + (48*I*a^2 - 96*I)* d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (8*b^3*c^3*d -

```
24*a*b^2*c^2*d^2 + 8*(b*x + a)^3*d^4 + 24*(a^2 - 2)*b*c*d^3 - 8*(a^3 - 6*a)
*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (
a^2 - 2)*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
+ 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(4*b*x + 4*a)
- 16*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 2)*b*c*d^3 -
 (a^3 - 6*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*b^3*c^3*d - 24*I
*a*b^2*c^2*d^2 + 8*I*(b*x + a)^3*d^4 + (24*I*a^2 - 48*I)*b*c*d^3 + (-8*I*a^
3 + 48*I*a)*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d
^2 - 48*I*a*b*c*d^3 + (24*I*a^2 - 48*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) +
(-16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 - 16*I*(b*x + a)^3*d^4 + (-48*I*a^2 +
 96*I)*b*c*d^3 + (16*I*a^3 - 96*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*
x + a)^2 + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*d^3 + (-48*I*a^2 + 96*I)*d^4)*(b
*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^4*d^4 + 1
2*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4
)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 + (-6*I*a^2 + 12*I)*d^4)
*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 + (-12*I*a^2 + 24*I)*b*
c*d^3 + (4*I*a^3 - 24*I*a)*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + 12*I*b^2*
c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4 + (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x +
 a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 + (-6*I*a^2 + 12*I)*d^4)*(b*x +
a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 + (-12*I*a^2 + 24*I)*b*c*d^3 +
(4*I*a^3 - 24*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*d^4
- 24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4 + (8*I*b*c*d^3 - 8*I*a*d
^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + (12*I*a^2 - 24*I)*d^
4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + (24*I*a^2 - 48*I)*b*
c*d^3 + (-8*I*a^3 + 48*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^4
*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*
x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x +
a))*sin(4*b*x + 4*a) - 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 -
 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 -
2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^4*d^4 - 12*I*b^2*c
^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a
)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + (6*I*a^2 - 12*I)*d^4)*(b*x + a)^2
 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + (12*I*a^2 - 24*I)*b*c*d^3 + (-4*I*
a^3 + 24*I*a)*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 - 12*I*b^2*c^2*d^2 + 24*I
*a*b*c*d^3 - 12*I*a^2*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^
2*c^2*d^2 - 12*I*a*b*c*d^3 + (6*I*a^2 - 12*I)*d^4)*(b*x + a)^2 + (4*I*b^3*c
^3*d - 12*I*a*b^2*c^2*d^2 + (12*I*a^2 - 24*I)*b*c*d^3 + (-4*I*a^3 + 24*I*a)
*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^4*d^4 + 24*I*b^2*c^2*d^
2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4 + (-8*I*b*c*d^3 + 8*I*a*d^4)*(b*x + a)^3
+ (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 + (-12*I*a^2 + 24*I)*d^4)*(b*x + a)^2
```

```
 + (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 + (-24*I*a^2 + 48*I)*b*c*d^3 + (8*I
*a^3 - 48*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 - 12*b^2
*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*
a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a^3 - 6*a)*d^4)*(b*x + a))*sin(4*b*x
 + 4*a) + 2*((b*x + a)^4*d^4 - 12*b^2*c^2*d^2 + 24*a*b*c*d^3 - 12*a^2*d^4 +
 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)
*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 -
(a^3 - 6*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1) + (48*I*d^4*cos(4*b*x + 4*a) - 96*I*d^4*cos(2*
b*x + 2*a) - 48*d^4*sin(4*b*x + 4*a) + 96*d^4*sin(2*b*x + 2*a) + 48*I*d^4)*
polylog(5, -e^(I*b*x + I*a)) + (-48*I*d^4*cos(4*b*x + 4*a) + 96*I*d^4*cos(2
*b*x + 2*a) + 48*d^4*sin(4*b*x + 4*a) - 96*d^4*sin(2*b*x + 2*a) - 48*I*d^4)
*polylog(5, e^(I*b*x + I*a)) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 +
48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 96*(b*c*d^3 + (b*x
+ a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 4
8*I*a*d^4)*sin(4*b*x + 4*a) + (-96*I*b*c*d^3 - 96*I*(b*x + a)*d^4 + 96*I*a*
d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*b*x + I*a)) - (48*b*c*d^3 + 48*(b*x
 + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a
) - 96*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-48*I*b*c*d^3
- 48*I*(b*x + a)*d^4 + 48*I*a*d^4)*sin(4*b*x + 4*a) - (96*I*b*c*d^3 + 96*I*
(b*x + a)*d^4 - 96*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) +
 (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 + (-24*I*a^2 +
48*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-24*I*b^2*c^2*d^2 + 4
8*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 + (-24*I*a^2 + 48*I)*d^4 + (-48*I*b*c*
d^3 + 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (48*I*b^2*c^2*d^2 - 96*I*a*
b*c*d^3 + 48*I*(b*x + a)^2*d^4 + (48*I*a^2 - 96*I)*d^4 + (96*I*b*c*d^3 - 96
*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*
x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4
*a) - 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(
b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a))
+ (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + (24*I*a^2 - 4
8*I)*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x + a) + (24*I*b^2*c^2*d^2 - 48*I
*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + (24*I*a^2 - 48*I)*d^4 + (48*I*b*c*d^3 -
 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-48*I*b^2*c^2*d^2 + 96*I*a*b*c*
d^3 - 48*I*(b*x + a)^2*d^4 + (-48*I*a^2 + 96*I)*d^4 + (-96*I*b*c*d^3 + 96*I
*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a
) + 48*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 2)*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (
4*(b*x + a)^4*d^4 - 16*I*b^3*c^3*d + 48*I*a*b^2*c^2*d^2 - 48*I*a^2*b*c*d^3
+ 16*I*a^3*d^4 + (16*b*c*d^3 - (16*a + 16*I)*d^4)*(b*x + a)^3 + (24*b^2*c^2
*d^2 - (48*a + 48*I)*b*c*d^3 + 24*(a^2 + 2*I*a)*d^4)*(b*x + a)^2 + (16*b^3*
c^3*d - (48*a + 48*I)*b^2*c^2*d^2 + 48*(a^2 + 2*I*a)*b*c*d^3 - 16*(a^3 + 3*
I*a^2)*d^4)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x + a)^4*d^4 + 16*I*b^3*c^3
```

```
*d - 48*I*a*b^2*c^2*d^2 + 48*I*a^2*b*c*d^3 - 16*I*a^3*d^4 + (16*b*c*d^3 - (
16*a - 16*I)*d^4)*(b*x + a)^3 + (24*b^2*c^2*d^2 - (48*a - 48*I)*b*c*d^3 + 2
4*(a^2 - 2*I*a)*d^4)*(b*x + a)^2 + (16*b^3*c^3*d - (48*a - 48*I)*b^2*c^2*d^
2 + 48*(a^2 - 2*I*a)*b*c*d^3 - 16*(a^3 - 3*I*a^2)*d^4)*(b*x + a))*sin(b*x +
 a))/(-4*I*b^4*cos(4*b*x + 4*a) + 8*I*b^4*cos(2*b*x + 2*a) + 4*b^4*sin(4*b*
x + 4*a) - 8*b^4*sin(2*b*x + 2*a) - 4*I*b^4))/b
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^2(a+bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**2*csc(a + b*x), x)
```

3.113 $\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=308

$$\frac{3id^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c}{}$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^3+(d*x+c)^3*arctanh(exp(I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)*csc(b*x+a)/b+3*
I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*
x+a)))/b^2-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(
2,exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3-3*d^2*(d
*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-
3*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4
```

Rubi [A] time = 0.34, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4415, 4183, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{3id(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^3 + ((c + d*x)^3*ArcTanh[E^(I
*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot
[a + b*x]*Csc[a + b*x])/(2*b) + ((3*I)*d^3*PolyLog[2, -E^(I*(a + b*x))])/b^
4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d^3
*PolyLog[2, E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^(
I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 - (
3*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[4, -E
^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
```

```
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^3 \csc(a+bx)\,dx + \int (c+dx)^3 \csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^3 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\csc(a+bx)}{2b^2} - \frac{(c+dx)^3 \cot(}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+a}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+a}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+a}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+a}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+a}{}$$

Mathematica [A] time = 4.74, size = 528, normalized size = 1.71

$$-\frac{b^3c^3\log\left(1-e^{i(a+bx)}\right) - b^3c^3\log\left(1+e^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1-e^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1+e^{i(a+bx)}\right) + 3b^3cd^2x}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/2*(b^2*(c + d*x)^2*(3*d + b*(c + d*x)*Cot[a + b*x])*Csc[a + b*x] + b^3*c
^3*Log[1 - E^(I*(a + b*x))] - 6*b*c*d^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^
```

```
2*d*x*Log[1 - E^(I*(a + b*x))] - 6*b*d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3
*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))]
- b^3*c^3*Log[1 + E^(I*(a + b*x))] + 6*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3
*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 + E^(I*(a + b*x))]
- 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a +
b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, -E^(I*(a + b*x))] -
(3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, E^(I*(a + b*x))] - 6*b*c*d^2*
PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, -E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, E^(I*(a + b*x))]
- (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4
```

fricas [C] time = 1.19, size = 1734, normalized size = 5.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x +
a) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b
^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*
I*b^2*c^2*d - 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^
2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3 + (3*I*b^2*d^3*x^2 +
6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x +
 a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
- 6*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*
cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^
3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d -
2*b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 -
 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b
*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/
2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 -
(a^3 - 6*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6
*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)
+ (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6
*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
(a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d
- 2*b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^
```

```
3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(
b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*log(-co
s(b*x + a) - I*sin(b*x + a) + 1) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*poly
log(4, cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*
polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*
d^3)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 -
 6*I*d^3)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2
 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x
+ a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(
3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d
^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x
 + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(b*x + a)
)/(b^4*cos(b*x + a)^2 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \cot (bx + a)^2 \csc (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^2*csc(b*x + a), x)
```

maple [B] time = 0.14, size = 1056, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 3*I*d^3*polylog(2,-exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-exp(I*(b*x+a)))/b
^4+3/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-3/b^3*c*d^2*polylog(3,exp(I*(b*x+
a)))-3/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+3/b^3*d^3*polylog(3,-exp(I*(b*x+
a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,exp(I*(b*x+a
)))/b^4+1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^3*x^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^
2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))+d^3*x^3*b*exp(I*(b*x+a))+
c^3*b*exp(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(3*I*(b*
x+a))+3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*I*(b*x+a))+c^3*b*exp(I*(
b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a))+6*I*c*d^2*x*e
xp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*x+a)))+1/b*c^3*arctanh(exp(I*(b*x+a)))-3/b
^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln
(1-exp(I*(b*x+a)))*a-1/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))+3/2/b^2*c^2*d*ln
```

```
(exp(I*(b*x+a))+1)*a-3/2/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)+1/2/b^4*d^3*ln(
exp(I*(b*x+a))+1)*a^3+3/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+3/b^3*c*d
^2*a^2*arctanh(exp(I*(b*x+a)))-3/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-3/2*I/
b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*
x+a)))+3/2*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+3/2/b*c^2*d*ln(exp(I*(b*x+
a))+1)*x-3/2/b*c^2*d*ln(1-exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln(1-exp(I*(b*x+a
)))*a+3/2/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-3/2/b*c*d^2*ln(1-exp(I*(b*x+a)
))*x^2+3/2/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-1/2/b*d^3*ln(1-exp(I*(b*x+a)))*
x^3-1/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/2/b*d^3*ln(exp(I*(b*x+a))+1)*x^3
-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a+6/b^4*d^3*a*arctanh(exp(I*(b*x+a)))-6/b^3
*c*d^2*arctanh(exp(I*(b*x+a)))-3*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x+3
*I/b^2*polylog(2,exp(I*(b*x+a)))*c*d^2*x
```

maxima [B] time = 1.92, size = 3872, normalized size = 12.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(
cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*cos(b*x + a)/
(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 -
 a^3*d^3*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1))/b^3 + 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 6
*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3
)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(
4*b*x + 4*a) - 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos
(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a^2
- 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3 + 24*I*b*c
*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c
^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*
c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) + (
12*I*b*c*d^2 - 12*I*a*d^3)*sin(4*b*x + 4*a) + (-24*I*b*c*d^2 + 24*I*a*d^3)*
sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^3*
d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 -
 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b
*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
```

```
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
 (6*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^3*d^3
+ (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*
d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), -cos(b*x + a) + 1) + (-4*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*
d^2 - 12*a^2*d^3 - 12*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a)^2 + (-12*I*b^2
*c^2*d - 24*(-I*a + 1)*b*c*d^2 + (-12*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(3*b
*x + 3*a) + (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^
3 + (-12*I*b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24
*(-I*a - 1)*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*cos(b*x + a) - (6*
b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^
2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
- 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2
*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(
4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (1
2*I*a^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*dilog(-e^(I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d
^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*
b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2
)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d -
12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2
 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*
d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2 + 24*I
*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)
^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 +
 (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b
*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x +
 a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*
cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (6*I*
b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6*I*a
^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2
+ 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
(a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2
 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
 (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3
+ (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 +
(3*I*a^2 - 6*I)*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d
^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2
 + (3*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^3*d^3
 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6
*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(2*b*x
```

```
 + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
 + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x
+ 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x
 + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (12*d^
3*cos(4*b*x + 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) -
24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(I*b*x + I*a)) - (12*d^3*
cos(4*b*x + 4*a) - 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) - 24
*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, e^(I*b*x + I*a)) + (-12*I*b*c*
d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3
 + 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I
*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x +
 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -
e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*
b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(4*b*x + 4*a) + (-24*I*b*c*d^
2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x
+ a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*s
in(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 - 12*I*b^
2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a + 12*I)*d^3)*
(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 + 12*(a^2 + 2*I*a)*d^3)
*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a
*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a)^2 + (1
2*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a)*d^3)*(b*x + a))*sin(
b*x + a))/(-4*I*b^3*cos(4*b*x + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin
(4*b*x + 4*a) - 8*b^3*sin(2*b*x + 2*a) - 4*I*b^3))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^3)/sin(a + b*x),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \cot^2(a + bx) \csc(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**2*csc(b*x+a),x)
[Out] Integral((c + d*x)**3*cot(a + b*x)**2*csc(a + b*x), x)
```

3.114 $\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=179

$$\frac{d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d}{}$$

```
[Out] (d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-d^2*arctanh(cos(b*x+a))/b^3-d*(d*x+c)*c
sc(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)*csc(b*x+a)/b-I*d*(d*x+c)*polylog(2,-
exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2+d^2*polylog(3
,-exp(I*(b*x+a)))/b^3-d^2*polylog(3,exp(I*(b*x+a)))/b^3
```

Rubi [A] time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4415, 4183, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,e^{i(}\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3
- (d*(c + d*x)*Csc[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]*Csc[a + b*x])/
(2*b) - (I*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*P
olyLog[2, E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 -
(d^2*PolyLog[3, E^(I*(a + b*x))])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)^2\csc(a+bx)\,dx + \int (c+dx)^2\csc^3(a+bx)\,dx$$

$$= \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\csc(a+bx)}{b^2} - \frac{(c+dx)^2\cot(a}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a}{b^2}$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a}{b^2}$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a}{b^2}$$

$$= \frac{(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3} - \frac{d(c+dx)\csc(a}{b^2}$$

Mathematica [B] time = 7.58, size = 471, normalized size = 2.63

$$\frac{\csc\left(\frac{a}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)\left(cd\sin\left(\frac{bx}{2}\right)+d^2x\sin\left(\frac{bx}{2}\right)\right)}{2b^2} + \frac{\sec\left(\frac{a}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)\left(d^2(-x)\sin\left(\frac{bx}{2}\right)-cd\sin\left(\frac{bx}{2}\right)\right)}{2b^2} - \frac{d\csc(a)(}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -((d*(c + d*x)*Csc[a])/b^2) + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2
]^2)/(8*b) + (-(b^2*c^2*Log[1 - E^(I*(a + b*x))]) + 2*d^2*Log[1 - E^(I*(a +
 b*x))] - 2*b^2*c*d*x*Log[1 - E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 - E^(I*(
a + b*x))] + b^2*c^2*Log[1 + E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x
))] + 2*b^2*c*d*x*Log[1 + E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 + E^(I*(a +
b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + (2*I)*b*d*(c +
d*x)*PolyLog[2, E^(I*(a + b*x))] + 2*d^2*PolyLog[3, -E^(I*(a + b*x))] - 2*d
^2*PolyLog[3, E^(I*(a + b*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/
2 + (b*x)/2]^2)/(8*b) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) -
 d^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)
/2] + d^2*x*Sin[(b*x)/2]))/(2*b^2)
```

fricas [C] time = 0.71, size = 966, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a) + (-2*I*b*d^2*x -
 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*co
s(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin
(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x
+ a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 -
2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 -
2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + (a^2
 - 2)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*
cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)
*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b
*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a
)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 2*(d^2*cos(b*x + a)^2
 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 -
 d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^2 -
d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 4*(b*d^2*x + b*c*d)*sin(b
*x + a))/(b^3*cos(b*x + a)^2 - b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cot (bx+a)^2 \csc (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^2*csc(b*x + a), x)
```

maple [B] time = 0.12, size = 546, normalized size = 3.05

$$\frac{d^2x^2b\, e^{3i(bx+a)} + 2cdxb\, e^{3i(bx+a)} + c^2b\, e^{3i(bx+a)} + d^2x^2b\, e^{i(bx+a)} + 2cdxb\, e^{i(bx+a)} - 2id^2x\, e^{3i(bx+a)} + c^2b\, e^{i(bx+a)} - 2idc}{b^2\left(e^{2i(bx+a)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I*
(b*x+a))+c^2*b*exp(3*I*(b*x+a))+d^2*x^2*b*exp(I*(b*x+a))+2*c*d*x*b*exp(I*(b
*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))+c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*x
+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))+1/b*c*d*ln(exp(I*(b*x
+a))+1)*x+1/b^2*c*d*ln(exp(I*(b*x+a))+1)*a-1/b*c*d*ln(1-exp(I*(b*x+a)))*x-1
/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+d^2*polylog(3,-exp(I*(b*x+a)))/b^3-d^2*poly
log(3,exp(I*(b*x+a)))/b^3-2/b^3*d^2*arctanh(exp(I*(b*x+a)))+1/b*c^2*arctanh
(exp(I*(b*x+a)))+1/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))+1/2/b*d^2*ln(exp(I*(
b*x+a))+1)*x^2-I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-1/2/b*d^2*ln(1-exp(I*(b
*x+a)))*x^2+I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x-2/b^2*c*d*a*arctanh(exp(I
*(b*x+a)))-I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x+I/b^2*c*d*polylog(2,exp(I
*(b*x+a)))-1/2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+1/2/b^3*d^2*ln(1-exp(I*(b*x
+a)))*a^2
```

maxima [B] time = 0.81, size = 1932, normalized size = 10.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log
(cos(b*x + a) - 1)) - 2*a*c*d*(2*cos(b*x + a)/(cos(b*x + a)^2 - 1) + log(co
s(b*x + a) + 1) - log(cos(b*x + a) - 1))/b + a^2*d^2*(2*cos(b*x + a)/(cos(b
*x + a)^2 - 1) + log(cos(b*x + a) + 1) - log(cos(b*x + a) - 1))/b^2 + 4*((2
*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)
^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (-
4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b
*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*d^2*cos(4*b*x + 4*a
) - 8*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x +
 2*a) + 4*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2
 + 4*(b*c*d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a))*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a)
)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a))*sin(4*b*x + 4*a) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(I*(
b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(I*b*c*d + (-I*a + 1)*d^2)*(b*x + a)
)*cos(3*b*x + 3*a) + (-4*I*(b*x + a)^2*d^2 + 8*b*c*d - 8*a*d^2 + (-8*I*b*c*
d - 8*(-I*a - 1)*d^2)*(b*x + a))*cos(b*x + a) - (4*b*c*d + 4*(b*x + a)*d^2
- 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2
+ 4*I*a*d^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)
*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4*b*c*d + 4*(b*x + a)*d^2 - 4
```

```
*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b
*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*
I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*si
n(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d
+ 2*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*
a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*
b*c*d - 4*I*a*d^2)*(b*x + a) - 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2
 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a
)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*
d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*
a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*
I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d
^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2
*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-4*I*d^2*cos(4*b*x + 4*a) +
 8*I*d^2*cos(2*b*x + 2*a) + 4*d^2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a)
 - 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*
d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*
I*d^2)*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a
*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*sin(3*b*x + 3*a) + (4*(b*x +
a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))*s
in(b*x + a))/(-4*I*b^2*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*
sin(4*b*x + 4*a) - 8*b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x)^2)/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \cot^2 (a + bx) \csc (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cot(a + b*x)**2*csc(a + b*x), x)
```

3.115 $\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx$

Optimal. Leaf size=108

$$-\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+bx)\csc(a+bx)}{2b}$$

```
[Out] (d*x+c)*arctanh(exp(I*(b*x+a)))/b-1/2*d*csc(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+
a)*csc(b*x+a)/b-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/2*I*d*polylog(2,ex
p(I*(b*x+a)))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4415, 4183, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}+\frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{(c+dx)\cot(a+b}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] ((c + d*x)*ArcTanh[E^(I*(a + b*x))])/b - (d*Csc[a + b*x])/(2*b^2) - ((c + d
*x)*Cot[a + b*x]*Csc[a + b*x])/(2*b) - ((I/2)*d*PolyLog[2, -E^(I*(a + b*x))
])/b^2 + ((I/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4415

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_)*Csc[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Csc[a + b*x]*Cot[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Csc[a + b*x]^3*Cot[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int (c+dx)\cot^2(a+bx)\csc(a+bx)\,dx = -\int (c+dx)\csc(a+bx)\,dx + \int (c+dx)\csc^3(a+bx)\,dx$$
$$= \frac{2(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$
$$= \frac{(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\csc(a+bx)}{2b^2} - \frac{(c+dx)\cot(a+bx)\csc(a+$$

Mathematica [B] time = 1.59, size = 260, normalized size = 2.41

$$-\frac{d\left(i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{2b^2}-\frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}-\frac{d\cot\left(\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^2*Csc[a + b*x],x]
```

```
[Out] -1/4*(d*Cot[(a + b*x)/2])/b^2 - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a
+ b*x)/2]^2)/(8*b) + (c*Log[Cos[(a + b*x)/2]])/(2*b) - (c*Log[Sin[(a + b*x)
/2]])/(2*b) + (a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) - (d*((a + b*x)*(Log[1 -
```

```
E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x
))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b)
 + (d*x*Sec[(a + b*x)/2]^2)/(8*b) - (d*Tan[(a + b*x)/2])/(4*b^2)
```

fricas [B] time = 0.57, size = 454, normalized size = 4.20

$$2\,(bdx + bc)\cos{(bx + a)} + \left(i\,d\cos{(bx + a)}^{2} - i\,d\right){\rm Li}_2\left(\cos{(bx + a)} + i\,\sin{(bx + a)}\right) + \left(-i\,d\cos{(bx + a)}^{2} + i\,d\right){\rm L}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b*d*x + b*c)*cos(b*x + a) + (I*d*cos(b*x + a)^2 - I*d)*dilog(cos(b*
x + a) + I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(cos(b*x + a) -
 I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b
*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)
) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a
) - I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((b*c - a*d)*cos(b*x + a)^2 - b
*c + a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x - (b*d
*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (
b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1) + 2*d*sin(b*x + a))/(b^2*cos(b*x + a)^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\cot{(bx + a)}^{2}\csc{(bx + a)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^2*csc(b*x + a), x)
```

maple [B] time = 0.10, size = 246, normalized size = 2.28

$$\frac{bdx\,{\rm e}^{3i(bx+a)} + cb\,{\rm e}^{3i(bx+a)} + bdx\,{\rm e}^{i(bx+a)} + cb\,{\rm e}^{i(bx+a)} - id\,{\rm e}^{3i(bx+a)} + id\,{\rm e}^{i(bx+a)}}{b^{2}\left({\rm e}^{2i(bx+a)} - 1\right)^{2}} + \frac{c\,\mathrm{arctanh}\left({\rm e}^{i(bx+a)}\right)}{b} + \frac{d\ln\left({\rm e}^{i(bx+a)}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2*(b*d*x*exp(3*I*(b*x+a))+c*b*exp(3*I*(b*x+a))+b
*d*x*exp(I*(b*x+a))+c*b*exp(I*(b*x+a))-I*d*exp(3*I*(b*x+a))+I*d*exp(I*(b*x+
a)))+1/b*c*arctanh(exp(I*(b*x+a)))+1/2/b*d*ln(exp(I*(b*x+a))+1)*x+1/2/b^2*d
*ln(exp(I*(b*x+a))+1)*a-1/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-1/2/b*d*ln(1
-exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1-exp(I*(b*x+a)))*a+1/2*I*d*polylog(2,exp(I
*(b*x+a)))/b^2-1/b^2*d*a*arctanh(exp(I*(b*x+a)))
```

maxima [B] time = 0.54, size = 770, normalized size = 7.13

$$(2\,bdx + 2\,bc + 2\,(bdx + bc)\cos\left(4\,bx + 4\,a\right) - 4\,(bdx + bc)\cos\left(2\,bx + 2\,a\right) + (2i\,bdx + 2i\,bc)\sin\left(4\,bx + 4\,a\right) + (-4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^2*csc(b*x+a),x, algorithm="maxima")
```

```
[Out] ((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(
2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(
4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*
sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*
x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a)
- 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (-4*I*b*d*x - 4*I*b*c - 4*d)*cos(3*b*x + 3*a) + (-4*I*b*d*x - 4*I*b
*c + 4*d)*cos(b*x + a) - (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I
*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a))
 + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) -
4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c +
(-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a
) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c +
(I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a
) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (4*b*d*x + 4*b*c -
4*I*d)*sin(3*b*x + 3*a) + (4*b*d*x + 4*b*c + 4*I*d)*sin(b*x + a))/(-4*I*b^2
*cos(4*b*x + 4*a) + 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) - 8*b
^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(a + b*x)^2*(c + d*x))/sin(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \cot^2 (a + bx) \csc (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**2*csc(b*x+a),x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**2*csc(a + b*x), x)
```

3.116 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\csc^3(a+bx)}{c+dx},x\right)-\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable(csc(b*x+a)/(d*x+c),x)+Unintegrable(csc(b*x+a)^3/(d*x+c),x)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x), x] + Defer[Int][Csc[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx=-\int \frac{\csc(a+bx)}{c+dx}\,dx+\int \frac{\csc^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 35.89, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.73, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot\left(bx+a\right)^2\csc\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot (bx+a)^2 \csc (bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c), x)
```

maple [A] time = 2.76, size = 0, normalized size = 0.00

$$\int \frac{\left(\cot^2 (bx+a)\right) \csc (bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - d*sin(3*b*x
 + 3*a) + d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x + b*c)
*cos(2*b*x + 2*a) - 2*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x + b*
c)*cos(b*x + a) + d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*cos(b*x
+ a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 -
4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) +
 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2
*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*si
n(b*x + a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*
```

```
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 + 2*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) -
(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*
a))*cos(4*b*x + 4*a) - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x +
2*a))*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x +
 a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)^2 + (b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(b*x + a)^2 - 2*(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(b*x + a)), x) + (d*cos(
3*b*x + 3*a) - d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*x + b
*c)*sin(b*x + a))*sin(4*b*x + 4*a) + (2*d*cos(2*b*x + 2*a) - 2*(b*d*x + b*c
)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(d*cos(b*x + a) - (b*d*x + b*c
)*sin(b*x + a))*sin(2*b*x + 2*a) + d*sin(b*x + a))/(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+bx)^2}{\sin(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x), x)
```

3.117 $\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\csc^3(a+bx)}{(c+dx)^2},x\right)-\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable(csc(b*x+a)/(d*x+c)^2,x)+Unintegrable(csc(b*x+a)^3/(d*x+c)^2,x
)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Csc[a + b*x]/(c + d*x)^2, x] + Defer[Int][Csc[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx=-\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx+\int \frac{\csc^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 41.72, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Cot[a + b*x]^2*Csc[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^2\csc(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^2*csc(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^2 \csc(bx+a)}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^2*csc(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 4.19, size = 0, normalized size = 0.00

$$\int \frac{\left(\cot^2(bx+a)\right)\csc(bx+a)}{(dx+c)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^2*csc(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (((b*d*x + b*c)*cos(3*b*x + 3*a) + (b*d*x + b*c)*cos(b*x + a) - 2*d*sin(3*b
*x + 3*a) + 2*d*sin(b*x + a))*cos(4*b*x + 4*a) + (b*d*x + b*c - 2*(b*d*x +
b*c)*cos(2*b*x + 2*a) - 4*d*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) - 2*((b*d*x
+ b*c)*cos(b*x + a) + 2*d*sin(b*x + a))*cos(2*b*x + 2*a) + (b*d*x + b*c)*co
s(b*x + a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 +
 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*
a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x
+ 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(
4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
```

```
b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(1/2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/(b^2*d^4*x^4 + 4*b^2*c*d^3*x
^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d^4*x^4 + 4*b^2*c*d
^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)^2 + (b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*s
in(b*x + a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^
2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) - 4
*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)
)*integrate(1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 6*d^2)*sin(b*x + a)/
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^
2*c^4)*cos(b*x + a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(b*x + a)^2 - 2*(b^2*d^4*x^4 + 4*b^2*c*d^3*x^
3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(b*x + a)), x) + (2*d*c
os(3*b*x + 3*a) - 2*d*cos(b*x + a) + (b*d*x + b*c)*sin(3*b*x + 3*a) + (b*d*
x + b*c)*sin(b*x + a))*sin(4*b*x + 4*a) + 2*(2*d*cos(2*b*x + 2*a) - (b*d*x
+ b*c)*sin(2*b*x + 2*a) - d)*sin(3*b*x + 3*a) + 2*(2*d*cos(b*x + a) - (b*d*
x + b*c)*sin(b*x + a))*sin(2*b*x + 2*a) + 2*d*sin(b*x + a))/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^
2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 - 4*(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a
) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x +
 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 - 2*(b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*c
os(4*b*x + 4*a) - 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(a+b\,x)^2}{\sin(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(cot(a + b*x)^2/(sin(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**2*csc(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**2*csc(a + b*x)/(c + d*x)**2, x)
```

3.118 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=406

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)S}{144b^{7/2}}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*(
d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864*d
^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/32*
d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16*d^2*cos
(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.67, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int (c+d}{}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+dx)^{3/2}}{8b}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,c}{144}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,c}{144}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,c}{144}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,c}{144}$$

Mathematica [C] time = 15.95, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
```

```
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

fricas [A] time = 0.54, size = 341, normalized size = 0.84

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-40\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 3.08, size = 2465, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(72*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((I*sqrt(6)*sqrt(p
i)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*
b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2
)/d^2 + 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I
*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d -
 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2
*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2
)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^
2) + d^3*((-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 -
5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt
(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 -
 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d
^3 + 27*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*
I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(
4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x +
c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(I*s
qrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
 c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2
*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*sqrt(pi
)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*s
```

```
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)
*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^
((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3) + 36*(-I*sqrt(6)*sqrt(p
i)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b
*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqr
t(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x +
 c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.04, size = 476, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-}{d}\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{4b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
```

```
*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.53, size = 543, normalized size = 1.34

$$\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+2160\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^4}{d}-5\,\sqrt{dx+c}\,b^2d\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/2
)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*(
4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
a*d)/d) + ((5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*
(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*s
in(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + ((405*I - 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (405*I + 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-
(b*c - a*d)/d) - (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(5*I + 5)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-3*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.119 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\ldots}{}\right.}{8b^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d
^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*sin(b*x+a
)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.53, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$
$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{d\int\sqrt{c+dx}$$
$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ \mathrm{s}}{8b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ \mathrm{s}}{8b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ \mathrm{s}}{8b^2}$$
$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\ \mathrm{cos}}{}$$

Mathematica [C] time = 9.04, size = 676, normalized size = 1.92

$$-\frac{d\sqrt{\frac{b}{d}}\left(\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\frac{bc}{d}\right)+2bc\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\cos\left(a-\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
```

```
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

fricas [A] time = 0.70, size = 280, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

giac [C] time = 4.78, size = 1538, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(12*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d
+ sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2
+ 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*
```

```
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d -
4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c
^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - s
qrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) + 4
*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c
- I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sq
rt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I
*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 1
8*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c
)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.04, size = 384, normalized size = 1.09

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+c}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
```

```
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.51, size = 499, normalized size = 1.41

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{144\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-24\,\sqrt{dx+c}\,b^2\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\,\sqrt{dx+c}\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^
2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x + c)
*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (-(27*I + 27)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (27*I - 27)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) - ((27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) - (27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

3.120 $\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\cdots}{\sqrt{d}}\right)}{12b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1
/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(b*x+
a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.42, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a\cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\,\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\,\sin(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\,\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\,\sin(3a+3bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}dx}{24b}+$$
$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)}{24b}$$
$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Sub}}{}$$
$$= -\frac{\sqrt{c+dx}\,\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right.}{4b^3}$$

Mathematica [C] time = 6.62, size = 278, normalized size = 0.91

$$-\frac{-\sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)+2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(}{24\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (2*Sqrt[3]*Sqrt[b/d]*Sqr
t[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*
Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(24*Sqrt[3]*b*Sqrt[b/d])
```

fricas [A] time = 0.73, size = 235, normalized size = 0.77

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)\mathrm{C}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

giac [C] time = 2.78, size = 842, normalized size = 2.77

$$-\frac{-\frac{i\sqrt{6}\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9i\sqrt{2}\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9i\sqrt{2}\sqrt{\pi}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1
/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(
2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
+ I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 6*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 6*sqrt(d*x
+ c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*
c - 3*I*a*d)/d)/b)/d
```

maple [A] time = 0.03, size = 296, normalized size = 0.97

$$-\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.50, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i+1)\cdot 9\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + ((9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d
) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) + (-(9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

3.121 $\int \sqrt{c+dx} \cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=304

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}$$

```
[Out] 1/72*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/72*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1
/8*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^
(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/8*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/4*cos(b*x+
a)*(d*x+c)^(1/2)/b-1/12*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.42, antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{12b^{3/2}}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(4*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(12*
b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]])/(4*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]
*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(12*b^(3/2)) - (Sqrt
[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a
 - (3*b*c)/d])/(12*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(4*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(a+bx)+\frac{1}{4}\sqrt{c+dx}\sin(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int \sqrt{c+dx}\sin(a+bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(3a+3bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{d\int \frac{\cos(3a+3bx)}{\sqrt{c+dx}}dx}{24b}+$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\left(d\cos\left(3a-\frac{3bc}{d}\right)\right)}{24b}$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Sub}}{}$$
$$= -\frac{\sqrt{c+dx}\cos(a+bx)}{4b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{12b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right.}{4b^3}$$

Mathematica [C] time = 6.39, size = 264, normalized size = 0.87

$$\frac{\frac{\sqrt{6\pi}\cos\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-\sqrt{6\pi}\sin\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right)-6\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(3(a+bx))}{\sqrt{\frac{b}{d}}}+9\sqrt{c+dx}\,e^{-\frac{i(ad+bc)}{d}}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(}{}\right.}{\sqrt{-\frac{ib(c+}{d}}}\right.}{72b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] ((9*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((
-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sq
rt[(I*b*(c + d*x))/d]))/E^((I*(b*c + a*d))/d) + (-6*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[3*(a + b*x)] + Sqrt[6*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt
[6/Pi]*Sqrt[c + d*x]] - Sqrt[6*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*Sin[3*a - (3*b*c)/d])/Sqrt[b/d])/(72*b)
```

fricas [A] time = 0.72, size = 235, normalized size = 0.77

$$\frac{\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+9\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-9\sqrt{2}\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/72*(sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 9*sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c -
a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 9*sqrt(2)*pi*d*
sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c
- a*d)/d) - sqrt(6)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*sqrt(d*x + c)*b*cos(b*x + a)^3)/b
^2
```

giac [C] time = 4.92, size = 842, normalized size = 2.77

$$-\frac{-\frac{i\sqrt{6}\sqrt{\pi}\,(6\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{6}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{3i\,bc-3i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}-\frac{9i\sqrt{2}\sqrt{\pi}\,(2\,bc+i\,d)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\,\sqrt{bd}\,\sqrt{dx+c}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)}{2\,d}\right)e^{\left(\frac{i\,bc-i\,ad}{d}\right)}}{\sqrt{bd}\left(\frac{i\,bd}{\sqrt{b^2d^2}}+1\right)b}+\frac{9i\sqrt{2}\sqrt{\pi}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/144*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)
*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1
/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c -
 I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(
2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
+ I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)*b) + 6*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(
2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*
I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c + 6*sqrt(d*x
+ c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((-I*(d*x + c
)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*
c - 3*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 296, normalized size = 0.97

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{12b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/16/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/24/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/144/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)
/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.50, size = 422, normalized size = 1.39

$$-\frac{\left(\frac{24\sqrt{dx+c}\,b^2\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{72\sqrt{dx+c}\,b^2\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}+\left((i-1)\cdot 9^{\frac{1}{4}}\sqrt{2}\,\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{3(bc-ad)}{d}\right)+(i+1)\cdot 9\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/288*(24*sqrt(d*x + c)*b^2*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 72*sqrt
(d*x + c)*b^2*cos(((d*x + c)*b - b*c + a*d)/d)/d + ((I - 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (I + 1)*9^(1/4)*sqrt(2)
*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3
*I*b/d)) + ((9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d
) + (9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(I*b/d)) + (-(9*I + 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) - (9*I - 9)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-(b*
c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(I + 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I - 1)*9^(1/4)*sqrt(2)*sq
rt(pi)*b*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I
*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \sin{(a + bx)} \cos^2{(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**2, x)
```

3.122 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=353

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\ }}{}\right.}{8b^{5/2}}$$

```
[Out] -1/4*(d*x+c)^(3/2)*cos(b*x+a)/b-1/12*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b-1/144*d
^(3/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/144*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d
^(3/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2)
)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/16*d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/8*d*sin(b*x+a
)*(d*x+c)^(1/2)/b^2+1/24*d*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.53, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{24b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(4*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(12*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi
]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b
*c)/d]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(24*b^(5/2)) -
 (d^(3/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[3*a - (3*b*c)/d])/(24*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(5/2)) + (3*d*Sq
rt[c + d*x]*Sin[a + b*x])/(8*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(24*
b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{3/2}\sin(3a+3bx)\right)dx$$

$$= \frac{1}{4}\int (c+dx)^{3/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{d\int\sqrt{c+dx}\ \mathrm{c}}{8}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ \mathrm{si}}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ \mathrm{si}}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}+\frac{3d\sqrt{c+dx}\ \mathrm{si}}{8b^2}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{12b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\Big(}{}$$

Mathematica [C] time = 8.93, size = 676, normalized size = 1.92

$$-\frac{d\sqrt{\frac{b}{d}}\left(\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\sin\left(a-\frac{bc}{d}\right)+2bc\cos\left(a-\frac{bc}{d}\right)\right)+\sqrt{2\pi}\,S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(3d\cos\left(a-\right.\right.\right.}{16b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c*(2*Sqrt[3]*Sqrt[b/d
]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelC
[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6
/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (Sqrt
[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d*Cos[a
 - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[
2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*Sq
rt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(16*b^3) -
```

```
(Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Co
s[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (
3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a + b*x)] -
Sin[3*(a + b*x)])))/(48*Sqrt[3]*b^3)
```

fricas [A] time = 0.72, size = 280, normalized size = 0.79

$$-\frac{\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{bc-ad}{d}\right)\mathrm{S}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+27\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/144*(sqrt(6)*pi*d^2*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqr
t(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-
(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 27*sqrt(
2)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-(b*c - a*d)/d) + sqrt(6)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqr
t(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) + 24*(2*(b^2*d*x + b^2*c)*
cos(b*x + a)^3 - (b*d*cos(b*x + a)^2 + 2*b*d)*sin(b*x + a))*sqrt(d*x + c))/
b^3
```

giac [C] time = 1.93, size = 1538, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/288*(12*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
t(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d
*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d
)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*((I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d
+ sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2
+ 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*
```

```
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/
(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d -
4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c -
 I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*
I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((
I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12*b^2*c
^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*
d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - s
qrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2) + 4
*(-I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqr
t(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c
- I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1
)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sq
rt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b) + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I
*a*d)/d)/b + 18*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 1
8*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c
)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 384, normalized size = 1.09

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{4b}-\frac{d(dx+c)^{\frac{3}{2}}}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/8/b*d*(1/2/b*d
*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)))-1/24/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/8/b*d*(1
/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(
```

```
1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.53, size = 499, normalized size = 1.41

$$-\frac{\left(\frac{48\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{144\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d}-24\,\sqrt{dx+c}\,b^2\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)-216\,\sqrt{dx+c}\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/576*(48*(d*x + c)^(3/2)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)/d + 144*(
d*x + c)^(3/2)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d - 24*sqrt(d*x + c)*b^
2*sin(3*((d*x + c)*b - b*c + a*d)/d) - 216*sqrt(d*x + c)*b^2*sin(((d*x + c)
*b - b*c + a*d)/d) - (-(I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)
*cos(-3*(b*c - a*d)/d) + (I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/
4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - (-(27*I + 27)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (27*I - 27)*sqrt
(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqr
t(I*b/d)) - ((27*I - 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) - (27*I + 27)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - ((I - 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (I + 1)*9^(1/4)*sqrt(2)*sqrt(pi)*b
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
)*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\sin(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a),x)
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)*cos(a + b*x)**2, x)
```

3.123 $\int (c+dx)^{5/2} \cos^2(a+bx) \sin(a+bx)\, dx$

Optimal. Leaf size=406

$$-\frac{15\sqrt{\frac{\pi}{2}}\, d^{5/2} \cos\left(a-\frac{bc}{d}\right) C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\, d^{5/2} \cos\left(3a-\frac{3bc}{d}\right) C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\, d^{5/2} \sin\left(3a-\frac{3bc}{d}\right) S}{144b^{7/2}}$$

```
[Out] -1/4*(d*x+c)^(5/2)*cos(b*x+a)/b-1/12*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+5/8*d*(
d*x+c)^(3/2)*sin(b*x+a)/b^2+5/72*d*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-5/864*d
^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/864*d^(5/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(7/2)-15/32*
d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/32*d^(5/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/16*d^2*cos
(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.63, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\, d^{5/2} \cos\left(a-\frac{bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\, d^{5/2} \cos\left(3a-\frac{3bc}{d}\right) \text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{144b^{7/2}}+\frac{5\sqrt{\frac{\pi}{6}}\, d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(4*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(144*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(12*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*Fres
nelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (5*d^(5/2)
*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x]
)/Sqrt[d]])/(144*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(144*b^(7/2)) + (15*d^(5/
2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a -
(b*c)/d])/(16*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[a + b*x])/(8*b^2) + (5*d*
(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(72*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{4}(c+dx)^{5/2}\sin(3a+3bx)\right)dx$$
$$= \frac{1}{4}\int (c+dx)^{5/2}\sin(a+bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{(5d)\int (c+d\ldots}{\ldots}$$
$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{12b}+\frac{5d(c+dx)^{3/2}\ldots}{8b\ldots}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{c}\ldots}{144\ldots}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{c}\ldots}{144\ldots}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{c}\ldots}{144\ldots}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{16b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{4b}+\frac{5d^2\sqrt{c+dx}\,\mathrm{c}\ldots}{144\ldots}$$

Mathematica [C] time = 15.31, size = 1168, normalized size = 2.88

$$\frac{e^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)c^2}{8b}-\frac{\left(2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\,\cos(3(a+bx))-\sqrt{2\pi}\,\cos\left(3a-\frac{3bc}{d}\right)\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x],x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(8*b*E^((I*(b*c + a*d))/d)) - (c^2*(2*Sqrt[3]*Sqrt
[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*Fres
nelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(24*Sqrt[3]*b*Sqrt[b/d]) - (
c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(3*d
```

```
*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]
*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d])
+ 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a + b*x])))/(8*b^
3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d]) - S
qrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (
b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) - 2*Sqrt[b/d]*d*Sqrt[c + d
*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*d*x)*Sin[a + b*x])))/(32
*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d
*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*c)/d] + d*S
in[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[3*(a
+ b*x)] - Sin[3*(a + b*x)])))/(24*Sqrt[3]*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*P
i]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2*c^2 - 5*d^2)*Cos[3
*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3*b*c)/d] + (12*b^2*c
^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*
(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)*Sin[3*(a + b*x)])))/(28
8*Sqrt[3]*b^5)
```

fricas [A] time = 0.52, size = 341, normalized size = 0.84

$$-\frac{5\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+405\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{bc-ad}{d}\right)C\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-40}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/864*(5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*cos(-3*(b*c - a*d)/d)*fresnel_cos(s
qrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 405*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*co
s(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 405*s
qrt(2)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-(b*c - a*d)/d) - 5*sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(
6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d)/d) - 24*(30*b*d^2*cos(b
*x + a) - (12*b^3*d^2*x^2 + 24*b^3*c*d*x + 12*b^3*c^2 - 5*b*d^2)*cos(b*x +
a)^3 + 10*(2*b^2*d^2*x + 2*b^2*c*d + (b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*
sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 4.90, size = 2465, normalized size = 6.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1728*(72*(I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) - 3*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*d^2*((I*sqrt(6)*sqrt(p
i)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c
)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*
b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2
)/d^2 + 9*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*
d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*
b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I
*b*c - I*a*d)/d)/b^2)/d^2 + 9*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d -
 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2
*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2
)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + (-I*sqrt(6)*sqrt(pi)*(12
*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c
*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^
2) + d^3*((-I*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 -
5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) -
 6*I*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt
(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 -
 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d
^3 + 27*(-I*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*
I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(
4*I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x +
c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*s
qrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 27*(I*s
qrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(
-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b
*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
 c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2
*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + (I*sqrt(6)*sqrt(pi
)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*s
```

```
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d
)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)
*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^
((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3) + 36*(-I*sqrt(6)*sqrt(p
i)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b) - 9*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(6)*sqrt(pi)*(6*b
*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b)
 + 6*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 18*sqr
t(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 18*sqrt(d*x + c)*d*e
^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 6*sqrt(d*x + c)*d*e^((-3*I*(d*x +
 c)*b + 3*I*b*c - 3*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 476, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{4b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-\cdots}{d}\right)\cdots\right.}{4b\sqrt{\frac{b}{d}}}\cdots\right.}{2b}\cdots\right.}{4b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/8/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/8/b*d*(1/2/b*d
*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(1/
2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(
(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/
24/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/24/b*d*(1/6/b*d*(d*
x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/2)
```

```
*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.52, size = 543, normalized size = 1.34

$$\frac{\left(240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)+2160\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)-24\left(\frac{12\,(dx+c)^{\frac{5}{2}}b^4}{d}-5\,\sqrt{dx+c}\,b^2d\right)c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/3456*(240*(d*x + c)^(3/2)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d) + 2160*(
d*x + c)^(3/2)*b^3*sin(((d*x + c)*b - b*c + a*d)/d) - 24*(12*(d*x + c)^(5/2
)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(3*((d*x + c)*b - b*c + a*d)/d) - 216*(
4*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(((d*x + c)*b - b*c +
a*d)/d) + ((5*I - 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*
(b*c - a*d)/d) + (5*I + 5)*9^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*s
in(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + ((405*I - 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) + (405*I + 405)*sqr
t(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*
sqrt(I*b/d)) + (-(405*I + 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-
(b*c - a*d)/d) - (405*I - 405)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(5*I + 5)*9^(1/4)*sqrt(
2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (5*I - 5)*9^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-3*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.124 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sin(4a}{2048b^3}$$

```
[Out] 1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)^
(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^
(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*
b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^
3
```

Rubi [A] time = 0.40, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(5d)\int (c+dx)^{3/2}\sin(4a+}{64b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{5/2}\sin(4a+4b}{32b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4b}{d}\right.}{4096b}$$

Mathematica [A] time = 3.42, size = 206, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{d}}\left(4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(4(a+bx))\left(64b^2(c+dx)^2-15d^2\right)-280bd^2(c+dx)\cos(4(a+bx))+512b^3(c+dx\right.\right.}{57344b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

fricas [A] time = 0.51, size = 347, normalized size = 1.52

$$-\frac{105\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

giac [C] time = 3.30, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/573440*(17920*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^3 + 56*c*d^2*(512*(3*(d*x + c)^(5/
2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi
)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)
/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2
)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b
 - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2) + d^3*(4096*(5*(d*x + c)^(7/2) - 21*(d*x
 + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sq
rt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
 - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(64*I*(d*x + c
)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2
*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 - 35*(sqrt(2)
*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c +
```

```
4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(-64*I*(d*x + c)
^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*
d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c
)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3) - 2240*(3*sqrt
(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c
 - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*
I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.05, size = 251, normalized size = 1.10

$$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28}-\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\cdots}{\sqrt{\cdots}}\right.\right.}{\cdots}\right.}{8b}\right.}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.47, size = 285, normalized size = 1.25

$$\frac{\sqrt{2}\left(\frac{4096\sqrt{2}\,(dx+c)^{\frac{7}{2}}b^4}{d}-2240\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-\left((105i+105)\,\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)-\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I + 105)*sqrt(pi)
*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (105*I - 105)*sqrt(pi)*d^3*(b^
2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(1
05*I - 105)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (105*I + 1
05)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)
*sqrt(-I*b/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x +
 c)*b*d^2)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^2 \sin(a+bx)^2 (c+dx)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.125 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4a+4bx}{256b^2}$$

```
[Out] 1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*cos
(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d*cos
(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.33, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\cos(4}{256b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\,\sin(4a+$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$
$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}}$$

Mathematica [A] time = 2.80, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{2\pi}\,d^2\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 15\sqrt{2\pi}\,d^2\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}}\right.}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

fricas [A] time = 0.74, size = 249, normalized size = 1.24

$$15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

giac [C] time = 1.99, size = 842, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(960*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^2 + d^2*(512*(3*(d*x + c)^(5/2) - 10*
(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^
2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + 1
5*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d +
16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^2)/d^2) - 80*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b
*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6
4*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x
 + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)
*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.05, size = 206, normalized size = 1.03

$$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{}}{}\right.\right.}{32b\sqrt{\frac{b}{d}}}\right.}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.48, size = 264, normalized size = 1.32

$$\sqrt{2}\left(\frac{512\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^{3}}{d}-320\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^{2}\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-120\sqrt{2}\,\sqrt{dx+c}\,bd\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-\left((15i\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*
d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((15*I - 15)*sqrt(pi)*d^2*(b^2/d^2)^
(1/4)*cos(-4*(b*c - a*d)/d) + (15*I + 15)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(15*I + 15)*sqrt(pi
)*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (15*I - 15)*sqrt(pi)*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{\frac{3}{2}} \sin^2(a + bx) \cos^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

3.126 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c}{}$$

```
[Out] 1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelC
(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)
*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.27, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx} - \frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{3/2}}{12d} - \frac{1}{8}\int \sqrt{c+dx}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$

$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4\right.}{\sqrt{c+dx}}}{64b}$$

$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int \sin\left(\right.\right.}{32b}$$

$$= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C\left(\right.}{}$$

Mathematica [A] time = 0.82, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(8\ldots}{384d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

fricas [A] time = 0.52, size = 175, normalized size = 1.01

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) + 3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right) + \ldots}{384\,b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

giac [C] time = 0.99, size = 452, normalized size = 2.60

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-24\left(\frac{\sqrt{2}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 24*(sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
- 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b
*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*
c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I
*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

maple [A] time = 0.05, size = 159, normalized size = 0.91

$$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
```

```
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.46, size = 219, normalized size = 1.26

$$\sqrt{2}\left(\frac{64\,\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2}{d} - 24\,\sqrt{2}\,\sqrt{dx+c}\,b\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right) - \left(-(3i+3)\,\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right) + (3i-3)\,\sqrt{\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(4*((d*x + c)*b - b*c + a*d)/d) - (-(3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1
/4)*cos(-4*(b*c - a*d)/d) + (3*I - 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((3*I - 3)*sqrt(pi)*d*(b^2/
d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\sin^2(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

3.127 $\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=174

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4bx)}{32b}+\frac{(c+}{1}$$

```
[Out] 1/12*(d*x+c)^(3/2)/d+1/128*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/128*FresnelC
(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)
*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.25, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}-\frac{\sqrt{c+dx}\sin(4a+4b}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(12*d) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt
[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*
b*c)/d])/(64*b^(3/2)) - (Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx} - \frac{1}{8}\sqrt{c+dx}\cos(4a+4bx)\right)dx$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{1}{8}\int \sqrt{c+dx}\cos(4a+4bx)\,dx$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{d\int \frac{\sin(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{\sqrt{c+dx}}}{64b}$$
$$= \frac{(c+dx)^{3/2}}{12d} - \frac{\sqrt{c+dx}\sin(4a+4bx)}{32b} + \frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Subst}\left(\int \sin\left(\right.\right.}{32b}$$
$$= \frac{(c+dx)^{3/2}}{12d} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{2}}C\left(\right.}{}$$

Mathematica [A] time = 0.11, size = 161, normalized size = 0.93

$$\frac{3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+4\sqrt{\frac{b}{d}}\sqrt{c+dx}\,(8b(}{384d^2\left(\frac{b}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c
 + d*x]] + 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Si
n[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(8*b*(c + d*x) - 3*d*Sin[4*(
a + b*x)]))/(384*(b/d)^(3/2)*d^2)
```

fricas [A] time = 0.75, size = 175, normalized size = 1.01

$$\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\text{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\text{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+1}{384\,b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(2*
sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fre
snel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 16
*(2*b^2*d*x + 2*b^2*c - 3*(2*b*d*cos(b*x + a)^3 - b*d*cos(b*x + a))*sin(b*x
 + a))*sqrt(d*x + c))/(b^2*d)
```

giac [C] time = 1.01, size = 452, normalized size = 2.60

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{3\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-24\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/768*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 24*(sqrt(2)*sqrt(pi)*d*erf
(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
- 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b
*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*
c - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I
*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x
 + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 159, normalized size = 0.91

$$\frac{\frac{(dx+c)^{\frac{3}{2}}}{12}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/24*(d*x+c)^(3/2)-1/64/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+1/256/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*
```

```
2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC
(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.46, size = 219, normalized size = 1.26

$$\sqrt{2}\left(\frac{64\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2}{d} - 24\sqrt{2}\,\sqrt{dx+c}\,b\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right) - \left(-(3i+3)\,\sqrt{\pi}\,d\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right) + (3i-3)\,\sqrt{\pi}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/1536*sqrt(2)*(64*sqrt(2)*(d*x + c)^(3/2)*b^2/d - 24*sqrt(2)*sqrt(d*x + c)
*b*sin(4*((d*x + c)*b - b*c + a*d)/d) - (-(3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1
/4)*cos(-4*(b*c - a*d)/d) + (3*I - 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin(-4*(b*
c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - ((3*I - 3)*sqrt(pi)*d*(b^2/
d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (3*I + 3)*sqrt(pi)*d*(b^2/d^2)^(1/4)*sin
(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

3.128 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=200

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\,\cos(4a+4bx}{256b^2}$$

```
[Out] 1/20*(d*x+c)^(5/2)/d-1/32*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b+3/1024*d^(3/2)*cos
(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^
(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5/2)-3/256*d*cos
(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.32, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3d\sqrt{c+dx}\,\cos(4}{256b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(5/2)/(20*d) - (3*d*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(256*b^2) + (
3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(2*Sq
rt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)
) - ((c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2} - \frac{1}{8}(c+dx)^{3/2}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{1}{8}\int (c+dx)^{3/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+dx}\,\sin(4a+ }{64b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{3/2}\sin(4a+4bx)}{32b}$$

$$= \frac{(c+dx)^{5/2}}{20d} - \frac{3d\sqrt{c+dx}\,\cos(4a+4bx)}{256b^2} + \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4bc}{d}\right)}{512b^{5/2}}$$

Mathematica [A] time = 1.27, size = 187, normalized size = 0.94

$$\frac{\sqrt{\frac{b}{d}}\left(15\sqrt{2\pi}\,d^2\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) - 15\sqrt{2\pi}\,d^2\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right) + 4\sqrt{\frac{b}{d}}\sqrt{\right.}{5120b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(15*d^2*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqr
t[2/Pi]*Sqrt[c + d*x]] - 15*d^2*Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*
Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(-15*d^2*Co
s[4*(a + b*x)] + 8*b*(c + d*x)*(8*b*(c + d*x) - 5*d*Sin[4*(a + b*x)]))))/(5
120*b^3)
```

fricas [A] time = 0.52, size = 249, normalized size = 1.24

$$15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - 15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/5120*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*
fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
 4*(64*b^3*d^2*x^2 - 120*b*d^2*cos(b*x + a)^4 + 128*b^3*c*d*x + 64*b^3*c^2
+ 120*b*d^2*cos(b*x + a)^2 - 15*b*d^2 - 160*(2*(b^2*d^2*x + b^2*c*d)*cos(b*
x + a)^3 - (b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))
/(b^3*d)
```

giac [C] time = 4.12, size = 842, normalized size = 4.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(960*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqr
t(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^2 + d^2*(512*(3*(d*x + c)^(5/2) - 10*
(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^
2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*s
qrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + 1
5*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2)*b*d +
16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b
*c + 4*I*a*d)/d)/b^2)/d^2) - 80*(3*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b
*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6
4*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x
 + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)
*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 206, normalized size = 1.03

$$\frac{\frac{(dx+c)^{\frac{5}{2}}}{20}-\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{4da-4cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{\ldots}}{\ldots}\right.\ldots}{32b\sqrt{\frac{b}{d}}}\right.\ldots}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/40*(d*x+c)^(5/2)-1/64/b*d*(d*x+c)^(3/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+3/64/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/32
/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)
/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.78, size = 264, normalized size = 1.32

$$\frac{\sqrt{2}\left(\frac{512\sqrt{2}\,(dx+c)^{\frac{5}{2}}b^{3}}{d}-320\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^{2}\sin\left(\frac{4\left((dx+c)b-bc+ad\right)}{d}\right)-120\sqrt{2}\,\sqrt{dx+c}\,bd\cos\left(\frac{4\left((dx+c)b-bc+ad\right)}{d}\right)-\left((15i\ldots\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/20480*sqrt(2)*(512*sqrt(2)*(d*x + c)^(5/2)*b^3/d - 320*sqrt(2)*(d*x + c)^
(3/2)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 120*sqrt(2)*sqrt(d*x + c)*b*
d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((15*I - 15)*sqrt(pi)*d^2*(b^2/d^2)^
(1/4)*cos(-4*(b*c - a*d)/d) + (15*I + 15)*sqrt(pi)*d^2*(b^2/d^2)^(1/4)*sin(
-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(15*I + 15)*sqrt(pi
)*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (15*I - 15)*sqrt(pi)*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2\left(c+d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^{\frac{3}{2}} \sin^2 (a + bx) \cos^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**(3/2)*sin(a + b*x)**2*cos(a + b*x)**2, x)
```

3.129 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=228

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sin(4a}{2048b^3}$$

```
[Out] 1/28*(d*x+c)^(7/2)/d-5/256*d*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b^2-1/32*(d*x+c)^
(5/2)*sin(4*b*x+4*a)/b-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*
2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)-15/8192*d^
(5/2)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*
b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)+15/2048*d^2*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^
3
```

Rubi [A] time = 0.38, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}+\frac{15d^2\sqrt{c+dx}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(7/2)/(28*d) - (5*d*(c + d*x)^(3/2)*Cos[4*a + 4*b*x])/(256*b^2) -
 (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]
*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[
(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b
^(7/2)) + (15*d^2*Sqrt[c + d*x]*Sin[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(
5/2)*Sin[4*a + 4*b*x])/(32*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
```

```
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2} - \frac{1}{8}(c+dx)^{5/2}\cos(4a+4bx)\right)dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{1}{8}\int (c+dx)^{5/2}\cos(4a+4bx)\,dx$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{(c+dx)^{5/2}\sin(4a+4bx)}{32b} + \frac{(5d)\int (c+dx)^{3/2}\sin(4a+}{64b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{(c+dx)^{5/2}\sin(4a+4b}{32b}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} + \frac{15d^2\sqrt{c+dx}\,\sin(4a+}{2048b^3}$$

$$= \frac{(c+dx)^{7/2}}{28d} - \frac{5d(c+dx)^{3/2}\cos(4a+4bx)}{256b^2} - \frac{15d^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(4a-\frac{4b}{d}\right.}{4096b}$$

Mathematica [A] time = 2.32, size = 206, normalized size = 0.90

$$\frac{\sqrt{\frac{b}{d}}\left(4\sqrt{\frac{b}{d}}\sqrt{c+dx}\left(-7d\sin(4(a+bx))\left(64b^2(c+dx)^2-15d^2\right)-280bd^2(c+dx)\cos(4(a+bx))+512b^3(c+dx)\right.\right.}{57344b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^2,x]
```

```
[Out] (Sqrt[b/d]*(-105*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*S
qrt[2/Pi]*Sqrt[c + d*x]] - 105*d^3*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 4*Sqrt[b/d]*Sqrt[c + d*x]*(512*b^3
*(c + d*x)^3 - 280*b*d^2*(c + d*x)*Cos[4*(a + b*x)] - 7*d*(-15*d^2 + 64*b^2
*(c + d*x)^2)*Sin[4*(a + b*x)])))/(57344*b^4)
```

fricas [A] time = 0.61, size = 347, normalized size = 1.52

$$-\frac{105\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+105\sqrt{2}\,\pi d^4\sqrt{\frac{b}{\pi d}}\,C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/57344*(105*sqrt(2)*pi*d^4*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_s
in(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 105*sqrt(2)*pi*d^4*sqrt(b/(pi*
d))*fresnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/
d) - 16*(128*b^4*d^3*x^3 + 384*b^4*c*d^2*x^2 + 128*b^4*c^3 - 70*b^2*c*d^2 -
 560*(b^2*d^3*x + b^2*c*d^2)*cos(b*x + a)^4 + 560*(b^2*d^3*x + b^2*c*d^2)*c
os(b*x + a)^2 + 2*(192*b^4*c^2*d - 35*b^2*d^3)*x - 7*(2*(64*b^3*d^3*x^2 + 1
28*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a)^3 - (64*b^3*d^3*x^2
+ 128*b^3*c*d^2*x + 64*b^3*c^2*d - 15*b*d^3)*cos(b*x + a))*sin(b*x + a))*sq
rt(d*x + c))/(b^4*d)
```

giac [C] time = 4.93, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/573440*(17920*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 8*sqrt(d*x + c))*c^3 + 56*c*d^2*(512*(3*(d*x + c)^(5/
2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)/d^2 + 15*(sqrt(2)*sqrt(pi
)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)*b^2) - 4*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)
/d^2 + 15*(sqrt(2)*sqrt(pi)*(64*b^2*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*
a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*(-8*I*(d*x + c)^(3/2
)*b*d + 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b
 - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2) + d^3*(4096*(5*(d*x + c)^(7/2) - 21*(d*x
 + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)/d^3 - 35*(sq
rt(2)*sqrt(pi)*(512*b^3*c^3 + 192*I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*er
f(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c
 - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(64*I*(d*x + c
)^(5/2)*b^2*d - 192*I*(d*x + c)^(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2
*d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x +
c)*d^3)*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^3)/d^3 - 35*(sqrt(2)
*sqrt(pi)*(512*b^3*c^3 - 192*I*b^2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sq
rt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c +
```

```
4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 4*(-64*I*(d*x + c)
^(5/2)*b^2*d + 192*I*(d*x + c)^(3/2)*b^2*c*d - 192*I*sqrt(d*x + c)*b^2*c^2*
d + 40*(d*x + c)^(3/2)*b*d^2 - 72*sqrt(d*x + c)*b*c*d^2 + 15*I*sqrt(d*x + c
)*d^3)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^3)/d^3) - 2240*(3*sqrt
(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b) + 3*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt
(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*
d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 64*(d*x + c)^(3/2) + 192*sqrt(d*x + c)*c
 - 12*I*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 12*
I*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 251, normalized size = 1.10

$$\frac{\frac{(dx+c)^{\frac{7}{2}}}{28} - \frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b} + \frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b} + \frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{8b} - \frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{4da-4cb}{d}\right)S\left(\frac{2\sqrt{2}}{\sqrt{}}\right.\right.}{}\right.}{8b}\right)}{32b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/56*(d*x+c)^(7/2)-1/64/b*d*(d*x+c)^(5/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c
)/d)+5/64/b*d*(-1/8/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/8/
b*d*(1/8/b*d*(d*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d
)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(
b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 2.02, size = 285, normalized size = 1.25

$$\sqrt{2}\left(\frac{4096\sqrt{2}\,(dx+c)^{\frac{7}{2}}b^4}{d} - 2240\sqrt{2}\,(dx+c)^{\frac{3}{2}}b^2d\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right) - \left((105i+105)\,\sqrt{\pi}\,d^3\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{4\,(bc-ad)}{d}\right) - \right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/229376*sqrt(2)*(4096*sqrt(2)*(d*x + c)^(7/2)*b^4/d - 2240*sqrt(2)*(d*x +
c)^(3/2)*b^2*d*cos(4*((d*x + c)*b - b*c + a*d)/d) - ((105*I + 105)*sqrt(pi)
*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (105*I - 105)*sqrt(pi)*d^3*(b^
2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) - (-(1
05*I - 105)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (105*I + 1
05)*sqrt(pi)*d^3*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)
*sqrt(-I*b/d)) - 56*(64*sqrt(2)*(d*x + c)^(5/2)*b^3 - 15*sqrt(2)*sqrt(d*x +
 c)*b*d^2)*sin(4*((d*x + c)*b - b*c + a*d)/d))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.130 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)\cdots}{1600b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d
*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+3/1
6000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2)*10^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^
(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*FresnelS(b^(1/2
)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)
/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelS(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2
)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3*a)*(d*x+c)^(
1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 1.15, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
```

```
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+\right.$$

$$= \frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5bx)$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}}{8}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}}{8}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

Mathematica [C] time = 24.10, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
```

```
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
```

```
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b
/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(
7/2)*d^3) - (Sin[(5*b*c)/d]*(25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*
b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c +
d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]])
+ Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqr
t[5]*(b/d)^(7/2)*d^3))))/16
```

fricas [A] time = 1.01, size = 521, normalized size = 0.85

$$81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)C\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-625\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

giac [C] time = 15.99, size = 3689, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/864000*(1800*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(1
0)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^3 + 18*c*d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^
2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I
*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^
2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*s
qrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
+ c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d
)/b^2)/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c
)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 -
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d
*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(
6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sq
rt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*
a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
```

```
*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10
*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*
d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + d^3*(27*(I*sqr
t(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(
-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*
b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(
d*x + c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^
2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*
x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^3 + 125*(-I
*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I
*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d
*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2
*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x
 + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 + 6750*(-I
*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*
d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c
)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(I*sqrt(2)*sq
rt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)
*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(I*sqrt(6)*sqrt(pi)*(72
*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*
d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x +
 c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(10)*sqrt(pi)*(2
00*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)
*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^
((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) + 180*(9*I*sqrt(10)*sqr
t(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
```

```
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.05, size = 719, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-}{}\right.\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*(d*x+c)*b+5*(a
```

```
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.90, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d - 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*x
+ c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x + c)^(
5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c + a*d)
/d) + 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x + c)*b
^3)*cos(((d*x + c)*b - b*c + a*d)/d) + ((162*I - 162)*25^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(1/4)*sqrt(pi)*
b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c -
 a*d)/d) - (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(202500*I - 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt(p
i)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)
) + ((202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
 + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
 + (-(162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*
d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.131 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{}}{}\right.}{16b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/
2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/800
0*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a
-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1
/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3
*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/32*
d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/
d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+1/96*d*sin(
3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.88, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-3\sqrt{\frac{\pi}{2}}\,d^{3/2}\text{s}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
```

```
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c+$$

$$= \frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx - \frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5bx)$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}}{8}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}}{8}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}}{8}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}}{8}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b} - \frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b} + \frac{(c+dx)^{3/2}}{8}$$

Mathematica [C] time = 11.68, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right.\right.}{1}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
```

```
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

fricas [A] time = 0.99, size = 427, normalized size = 0.80

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

giac [C] time = 4.63, size = 2300, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

[Out] -1/144000*(300*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + 20*(9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*

```
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.04, size = 580, normalized size = 1.09

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+c)^{\frac{3}{2}}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d
*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*
b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)))
```

maxima [C] time = 0.92, size = 760, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
+ a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d^2 - 1080*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c + a*d
)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/
d + 54000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d - (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) - ((250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - ((13500*
I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I - 13
500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - (-(13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (-(250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - ((
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (5
4*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.132 $\int \sqrt{c+dx} \cos^2(a+bx) \sin^3(a+bx) \, dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{1}{\pi}}}{}\right.}{80b^{3/2}}$$

```
[Out] -1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^
(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b
^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/
8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+1/80*cos(5
*b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.67, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int\left(\frac{1}{8}\sqrt{c+dx}\,\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\,\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\,\mathrm{s}\right.$$

$$=\frac{1}{16}\int\sqrt{c+dx}\,\sin(3a+3bx)\,dx-\frac{1}{16}\int\sqrt{c+dx}\,\sin(5a+5bx)\,dx+$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5a+}{80b}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5a+}{80b}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5a+}{80b}$$

$$=-\frac{\sqrt{c+dx}\,\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\,\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\,\cos(5a+}{80b}$$

Mathematica [C] time = 7.34, size = 432, normalized size = 0.94

$$\frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(5a-\frac{5bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

fricas [A] time = 1.01, size = 356, normalized size = 0.78

$$-\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

giac [C] time = 2.74, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(
2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt
(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b) + 30*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt
```

```
(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*
sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)))*c - 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d
)/d)/b + 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b
+ 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*
x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(
(-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(
d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

maple [A] time = 0.04, size = 447, normalized size = 0.97

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.84, size = 674, normalized size = 1.47

$$\sqrt{2}\left(\frac{360\,\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{5\,((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{600\,\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{3\,((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\,\sqrt{2}\,\sqrt{dx+c}\,b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(\frac{(18i-18)\cdot 25\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a
*d)/d)/d^2 - 600*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/
d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d^
2 + ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d
)/d + (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I - 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d))
+ (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (900
*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
+ c)*sqrt(I*b/d)) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d)/d + (900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (
-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
- (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d
)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3(a+bx)\cos^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

3.133 $\int \sqrt{c+dx}\,\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5a-\frac{5bc}{d}\right)C\left(\sqrt{b}\ \cdots\right.}{80b^{3/2}}$$

```
[Out] -1/800*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelS(b^(1/2)*10^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^
(3/2)+1/288*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)-1/288*FresnelS(b^(1/2)*6^(1/2)/
Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b
^(3/2)+1/16*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(b^(1/2)*2^(1/2)/Pi^(1
/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/
8*cos(b*x+a)*(d*x+c)^(1/2)/b-1/48*cos(3*b*x+3*a)*(d*x+c)^(1/2)/b+1/80*cos(5
*b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.66, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\cos\left(5\ \cdots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[a + b*x])/(8*b) - (Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(48*
b) + (Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(80*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[a -
(b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(3/2))
+ (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqr
t[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/10]*Cos[5*a - (5*b*c)
/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(80*b^(3/2)) + (
Sqrt[d]*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*S
in[5*a - (5*b*c)/d])/(80*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/6]*FresnelS[(Sqrt[b]*S
qrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(48*b^(3/2)) - (Sqr
t[d]*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a
- (b*c)/d])/(8*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\sin(a+bx)+\frac{1}{16}\sqrt{c+dx}\sin(3a+3bx)-\frac{1}{16}\sqrt{c+dx}\right.$$

$$=\frac{1}{16}\int\sqrt{c+dx}\sin(3a+3bx)\,dx-\frac{1}{16}\int\sqrt{c+dx}\sin(5a+5bx)\,dx+$$

$$=-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a}{80b}$$

$$=-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a}{80b}$$

$$=-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a}{80b}$$

$$=-\frac{\sqrt{c+dx}\cos(a+bx)}{8b}-\frac{\sqrt{c+dx}\cos(3a+3bx)}{48b}+\frac{\sqrt{c+dx}\cos(5a}{80b}$$

Mathematica [C] time = 7.26, size = 432, normalized size = 0.94

$$\frac{-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+\sqrt{2\pi}\sin\left(5a-\frac{5bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}\right)+2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos}{160\sqrt{5}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-I)
*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqrt[
(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (2*Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelC[Sqr
t[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/P
i]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d])/(160*Sqrt[5]*b*Sqrt[b/d]) - (2*Sqrt
[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a - (3*b*c)
/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt
[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d])/(96*Sqrt[3]*b*Sqrt[b/
d])
```

fricas [A] time = 0.67, size = 356, normalized size = 0.78

$$-\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)C\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-25\,\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_cos(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(
-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*s
qrt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)*sqrt(d*x
 + c)*sqrt(b/(pi*d))) + 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b
/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*
d)/d) - 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_sin(sqrt(10)*sqrt(d*x + c)*s
qrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^5 - 5*b*cos(b*
x + a)^3)*sqrt(d*x + c))/b^2
```

giac [C] time = 2.85, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/14400*(9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(
2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d
/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) + 450*I*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*
d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt
(10)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/s
qrt(b^2*d^2) + 1)*b) + 30*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt
```

```
(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*
a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)
*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*
sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) +
3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)))*c - 90*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d
)/d)/b + 150*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b
+ 900*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*
x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^(
(-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(
d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 447, normalized size = 0.97

$$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/32/b*d*2^(1/2
)*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1
/2)*(d*x+c)^(1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)
/d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fres
nelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c
)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))+1/16
0/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/1600/b*d*2^(1/2)*Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^
(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

maxima [C] time = 0.57, size = 674, normalized size = 1.47

$$\frac{\sqrt{2}\left(\frac{360\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{600\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\sqrt{2}\sqrt{dx+c}\,b^3\cos\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(\frac{(18i-18)\cdot 25^{\frac{1}{4}}}{\cdots}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a
*d)/d)/d^2 - 600*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/
d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d^
2 + ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d
)/d + (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I - 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2
*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d))
+ (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (900
*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
+ c)*sqrt(I*b/d)) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c -
 a*d)/d)/d + (900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(b^
2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (
-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
- (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/d
)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^3\left(a+bx\right)\cos^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**3*cos(a + b*x)**2, x)
```

3.134 $\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=534

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}}{}\right.}{16b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(b*x+a)/b-1/48*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(3/2)*cos(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/
2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)+3/800
0*d^(3/2)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a
-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)-1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelS
(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1
/576*d^(3/2)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3
*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelS(b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/32*
d^(3/2)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/
d)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*sin(b*x+a)*(d*x+c)^(1/2)/b^2+1/96*d*sin(
3*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.80, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{800b^{5/2}}-\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[3*a + 3*b*x])/
(48*b) + ((c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(80*b) - (3*d^(3/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(16*
b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[
6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*Cos[5*
a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(800*
b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*
x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt[Pi/6]*Fres
nelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d])/(96*
b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x]
)/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[a + b*x]
```

```
)/(16*b^2) + (d*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(96*b^2) - (3*d*Sqrt[c + d*
x]*Sin[5*a + 5*b*x])/(800*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{3/2}\sin(3a+3bx)-\frac{1}{16}(c+$$

$$= \frac{1}{16}\int (c+dx)^{3/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{3/2}\sin(5a+5bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\text{co}}{80}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\text{co}}{80}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\text{co}}{80}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\text{co}}{80}$$

$$= -\frac{(c+dx)^{3/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{3/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{3/2}\text{co}}{80}$$

Mathematica [C] time = 11.45, size = 1041, normalized size = 1.95

$$\frac{ce^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(-\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}}-\frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b}+\frac{c\left(2\sqrt{5}\sqrt{\frac{b}{d}}\sqrt{c+dx}\cos(5(a+bx))-\sqrt{2\pi}\cos\left(5a-\frac{5bc}{d}\right)\right.}{16}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[((-
I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/Sqr
t[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c*(2*Sqrt[5]*Sqrt[b/
d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fresnel
C[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (
c*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Cos[3*a
- (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*Fres
```

```
nelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*Sqrt[3]
*b*Sqrt[b/d]) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt
[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2*Pi]*Fre
snelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin
[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] - 3*Sin[a
+ b*x])))/(32*b^3) - (Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]
*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b*
c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*
x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) + (Sqrt[b/d]*d*(
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*
b*c)/d] - 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)
/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin
[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

fricas [A] time = 0.63, size = 427, normalized size = 0.80

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_si
n(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) - 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) + 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(30*(b^2*d*x +
 b^2*c)*cos(b*x + a)^5 - 50*(b^2*d*x + b^2*c)*cos(b*x + a)^3 - (9*b*d*cos(b
*x + a)^4 - 13*b*d*cos(b*x + a)^2 - 26*b*d)*sin(b*x + a))*sqrt(d*x + c))/b^
3
```

giac [C] time = 4.21, size = 2300, normalized size = 4.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

[Out] -1/144000*(300*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + 20*(9*I*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*

```
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 580, normalized size = 1.09

$$-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/16/b*d*(1/2/b
*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(1/6/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3
^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/P
i^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(3/2)*co
s(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-3/160/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d
*x+c)*b+5*(a*d-b*c)/d)-1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(
5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*
b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+
c)^(1/2)*b/d)))
```

maxima [C] time = 0.80, size = 760, normalized size = 1.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*c
+ a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
a*d)/d)/d^2 - 1080*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c + a*d
)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)/d)/
d + 54000*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d - (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) - ((250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) - ((13500*
I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I - 13
500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sq
rt(I*b/d)) - (-(13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) - (-(250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) - ((
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (5
4*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.135 $\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\cos\left(5a-\frac{5bc}{d}\right)(}{1600b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(b*x+a)/b-1/48*(d*x+c)^(5/2)*cos(3*b*x+3*a)/b+1/80*(d
*x+c)^(5/2)*cos(5*b*x+5*a)/b+5/16*d*(d*x+c)^(3/2)*sin(b*x+a)/b^2+5/288*d*(d
*x+c)^(3/2)*sin(3*b*x+3*a)/b^2-1/160*d*(d*x+c)^(3/2)*sin(5*b*x+5*a)/b^2+3/1
6000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelS(b^(1/2)*10^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^
(7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelC(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)+5/3456*d^(5/2)*FresnelS(b^(1/2
)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)
/b^(7/2)-15/64*d^(5/2)*cos(a-b*c/d)*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+
c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelS(b^(1/2)*2
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2
)+15/32*d^2*cos(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*cos(3*b*x+3*a)*(d*x+c)^(
1/2)/b^3-3/1600*d^2*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.95, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[a + b*x])/(32*b^3) - ((c + d*x)^(5/2)*Cos[a + b*x
])/(8*b) + (5*d^2*Sqrt[c + d*x]*Cos[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5
/2)*Cos[3*a + 3*b*x])/(48*b) - (3*d^2*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(1600
*b^3) + ((c + d*x)^(5/2)*Cos[5*a + 5*b*x])/(80*b) - (15*d^(5/2)*Sqrt[Pi/2]*
Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(32*
b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sqrt[b]*Sqr
t[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (3*d^(5/2)*Sqrt[Pi/10]*Cos
[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1
600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c
```

```
+ d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) + (5*d^(5/2)*Sqrt[Pi/
6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*c)/d
])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt
[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) + (5*d*(c + d*x)^(3/2)*S
in[a + b*x])/(16*b^2) + (5*d*(c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(288*b^2) -
(d*(c + d*x)^(3/2)*Sin[5*a + 5*b*x])/(160*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^2(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\sin(a+bx)+\frac{1}{16}(c+dx)^{5/2}\sin(3a+3bx)-\frac{1}{16}(c+\right.$$

$$= \frac{1}{16}\int (c+dx)^{5/2}\sin(3a+3bx)\,dx-\frac{1}{16}\int (c+dx)^{5/2}\sin(5a+5bx)$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}}{8}$$

$$= -\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}-\frac{(c+dx)^{5/2}\cos(3a+3bx)}{48b}+\frac{(c+dx)^{5/2}}{8}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

$$= \frac{15d^2\sqrt{c+dx}\,\cos(a+bx)}{32b^3}-\frac{(c+dx)^{5/2}\cos(a+bx)}{8b}+\frac{5d^2\sqrt{c+dx}}{57}$$

Mathematica [C] time = 22.97, size = 3348, normalized size = 5.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^2*Sin[a + b*x]^3,x]
```

```
[Out] (c^2*Sqrt[c + d*x]*(-((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sqrt[(
(-I)*b*(c + d*x))/d]) - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d])/S
qrt[(I*b*(c + d*x))/d]))/(16*b*E^((I*(b*c + a*d))/d)) + (c^2*(2*Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Cos[5*(a + b*x)] - Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*Fre
snelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*
Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d]))/(160*Sqrt[5]*b*Sqrt[b/d])
```

```
 - (c^2*(2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[3*(a + b*x)] - Sqrt[2*Pi]*Co
s[3*a - (3*b*c)/d]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]] + Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/
Pi]*Sqrt[c + d*x]]*(3*d*Cos[a - (b*c)/d] - 2*b*c*Sin[a - (b*c)/d]) + Sqrt[2
*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] +
 3*d*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(2*b*x*Cos[a + b*x] -
3*Sin[a + b*x])))/(16*b^3) + ((b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d
]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c
*d*Sin[a - (b*c)/d]) - Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*
x]]*(-12*b*c*d*Cos[a - (b*c)/d] + (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) -
2*Sqrt[b/d]*d*Sqrt[c + d*x]*(d*(-15 + 4*b^2*x^2)*Cos[a + b*x] + 2*b*(c - 5*
d*x)*Sin[a + b*x])))/(64*b^5) - (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*FresnelS[Sqrt[b/
d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(d*Cos[3*a - (3*b*c)/d] - 2*b*c*Sin[3*a - (3*b
*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Co
s[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]*d*Sqrt[c
 + d*x]*(2*b*x*Cos[3*(a + b*x)] - Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) + ((
b/d)^(3/2)*d^2*(Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((1
2*b^2*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d]) -
Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a
- (3*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[
b/d]*d*Sqrt[c + d*x]*(d*(5 - 12*b^2*x^2)*Cos[3*(a + b*x)] - 2*b*(c - 5*d*x)
*Sin[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) + (c*Sqrt[b/d]*d*(Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(3*d*Cos[5*a - (5*b*c)/d] - 10*b*c*
Sin[5*a - (5*b*c)/d]) + Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c +
d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*
Sqrt[b/d]*d*Sqrt[c + d*x]*(10*b*x*Cos[5*(a + b*x)] - 3*Sin[5*(a + b*x)])))/
(800*Sqrt[5]*b^3) - (d^2*(Sin[5*a]*((c^2*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]
*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c
 + d*x]])*Sin[(5*b*c)/d])/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (c^2*Cos[(5*b*c)/d]
*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqr
t[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) -
 (2*c*Cos[(5*b*c)/d]*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c +
d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 +
5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*
(b/d)^(5/2)*d^3) - (2*c*Sin[(5*b*c)/d]*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3
/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi
]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d])
)/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (Sin[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/
2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sq
rt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10
/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c
 + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3) + (Cos[(5*b*c)/d]*(25*Sqrt[
5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d
)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[
```

```
Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin
[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqrt[5]*(b/d)^(7/2)*d^3)) + Cos[5*a]*((c
^2*Cos[(5*b*c)/d]*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]
) + Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/(5*Sqrt[5]*(
b/d)^(3/2)*d^3) - (c^2*Sin[(5*b*c)/d]*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt
[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x
))/d]))/(5*Sqrt[5]*(b/d)^(3/2)*d^3) + (2*c*Sin[(5*b*c)/d]*((-3*(-(Sqrt[5]*S
qrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) + Sqrt[Pi/2]*FresnelC[Sqrt[b
/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*
Sin[(5*b*(c + d*x))/d]))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) - (2*c*Cos[(5*b*c)/d]
*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c + d*x))/d] + (3*(-(Sqr
t[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) + Sqrt[5]*Sqrt[b/d]*
Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/(25*Sqrt[5]*(b/d)^(5/2)*d^3) + (
Cos[(5*b*c)/d]*(-25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Cos[(5*b*(c + d*x))
/d] + (5*((-3*(-(Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Cos[(5*b*(c + d*x))/d]) +
Sqrt[Pi/2]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]))/2 + 5*Sqrt[5]*(b
/d)^(3/2)*(c + d*x)^(3/2)*Sin[(5*b*(c + d*x))/d]))/2))/(125*Sqrt[5]*(b/d)^(
7/2)*d^3) - (Sin[(5*b*c)/d]*(25*Sqrt[5]*(b/d)^(5/2)*(c + d*x)^(5/2)*Sin[(5*
b*(c + d*x))/d] - (5*(-5*Sqrt[5]*(b/d)^(3/2)*(c + d*x)^(3/2)*Cos[(5*b*(c +
d*x))/d] + (3*(-(Sqrt[Pi/2]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]])
+ Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[(5*b*(c + d*x))/d]))/2))/2))/(125*Sqr
t[5]*(b/d)^(7/2)*d^3))))/16
```

fricas [A] time = 0.82, size = 521, normalized size = 0.85

$$\frac{81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-625\,\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{C}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_c
os(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 625*sqrt(6)*pi*d^3*sqrt(b/(pi*d
))*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
- 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*fr
esnel_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*s
qrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d
)))*sin(-3*(b*c - a*d)/d) - 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(s
qrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(9*(20*b^
3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*cos(b*x + a)^5 + 390*b*d^2
*cos(b*x + a) - 5*(60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 - 13*b*d^2)*
cos(b*x + a)^3 + 10*(26*b^2*d^2*x - 9*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^4
+ 26*b^2*c*d + 13*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(
d*x + c))/b^4
```

giac [C] time = 7.58, size = 3689, normalized size = 6.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/864000*(1800*(-3*I*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(
d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*
(I*b*d/sqrt(b^2*d^2) + 1)) + 5*I*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(s
qrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*I*sqrt(2)*sqrt(pi)*d*erf(-1/
2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c
+ I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) - 5*I*sqrt(6)*sqrt(pi)*d
*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*I*sqrt(1
0)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^3 + 18*c*d^2*(9*(-I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^
2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*I
*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^
2)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(I*sqrt(6)*s
qrt(pi)*(12*b^2*c^2 + 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x
+ c)*b*c*d + sqrt(d*x + c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d
)/b^2)/d^2 + 2250*(I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf
(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*
c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(2*I*(d*x + c
)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x +
c)*b + I*b*c - I*a*d)/d)/b^2)/d^2 + 2250*(-I*sqrt(2)*sqrt(pi)*(4*b^2*c^2 -
4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b^2) - 2*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d
*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(-I*sqrt(
6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b
*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sq
rt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*
a*d)/d)/b^2)/d^2 + 9*(I*sqrt(10)*sqrt(pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2
)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)
```

```
*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10
*I*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*
d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^2)/d^2) + d^3*(27*(I*sqr
t(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(
-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*
b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(
d*x + c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^
2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*
x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b^3)/d^3 + 125*(-I
*sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d - 18*b*c*d^2 - 5*I*d^3)*d*er
f(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I
*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d
*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2
*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x
 + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^3)/d^3 + 6750*(-I
*sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*c*d^2 - 15*I*d^3)*d*er
f(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b
*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x +
c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*
d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c
)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3 + 6750*(I*sqrt(2)*sq
rt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d^3)*d*erf(-1/2*sqrt
(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(4*I*(d*x + c)^(5/2)
*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)*b^2*c^2*d - 10*(
d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e
^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(I*sqrt(6)*sqrt(pi)*(72
*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/
(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 6*I*(12*I*(d*x + c)^(5/2)*b^2*
d - 36*I*(d*x + c)^(3/2)*b^2*c*d + 36*I*sqrt(d*x + c)*b^2*c^2*d - 10*(d*x +
 c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((3*I
*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(10)*sqrt(pi)*(2
00*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 10*I*(-20*I*(d*x + c)^(5/2)
*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(
d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c)*d^3)*e^
((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) + 180*(9*I*sqrt(10)*sqr
t(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
t(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b) - 25*I*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*I*sqrt(2)*sqrt(pi)*(2*b*c + I*d)*
d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
```

```
(I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 450*I*sqrt(2)*
sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2
) + 1)*b) + 25*I*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d
)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(s
qrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(10)*sqrt(pi)*(10*b*c - I*
d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*
sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b + 150*sqrt(d*
x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*sqrt(d*x + c)*
d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b + 900*sqrt(d*x + c)*d*e^((-I*(d*x
 + c)*b + I*b*c - I*a*d)/d)/b + 150*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)*b +
3*I*b*c - 3*I*a*d)/d)/b - 90*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c
 - 5*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 719, normalized size = 1.17

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-\cdots}{\cdots}\right)\cdots\right)}{4b\sqrt{\frac{b}{d}}}\right)}{2b}\cdots\right)}{8b}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+5/16/b*d*(1/2/b
*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/2/b*d*(-1/2/b*d*(d*x+c)^(
1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(c
os((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-si
n((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-
1/96/b*d*(d*x+c)^(5/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(1/6/b*d*(
d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/2/b*d*(-1/6/b*d*(d*x+c)^(1/
2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d
*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)
^(1/2)*(d*x+c)^(1/2)*b/d)))+1/160/b*d*(d*x+c)^(5/2)*cos(5/d*(d*x+c)*b+5*(a
```

```
*d-b*c)/d)-1/32/b*d*(1/10/b*d*(d*x+c)^(3/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)-3/10/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/100/
b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(5*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 1.79, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^2*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b -
b*c + a*d)/d)/d - 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d - 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sqrt(d*x
+ c)*b^3)*cos(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x + c)^(
5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*cos(3*((d*x + c)*b - b*c + a*d)
/d) + 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x + c)*b
^3)*cos(((d*x + c)*b - b*c + a*d)/d) + ((162*I - 162)*25^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(1/4)*sqrt(pi)*
b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d
)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c -
 a*d)/d) - (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b
*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(202500*I - 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt(p
i)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)
) + ((202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)
 + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*
erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) + (1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2
*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d))
 + (-(162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*
d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c -
 a*d)/d))*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\,\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**2*sin(b*x+a)**3,x)
[Out] Timed out
```

3.136 $\int (c+dx)^m \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=273

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\right.}{b}$$

```
[Out] -2^(-4-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b
*(d*x+c)/d)^m)-2^(-4-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b
*c/d))/((I*b*(d*x+c)/d)^m)-exp(4*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-4*I*b*(d
*x+c)/d)/(2^(6+2*m))/b/((-I*b*(d*x+c)/d)^m)-(d*x+c)^m*GAMMA(1+m,4*I*b*(d*x+
c)/d)/(2^(6+2*m))/b/exp(4*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.29, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3308, 2181}

$$-\frac{2^{-m-4}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-2(m+3)}e^{4i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gar}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((2^(-4 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c
 + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m)) - (2^(-4 - m)*(c + d*x)^m*Gamma[
1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/
d)^m) - (E^((4*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-4*I)*b*(c + d*
x))/d])/(2^(2*(3 + m))*b*(((-I)*b*(c + d*x))/d)^m) - ((c + d*x)^m*Gamma[1 +
 m, ((4*I)*b*(c + d*x))/d])/(2^(2*(3 + m))*b*E^((4*I)*(a - (b*c)/d))*((I*b*
(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\cos^3(a+bx)\sin(a+bx)\,dx &= \int\left(\frac{1}{4}(c+dx)^m\sin(2a+2bx)+\frac{1}{8}(c+dx)^m\sin(4a+4bx)\right)dx\\ &= \frac{1}{8}\int(c+dx)^m\sin(4a+4bx)\,dx+\frac{1}{4}\int(c+dx)^m\sin(2a+2bx)\,dx\\ &= \frac{1}{16}i\int e^{-i(4a+4bx)}(c+dx)^m\,dx-\frac{1}{16}i\int e^{i(4a+4bx)}(c+dx)^m\,dx+\frac{1}{8}i\int e^{-i}\\ &= -\frac{2^{-4-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}-\frac{2^{-4-m}e^{-}}{}\end{aligned}$$

Mathematica [A] time = 0.24, size = 245, normalized size = 0.90

$$-\frac{4^{-m-3}e^{-\frac{4i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(2^{m+2}e^{2i\left(a+\frac{3bc}{d}\right)}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right)+2^{m+2}e^{2i\left(3a+\frac{bc}{d}\right)}\left(\frac{ib(c+dx)}{d}\right)^m}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((4^(-3 - m)*(c + d*x)^m*(2^(2 + m)*E^((2*I)*(3*a + (b*c)/d))*((I*b*(c + d
*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d] + 2^(2 + m)*E^((2*I)*(a + (3
*b*c)/d))*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c + d*x))/d] + E^
((8*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-4*I)*b*(c + d*x))/d] + E^((
(8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((4*I)*b*(c + d*x))/d])
)/(b*E^(((4*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m))
```

fricas [A] time = 0.60, size = 184, normalized size = 0.67

$$-\frac{e^{\left(-\frac{dm\log\left(\frac{4ib}{d}\right)-4ibc+4iad}{d}\right)}\Gamma\left(m+1,\frac{4ibdx+4ibc}{d}\right)+4e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right)+4e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/64*(e^(-(d*m*log(4*I*b/d) - 4*I*b*c + 4*I*a*d)/d)*gamma(m + 1, (4*I*b*d*
x + 4*I*b*c)/d) + 4*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) + 4*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d
)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-4*I*b/d) + 4*I*
b*c - 4*I*a*d)/d)*gamma(m + 1, (-4*I*b*d*x - 4*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right) \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.137 $\int (c+dx)^4 \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=260

$$-\frac{3d^4\cos^4(a+bx)}{128b^5}-\frac{45d^4\cos^2(a+bx)}{128b^5}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a+bx)\cos(a+$$

```
[Out] -45/64*c*d^3*x/b^3-45/128*d^4*x^2/b^3+3/32*(d*x+c)^4/b-45/128*d^4*cos(b*x+a
)^2/b^5+9/16*d^2*(d*x+c)^2*cos(b*x+a)^2/b^3-3/128*d^4*cos(b*x+a)^4/b^5+3/16
*d^2*(d*x+c)^2*cos(b*x+a)^4/b^3-1/4*(d*x+c)^4*cos(b*x+a)^4/b-45/64*d^3*(d*x
+c)*cos(b*x+a)*sin(b*x+a)/b^4+3/8*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/3
2*d^3*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^4+1/4*d*(d*x+c)^3*cos(b*x+a)^3*sin(
b*x+a)/b^2
```

Rubi [A] time = 0.23, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4405, 3311, 32, 3310}

$$\frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^3(c+dx)\sin(a+bx)\cos^3(a+bx)}{32b^4}-\frac{45d^3(c+dx)\sin(a}{64}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-45*c*d^3*x)/(64*b^3) - (45*d^4*x^2)/(128*b^3) + (3*(c + d*x)^4)/(32*b) -
(45*d^4*Cos[a + b*x]^2)/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[a + b*x]^2)/(16*
b^3) - (3*d^4*Cos[a + b*x]^4)/(128*b^5) + (3*d^2*(c + d*x)^2*Cos[a + b*x]^4
)/(16*b^3) - ((c + d*x)^4*Cos[a + b*x]^4)/(4*b) - (45*d^3*(c + d*x)*Cos[a +
 b*x]*Sin[a + b*x])/(64*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/
(8*b^2) - (3*d^3*(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x])/(32*b^4) + (d*(c +
d*x)^3*Cos[a + b*x]^3*Sin[a + b*x])/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^4\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d\int (c+dx)^3\cos^4(a+bx)\,dx}{b}$$

$$= \frac{3d^2(c+dx)^2\cos^4(a+bx)}{16b^3}-\frac{(c+dx)^4\cos^4(a+bx)}{4b}+\frac{d(c+dx)^3\cos^3(a}{4}$$

$$= \frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos^4(a+bx)}{128b^5}+\frac{3d^2(c+dx)^2\cos^4(a+}{16b^3}$$

$$= \frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos^2(a+bx)}{16b^3}-\frac{3d^4\cos}{12}$$

$$= -\frac{45cd^3x}{64b^3}-\frac{45d^4x^2}{128b^3}+\frac{3(c+dx)^4}{32b}-\frac{45d^4\cos^2(a+bx)}{128b^5}+\frac{9d^2(c+dx)^2\mathrm{c}}{16b^3}$$

Mathematica [A] time = 1.85, size = 158, normalized size = 0.61

$$-\frac{-8bd(c+dx)\sin(2(a+bx))\left(\cos(2(a+bx))\left(8b^2(c+dx)^2-3d^2\right)+16\left(2b^2(c+dx)^2-3d^2\right)\right)+64\cos(2(a+bx)}{1024b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/1024*(64*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a +
b*x)] + (3*d^4 - 24*b^2*d^2*(c + d*x)^2 + 32*b^4*(c + d*x)^4)*Cos[4*(a + b*
```

```
x)] - 8*b*d*(c + d*x)*(16*(-3*d^2 + 2*b^2*(c + d*x)^2) + (-3*d^2 + 8*b^2*(c
 + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)])/b^5
```

fricas [A] time = 0.70, size = 378, normalized size = 1.45

$$\frac{12\,b^4d^4x^4 + 48\,b^4cd^3x^3 - \left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 32\,b^4c^4 - 24\,b^2c^2d^2 + 3\,d^4 + 24\left(8\,b^4c^2d^2 - b^2d^4\right)x^2 + 16\,(\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(12*b^4*d^4*x^4 + 48*b^4*c*d^3*x^3 - (32*b^4*d^4*x^4 + 128*b^4*c*d^3*
x^3 + 32*b^4*c^4 - 24*b^2*c^2*d^2 + 3*d^4 + 24*(8*b^4*c^2*d^2 - b^2*d^4)*x^
2 + 16*(8*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^4 + 9*(8*b^4*c^2*d^2 - 5
*b^2*d^4)*x^2 + 9*(8*b^2*d^4*x^2 + 16*b^2*c*d^3*x + 8*b^2*c^2*d^2 - 5*d^4)*
cos(b*x + a)^2 + 6*(8*b^4*c^3*d - 15*b^2*c*d^3)*x + 2*(2*(8*b^3*d^4*x^3 + 2
4*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 3*b*c*d^3 + 3*(8*b^3*c^2*d^2 - b*d^4)*x)*co
s(b*x + a)^3 + 3*(8*b^3*d^4*x^3 + 24*b^3*c*d^3*x^2 + 8*b^3*c^3*d - 15*b*c*d
^3 + 3*(8*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(b*x + a))*sin(b*x + a))/b^5
```

giac [A] time = 0.26, size = 361, normalized size = 1.39

$$-\frac{\left(32\,b^4d^4x^4 + 128\,b^4cd^3x^3 + 192\,b^4c^2d^2x^2 + 128\,b^4c^3dx + 32\,b^4c^4 - 24\,b^2d^4x^2 - 48\,b^2cd^3x - 24\,b^2c^2d^2 + 3\,d^4\right)\,c\ldots}{1024\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/1024*(32*b^4*d^4*x^4 + 128*b^4*c*d^3*x^3 + 192*b^4*c^2*d^2*x^2 + 128*b^4
*c^3*d*x + 32*b^4*c^4 - 24*b^2*d^4*x^2 - 48*b^2*c*d^3*x - 24*b^2*c^2*d^2 +
3*d^4)*cos(4*b*x + 4*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^
4*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x
- 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 + 1/256*(8*b^3*d^4*x^3 + 24*b
^3*c*d^3*x^2 + 24*b^3*c^2*d^2*x + 8*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(
4*b*x + 4*a)/b^5 + 1/8*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x +
 2*b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

maple [B] time = 0.09, size = 1150, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^4*d^4*(-1/4*(b*x+a)^4*cos(b*x+a)^4+(b*x+a)^3*(1/4*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/16*(b*x+a)^2*cos(b*x+a)^4-3/8*(b*
x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+45/128*(b
*x+a)^2-3/128*cos(b*x+a)^4-9/128*cos(b*x+a)^2+9/16*(b*x+a)^2*cos(b*x+a)^2-9
/8*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+9/32*sin(b*x+a)^2-9/32
*(b*x+a)^4)-4/b^4*a*d^4*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(co
s(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a
)^4-3/128*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32
*(b*x+a)*cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+4/b^3*c*d^
3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^3+3/2*cos(b*x
+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*cos(b*x+a)^2-
9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)+6/b^4*a^2*d^4*(-1/4*(b*x+a)^2*co
s(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*
x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)-12/b^3*a*c*d^3
*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a)
)*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a
)^2)+6/b^2*c^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a
)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a
)^4+3/32*cos(b*x+a)^2)-4/b^4*a^3*d^4*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b
*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+12/b^3*a^2*c*d^3*(-1/4*
(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x
+3/32*a)-12/b^2*a*c^2*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2
*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+4/b*c^3*d*(-1/4*(b*x+a)*cos(b*x+a)
^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4/b^4*a
^4*d^4*cos(b*x+a)^4+1/b^3*a^3*c*d^3*cos(b*x+a)^4-3/2/b^2*a^2*c^2*d^2*cos(b*
x+a)^4+1/b*a*c^3*d*cos(b*x+a)^4-1/4*c^4*cos(b*x+a)^4)
```

maxima [B] time = 0.39, size = 967, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^4*cos(b*x + a)^4 - 1024*a*c^3*d*cos(b*x + a)^4/b + 1536*a^2*
c^2*d^2*cos(b*x + a)^4/b^2 - 1024*a^3*c*d^3*cos(b*x + a)^4/b^3 + 256*a^4*d^
4*cos(b*x + a)^4/b^4 + 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(
2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c^3*d/b - 96*(4*(b*x
+ a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) -
8*sin(2*b*x + 2*a))*a*c^2*d^2/b^2 + 96*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(
b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^2*c*d^
3/b^3 - 32*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) -
sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a^3*d^4/b^4 + 24*((8*(b*x + a)^2 - 1
)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*
```

```
sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 - 48*((8*(b*x
 + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4
*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 +
24*((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x
 + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a^2
*d^4/b^4 + 4*(4*(8*(b*x + a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x
 + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x +
 4*a) - 96*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 - 4*(4*(8*(b*x +
 a)^3 - 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*a*d^4/b^4 + ((32*(b*x + a)^4 - 24*(b*x + a)^2 + 3)*
cos(4*b*x + 4*a) + 64*(2*(b*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a)
- 4*(8*(b*x + a)^3 - 3*b*x - 3*a)*sin(4*b*x + 4*a) - 128*(2*(b*x + a)^3 - 3
*b*x - 3*a)*sin(2*b*x + 2*a))*d^4/b^4)/b
```

mupad [B] time = 1.34, size = 576, normalized size = 2.22

$$-\frac{192\,d^4\,\cos\left(2\,a+2\,b\,x\right)+3\,d^4\,\cos\left(4\,a+4\,b\,x\right)+128\,b^4\,c^4\,\cos\left(2\,a+2\,b\,x\right)+32\,b^4\,c^4\,\cos\left(4\,a+4\,b\,x\right)-256\,b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^4,x)
```

```
[Out] -(192*d^4*cos(2*a + 2*b*x) + 3*d^4*cos(4*a + 4*b*x) + 128*b^4*c^4*cos(2*a +
 2*b*x) + 32*b^4*c^4*cos(4*a + 4*b*x) - 256*b^3*c^3*d*sin(2*a + 2*b*x) - 32
*b^3*c^3*d*sin(4*a + 4*b*x) - 384*b^2*c^2*d^2*cos(2*a + 2*b*x) - 24*b^2*c^2
*d^2*cos(4*a + 4*b*x) - 384*b^2*d^4*x^2*cos(2*a + 2*b*x) - 24*b^2*d^4*x^2*c
os(4*a + 4*b*x) + 128*b^4*d^4*x^4*cos(2*a + 2*b*x) + 32*b^4*d^4*x^4*cos(4*a
 + 4*b*x) - 256*b^3*d^4*x^3*sin(2*a + 2*b*x) - 32*b^3*d^4*x^3*sin(4*a + 4*b
*x) + 384*b*c*d^3*sin(2*a + 2*b*x) + 12*b*c*d^3*sin(4*a + 4*b*x) + 384*b*d^
4*x*sin(2*a + 2*b*x) + 12*b*d^4*x*sin(4*a + 4*b*x) + 768*b^4*c^2*d^2*x^2*co
s(2*a + 2*b*x) + 192*b^4*c^2*d^2*x^2*cos(4*a + 4*b*x) - 768*b^2*c*d^3*x*cos
(2*a + 2*b*x) + 512*b^4*c^3*d*x*cos(2*a + 2*b*x) - 48*b^2*c*d^3*x*cos(4*a +
 4*b*x) + 128*b^4*c^3*d*x*cos(4*a + 4*b*x) + 512*b^4*c*d^3*x^3*cos(2*a + 2*
b*x) + 128*b^4*c*d^3*x^3*cos(4*a + 4*b*x) - 768*b^3*c^2*d^2*x*sin(2*a + 2*b
*x) - 768*b^3*c*d^3*x^2*sin(2*a + 2*b*x) - 96*b^3*c^2*d^2*x*sin(4*a + 4*b*x
) - 96*b^3*c*d^3*x^2*sin(4*a + 4*b*x))/(1024*b^5)
```

sympy [A] time = 13.23, size = 935, normalized size = 3.60

$$\begin{cases} -\frac{c^4\cos^4(a+bx)}{4b} + \frac{3c^3dx\sin^4(a+bx)}{8b} + \frac{3c^3dx\sin^2(a+bx)\cos^2(a+bx)}{4b} - \frac{5c^3dx\cos^4(a+bx)}{8b} + \frac{9c^2d^2x^2\sin^4(a+bx)}{16b} + \frac{9c^2d^2x^2\sin^2(a+bx)\cos}{8b} \\ \left(c^4x + 2c^3dx^2 + 2c^2d^2x^3 + cd^3x^4 + \frac{d^4x^5}{5}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**4*cos(a + b*x)**4/(4*b) + 3*c**3*d*x*sin(a + b*x)**4/(8*b) +
 3*c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(4*b) - 5*c**3*d*x*cos(a + b*x)
**4/(8*b) + 9*c**2*d**2*x**2*sin(a + b*x)**4/(16*b) + 9*c**2*d**2*x**2*sin(
a + b*x)**2*cos(a + b*x)**2/(8*b) - 15*c**2*d**2*x**2*cos(a + b*x)**4/(16*b
) + 3*c*d**3*x**3*sin(a + b*x)**4/(8*b) + 3*c*d**3*x**3*sin(a + b*x)**2*cos
(a + b*x)**2/(4*b) - 5*c*d**3*x**3*cos(a + b*x)**4/(8*b) + 3*d**4*x**4*sin(
a + b*x)**4/(32*b) + 3*d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5
*d**4*x**4*cos(a + b*x)**4/(32*b) + 3*c**3*d*sin(a + b*x)**3*cos(a + b*x)/(
8*b**2) + 5*c**3*d*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 9*c**2*d**2*x*si
n(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*c**2*d**2*x*sin(a + b*x)*cos(a + b
*x)**3/(8*b**2) + 9*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(8*b**2) + 15*
c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(8*b**2) + 3*d**4*x**3*sin(a + b*x
)**3*cos(a + b*x)/(8*b**2) + 5*d**4*x**3*sin(a + b*x)*cos(a + b*x)**3/(8*b*
*2) - 9*c**2*d**2*sin(a + b*x)**4/(32*b**3) + 15*c**2*d**2*cos(a + b*x)**4/
(32*b**3) - 45*c*d**3*x*sin(a + b*x)**4/(64*b**3) - 9*c*d**3*x*sin(a + b*x)
**2*cos(a + b*x)**2/(32*b**3) + 51*c*d**3*x*cos(a + b*x)**4/(64*b**3) - 45*
d**4*x**2*sin(a + b*x)**4/(128*b**3) - 9*d**4*x**2*sin(a + b*x)**2*cos(a +
b*x)**2/(64*b**3) + 51*d**4*x**2*cos(a + b*x)**4/(128*b**3) - 45*c*d**3*sin
(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*c*d**3*sin(a + b*x)*cos(a + b*x)**
3/(64*b**4) - 45*d**4*x*sin(a + b*x)**3*cos(a + b*x)/(64*b**4) - 51*d**4*x*
sin(a + b*x)*cos(a + b*x)**3/(64*b**4) + 45*d**4*sin(a + b*x)**4/(256*b**5)
 - 51*d**4*cos(a + b*x)**4/(256*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2
+ 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)*cos(a)**3, True))
```

3.138 $\int (c+dx)^3 \cos^3(a+bx) \sin(a+bx)\,dx$

Optimal. Leaf size=196

$$-\frac{3d^3 \sin(a+bx)\cos^3(a+bx)}{128b^4}-\frac{45d^3 \sin(a+bx)\cos(a+bx)}{256b^4}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+b$$

```
[Out] -45/256*d^3*x/b^3+3/32*(d*x+c)^3/b+9/32*d^2*(d*x+c)*cos(b*x+a)^2/b^3+3/32*d
^2*(d*x+c)*cos(b*x+a)^4/b^3-1/4*(d*x+c)^3*cos(b*x+a)^4/b-45/256*d^3*cos(b*x
+a)*sin(b*x+a)/b^4+9/32*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3/128*d^3*cos
(b*x+a)^3*sin(b*x+a)/b^4+3/16*d*(d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Rubi [A] time = 0.16, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4405, 3311, 32, 2635, 8}

$$\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d(c+dx)^2\sin(a+bx)\cos^3(a+bx)}{16b^2}+\frac{9d(c+dx)^2\sin(a+}{32b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-45*d^3*x)/(256*b^3) + (3*(c + d*x)^3)/(32*b) + (9*d^2*(c + d*x)*Cos[a + b
*x]^2)/(32*b^3) + (3*d^2*(c + d*x)*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^3*
Cos[a + b*x]^4)/(4*b) - (45*d^3*Cos[a + b*x]*Sin[a + b*x])/(256*b^4) + (9*d
*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])/(32*b^2) - (3*d^3*Cos[a + b*x]^3*Si
n[a + b*x])/(128*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x])/(16*b
^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^3\cos^4(a+bx)}{4b}+\frac{(3d)\int (c+dx)^2\cos^4(a+bx)\,dx}{4b}$$

$$= \frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\cos^4(a+bx)}{4b}+\frac{3d(c+dx)^2\cos^3(a}{16}$$

$$= \frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^3\cos^4(a}{4b}$$

$$= \frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a+bx)}{32b^3}-\frac{(c}{}$$

$$= -\frac{45d^3x}{256b^3}+\frac{3(c+dx)^3}{32b}+\frac{9d^2(c+dx)\cos^2(a+bx)}{32b^3}+\frac{3d^2(c+dx)\cos^4(a}{32b^3}$$

Mathematica [A] time = 0.92, size = 135, normalized size = 0.69

$$\frac{-64b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)-4b(c+dx)\cos(4(a+bx))\left(8b^2(c+dx)^2-3d^2\right)+6d\sin(2(a+}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-64*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] - 4*b*(c + d
*x)*(-3*d^2 + 8*b^2*(c + d*x)^2)*Cos[4*(a + b*x)] + 6*d*(16*(-d^2 + 2*b^2*(
```

```
c + d*x)^2) + (-d^2 + 8*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])*Sin[2*(a + b*x)]
)/(1024*b^4)
```

fricas [A] time = 0.67, size = 238, normalized size = 1.21

$$\frac{24\,b^3d^3x^3+72\,b^3cd^2x^2-8\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+8\,b^3c^3-3\,bcd^2+3\left(8\,b^3c^2d-bd^3\right)x\right)\cos\left(bx+a\right)^4+72\left(bc\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/256*(24*b^3*d^3*x^3 + 72*b^3*c*d^2*x^2 - 8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*
x^2 + 8*b^3*c^3 - 3*b*c*d^2 + 3*(8*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^4 + 7
2*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 9*(8*b^3*c^2*d - 5*b*d^3)*x + 3*(2*(
8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 + 3*(8*b
^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - 5*d^3)*cos(b*x + a))*sin(b*x +
a))/b^4
```

giac [A] time = 4.51, size = 241, normalized size = 1.23

$$-\frac{\left(8\,b^3d^3x^3+24\,b^3cd^2x^2+24\,b^3c^2dx+8\,b^3c^3-3\,bd^3x-3\,bcd^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^4}-\frac{\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 - 3*b
*d^3*x - 3*b*c*d^2)*cos(4*b*x + 4*a)/b^4 - 1/16*(2*b^3*d^3*x^3 + 6*b^3*c*d^
2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)
/b^4 + 3/1024*(8*b^2*d^3*x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*sin(4*b*
x + 4*a)/b^4 + 3/32*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

maple [B] time = 0.02, size = 594, normalized size = 3.03

$$\frac{\frac{d^3\left(-\frac{(bx+a)^3\left(\cos^4(bx+a)\right)}{4}+\frac{3(bx+a)^2\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{4}+\frac{3(bx+a)\left(\cos^4(bx+a)\right)}{32}-\frac{3\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{128}-\frac{45bx}{256}-\frac{45a}{256}+\frac{9(bx+\cdots}{}\right.}{b^3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^3*d^3*(-1/4*(b*x+a)^3*cos(b*x+a)^4+3/4*(b*x+a)^2*(1/4*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)+3/32*(b*x+a)*cos(b*x+a)^4-3/128
*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)-45/256*b*x-45/256*a+9/32*(b*x+a)*
cos(b*x+a)^2-9/64*cos(b*x+a)*sin(b*x+a)-3/16*(b*x+a)^3)-3/b^3*a*d^3*(-1/4*(
b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*
x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*cos(b*x+a)^2)+3/b
^2*c*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+3/2*co
s(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+3/32*c
os(b*x+a)^2)+3/b^3*a^2*d^3*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/
2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-6/b^2*a*c*d^2*(-1/4*(b*x+a)*cos(b
*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+3/b*
c^2*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x
+a)+3/32*b*x+3/32*a)+1/4/b^3*a^3*d^3*cos(b*x+a)^4-3/4/b^2*a^2*c*d^2*cos(b*x
+a)^4+3/4/b*a*c^2*d*cos(b*x+a)^4-1/4*c^3*cos(b*x+a)^4)
```

maxima [B] time = 0.36, size = 549, normalized size = 2.80

$$-\frac{256\,c^3\cos\left(bx+a\right)^4-\frac{768\,ac^2d\cos(bx+a)^4}{b}+\frac{768\,a^2cd^2\cos(bx+a)^4}{b^2}-\frac{256\,a^3d^3\cos(bx+a)^4}{b^3}+\frac{24\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(256*c^3*cos(b*x + a)^4 - 768*a*c^2*d*cos(b*x + a)^4/b + 768*a^2*c*
d^2*cos(b*x + a)^4/b^2 - 256*a^3*d^3*cos(b*x + a)^4/b^3 + 24*(4*(b*x + a)*c
os(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(
2*b*x + 2*a))*c^2*d/b - 48*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos
(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*a*c*d^2/b^2 + 24*(4*
(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*
a) - 8*sin(2*b*x + 2*a))*a^2*d^3/b^3 + 12*((8*(b*x + a)^2 - 1)*cos(4*b*x +
4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*
a) - 32*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 - 12*((8*(b*x + a)^2 - 1)*cos
(4*b*x + 4*a) + 16*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4
*b*x + 4*a) - 32*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/b^3 + (4*(8*(b*x + a)^3
- 3*b*x - 3*a)*cos(4*b*x + 4*a) + 64*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*
x + 2*a) - 3*(8*(b*x + a)^2 - 1)*sin(4*b*x + 4*a) - 96*(2*(b*x + a)^2 - 1)*
sin(2*b*x + 2*a))*d^3/b^3)/b
```

mupad [B] time = 2.06, size = 366, normalized size = 1.87

$$-\frac{24\,d^3\,\sin\left(2\,a+2\,b\,x\right)+\frac{3\,d^3\,\sin(4\,a+4\,b\,x)}{4}+32\,b^3\,c^3\,\cos\left(2\,a+2\,b\,x\right)+8\,b^3\,c^3\,\cos\left(4\,a+4\,b\,x\right)-48\,b^2\,c^2\,d\,\sin\left(2\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^3,x)
```

```
[Out] -(24*d^3*sin(2*a + 2*b*x) + (3*d^3*sin(4*a + 4*b*x))/4 + 32*b^3*c^3*cos(2*a
 + 2*b*x) + 8*b^3*c^3*cos(4*a + 4*b*x) - 48*b^2*c^2*d*sin(2*a + 2*b*x) - 6*
b^2*c^2*d*sin(4*a + 4*b*x) + 32*b^3*d^3*x^3*cos(2*a + 2*b*x) + 8*b^3*d^3*x^
3*cos(4*a + 4*b*x) - 48*b^2*d^3*x^2*sin(2*a + 2*b*x) - 6*b^2*d^3*x^2*sin(4*
a + 4*b*x) - 48*b*c*d^2*cos(2*a + 2*b*x) - 3*b*c*d^2*cos(4*a + 4*b*x) - 48*
b*d^3*x*cos(2*a + 2*b*x) - 3*b*d^3*x*cos(4*a + 4*b*x) + 96*b^3*c^2*d*x*cos(
2*a + 2*b*x) + 24*b^3*c^2*d*x*cos(4*a + 4*b*x) - 96*b^2*c*d^2*x*sin(2*a + 2
*b*x) - 12*b^2*c*d^2*x*sin(4*a + 4*b*x) + 96*b^3*c*d^2*x^2*cos(2*a + 2*b*x)
 + 24*b^3*c*d^2*x^2*cos(4*a + 4*b*x))/(256*b^4)
```

sympy [A] time = 7.63, size = 602, normalized size = 3.07

$$\begin{cases} -\frac{c^3\cos^4(a+bx)}{4b} + \frac{9c^2dx\sin^4(a+bx)}{32b} + \frac{9c^2dx\sin^2(a+bx)\cos^2(a+bx)}{16b} - \frac{15c^2dx\cos^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^4(a+bx)}{32b} + \frac{9cd^2x^2\sin^2(a+bx)\cos^2}{16b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**3*cos(a + b*x)**4/(4*b) + 9*c**2*d*x*sin(a + b*x)**4/(32*b)
+ 9*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 15*c**2*d*x*cos(a + b
*x)**4/(32*b) + 9*c*d**2*x**2*sin(a + b*x)**4/(32*b) + 9*c*d**2*x**2*sin(a
+ b*x)**2*cos(a + b*x)**2/(16*b) - 15*c*d**2*x**2*cos(a + b*x)**4/(32*b) +
3*d**3*x**3*sin(a + b*x)**4/(32*b) + 3*d**3*x**3*sin(a + b*x)**2*cos(a + b*
x)**2/(16*b) - 5*d**3*x**3*cos(a + b*x)**4/(32*b) + 9*c**2*d*sin(a + b*x)**
3*cos(a + b*x)/(32*b**2) + 15*c**2*d*sin(a + b*x)*cos(a + b*x)**3/(32*b**2)
 + 9*c*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 15*c*d**2*x*sin(a +
b*x)*cos(a + b*x)**3/(16*b**2) + 9*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)/(
32*b**2) + 15*d**3*x**2*sin(a + b*x)*cos(a + b*x)**3/(32*b**2) - 9*c*d**2*s
in(a + b*x)**4/(64*b**3) + 15*c*d**2*cos(a + b*x)**4/(64*b**3) - 45*d**3*x*
sin(a + b*x)**4/(256*b**3) - 9*d**3*x*sin(a + b*x)**2*cos(a + b*x)**2/(128*
b**3) + 51*d**3*x*cos(a + b*x)**4/(256*b**3) - 45*d**3*sin(a + b*x)**3*cos(
a + b*x)/(256*b**4) - 51*d**3*sin(a + b*x)*cos(a + b*x)**3/(256*b**4), Ne(b
, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)*cos(a
)**3, True))
```

3.139 $\int (c+dx)^2 \cos^3(a+bx) \sin(a+bx) \, dx$

Optimal. Leaf size=134

$$\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}-\frac{(c+}{}$$

```
[Out] 3/16*c*d*x/b+3/32*d^2*x^2/b+3/32*d^2*cos(b*x+a)^2/b^3+1/32*d^2*cos(b*x+a)^4
/b^3-1/4*(d*x+c)^2*cos(b*x+a)^4/b+3/16*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+
1/8*d*(d*x+c)*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Rubi [A] time = 0.09, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {4405, 3310}

$$\frac{d(c+dx)\sin(a+bx)\cos^3(a+bx)}{8b^2}+\frac{3d(c+dx)\sin(a+bx)\cos(a+bx)}{16b^2}+\frac{d^2\cos^4(a+bx)}{32b^3}+\frac{3d^2\cos^2(a+bx)}{32b^3}-\frac{(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*c*d*x)/(16*b) + (3*d^2*x^2)/(32*b) + (3*d^2*Cos[a + b*x]^2)/(32*b^3) + (
d^2*Cos[a + b*x]^4)/(32*b^3) - ((c + d*x)^2*Cos[a + b*x]^4)/(4*b) + (3*d*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/(16*b^2) + (d*(c + d*x)*Cos[a + b*x]^3*S
in[a + b*x])/(8*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)^2\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{d\int(c+dx)\cos^4(a+bx)\,dx}{2b}$$
$$=\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{d(c+dx)\cos^3(a+bx)\sin}{8b^2}$$
$$=\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4(a+bx)}{4b}+\frac{3d(c-}{}$$
$$=\frac{3cdx}{16b}+\frac{3d^2x^2}{32b}+\frac{3d^2\cos^2(a+bx)}{32b^3}+\frac{d^2\cos^4(a+bx)}{32b^3}-\frac{(c+dx)^2\cos^4}{4b}$$

Mathematica [A] time = 0.46, size = 89, normalized size = 0.66

$$\frac{-16\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(4(a+bx))\left(d^2-8b^2(c+dx)^2\right)+4bd(c+dx)(8\sin(2(a+bx))+\sin}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (d^2 - 8*b^2*(c + d*x)^2
)*Cos[4*(a + b*x)] + 4*b*d*(c + d*x)*(8*Sin[2*(a + b*x)] + Sin[4*(a + b*x)]
))/(256*b^3)
```

fricas [A] time = 0.64, size = 130, normalized size = 0.97

$$\frac{3\,b^2d^2x^2+6\,b^2cdx-\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(bx+a\right)^4+3\,d^2\cos\left(bx+a\right)^2+2\left(2\left(bd^2x+bcd\right)c\right.}{32\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/32*(3*b^2*d^2*x^2 + 6*b^2*c*d*x - (8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c
^2 - d^2)*cos(b*x + a)^4 + 3*d^2*cos(b*x + a)^2 + 2*(2*(b*d^2*x + b*c*d)*co
s(b*x + a)^3 + 3*(b*d^2*x + b*c*d)*cos(b*x + a))*sin(b*x + a))/b^3
```

giac [A] time = 2.96, size = 145, normalized size = 1.08

$$-\frac{\left(8\,b^2d^2x^2+16\,b^2cdx+8\,b^2c^2-d^2\right)\cos\left(4\,bx+4\,a\right)}{256\,b^3}-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{16\,b^3}+\frac{\left(bd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(8*b^2*d^2*x^2 + 16*b^2*c*d*x + 8*b^2*c^2 - d^2)*cos(4*b*x + 4*a)/b^
3 - 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a)/b
^3 + 1/64*(b*d^2*x + b*c*d)*sin(4*b*x + 4*a)/b^3 + 1/8*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a)/b^3
```

maple [B] time = 0.02, size = 260, normalized size = 1.94

$$\frac{d^2\left(-\frac{(bx+a)^2\left(\cos^4(bx+a)\right)}{4}+\frac{(bx+a)\left(\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{2}-\frac{3(bx+a)^2}{32}+\frac{\left(\cos^4(bx+a)\right)}{32}+\frac{3\left(\cos^2(bx+a)\right)}{32}\right)}{b^2}-\frac{2a\,d^2\left(-\frac{(bx+a)\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^3(bx\right.}{}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b^2*d^2*(-1/4*(b*x+a)^2*cos(b*x+a)^4+1/2*(b*x+a)*(1/4*(cos(b*x+a)^3+
3/2*cos(b*x+a))*sin(b*x+a)+3/8*b*x+3/8*a)-3/32*(b*x+a)^2+1/32*cos(b*x+a)^4+
3/32*cos(b*x+a)^2)-2/b^2*a*d^2*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^
3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)+2/b*c*d*(-1/4*(b*x+a)*cos(b*x
+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*sin(b*x+a)+3/32*b*x+3/32*a)-1/4*d^
2/b^2*a^2*cos(b*x+a)^4+1/2*c*d/b*a*cos(b*x+a)^4-1/4*c^2*cos(b*x+a)^4)
```

maxima [B] time = 0.34, size = 263, normalized size = 1.96

$$-\frac{64\,c^2\cos(bx+a)^4-\frac{128\,acd\cos(bx+a)^4}{b}+\frac{64\,a^2d^2\cos(bx+a)^4}{b^2}+\frac{4\,(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2}{b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/256*(64*c^2*cos(b*x + a)^4 - 128*a*c*d*cos(b*x + a)^4/b + 64*a^2*d^2*cos
(b*x + a)^4/b^2 + 4*(4*(b*x + a)*cos(4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x
+ 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x + 2*a))*c*d/b - 4*(4*(b*x + a)*cos(
4*b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b
*x + 2*a))*a*d^2/b^2 + ((8*(b*x + a)^2 - 1)*cos(4*b*x + 4*a) + 16*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 4*(b*x + a)*sin(4*b*x + 4*a) - 32*(b*x + a)*s
in(2*b*x + 2*a))*d^2/b^2)/b
```

mupad [B] time = 1.63, size = 202, normalized size = 1.51

$$\frac{8\,d^2\,\cos(2\,a+2\,b\,x)+\frac{d^2\,\cos(4\,a+4\,b\,x)}{2}-16\,b^2\,c^2\,\cos(2\,a+2\,b\,x)-4\,b^2\,c^2\,\cos(4\,a+4\,b\,x)+16\,b\,c\,d\,\sin(2\,a+2\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (8*d^2*cos(2*a + 2*b*x) + (d^2*cos(4*a + 4*b*x))/2 - 16*b^2*c^2*cos(2*a + 2
*b*x) - 4*b^2*c^2*cos(4*a + 4*b*x) + 16*b*c*d*sin(2*a + 2*b*x) + 2*b*c*d*si
n(4*a + 4*b*x) - 16*b^2*d^2*x^2*cos(2*a + 2*b*x) - 4*b^2*d^2*x^2*cos(4*a +
4*b*x) + 16*b*d^2*x*sin(2*a + 2*b*x) + 2*b*d^2*x*sin(4*a + 4*b*x) - 32*b^2*
c*d*x*cos(2*a + 2*b*x) - 8*b^2*c*d*x*cos(4*a + 4*b*x))/(128*b^3)
```

sympy [A] time = 3.79, size = 320, normalized size = 2.39

$$\begin{cases} -\frac{c^2\cos^4(a+bx)}{4b} + \frac{3cdx\sin^4(a+bx)}{16b} + \frac{3cdx\sin^2(a+bx)\cos^2(a+bx)}{8b} - \frac{5cdx\cos^4(a+bx)}{16b} + \frac{3d^2x^2\sin^4(a+bx)}{32b} + \frac{3d^2x^2\sin^2(a+bx)\cos^2(a+bx)}{16b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c**2*cos(a + b*x)**4/(4*b) + 3*c*d*x*sin(a + b*x)**4/(16*b) + 3
*c*d*x*sin(a + b*x)**2*cos(a + b*x)**2/(8*b) - 5*c*d*x*cos(a + b*x)**4/(16*
b) + 3*d**2*x**2*sin(a + b*x)**4/(32*b) + 3*d**2*x**2*sin(a + b*x)**2*cos(a
 + b*x)**2/(16*b) - 5*d**2*x**2*cos(a + b*x)**4/(32*b) + 3*c*d*sin(a + b*x)
**3*cos(a + b*x)/(16*b**2) + 5*c*d*sin(a + b*x)*cos(a + b*x)**3/(16*b**2) +
 3*d**2*x*sin(a + b*x)**3*cos(a + b*x)/(16*b**2) + 5*d**2*x*sin(a + b*x)*co
s(a + b*x)**3/(16*b**2) - 3*d**2*sin(a + b*x)**4/(64*b**3) + 5*d**2*cos(a +
 b*x)**4/(64*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)*co
s(a)**3, True))
```

3.140 $\int (c + dx) \cos^3(a + bx) \sin(a + bx) \, dx$

Optimal. Leaf size=72

$$\frac{d \sin(a + bx) \cos^3(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} - \frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{3dx}{32b}$$

```
[Out] 3/32*d*x/b-1/4*(d*x+c)*cos(b*x+a)^4/b+3/32*d*cos(b*x+a)*sin(b*x+a)/b^2+1/16
*d*cos(b*x+a)^3*sin(b*x+a)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4405, 2635, 8}

$$\frac{d \sin(a + bx) \cos^3(a + bx)}{16b^2} + \frac{3d \sin(a + bx) \cos(a + bx)}{32b^2} - \frac{(c + dx) \cos^4(a + bx)}{4b} + \frac{3dx}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (3*d*x)/(32*b) - ((c + d*x)*Cos[a + b*x]^4)/(4*b) + (3*d*Cos[a + b*x]*Sin[a
 + b*x])/(32*b^2) + (d*Cos[a + b*x]^3*Sin[a + b*x])/(16*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 4405

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)], x_Symbol] :> -Simp[((c + d*x)^m*Cos[a + b*x]^(n + 1))/(b*(n + 1)
), x] + Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Cos[a + b*x]^(n + 1),
 x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin(a+bx)\,dx = -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{d\int \cos^4(a+bx)\,dx}{4b}$$
$$= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{d\cos^3(a+bx)\sin(a+bx)}{16b^2}+\frac{(3d)\int \cos^2(a+$$
$$= -\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a+bx)}{16b^2}$$
$$= \frac{3dx}{32b}-\frac{(c+dx)\cos^4(a+bx)}{4b}+\frac{3d\cos(a+bx)\sin(a+bx)}{32b^2}+\frac{d\cos^3(a-}{}$$

Mathematica [A] time = 0.14, size = 75, normalized size = 1.04

$$\frac{d(\sin(2(a+bx))-2bx\cos(2(a+bx)))}{16b^2}+\frac{d(\sin(4(a+bx))-4bx\cos(4(a+bx)))}{128b^2}-\frac{c\cos^4(a+bx)}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -1/4*(c*Cos[a + b*x]^4)/b + (d*(-2*b*x*Cos[2*(a + b*x)] + Sin[2*(a + b*x)])
)/(16*b^2) + (d*(-4*b*x*Cos[4*(a + b*x)] + Sin[4*(a + b*x)]))/(128*b^2)
```

fricas [A] time = 0.62, size = 58, normalized size = 0.81

$$-\frac{8\,(bdx+bc)\cos(bx+a)^4-3\,bdx-\left(2\,d\cos(bx+a)^3+3\,d\cos(bx+a)\right)\sin(bx+a)}{32\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/32*(8*(b*d*x + b*c)*cos(b*x + a)^4 - 3*b*d*x - (2*d*cos(b*x + a)^3 + 3*d
*cos(b*x + a))*sin(b*x + a))/b^2
```

giac [A] time = 0.20, size = 75, normalized size = 1.04

$$-\frac{(bdx+bc)\cos(4\,bx+4\,a)}{32\,b^2}-\frac{(bdx+bc)\cos(2\,bx+2\,a)}{8\,b^2}+\frac{d\sin(4\,bx+4\,a)}{128\,b^2}+\frac{d\sin(2\,bx+2\,a)}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/32*(b*d*x + b*c)*cos(4*b*x + 4*a)/b^2 - 1/8*(b*d*x + b*c)*cos(2*b*x + 2*
a)/b^2 + 1/128*d*sin(4*b*x + 4*a)/b^2 + 1/16*d*sin(2*b*x + 2*a)/b^2
```

maple [A] time = 0.02, size = 85, normalized size = 1.18

$$\frac{\frac{d\left(-\frac{(bx+a)\left(\cos^4(bx+a)\right)}{4}+\frac{\left(\cos^3(bx+a)+\frac{3\cos(bx+a)}{2}\right)\sin(bx+a)}{16}+\frac{3bx}{32}+\frac{3a}{32}\right)}{b}+\frac{da\left(\cos^4(bx+a)\right)}{4b}-\frac{c\left(\cos^4(bx+a)\right)}{4}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(-1/4*(b*x+a)*cos(b*x+a)^4+1/16*(cos(b*x+a)^3+3/2*cos(b*x+a))*si
n(b*x+a)+3/32*b*x+3/32*a)+1/4/b*d*a*cos(b*x+a)^4-1/4*c*cos(b*x+a)^4)
```

maxima [A] time = 0.32, size = 92, normalized size = 1.28

$$-\frac{32\,c\cos\left(bx+a\right)^4-\frac{32\,ad\cos(bx+a)^4}{b}+\frac{(4\,(bx+a)\cos(4\,bx+4\,a)+16\,(bx+a)\cos(2\,bx+2\,a)-\sin(4\,bx+4\,a)-8\,\sin(2\,bx+2\,a))d}{b}}{128\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/128*(32*c*cos(b*x + a)^4 - 32*a*d*cos(b*x + a)^4/b + (4*(b*x + a)*cos(4*
b*x + 4*a) + 16*(b*x + a)*cos(2*b*x + 2*a) - sin(4*b*x + 4*a) - 8*sin(2*b*x
 + 2*a))*d/b)/b
```

mupad [B] time = 0.31, size = 94, normalized size = 1.31

$$\frac{4\,d\,\sin\left(2\,a+2\,b\,x\right)+\frac{d\,\sin(4\,a+4\,b\,x)}{2}+4\,b\,c\,\sin\left(2\,a+2\,b\,x\right)^2+16\,b\,c\,\sin\left(a+b\,x\right)^2+8\,b\,d\,x\,\left(2\,\sin\left(a+b\,x\right)^2-1\right)+}{64\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x),x)
```

```
[Out] (4*d*sin(2*a + 2*b*x) + (d*sin(4*a + 4*b*x))/2 + 4*b*c*sin(2*a + 2*b*x)^2 +
 16*b*c*sin(a + b*x)^2 + 8*b*d*x*(2*sin(a + b*x)^2 - 1) + 2*b*d*x*(2*sin(2*
a + 2*b*x)^2 - 1))/(64*b^2)
```

sympy [A] time = 1.94, size = 138, normalized size = 1.92

$$\begin{cases} -\frac{c\cos^4\left(a+bx\right)}{4b}+\frac{3dx\sin^4\left(a+bx\right)}{32b}+\frac{3dx\sin^2\left(a+bx\right)\cos^2\left(a+bx\right)}{16b}-\frac{5dx\cos^4\left(a+bx\right)}{32b}+\frac{3d\sin^3\left(a+bx\right)\cos\left(a+bx\right)}{32b^2}+\frac{5d\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{32b^2} \\ \left(cx+\frac{dx^2}{2}\right)\sin\left(a\right)\cos^3\left(a\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Piecewise((-c*cos(a + b*x)**4/(4*b) + 3*d*x*sin(a + b*x)**4/(32*b) + 3*d*x*
sin(a + b*x)**2*cos(a + b*x)**2/(16*b) - 5*d*x*cos(a + b*x)**4/(32*b) + 3*d
*sin(a + b*x)**3*cos(a + b*x)/(32*b**2) + 5*d*sin(a + b*x)*cos(a + b*x)**3/
(32*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*sin(a)*cos(a)**3, True))
```

3.141 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\text{Si}\left(\frac{4b}{d}\right.}{8d}$$

```
[Out] 1/4*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+1/8*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*
b*x)/d+1/8*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d+1/4*Ci(2*b*c/d+2*b*x)*sin(2
*a-2*b*c/d)/d
```

Rubi [A] time = 0.21, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{8d}+\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{4d}+\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/(8*d) + (CosIntegral[
(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(4*d) + (Cos[2*a - (2*b*c)/d]*SinI
ntegral[(2*b*c)/d + 2*b*x])/(4*d) + (Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*
c)/d + 4*b*x])/(8*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{c+dx}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)} + \frac{\sin(4a+4bx)}{8(c+dx)}\right) dx$$
$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{c+dx}\,dx + \frac{1}{4}\int \frac{\sin(2a+2bx)}{c+dx}\,dx$$
$$= \frac{1}{8}\cos\left(4a-\frac{4bc}{d}\right)\int \frac{\sin\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx + \frac{1}{4}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right.}{c+dx}$$
$$= \frac{\mathrm{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{8d} + \frac{\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{4d} + \frac{\cos\left(2a-\frac{2bc}{d}\right.}{4}$$

Mathematica [A] time = 0.34, size = 110, normalized size = 0.85

$$\frac{\sin\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(c+dx)}{d}\right) + 2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right) + 2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(4a-\frac{4bc}{d}\right)\mathrm{Si}\left(\frac{4b}{}\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x),x]
```

```
[Out] (CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 2*CosIntegral[(2*b*(
c + d*x))/d]*Sin[2*a - (2*b*c)/d] + 2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b
*(c + d*x))/d] + Cos[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d])/(8*d)
```

fricas [A] time = 0.42, size = 155, normalized size = 1.20

$$\frac{2\left(\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right) + \mathrm{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right) + \left(\mathrm{Ci}\left(\frac{4\,(bdx+bc)}{d}\right) + \mathrm{Ci}\left(-\frac{4\,(bdx+bc)}{d}\right)\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right) + 2\,\cos\left(\right.}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/16*(2*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) + (cos_integral(4*(b*d*x + b*c)/d) + cos_integral(-
4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d) + 2*cos(-4*(b*c - a*d)/d)*sin_int
egral(4*(b*d*x + b*c)/d) + 4*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

giac [C] time = 1.92, size = 6046, normalized size = 46.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/16*(imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2
*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_in
tegral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*ta
n(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integra
l(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_
part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)*tan(b
*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(
2*b*c/d)^2*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2
*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x +
 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^
2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*
c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^
2*tan(2*b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*ta
n(2*b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(2*
b*c/d)^2 + 8*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan
(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*
a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*
tan(2*a)^2*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_integra
```

```
l(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b
*d*x + b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x +
 b*c)/d)*tan(2*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(4*b*x
+ 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 4*imag_part(cos_i
ntegral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 8*
sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2
 + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2*tan(b
*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d
)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(
2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d
))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*sin_integr
al(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(cos_
integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 2*real_part(c
os_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2*tan(2*b*c/d) + 4*real_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 + 4*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(2*b*c/d)^2 - 2*
real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)^2 -
 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*tan(2*b*c/d)
^2 + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b*c
/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2*tan(b
*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^
2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2
*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*
tan(2*b*c/d)^2*tan(b*c/d) + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(a)^2*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(2*a)^2*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c
/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*b*x - 4*b
*c/d))*tan(2*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_inte
gral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d)*tan(b*c/d)^2 - 2*real_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2*tan(b*c/d
)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(
b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^
2*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(a)
```

```
^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)^2 + 2*ima
g_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)^2 + imag_part(cos_
integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(a)^2 - 2*sin_integral(4*(b*d*x +
 b*c)/d)*tan(2*a)^2*tan(a)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2
*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(a)^2*ta
n(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^2*
tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(a)^2*tan(2*b*
c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 +
 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - 2*i
mag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 - imag_p
art(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d)^2 + 2*sin_integ
ral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 + 4*sin_integral(2*(b*d*x
+ b*c)/d)*tan(2*a)^2*tan(2*b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/
d))*tan(a)^2*tan(2*b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*ta
n(a)^2*tan(2*b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^
2*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2
*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 - 4*s
in_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(2*b*c/d)^2 + 8*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) - 8*imag_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(2*a)^2*tan(a)*tan(b*c/d) + 16*sin_integral(2*(
b*d*x + b*c)/d)*tan(2*a)^2*tan(a)*tan(b*c/d) + 8*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - 8*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) + 16*sin_integral(2*(b*
d*x + b*c)/d)*tan(a)*tan(2*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(4*b
*x + 4*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(2*a)^2*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(2*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan
(2*a)^2*tan(b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c
/d)^2 - 4*sin_integral(2*(b*d*x + b*c)/d)*tan(2*a)^2*tan(b*c/d)^2 + imag_pa
rt(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 2*imag_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x -
4*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^
2*tan(b*c/d)^2 + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/d)^2 +
4*imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(b*c/d)
^2 - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)*tan(
b*c/d)^2 + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b*c/d)*tan(b*c/
d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2
 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 +
imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d)^2 - 2*s
in_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 - 4*sin_integral
(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2*tan(b*c/d)^2 + 4*real_part(cos_integral(
2*b*x + 2*b*c/d))*tan(2*a)^2*tan(a) + 4*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(2*a)^2*tan(a) + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
```

```
*a)*tan(a)^2 + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)*tan(a)^
2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) + 2*
real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a)^2*tan(2*b*c/d) - 2*real_
part(cos_integral(4*b*x + 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos
_integral(-4*b*x - 4*b*c/d))*tan(a)^2*tan(2*b*c/d) - 2*real_part(cos_integr
al(4*b*x + 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 - 2*real_part(cos_integral(-4*
b*x - 4*b*c/d))*tan(2*a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(a)*tan(2*b*c/d)^2 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(a)*tan(2*b*c/d)^2 - 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(2
*a)^2*tan(b*c/d) - 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(2*a)^2*t
an(b*c/d) + 4*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)
 + 4*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) - 4*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) - 4*real_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2*tan(b*c/d) + 2*real_part(co
s_integral(4*b*x + 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 + 2*real_part(cos_integr
al(-4*b*x - 4*b*c/d))*tan(2*a)*tan(b*c/d)^2 - 4*real_part(cos_integral(2*b*
x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 4*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2
*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*
c/d)*tan(b*c/d)^2 - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*a)^2 + 2
*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(2*a)^2 - 2*imag_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(2*a)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/
d))*tan(2*a)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)^2 + 4*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(2*a)^2 + imag_part(cos_integral(4*b*x + 4*b*c/d)
)*tan(a)^2 - 2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 + 2*imag_p
art(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-4*b*
x - 4*b*c/d))*tan(a)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(a)^2 - 4*sin
_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(4*b*x + 4*
b*c/d))*tan(2*a)*tan(2*b*c/d) - 4*imag_part(cos_integral(-4*b*x - 4*b*c/d))
*tan(2*a)*tan(2*b*c/d) + 8*sin_integral(4*(b*d*x + b*c)/d)*tan(2*a)*tan(2*b
*c/d) - imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/d)^2 + 2*imag_pa
rt(cos_integral(2*b*x + 2*b*c/d))*tan(2*b*c/d)^2 - 2*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(2*b*c/d)^2 + imag_part(cos_integral(-4*b*x - 4*b*c/
d))*tan(2*b*c/d)^2 - 2*sin_integral(4*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 + 4*s
in_integral(2*(b*d*x + b*c)/d)*tan(2*b*c/d)^2 + 8*imag_part(cos_integral(2*
b*x + 2*b*c/d))*tan(a)*tan(b*c/d) - 8*imag_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(a)*tan(b*c/d) + 16*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/
d) + imag_part(cos_integral(4*b*x + 4*b*c/d))*tan(b*c/d)^2 - 2*imag_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 2*imag_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-4*b*x - 4*b*c/d))*tan(b
*c/d)^2 + 2*sin_integral(4*(b*d*x + b*c)/d)*tan(b*c/d)^2 - 4*sin_integral(2
*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(4*b*x + 4*b*c/d))
*tan(2*a) + 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*a) + 4*real_p
art(cos_integral(2*b*x + 2*b*c/d))*tan(a) + 4*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(4*b*x + 4*b*c/d))*tan(2*b*c/
```

```
d) - 2*real_part(cos_integral(-4*b*x - 4*b*c/d))*tan(2*b*c/d) - 4*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) - 4*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(4*b*x + 4*b*c/d)) + 2*ima
g_part(cos_integral(2*b*x + 2*b*c/d)) - 2*imag_part(cos_integral(-2*b*x - 2
*b*c/d)) - imag_part(cos_integral(-4*b*x - 4*b*c/d)) + 2*sin_integral(4*(b*
d*x + b*c)/d) + 4*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(2*a)^2*tan(a)^2*t
an(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2
*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*
tan(a)^2*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2*tan(a)^2 + d*tan(2*a)^2
*tan(2*b*c/d)^2 + d*tan(a)^2*tan(2*b*c/d)^2 + d*tan(2*a)^2*tan(b*c/d)^2 + d
*tan(a)^2*tan(b*c/d)^2 + d*tan(2*b*c/d)^2*tan(b*c/d)^2 + d*tan(2*a)^2 + d*t
an(a)^2 + d*tan(2*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

maple [A] time = 0.02, size = 178, normalized size = 1.38

$$\frac{\frac{b\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{32}+\frac{b\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da\cdots}{d}\right)\cdots}{d}\right)}{8}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] 1/b*(1/32*b*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d-4*Ci(4*b*
x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)+1/8*b*(2*Si(2*b*x+2*a+2*(-a*d+
b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*
c)/d)/d))
```

maxima [C] time = 0.49, size = 274, normalized size = 2.12

$$-\frac{b\left(2i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_1\left(-\cdots\right)\cdots\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/16*(b*(2*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) -
2*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/
d) - I*exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos(-4*
(b*c - a*d)/d) + 2*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
*(b*c - a*d)/d) + b*(exp_integral_e(1, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d
```

```
)/d) + exp_integral_e(1, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*sin(-4*
(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x), x)
```

3.142 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^2}-\frac{b\sin\left(4a-\frac{4bc}{d}\right)}{2d}$$

```
[Out] 1/2*b*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^2+1/2*b*Ci(2*b*c/d+2*b*x)*cos(2*
a-2*b*c/d)/d^2-1/2*b*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^2-1/2*b*Si(2*b*c/
d+2*b*x)*sin(2*a-2*b*c/d)/d^2-1/4*sin(2*b*x+2*a)/d/(d*x+c)-1/8*sin(4*b*x+4*
a)/d/(d*x+c)
```

Rubi [A] time = 0.27, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2\right.}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] (b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) + (b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(2*d^2) - Sin[2*a + 2*b*x]/(
4*d*(c + d*x)) - Sin[4*a + 4*b*x]/(8*d*(c + d*x)) - (b*Sin[2*a - (2*b*c)/d]
*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^2) - (b*Sin[4*a - (4*b*c)/d]*SinInteg
ral[(4*b*c)/d + 4*b*x])/(2*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^2}+\frac{\sin(4a+4bx)}{8(c+dx)^2}\right)dx \\
&= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^2}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{2d}+\frac{b\int \frac{\cos(4a+4bx)}{c+dx}\,dx}{2d} \\
&= -\frac{\sin(2a+2bx)}{4d(c+dx)}-\frac{\sin(4a+4bx)}{8d(c+dx)}+\frac{\left(b\cos\left(4a-\frac{4bc}{d}\right)\right)\int \frac{\cos\left(\frac{4bc}{d}+4bx\right)}{c+dx}\,dx}{2d}+\frac{\left(b\cos \right.}{} \\
&= \frac{b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^2}+\frac{b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)}{2d^2}-\frac{\sin(2a+2}{4d(c+d}
\end{aligned}$$

Mathematica [A] time = 1.63, size = 151, normalized size = 0.84

$$-\frac{-4b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right)-4b\cos\left(4a-\frac{4bc}{d}\right)\operatorname{Ci}\left(\frac{4b(c+dx)}{d}\right)+4b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right)+4b\sin\left(4a-\right.}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -1/8*(-4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 4*b*Cos[4*
a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d] + (2*d*Sin[2*(a + b*x)])/(c +
 d*x) + (d*Sin[4*(a + b*x)])/(c + d*x) + 4*b*Sin[2*a - (2*b*c)/d]*SinIntegr
al[(2*b*(c + d*x))/d] + 4*b*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x)
)/d])/d^2
```

fricas [A] time = 0.74, size = 235, normalized size = 1.31

$$-\frac{4\,d\cos\left(bx+a\right)^{3}\sin\left(bx+a\right)+2\left(bdx+bc\right)\sin\left(-\frac{4\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{4\left(bdx+bc\right)}{d}\right)+2\left(bdx+bc\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)\mathrm{Si}\left(\frac{2\left(bdx+\right.}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*d*cos(b*x + a)^3*sin(b*x + a) + 2*(b*d*x + b*c)*sin(-4*(b*c - a*d)/
d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*
sin_integral(2*(b*d*x + b*c)/d) - ((b*d*x + b*c)*cos_integral(2*(b*d*x + b*
c)/d) + (b*d*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/
d) - ((b*d*x + b*c)*cos_integral(4*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_int
egral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 256, normalized size = 1.43

$$\frac{\frac{b^{2}\left(-\frac{4\sin\left(4bx+4a\right)}{\left(\left(bx+a\right)d-da+cb\right)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{32}+\frac{b^{2}\left(-\frac{2\sin\left(2bx+2a\right)}{\left(\left(bx+a\right)d-da+cb\right)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\right.}{d}}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/32*b^2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+4*(4*Si(4*b*x+4*a+4*
(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-
a*d+b*c)/d)/d)/d)+1/8*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(
2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)
/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

maxima [C] time = 0.56, size = 301, normalized size = 1.68

$$-\frac{b^2\left(2i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/16*(b^2*(2*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(2, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(a+b\,x\right)}^3\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**2, x)
```

3.143 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=231

$$-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(4a-\right.}{}$$

```
[Out] -1/4*b*cos(2*b*x+2*a)/d^2/(d*x+c)-1/4*b*cos(4*b*x+4*a)/d^2/(d*x+c)-1/2*b^2*
cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3-b^2*cos(4*a-4*b*c/d)*Si(4*b*c/d+4*b*
x)/d^3-b^2*Ci(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^3-1/2*b^2*Ci(2*b*c/d+2*b*x)
*sin(2*a-2*b*c/d)/d^3-1/8*sin(2*b*x+2*a)/d/(d*x+c)^2-1/16*sin(4*b*x+4*a)/d/
(d*x+c)^2
```

Rubi [A] time = 0.33, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\sin\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{d^3}-\frac{b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{2d^3}-\frac{b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(4*d^2*(c + d*x)) - (b*Cos[4*a + 4*b*x])/(4*d^2*(c +
d*x)) - (b^2*CosIntegral[(4*b*c)/d + 4*b*x]*Sin[4*a - (4*b*c)/d])/d^3 - (b^
2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d^3) - Sin[2*a +
2*b*x]/(8*d*(c + d*x)^2) - Sin[4*a + 4*b*x]/(16*d*(c + d*x)^2) - (b^2*Cos[2
*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d^3) - (b^2*Cos[4*a - (4
*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/d^3
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^3}+\frac{\sin(4a+4bx)}{8(c+dx)^3}\right)dx$$

$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^3}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{4d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^2}\,dx}{4d}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{b^2\int}{}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{\sin(2a+2bx)}{8d(c+dx)^2}-\frac{\sin(4a+4bx)}{16d(c+dx)^2}-\frac{\left(b^2\,\mathrm{c}\right.}{}$$

$$= -\frac{b\cos(2a+2bx)}{4d^2(c+dx)}-\frac{b\cos(4a+4bx)}{4d^2(c+dx)}-\frac{b^2\operatorname{Ci}\left(\frac{4bc}{d}+4bx\right)\sin\left(4a-\frac{4bc}{d}\right)}{d^3}-\frac{b^2\mathrm{Ci}}{}$$

Mathematica [A] time = 3.75, size = 197, normalized size = 0.85

$$-\frac{16b^2\sin\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4b(c+dx)}{d}\right)+8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)+8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+16b^2\cos\left(4\right.}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^3,x]
```

```
[Out] -1/16*(16*b^2*CosIntegral[(4*b*(c + d*x))/d]*Sin[4*a - (4*b*c)/d] + 8*b^2*C
osIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (2*d*(2*b*(c + d*x)*Co
s[2*(a + b*x)] + d*Sin[2*(a + b*x)]))/(c + d*x)^2 + (d*(4*b*(c + d*x)*Cos[4
*(a + b*x)] + d*Sin[4*(a + b*x)]))/(c + d*x)^2 + 8*b^2*Cos[2*a - (2*b*c)/d]
*SinIntegral[(2*b*(c + d*x))/d] + 16*b^2*Cos[4*a - (4*b*c)/d]*SinIntegral[(
4*b*(c + d*x))/d])/d^3
```

fricas [A] time = 0.75, size = 397, normalized size = 1.72

$$-\frac{2\,d^2\cos\left(bx+a\right)^3\sin\left(bx+a\right)+8\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4-6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+4\left(b^2d^2x^2+2\,b^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cos(b*x + a)^3*sin(b*x + a) + 8*(b*d^2*x + b*c*d)*cos(b*x + a)^
4 - 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos(-4*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*
c)/d) + ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)
/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d
))*sin(-2*(b*c - a*d)/d) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_int
egral(4*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integr
al(-4*(b*d*x + b*c)/d))*sin(-4*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 329, normalized size = 1.42

$$\frac{\frac{b^3\left(-\frac{2\sin(4bx+4a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{8\cos(4bx+4a)}{((bx+a)d-da+cb)d}-\frac{8\left(\frac{4\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}-\frac{4\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}\right)}{d}}{d}\right)}{32}+\frac{b^3\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2 d}+\cdots\right)}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/32*b^3*(-2*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^2/d+2*(-4*cos(4*b*x+4*
a)/((b*x+a)*d-d*a+c*b)/d-4*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*cos(4*(-a*d+b*c)
/d)/d-4*Ci(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d)/d)/d)+1/8*b^3*(
-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c
*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*
a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

maxima [C] time = 0.69, size = 336, normalized size = 1.45

$$-\frac{b^3\left(2i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(b^3*(2*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(3, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(a+bx\right)\cos^{3}\left(a+bx\right)}{\left(c+dx\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**3, x)
```

3.144 $\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{Ci}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{4b^3\sin\left(\right.}{}$$

```
[Out] -4/3*b^3*Ci(4*b*c/d+4*b*x)*cos(4*a-4*b*c/d)/d^4-1/3*b^3*Ci(2*b*c/d+2*b*x)*c
os(2*a-2*b*c/d)/d^4-1/12*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2-1/12*b*cos(4*b*x+4*
a)/d^2/(d*x+c)^2+4/3*b^3*Si(4*b*c/d+4*b*x)*sin(4*a-4*b*c/d)/d^4+1/3*b^3*Si(
2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/12*sin(2*b*x+2*a)/d/(d*x+c)^3+1/6*b^2
*sin(2*b*x+2*a)/d^3/(d*x+c)-1/24*sin(4*b*x+4*a)/d/(d*x+c)^3+1/3*b^2*sin(4*b
*x+4*a)/d^3/(d*x+c)
```

Rubi [A] time = 0.45, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\cos\left(4a-\frac{4bc}{d}\right)\text{CosIntegral}\left(\frac{4bc}{d}+4bx\right)}{3d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\right.}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(12*d^2*(c + d*x)^2) - (b*Cos[4*a + 4*b*x])/(12*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) - (4*b^3*Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
- Sin[2*a + 2*b*x]/(12*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(6*d^3*(c +
d*x)) - Sin[4*a + 4*b*x]/(24*d*(c + d*x)^3) + (b^2*Sin[4*a + 4*b*x])/(3*d^3
*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*
d^4) + (4*b^3*Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*c)/d + 4*b*x])/(3*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\sin(2a+2bx)}{4(c+dx)^4}+\frac{\sin(4a+4bx)}{8(c+dx)^4}\right)dx$$

$$= \frac{1}{8}\int \frac{\sin(4a+4bx)}{(c+dx)^4}\,dx+\frac{1}{4}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}+\frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{6d}+\frac{b\int \frac{\cos(4a+4bx)}{(c+dx)^3}\,dx}{6d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}-\frac{\sin(4a+4bx)}{24d(c+dx)^3}-\frac{b^2\int \frac{\mathrm{si}}{}}{}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\frac{\mathrm{sin}}{24d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{\sin(2a+2bx)}{12d(c+dx)^3}+\frac{b^2\sin(2a+2bx)}{6d^3(c+dx)}-\frac{\mathrm{sin}}{24d}$$

$$= -\frac{b\cos(2a+2bx)}{12d^2(c+dx)^2}-\frac{b\cos(4a+4bx)}{12d^2(c+dx)^2}-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{4b^3\,\mathrm{c}}{}$$

Mathematica [A] time = 2.49, size = 316, normalized size = 1.10

$$-\frac{8b^3(c+dx)^3\left(\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right)-\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)\right)+32b^3(c+dx)^3\left(\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Ci}\left(\frac{4b(\cdots}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x])/(c + d*x)^4,x]
```

```
[Out] -1/24*(2*d*Cos[2*b*x]*(b*d*(c + d*x)*Cos[2*a] + (d^2 - 2*b^2*(c + d*x)^2)*S
in[2*a]) + d*Cos[4*b*x]*(2*b*d*(c + d*x)*Cos[4*a] + (d^2 - 8*b^2*(c + d*x)^
2)*Sin[4*a]) - 2*d*((-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*a] + b*d*(c + d*x)*Sin
[2*a])*Sin[2*b*x] - d*((-d^2 + 8*b^2*(c + d*x)^2)*Cos[4*a] + 2*b*d*(c + d*x
)*Sin[4*a])*Sin[4*b*x] + 8*b^3*(c + d*x)^3*(Cos[2*a - (2*b*c)/d]*CosIntegra
l[(2*b*(c + d*x))/d] - Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])
 + 32*b^3*(c + d*x)^3*(Cos[4*a - (4*b*c)/d]*CosIntegral[(4*b*(c + d*x))/d]
- Sin[4*a - (4*b*c)/d]*SinIntegral[(4*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

fricas [B] time = 0.70, size = 568, normalized size = 1.98

$$-\frac{4\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^4-3\left(bd^3x+bcd^2\right)\cos\left(bx+a\right)^2-8\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3\right)\sin\left(-\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(4*(b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)^2 - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-4
*(b*c - a*d)/d)*sin_integral(4*(b*d*x + b*c)/d) - 2*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*
d*x + b*c)/d) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^
2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) +
4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(4
*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*cos_integral(-4*(b*d*x + b*c)/d))*cos(-4*(b*c - a*d)/d) - 2*((8*b^2*d^3*
x^2 + 16*b^2*c*d^2*x + 8*b^2*c^2*d - d^3)*cos(b*x + a)^3 - 3*(b^2*d^3*x^2 +
 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*
x^2 + 3*c^2*d^5*x + c^3*d^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 404, normalized size = 1.41

$$\frac{\frac{b^4\left(-\frac{4\sin(4bx+4a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{8\cos(4bx+4a)}{3((bx+a)d-da+cb)^2 d}-\frac{8\left(-\frac{4\sin(4bx+4a)}{((bx+a)d-da+cb)d}+\frac{\frac{16\,\mathrm{Si}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\sin\left(\frac{-4da+4cb}{d}\right)}{d}+\frac{16\,\mathrm{Ci}\left(4bx+4a+\frac{-4da+4cb}{d}\right)\cos\left(\frac{-4da+4cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{32}+\frac{b^4 \cdots}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/32*b^4*(-4/3*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)^3/d+4/3*(-2*cos(4*b*
x+4*a)/((b*x+a)*d-d*a+c*b)^2/d-2*(-4*sin(4*b*x+4*a)/((b*x+a)*d-d*a+c*b)/d+4
*(4*Si(4*b*x+4*a+4*(-a*d+b*c)/d)*sin(4*(-a*d+b*c)/d)/d+4*Ci(4*b*x+4*a+4*(-a
*d+b*c)/d)*cos(4*(-a*d+b*c)/d)/d)/d)/d)/d)+1/8*b^4*(-2/3*sin(2*b*x+2*a)/((b
*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2
*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a
*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)
)
```

maxima [C] time = 1.90, size = 386, normalized size = 1.34

$$-\frac{b^4\left(2i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)-2i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(i\,E_4\left(\frac{4i\,bc+4i\,(bx+a)d-4i\,ad}{d}\right)-i\,E_4\left(\cdots\right.\right.}{16\left(b^3c^3d-3\,ab^2c^2d^2+3\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/16*(b^4*(2*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
- 2*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d - 4*I*a
*d)/d) - I*exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))*cos
(-4*(b*c - a*d)/d) + 2*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
```

```
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (4*I*b*c + 4*I*(b*x + a)*d -
 4*I*a*d)/d) + exp_integral_e(4, -(4*I*b*c + 4*I*(b*x + a)*d - 4*I*a*d)/d))
*sin(-4*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)}{{\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4,x)
[Out] int((cos(a + b*x)^3*sin(a + b*x))/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin{\left(a + bx \right)} \cos^{3}{\left(a + bx \right)}}{\left(c + dx\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)/(d*x+c)**4,x)
[Out] Integral(sin(a + b*x)*cos(a + b*x)**3/(c + d*x)**4, x)
```

3.145 $\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=419

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right)}{32b}$$

```
[Out] -1/16*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+
c)/d)^m)+1/16*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b
*(d*x+c)/d)^m)+1/32*I*3^(-1-m)*exp(3*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-3*I*
b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/32*I*3^(-1-m)*(d*x+c)^m*GAMMA(1+m,3*I
*b*(d*x+c)/d)/b/exp(3*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)+1/32*I*5^(-1-m)*exp(
5*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-5*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)
-1/32*I*5^(-1-m)*(d*x+c)^m*GAMMA(1+m,5*I*b*(d*x+c)/d)/b/exp(5*I*(a-b*c/d))/
((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.43, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3307, 2181}

$$-\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{i3^{-m-1}e^{3i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-I/16)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) + ((I/16)*(c + d*x)^m*Gamma[1 + m, (I*b*(c +
 d*x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + ((I/32)*3^(-1 -
m)*E^((3*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-3*I)*b*(c + d*x))/d]
)/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*3^(-1 - m)*(c + d*x)^m*Gamma[1 + m
, ((3*I)*b*(c + d*x))/d])/(b*E^((3*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m)
 + ((I/32)*5^(-1 - m)*E^((5*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-5
*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - ((I/32)*5^(-1 - m)*(c +
 d*x)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d])/(b*E^((5*I)*(a - (b*c)/d))*((I
*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&  !I
```

```
ntegerQ[m]
```

Rule 3307

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^m\cos(a+bx)-\frac{1}{16}(c+dx)^m\cos(3a+3bx)-\frac{1}{16}(c+d$$

$$= -\left(\frac{1}{16}\int (c+dx)^m\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^m\cos(5a+5bx$$

$$= -\left(\frac{1}{32}\int e^{-i(3a+3bx)}(c+dx)^m\,dx\right)-\frac{1}{32}\int e^{i(3a+3bx)}(c+dx)^m\,dx-\frac{1}{32}$$

$$= -\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{16b}+\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+$$

Mathematica [A] time = 0.57, size = 409, normalized size = 0.98

$$-\frac{i3^{-m-1}e^{-\frac{3i(ad+bc)}{d}}(c+dx)^m\left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{\frac{6ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,\frac{3ib(c+dx)}{d}\right)-e^{6ia}\left(\frac{ib(c+dx)}{d}\right)^m\Gamma\left(m+1,-\frac{3ib(c+dx)}{d}\right.\right.}{32b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*(c + d*x)^m*((E^((2*I)*a)*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/((
(-I)*b*(c + d*x))/d)^m - (E^(((2*I)*b*c)/d)*Gamma[1 + m, (I*b*(c + d*x))/d]
)/((I*b*(c + d*x))/d)^m))/(b*E^((I*(b*c + a*d))/d)) - ((I/32)*3^(-1 - m)*(c
```

```
 + d*x)^m*(-(E^((6*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-3*I)*b*(c +
d*x))/d]) + E^(((6*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((3*I)*
b*(c + d*x))/d]))/(b*E^(((3*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m) -
 ((I/32)*5^(-1 - m)*(c + d*x)^m*(-(E^((10*I)*a)*((I*b*(c + d*x))/d)^m*Gamma
[1 + m, ((-5*I)*b*(c + d*x))/d]) + E^(((10*I)*b*c)/d)*(((-I)*b*(c + d*x))/d
)^m*Gamma[1 + m, ((5*I)*b*(c + d*x))/d]))/(b*E^(((5*I)*(b*c + a*d))/d)*((b^
2*(c + d*x)^2)/d^2)^m)
```

fricas [A] time = 0.87, size = 276, normalized size = 0.66

$$-3i\,e^{\left(-\frac{dm\log\left(\frac{5ib}{d}\right)-5ibc+5iad}{d}\right)}\Gamma\left(m+1,\frac{5ibdx+5ibc}{d}\right)-5i\,e^{\left(-\frac{dm\log\left(\frac{3ib}{d}\right)-3ibc+3iad}{d}\right)}\Gamma\left(m+1,\frac{3ibdx+3ibc}{d}\right)+30i\,e^{\left(-\frac{dm\log\left(\frac{ib}{d}\right)-ibc+iad}{d}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(-3*I*e^(-(d*m*log(5*I*b/d) - 5*I*b*c + 5*I*a*d)/d)*gamma(m + 1, (5*I
*b*d*x + 5*I*b*c)/d) - 5*I*e^(-(d*m*log(3*I*b/d) - 3*I*b*c + 3*I*a*d)/d)*ga
mma(m + 1, (3*I*b*d*x + 3*I*b*c)/d) + 30*I*e^(-(d*m*log(I*b/d) - I*b*c + I*
a*d)/d)*gamma(m + 1, (I*b*d*x + I*b*c)/d) - 30*I*e^(-(d*m*log(-I*b/d) + I*b
*c - I*a*d)/d)*gamma(m + 1, (-I*b*d*x - I*b*c)/d) + 5*I*e^(-(d*m*log(-3*I*b
/d) + 3*I*b*c - 3*I*a*d)/d)*gamma(m + 1, (-3*I*b*d*x - 3*I*b*c)/d) + 3*I*e^
(-(d*m*log(-5*I*b/d) + 5*I*b*c - 5*I*a*d)/d)*gamma(m + 1, (-5*I*b*d*x - 5*I
*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

maple [F] time = 0.32, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right)\left(\sin^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \cos(bx + a)^3 \sin(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a + b x)^3 \sin(a + b x)^2 (c + d x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.146 $\int (c+dx)^4 \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=330

$$\frac{3d^4\sin(a+bx)}{b^5}-\frac{d^4\sin(3a+3bx)}{162b^5}-\frac{3d^4\sin(5a+5bx)}{6250b^5}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(c+dx)\cos(3a+3bx)}{54b^4}+\frac{3d^3(c}{}$$

```
[Out] -3*d^3*(d*x+c)*cos(b*x+a)/b^4+1/2*d*(d*x+c)^3*cos(b*x+a)/b^2+1/54*d^3*(d*x+
c)*cos(3*b*x+3*a)/b^4-1/36*d*(d*x+c)^3*cos(3*b*x+3*a)/b^2+3/1250*d^3*(d*x+c
)*cos(5*b*x+5*a)/b^4-1/100*d*(d*x+c)^3*cos(5*b*x+5*a)/b^2+3*d^4*sin(b*x+a)/
b^5-3/2*d^2*(d*x+c)^2*sin(b*x+a)/b^3+1/8*(d*x+c)^4*sin(b*x+a)/b-1/162*d^4*s
in(3*b*x+3*a)/b^5+1/36*d^2*(d*x+c)^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^4*sin(
3*b*x+3*a)/b-3/6250*d^4*sin(5*b*x+5*a)/b^5+3/500*d^2*(d*x+c)^2*sin(5*b*x+5*
a)/b^3-1/80*(d*x+c)^4*sin(5*b*x+5*a)/b
```

Rubi [A] time = 0.37, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2637}

$$-\frac{3d^2(c+dx)^2\sin(a+bx)}{2b^3}+\frac{d^2(c+dx)^2\sin(3a+3bx)}{36b^3}+\frac{3d^2(c+dx)^2\sin(5a+5bx)}{500b^3}-\frac{3d^3(c+dx)\cos(a+bx)}{b^4}+\frac{d^3(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d^3*(c + d*x)*Cos[a + b*x])/b^4 + (d*(c + d*x)^3*Cos[a + b*x])/(2*b^2)
+ (d^3*(c + d*x)*Cos[3*a + 3*b*x])/(54*b^4) - (d*(c + d*x)^3*Cos[3*a + 3*b*
x])/(36*b^2) + (3*d^3*(c + d*x)*Cos[5*a + 5*b*x])/(1250*b^4) - (d*(c + d*x)
^3*Cos[5*a + 5*b*x])/(100*b^2) + (3*d^4*Sin[a + b*x])/b^5 - (3*d^2*(c + d*x
)^2*Sin[a + b*x])/(2*b^3) + ((c + d*x)^4*Sin[a + b*x])/(8*b) - (d^4*Sin[3*a
 + 3*b*x])/(162*b^5) + (d^2*(c + d*x)^2*Sin[3*a + 3*b*x])/(36*b^3) - ((c +
d*x)^4*Sin[3*a + 3*b*x])/(48*b) - (3*d^4*Sin[5*a + 5*b*x])/(6250*b^5) + (3*
d^2*(c + d*x)^2*Sin[5*a + 5*b*x])/(500*b^3) - ((c + d*x)^4*Sin[5*a + 5*b*x]
)/(80*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^4\cos(a+bx) - \frac{1}{16}(c+dx)^4\cos(3a+3bx) - \frac{1}{16}(c+dx$$

$$= -\left(\frac{1}{16}\int (c+dx)^4\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^4\cos(5a+5bx)$$

$$= \frac{(c+dx)^4\sin(a+bx)}{8b} - \frac{(c+dx)^4\sin(3a+3bx)}{48b} - \frac{(c+dx)^4\sin(5a+}{80b}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(}{100b^2}$$

$$= \frac{d(c+dx)^3\cos(a+bx)}{2b^2} - \frac{d(c+dx)^3\cos(3a+3bx)}{36b^2} - \frac{d(c+dx)^3\cos(}{100b^2}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos}{54b^4}$$

$$= -\frac{3d^3(c+dx)\cos(a+bx)}{b^4} + \frac{d(c+dx)^3\cos(a+bx)}{2b^2} + \frac{d^3(c+dx)\cos}{54b^4}$$

Mathematica [A] time = 3.48, size = 563, normalized size = 1.71

$$-\frac{-506250b^4c^4\sin(a+bx) + 84375b^4c^4\sin(3(a+bx)) + 50625b^4c^4\sin(5(a+bx)) - 2025000b^3c^3d(bx\sin(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/4050000*(-506250*b^4*c^4*Sin[a + b*x] - 2025000*b^3*c^3*d*(Cos[a + b*x]
+ b*x*Sin[a + b*x]) - 2025000*b*c*d^3*(3*(-2 + b^2*x^2)*Cos[a + b*x] + b*x*
(-6 + b^2*x^2)*Sin[a + b*x]) - 3037500*b^2*c^2*d^2*(2*b*x*Cos[a + b*x] + (-
2 + b^2*x^2)*Sin[a + b*x]) - 506250*d^4*(4*b*x*(-6 + b^2*x^2)*Cos[a + b*x]
+ (24 - 12*b^2*x^2 + b^4*x^4)*Sin[a + b*x]) + 84375*b^4*c^4*Sin[3*(a + b*x)
] + 112500*b^3*c^3*d*(Cos[3*(a + b*x)] + 3*b*x*Sin[3*(a + b*x)]) + 37500*b*
c*d^3*((-2 + 9*b^2*x^2)*Cos[3*(a + b*x)] + 3*b*x*(-2 + 3*b^2*x^2)*Sin[3*(a
```

```
+ b*x)]) + 56250*b^2*c^2*d^2*(6*b*x*Cos[3*(a + b*x)] + (-2 + 9*b^2*x^2)*Sin
[3*(a + b*x)]) + 3125*d^4*(12*b*x*(-2 + 3*b^2*x^2)*Cos[3*(a + b*x)] + (8 -
36*b^2*x^2 + 27*b^4*x^4)*Sin[3*(a + b*x)]) + 50625*b^4*c^4*Sin[5*(a + b*x)]
 + 40500*b^3*c^3*d*(Cos[5*(a + b*x)] + 5*b*x*Sin[5*(a + b*x)]) + 1620*b*c*d
^3*((-6 + 75*b^2*x^2)*Cos[5*(a + b*x)] + 5*b*x*(-6 + 25*b^2*x^2)*Sin[5*(a +
 b*x)]) + 12150*b^2*c^2*d^2*(10*b*x*Cos[5*(a + b*x)] + (-2 + 25*b^2*x^2)*Si
n[5*(a + b*x)]) + 81*d^4*(20*b*x*(-6 + 25*b^2*x^2)*Cos[5*(a + b*x)] + (24 -
 300*b^2*x^2 + 625*b^4*x^4)*Sin[5*(a + b*x)]))/b^5
```

fricas [A] time = 1.20, size = 527, normalized size = 1.60

$$-\frac{1620\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+25\,b^3c^3d-6\,bcd^3+3\left(25\,b^3c^2d^2-2\,bd^4\right)x\right)\cos\left(bx+a\right)^5-300\left(75\,b^3d^4x^3+225\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/253125*(1620*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 25*b^3*c^3*d - 6*b*c*d
^3 + 3*(25*b^3*c^2*d^2 - 2*b*d^4)*x)*cos(b*x + a)^5 - 300*(75*b^3*d^4*x^3 +
 225*b^3*c*d^3*x^2 + 75*b^3*c^3*d + 22*b*c*d^3 + (225*b^3*c^2*d^2 + 22*b*d^
4)*x)*cos(b*x + a)^3 - 1800*(75*b^3*d^4*x^3 + 225*b^3*c*d^3*x^2 + 75*b^3*c^
3*d - 428*b*c*d^3 + (225*b^3*c^2*d^2 - 428*b*d^4)*x)*cos(b*x + a) - (33750*
b^4*d^4*x^4 + 135000*b^4*c*d^3*x^3 + 33750*b^4*c^4 - 385200*b^2*c^2*d^2 - 8
1*(625*b^4*d^4*x^4 + 2500*b^4*c*d^3*x^3 + 625*b^4*c^4 - 300*b^2*c^2*d^2 + 2
4*d^4 + 150*(25*b^4*c^2*d^2 - 2*b^2*d^4)*x^2 + 100*(25*b^4*c^3*d - 6*b^2*c*
d^3)*x)*cos(b*x + a)^4 + 760816*d^4 + 900*(225*b^4*c^2*d^2 - 428*b^2*d^4)*x
^2 + (16875*b^4*d^4*x^4 + 67500*b^4*c*d^3*x^3 + 16875*b^4*c^4 + 9900*b^2*c^
2*d^2 - 4792*d^4 + 450*(225*b^4*c^2*d^2 + 22*b^2*d^4)*x^2 + 900*(75*b^4*c^3
*d + 22*b^2*c*d^3)*x)*cos(b*x + a)^2 + 1800*(75*b^4*c^3*d - 428*b^2*c*d^3)*
x)*sin(b*x + a))/b^5
```

giac [A] time = 0.29, size = 531, normalized size = 1.61

$$-\frac{\left(25\,b^3d^4x^3+75\,b^3cd^3x^2+75\,b^3c^2d^2x+25\,b^3c^3d-6\,bd^4x-6\,bcd^3\right)\cos\left(5\,bx+5\,a\right)}{2500\,b^5}-\frac{\left(3\,b^3d^4x^3+9\,b^3cd^3x^2+9\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2500*(25*b^3*d^4*x^3 + 75*b^3*c*d^3*x^2 + 75*b^3*c^2*d^2*x + 25*b^3*c^3*
d - 6*b*d^4*x - 6*b*c*d^3)*cos(5*b*x + 5*a)/b^5 - 1/108*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 9*b^3*c^2*d^2*x + 3*b^3*c^3*d - 2*b*d^4*x - 2*b*c*d^3)*cos(
3*b*x + 3*a)/b^5 + 1/2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b
^3*c^3*d - 6*b*d^4*x - 6*b*c*d^3)*cos(b*x + a)/b^5 - 1/50000*(625*b^4*d^4*x
```

```
^4 + 2500*b^4*c*d^3*x^3 + 3750*b^4*c^2*d^2*x^2 + 2500*b^4*c^3*d*x + 625*b^4
*c^4 - 300*b^2*d^4*x^2 - 600*b^2*c*d^3*x - 300*b^2*c^2*d^2 + 24*d^4)*sin(5*
b*x + 5*a)/b^5 - 1/1296*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d
^2*x^2 + 108*b^4*c^3*d*x + 27*b^4*c^4 - 36*b^2*d^4*x^2 - 72*b^2*c*d^3*x - 3
6*b^2*c^2*d^2 + 8*d^4)*sin(3*b*x + 3*a)/b^5 + 1/8*(b^4*d^4*x^4 + 4*b^4*c*d^
3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*b^2*d^4*x^2 - 24*b
^2*c*d^3*x - 12*b^2*c^2*d^2 + 24*d^4)*sin(b*x + a)/b^5
```

maple [**B**] time = 0.07, size = 1842, normalized size = 5.58

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/3*(b*x+a)^4*(2+cos(b*x+a)^2)*sin(b*x+a)+8/15*(b*x+a)^3*co
s(b*x+a)-8/5*(b*x+a)^2*sin(b*x+a)+3424/1125*sin(b*x+a)-3424/1125*(b*x+a)*co
s(b*x+a)+4/45*(b*x+a)^3*cos(b*x+a)^3-4/45*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*
x+a)+88/3375*(b*x+a)*cos(b*x+a)^3-88/10125*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*
(b*x+a)^4*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-4/25*(b*x+a)^3*cos
(b*x+a)^5+12/125*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+2
4/625*(b*x+a)*cos(b*x+a)^5-24/3125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(
b*x+a))-4/b^4*a*d^4*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^
2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(
b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*
(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos
(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/62
5*cos(b*x+a)^5)+4/b^3*c*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*
(b*x+a)^2*cos(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a
)^2*cos(b*x+a)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a
)^3-1/5*(b*x+a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+
a)^2*cos(b*x+a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x
+a)+6/625*cos(b*x+a)^5)+6/b^4*a^2*d^4*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b
*x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/1
35*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+
a)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(
b*x+a)^2)*sin(b*x+a))-12/b^3*a*c*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*
x+a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/13
5*(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a
)^2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b
*x+a)^2)*sin(b*x+a))+6/b^2*c^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+
a)-4/15*sin(b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*
(2+cos(b*x+a)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^
2)*sin(b*x+a)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x
+a)^2)*sin(b*x+a))-4/b^4*a^3*d^4*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1
```

```
/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+
a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+12/b^3*a^2*c*d^3*(1/3*(b*x+a)*(2+cos(b*
x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b
*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-12/b^2*a*c^2*d^2*(1
/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/
5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)
+4/b*c^3*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+1/b^4*a^4*d^4*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a
)^2)*sin(b*x+a))-4/b^3*a^3*c*d^3*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(
b*x+a)^2)*sin(b*x+a))+6/b^2*a^2*c^2*d^2*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*
(2+cos(b*x+a)^2)*sin(b*x+a))-4/b*a*c^3*d*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15
*(2+cos(b*x+a)^2)*sin(b*x+a))+c^4*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos
(b*x+a)^2)*sin(b*x+a)))
```

maxima [B] time = 0.43, size = 1339, normalized size = 4.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4050000*(270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^4 - 1080000*(3*s
in(b*x + a)^5 - 5*sin(b*x + a)^3)*a*c^3*d/b + 1620000*(3*sin(b*x + a)^5 - 5
*sin(b*x + a)^3)*a^2*c^2*d^2/b^2 - 1080000*(3*sin(b*x + a)^5 - 5*sin(b*x +
a)^3)*a^3*c*d^3/b^3 + 270000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^4*d^4/
b^4 + 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 45
0*cos(b*x + a))*c^3*d/b - 13500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*
cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c^2*d^2/b^2 + 13500*(45*(b*x + a)*si
n(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a)
 + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*c*d^3/b
^3 - 4500*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) -
450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450
*cos(b*x + a))*a^3*d^4/b^4 + 450*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x
 + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2
- 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b
*x + a)^2 - 2)*sin(b*x + a))*c^2*d^2/b^2 - 900*(270*(b*x + a)*cos(5*b*x + 5
*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(2
5*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3
*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a*c*d^3/b^3 + 450*(270*(b*x + a)
*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*
x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)
*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*a^2*d^4/b^4 + 60*(
```

```
81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*
x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*
b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x
+ 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*c*d^3/b^3 - 60*(81
*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b*x + a)^2 - 2)*cos(3*b*x
+ 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 135*(25*(b*x + a)^3 - 6*b*
x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 - 2*b*x - 2*a)*sin(3*b*x +
3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x + a))*a*d^4/b^4 + (1620*(2
5*(b*x + a)^3 - 6*b*x - 6*a)*cos(5*b*x + 5*a) + 37500*(3*(b*x + a)^3 - 2*b*
x - 2*a)*cos(3*b*x + 3*a) - 2025000*((b*x + a)^3 - 6*b*x - 6*a)*cos(b*x + a
) + 81*(625*(b*x + a)^4 - 300*(b*x + a)^2 + 24)*sin(5*b*x + 5*a) + 3125*(27
*(b*x + a)^4 - 36*(b*x + a)^2 + 8)*sin(3*b*x + 3*a) - 506250*((b*x + a)^4 -
 12*(b*x + a)^2 + 24)*sin(b*x + a))*d^4/b^4)/b
```

mupad [B] time = 4.38, size = 816, normalized size = 2.47

$$-\frac{\frac{d^4\,\sin(3\,a+3\,b\,x)}{162}-3\,d^4\,\sin\left(a+b\,x\right)+\frac{3\,d^4\,\sin(5\,a+5\,b\,x)}{6250}-\frac{b^4\,c^4\,\sin(a+b\,x)}{8}+\frac{b^4\,c^4\,\sin(3\,a+3\,b\,x)}{48}+\frac{b^4\,c^4\,\sin(5\,a+5\,b\,x)}{80}+\frac{b^3\,c^3\,d\,\mathrm{co}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] -((d^4*sin(3*a + 3*b*x))/162 - 3*d^4*sin(a + b*x) + (3*d^4*sin(5*a + 5*b*x)
)/6250 - (b^4*c^4*sin(a + b*x))/8 + (b^4*c^4*sin(3*a + 3*b*x))/48 + (b^4*c^
4*sin(5*a + 5*b*x))/80 + (b^3*c^3*d*cos(3*a + 3*b*x))/36 + (b^3*c^3*d*cos(5
*a + 5*b*x))/100 + (3*b^2*c^2*d^2*sin(a + b*x))/2 - (b^3*d^4*x^3*cos(a + b*
x))/2 + (3*b^2*d^4*x^2*sin(a + b*x))/2 - (b^4*d^4*x^4*sin(a + b*x))/8 + 3*b
*c*d^3*cos(a + b*x) + 3*b*d^4*x*cos(a + b*x) - (b^2*c^2*d^2*sin(3*a + 3*b*x
))/36 - (3*b^2*c^2*d^2*sin(5*a + 5*b*x))/500 + (b^3*d^4*x^3*cos(3*a + 3*b*x
))/36 + (b^3*d^4*x^3*cos(5*a + 5*b*x))/100 - (b^2*d^4*x^2*sin(3*a + 3*b*x))
/36 - (3*b^2*d^4*x^2*sin(5*a + 5*b*x))/500 + (b^4*d^4*x^4*sin(3*a + 3*b*x))
/48 + (b^4*d^4*x^4*sin(5*a + 5*b*x))/80 - (b*c*d^3*cos(3*a + 3*b*x))/54 - (
3*b*c*d^3*cos(5*a + 5*b*x))/1250 - (b^3*c^3*d*cos(a + b*x))/2 - (b*d^4*x*co
s(3*a + 3*b*x))/54 - (3*b*d^4*x*cos(5*a + 5*b*x))/1250 + 3*b^2*c*d^3*x*sin(
a + b*x) - (b^4*c^3*d*x*sin(a + b*x))/2 + (b^4*c^2*d^2*x^2*sin(3*a + 3*b*x)
)/8 + (3*b^4*c^2*d^2*x^2*sin(5*a + 5*b*x))/40 - (3*b^3*c^2*d^2*x*cos(a + b*
x))/2 - (3*b^3*c*d^3*x^2*cos(a + b*x))/2 - (b^2*c*d^3*x*sin(3*a + 3*b*x))/1
8 + (b^4*c^3*d*x*sin(3*a + 3*b*x))/12 - (3*b^2*c*d^3*x*sin(5*a + 5*b*x))/25
0 + (b^4*c^3*d*x*sin(5*a + 5*b*x))/20 - (b^4*c*d^3*x^3*sin(a + b*x))/2 + (b
^3*c^2*d^2*x*cos(3*a + 3*b*x))/12 + (b^3*c*d^3*x^2*cos(3*a + 3*b*x))/12 + (
3*b^3*c^2*d^2*x*cos(5*a + 5*b*x))/100 + (3*b^3*c*d^3*x^2*cos(5*a + 5*b*x))/
100 + (b^4*c*d^3*x^3*sin(3*a + 3*b*x))/12 + (b^4*c*d^3*x^3*sin(5*a + 5*b*x)
)/20 - (3*b^4*c^2*d^2*x^2*sin(a + b*x))/4)/b^5
```

sympy [A] time = 20.95, size = 1098, normalized size = 3.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**4*sin(a + b*x)**5/(15*b) + c**4*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 8*c**3*d*x*sin(a + b*x)**5/(15*b) + 4*c**3*d*x*sin(a + b*x)**3
*cos(a + b*x)**2/(3*b) + 4*c**2*d**2*x**2*sin(a + b*x)**5/(5*b) + 2*c**2*d*
*2*x**2*sin(a + b*x)**3*cos(a + b*x)**2/b + 8*c*d**3*x**3*sin(a + b*x)**5/(
15*b) + 4*c*d**3*x**3*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*d**4*x**4*s
in(a + b*x)**5/(15*b) + d**4*x**4*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 8
*c**3*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*c**3*d*sin(a + b*x)**2*
cos(a + b*x)**3/(45*b**2) + 104*c**3*d*cos(a + b*x)**5/(225*b**2) + 8*c**2*
d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c**2*d**2*x*sin(a + b*x)*
*2*cos(a + b*x)**3/(15*b**2) + 104*c**2*d**2*x*cos(a + b*x)**5/(75*b**2) +
8*c*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b**2) + 52*c*d**3*x**2*sin(a
+ b*x)**2*cos(a + b*x)**3/(15*b**2) + 104*c*d**3*x**2*cos(a + b*x)**5/(75*b
**2) + 8*d**4*x**3*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 52*d**4*x**3*si
n(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 104*d**4*x**3*cos(a + b*x)**5/(22
5*b**2) - 1712*c**2*d**2*sin(a + b*x)**5/(1125*b**3) - 676*c**2*d**2*sin(a
+ b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*c**2*d**2*sin(a + b*x)*cos(a + b
*x)**4/(75*b**3) - 3424*c*d**3*x*sin(a + b*x)**5/(1125*b**3) - 1352*c*d**3*
x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 208*c*d**3*x*sin(a + b*x)*co
s(a + b*x)**4/(75*b**3) - 1712*d**4*x**2*sin(a + b*x)**5/(1125*b**3) - 676*
d**4*x**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 104*d**4*x**2*sin(a
+ b*x)*cos(a + b*x)**4/(75*b**3) - 3424*c*d**3*sin(a + b*x)**4*cos(a + b*x)
/(1125*b**4) - 20456*c*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 5
0272*c*d**3*cos(a + b*x)**5/(16875*b**4) - 3424*d**4*x*sin(a + b*x)**4*cos(
a + b*x)/(1125*b**4) - 20456*d**4*x*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b
**4) - 50272*d**4*x*cos(a + b*x)**5/(16875*b**4) + 760816*d**4*sin(a + b*x)
**5/(253125*b**5) + 303368*d**4*sin(a + b*x)**3*cos(a + b*x)**2/(50625*b**5
) + 50272*d**4*sin(a + b*x)*cos(a + b*x)**4/(16875*b**5), Ne(b, 0)), ((c**4
*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*sin(a)**
2*cos(a)**3, True))
```

3.147 $\int (c+dx)^3 \cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=259

$$-\frac{3d^3\cos(a+bx)}{4b^4}+\frac{d^3\cos(3a+3bx)}{216b^4}+\frac{3d^3\cos(5a+5bx)}{5000b^4}-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+-$$

```
[Out] -3/4*d^3*cos(b*x+a)/b^4+3/8*d*(d*x+c)^2*cos(b*x+a)/b^2+1/216*d^3*cos(3*b*x+
3*a)/b^4-1/48*d*(d*x+c)^2*cos(3*b*x+3*a)/b^2+3/5000*d^3*cos(5*b*x+5*a)/b^4-
3/400*d*(d*x+c)^2*cos(5*b*x+5*a)/b^2-3/4*d^2*(d*x+c)*sin(b*x+a)/b^3+1/8*(d*
x+c)^3*sin(b*x+a)/b+1/72*d^2*(d*x+c)*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^3*sin(
3*b*x+3*a)/b+3/1000*d^2*(d*x+c)*sin(5*b*x+5*a)/b^3-1/80*(d*x+c)^3*sin(5*b*x
+5*a)/b
```

Rubi [A] time = 0.27, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{3d^2(c+dx)\sin(a+bx)}{4b^3}+\frac{d^2(c+dx)\sin(3a+3bx)}{72b^3}+\frac{3d^2(c+dx)\sin(5a+5bx)}{1000b^3}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (-3*d^3*Cos[a + b*x])/(4*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x])/(8*b^2) + (d
^3*Cos[3*a + 3*b*x])/(216*b^4) - (d*(c + d*x)^2*Cos[3*a + 3*b*x])/(48*b^2)
+ (3*d^3*Cos[5*a + 5*b*x])/(5000*b^4) - (3*d*(c + d*x)^2*Cos[5*a + 5*b*x])/
(400*b^2) - (3*d^2*(c + d*x)*Sin[a + b*x])/(4*b^3) + ((c + d*x)^3*Sin[a + b
*x])/(8*b) + (d^2*(c + d*x)*Sin[3*a + 3*b*x])/(72*b^3) - ((c + d*x)^3*Sin[3
*a + 3*b*x])/(48*b) + (3*d^2*(c + d*x)*Sin[5*a + 5*b*x])/(1000*b^3) - ((c +
 d*x)^3*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^3\cos(a+bx)-\frac{1}{16}(c+dx)^3\cos(3a+3bx)-\frac{1}{16}(c+dx)^3$$

$$= -\left(\frac{1}{16}\int (c+dx)^3\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^3\cos(5a+5bx)\,d$$

$$= \frac{(c+dx)^3\sin(a+bx)}{8b}-\frac{(c+dx)^3\sin(3a+3bx)}{48b}-\frac{(c+dx)^3\sin(5a+5}{80b}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\cos}{400b}$$

$$= \frac{3d(c+dx)^2\cos(a+bx)}{8b^2}-\frac{d(c+dx)^2\cos(3a+3bx)}{48b^2}-\frac{3d(c+dx)^2\cos}{400b}$$

$$= -\frac{3d^3\cos(a+bx)}{4b^4}+\frac{3d(c+dx)^2\cos(a+bx)}{8b^2}+\frac{d^3\cos(3a+3bx)}{216b^4}-\frac{d(c}{}$$

Mathematica [A] time = 2.18, size = 195, normalized size = 0.75

$$-\frac{30b(c+dx)\sin(a+bx)\left(8\cos(2(a+bx))\left(75b^2(c+dx)^2-38d^2\right)+9\cos(4(a+bx))\left(25b^2(c+dx)^2-6d^2\right)-825}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/270000*(-101250*d*(-2*d^2 + b^2*(c + d*x)^2)*Cos[a + b*x] + 625*d*(-2*d^
2 + 9*b^2*(c + d*x)^2)*Cos[3*(a + b*x)] + 81*d*(-2*d^2 + 25*b^2*(c + d*x)^2
)*Cos[5*(a + b*x)] + 30*b*(c + d*x)*(-825*b^2*c^2 + 6598*d^2 - 1650*b^2*c*d
*x - 825*b^2*d^2*x^2 + 8*(-38*d^2 + 75*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] +
9*(-6*d^2 + 25*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[a + b*x])/b^4
```

fricas [A] time = 0.77, size = 342, normalized size = 1.32

$$-\frac{81\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^5-5\left(225\,b^2d^3x^2+450\,b^2cd^2x+225\,b^2c^2d+22\,d^3\right)\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/16875*(81*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^5 - 5*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d + 22*d^3)*
cos(b*x + a)^3 - 30*(225*b^2*d^3*x^2 + 450*b^2*c*d^2*x + 225*b^2*c^2*d - 42
8*d^3)*cos(b*x + a) - 15*(150*b^3*d^3*x^3 + 450*b^3*c*d^2*x^2 + 150*b^3*c^3
 - 9*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 25*b^3*c^3 - 6*b*c*d^2 + 3*(25*b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^4 - 856*b*c*d^2 + (75*b^3*d^3*x^3 + 225*
b^3*c*d^2*x^2 + 75*b^3*c^3 + 22*b*c*d^2 + (225*b^3*c^2*d + 22*b*d^3)*x)*cos
(b*x + a)^2 + 2*(225*b^3*c^2*d - 428*b*d^3)*x)*sin(b*x + a))/b^4
```

giac [A] time = 0.36, size = 351, normalized size = 1.36

$$-\frac{3\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(5\,bx+5\,a\right)}{10000\,b^4}-\frac{\left(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d-2\,d^3\right)\cos\left(3\,bx\right.}{432\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -3/10000*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(5*b*x
 + 5*a)/b^4 - 1/432*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 2*d^3)*
cos(3*b*x + 3*a)/b^4 + 3/8*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3
)*cos(b*x + a)/b^4 - 1/2000*(25*b^3*d^3*x^3 + 75*b^3*c*d^2*x^2 + 75*b^3*c^2
*d*x + 25*b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(5*b*x + 5*a)/b^4 - 1/144*(3*
b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x + 3*b^3*c^3 - 2*b*d^3*x - 2*b
*c*d^2)*sin(3*b*x + 3*a)/b^4 + 1/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c
^2*d*x + b^3*c^3 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x + a)/b^4
```

maple [B] time = 0.02, size = 1016, normalized size = 3.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/3*(b*x+a)^3*(2+cos(b*x+a)^2)*sin(b*x+a)+2/5*(b*x+a)^2*cos
(b*x+a)-856/1125*cos(b*x+a)-4/5*(b*x+a)*sin(b*x+a)+1/15*(b*x+a)^2*cos(b*x+a
)^3-2/45*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+22/3375*cos(b*x+a)^3-1/5*(b*x+
a)^3*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-3/25*(b*x+a)^2*cos(b*x+
a)^5+6/125*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)+6/625*cos
(b*x+a)^5)-3/b^3*a*d^3*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(
b*x+a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a
)^2)*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
)-2/25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b
*x+a))+3/b^2*c*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+
```

```
a)+4/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)
*sin(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/
25*(b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a
))+3/b^3*a^2*d^3*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3
+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)
-1/25*cos(b*x+a)^5)-6/b^2*a*c*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+
1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x
+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)+3/b*c^2*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2
)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^
4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-1/b^3*a^3*d^3*(-1/5*cos(b
*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+3/b^2*a^2*c*d^2*(-1/5*
cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))-3/b*a*c^2*d*(-1/5
*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+c^3*(-1/5*cos(b*
x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a)))
```

maxima [B] time = 0.85, size = 766, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/270000*(18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^3 - 54000*(3*sin(b
*x + a)^5 - 5*sin(b*x + a)^3)*a*c^2*d/b + 54000*(3*sin(b*x + a)^5 - 5*sin(b
*x + a)^3)*a^2*c*d^2/b^2 - 18000*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a^3*
d^3/b^3 + 225*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a
) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) -
 450*cos(b*x + a))*c^2*d/b - 450*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x +
 a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25
*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*c*d^2/b^2 + 225*(45*(b*x + a)*sin(5
*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) +
9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a^2*d^3/b^3 +
15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500
*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9
*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*c
*d^2/b^2 - 15*(270*(b*x + a)*cos(5*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3
*a) - 13500*(b*x + a)*cos(b*x + a) + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*
a) + 125*(9*(b*x + a)^2 - 2)*sin(3*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(
b*x + a))*a*d^3/b^3 + (81*(25*(b*x + a)^2 - 2)*cos(5*b*x + 5*a) + 625*(9*(b
*x + a)^2 - 2)*cos(3*b*x + 3*a) - 101250*((b*x + a)^2 - 2)*cos(b*x + a) + 1
35*(25*(b*x + a)^3 - 6*b*x - 6*a)*sin(5*b*x + 5*a) + 1875*(3*(b*x + a)^3 -
2*b*x - 2*a)*sin(3*b*x + 3*a) - 33750*((b*x + a)^3 - 6*b*x - 6*a)*sin(b*x +
 a))*d^3/b^3)/b
```

mupad [B] time = 2.43, size = 516, normalized size = 1.99

$$-\frac{\frac{3\,d^3\,\cos(a+b\,x)}{4}-\frac{d^3\,\cos(3\,a+3\,b\,x)}{216}-\frac{3\,d^3\,\cos(5\,a+5\,b\,x)}{5000}-\frac{b^3\,c^3\,\sin(a+b\,x)}{8}+\frac{b^3\,c^3\,\sin(3\,a+3\,b\,x)}{48}+\frac{b^3\,c^3\,\sin(5\,a+5\,b\,x)}{80}+\frac{b^2\,c^2\,d\,\cos(3\,a+ }{48}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] -((3*d^3*cos(a + b*x))/4 - (d^3*cos(3*a + 3*b*x))/216 - (3*d^3*cos(5*a + 5*
b*x))/5000 - (b^3*c^3*sin(a + b*x))/8 + (b^3*c^3*sin(3*a + 3*b*x))/48 + (b^
3*c^3*sin(5*a + 5*b*x))/80 + (b^2*c^2*d*cos(3*a + 3*b*x))/48 + (3*b^2*c^2*d
*cos(5*a + 5*b*x))/400 - (3*b^2*d^3*x^2*cos(a + b*x))/8 - (b^3*d^3*x^3*sin(
a + b*x))/8 + (3*b*c*d^2*sin(a + b*x))/4 + (3*b*d^3*x*sin(a + b*x))/4 + (b^
2*d^3*x^2*cos(3*a + 3*b*x))/48 + (3*b^2*d^3*x^2*cos(5*a + 5*b*x))/400 + (b^
3*d^3*x^3*sin(3*a + 3*b*x))/48 + (b^3*d^3*x^3*sin(5*a + 5*b*x))/80 - (3*b^2
*c^2*d*cos(a + b*x))/8 - (b*c*d^2*sin(3*a + 3*b*x))/72 - (3*b*c*d^2*sin(5*a
 + 5*b*x))/1000 - (b*d^3*x*sin(3*a + 3*b*x))/72 - (3*b*d^3*x*sin(5*a + 5*b*
x))/1000 - (3*b^2*c*d^2*x*cos(a + b*x))/4 - (3*b^3*c^2*d*x*sin(a + b*x))/8
+ (b^2*c*d^2*x*cos(3*a + 3*b*x))/24 + (3*b^2*c*d^2*x*cos(5*a + 5*b*x))/200
+ (b^3*c^2*d*x*sin(3*a + 3*b*x))/16 + (3*b^3*c^2*d*x*sin(5*a + 5*b*x))/80 -
 (3*b^3*c*d^2*x^2*sin(a + b*x))/8 + (b^3*c*d^2*x^2*sin(3*a + 3*b*x))/16 + (
3*b^3*c*d^2*x^2*sin(5*a + 5*b*x))/80)/b^4
```

sympy [A] time = 11.35, size = 690, normalized size = 2.66

$$\begin{cases} \frac{2c^3\sin^5(a+bx)}{15b}+\frac{c^3\sin^3(a+bx)\cos^2(a+bx)}{3b}+\frac{2c^2dx\sin^5(a+bx)}{5b}+\frac{c^2dx\sin^3(a+bx)\cos^2(a+bx)}{b}+\frac{2cd^2x^2\sin^5(a+bx)}{5b}+\frac{cd^2x^2\sin^3(a+bx)\,\mathrm{c}}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\sin^2(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**3*sin(a + b*x)**5/(15*b) + c**3*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 2*c**2*d*x*sin(a + b*x)**5/(5*b) + c**2*d*x*sin(a + b*x)**3*co
s(a + b*x)**2/b + 2*c*d**2*x**2*sin(a + b*x)**5/(5*b) + c*d**2*x**2*sin(a +
 b*x)**3*cos(a + b*x)**2/b + 2*d**3*x**3*sin(a + b*x)**5/(15*b) + d**3*x**3
*sin(a + b*x)**3*cos(a + b*x)**2/(3*b) + 2*c**2*d*sin(a + b*x)**4*cos(a + b
*x)/(5*b**2) + 13*c**2*d*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*c**
2*d*cos(a + b*x)**5/(75*b**2) + 4*c*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(5*
b**2) + 26*c*d**2*x*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 52*c*d**2*x
*cos(a + b*x)**5/(75*b**2) + 2*d**3*x**2*sin(a + b*x)**4*cos(a + b*x)/(5*b*
*2) + 13*d**3*x**2*sin(a + b*x)**2*cos(a + b*x)**3/(15*b**2) + 26*d**3*x**2
```

```
*cos(a + b*x)**5/(75*b**2) - 856*c*d**2*sin(a + b*x)**5/(1125*b**3) - 338*c
*d**2*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*c*d**2*sin(a + b*x)*c
os(a + b*x)**4/(75*b**3) - 856*d**3*x*sin(a + b*x)**5/(1125*b**3) - 338*d**
3*x*sin(a + b*x)**3*cos(a + b*x)**2/(225*b**3) - 52*d**3*x*sin(a + b*x)*cos
(a + b*x)**4/(75*b**3) - 856*d**3*sin(a + b*x)**4*cos(a + b*x)/(1125*b**4)
- 5114*d**3*sin(a + b*x)**2*cos(a + b*x)**3/(3375*b**4) - 12568*d**3*cos(a
+ b*x)**5/(16875*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3
 + d**3*x**4/4)*sin(a)**2*cos(a)**3, True))
```

3.148 $\int (c+dx)^2 \cos^3(a+bx) \sin^2(a+bx)\,dx$

Optimal. Leaf size=184

$$-\frac{d^2 \sin(a+bx)}{4b^3}+\frac{d^2 \sin(3a+3bx)}{216b^3}+\frac{d^2 \sin(5a+5bx)}{1000b^3}+\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+d}{}$$

```
[Out] 1/4*d*(d*x+c)*cos(b*x+a)/b^2-1/72*d*(d*x+c)*cos(3*b*x+3*a)/b^2-1/200*d*(d*x
+c)*cos(5*b*x+5*a)/b^2-1/4*d^2*sin(b*x+a)/b^3+1/8*(d*x+c)^2*sin(b*x+a)/b+1/
216*d^2*sin(3*b*x+3*a)/b^3-1/48*(d*x+c)^2*sin(3*b*x+3*a)/b+1/1000*d^2*sin(5
*b*x+5*a)/b^3-1/80*(d*x+c)^2*sin(5*b*x+5*a)/b
```

Rubi [A] time = 0.19, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+5bx)}{200b^2}-\frac{d^2 \sin(a+bx)}{4b^3}+\frac{d^2 \sin(3a+3bx)}{216b^3}+\frac{d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*(c + d*x)*Cos[a + b*x])/(4*b^2) - (d*(c + d*x)*Cos[3*a + 3*b*x])/(72*b^2
) - (d*(c + d*x)*Cos[5*a + 5*b*x])/(200*b^2) - (d^2*Sin[a + b*x])/(4*b^3) +
 ((c + d*x)^2*Sin[a + b*x])/(8*b) + (d^2*Sin[3*a + 3*b*x])/(216*b^3) - ((c
+ d*x)^2*Sin[3*a + 3*b*x])/(48*b) + (d^2*Sin[5*a + 5*b*x])/(1000*b^3) - ((c
 + d*x)^2*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^2\cos(a+bx)-\frac{1}{16}(c+dx)^2\cos(3a+3bx)-\frac{1}{16}(c+dx)^2\right.$$
$$= -\left(\frac{1}{16}\int (c+dx)^2\cos(3a+3bx)\,dx\right)-\frac{1}{16}\int (c+dx)^2\cos(5a+5bx)\,d$$
$$= \frac{(c+dx)^2\sin(a+bx)}{8b}-\frac{(c+dx)^2\sin(3a+3bx)}{48b}-\frac{(c+dx)^2\sin(5a+5}{80b}$$
$$= \frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+}{200b^2}$$
$$= \frac{d(c+dx)\cos(a+bx)}{4b^2}-\frac{d(c+dx)\cos(3a+3bx)}{72b^2}-\frac{d(c+dx)\cos(5a+}{200b^2}$$

Mathematica [A] time = 0.96, size = 252, normalized size = 1.37

$$-\frac{-6750b^2c^2\sin(a+bx)+1125b^2c^2\sin(3(a+bx))+675b^2c^2\sin(5(a+bx))-13500b^2cdx\sin(a+bx)+2250b^2c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/54000*(-13500*b*d*(c + d*x)*Cos[a + b*x] + 750*b*d*(c + d*x)*Cos[3*(a +
b*x)] + 270*b*c*d*Cos[5*(a + b*x)] + 270*b*d^2*x*Cos[5*(a + b*x)] - 6750*b^
2*c^2*Sin[a + b*x] + 13500*d^2*Sin[a + b*x] - 13500*b^2*c*d*x*Sin[a + b*x]
- 6750*b^2*d^2*x^2*Sin[a + b*x] + 1125*b^2*c^2*Sin[3*(a + b*x)] - 250*d^2*S
in[3*(a + b*x)] + 2250*b^2*c*d*x*Sin[3*(a + b*x)] + 1125*b^2*d^2*x^2*Sin[3*
(a + b*x)] + 675*b^2*c^2*Sin[5*(a + b*x)] - 54*d^2*Sin[5*(a + b*x)] + 1350*
b^2*c*d*x*Sin[5*(a + b*x)] + 675*b^2*d^2*x^2*Sin[5*(a + b*x)])/b^3
```

fricas [A] time = 0.51, size = 193, normalized size = 1.05

$$-\frac{270\left(bd^2x+bcd\right)\cos\left(bx+a\right)^5-150\left(bd^2x+bcd\right)\cos\left(bx+a\right)^3-900\left(bd^2x+bcd\right)\cos\left(bx+a\right)-\left(450\,b^2d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/3375*(270*(b*d^2*x + b*c*d)*cos(b*x + a)^5 - 150*(b*d^2*x + b*c*d)*cos(b
*x + a)^3 - 900*(b*d^2*x + b*c*d)*cos(b*x + a) - (450*b^2*d^2*x^2 + 900*b^2
*c*d*x - 27*(25*b^2*d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*cos(b*x +
a)^4 + 450*b^2*c^2 + (225*b^2*d^2*x^2 + 450*b^2*c*d*x + 225*b^2*c^2 + 22*d^
2)*cos(b*x + a)^2 - 856*d^2)*sin(b*x + a))/b^3
```

giac [A] time = 2.11, size = 209, normalized size = 1.14

$$-\frac{\left(bd^2x+bcd\right)\cos\left(5\,bx+5\,a\right)}{200\,b^3}-\frac{\left(bd^2x+bcd\right)\cos\left(3\,bx+3\,a\right)}{72\,b^3}+\frac{\left(bd^2x+bcd\right)\cos\left(bx+a\right)}{4\,b^3}-\frac{\left(25\,b^2d^2x^2+50\,b^2cd\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/200*(b*d^2*x + b*c*d)*cos(5*b*x + 5*a)/b^3 - 1/72*(b*d^2*x + b*c*d)*cos(
3*b*x + 3*a)/b^3 + 1/4*(b*d^2*x + b*c*d)*cos(b*x + a)/b^3 - 1/2000*(25*b^2*
d^2*x^2 + 50*b^2*c*d*x + 25*b^2*c^2 - 2*d^2)*sin(5*b*x + 5*a)/b^3 - 1/432*(
9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 2*d^2)*sin(3*b*x + 3*a)/b^3 + 1/
8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*d^2)*sin(b*x + a)/b^3
```

maple [B] time = 0.02, size = 484, normalized size = 2.63

$$\frac{d^2\left(\frac{(bx+a)^2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}-\frac{4\sin(bx+a)}{15}+\frac{4(bx+a)\cos(bx+a)}{15}+\frac{2(bx+a)\left(\cos^3(bx+a)\right)}{45}-\frac{2\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{135}-\frac{(bx+a)^2\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(b}{5}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/3*(b*x+a)^2*(2+cos(b*x+a)^2)*sin(b*x+a)-4/15*sin(b*x+a)+4
/15*(b*x+a)*cos(b*x+a)+2/45*(b*x+a)*cos(b*x+a)^3-2/135*(2+cos(b*x+a)^2)*sin
(b*x+a)-1/5*(b*x+a)^2*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-2/25*(
b*x+a)*cos(b*x+a)^5+2/125*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a))-2
/b^2*a*d^2*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*
cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*
cos(b*x+a)^5)+2/b*c*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x
+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b
*x+a)-1/25*cos(b*x+a)^5)+d^2/b^2*a^2*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+
cos(b*x+a)^2)*sin(b*x+a))-2*c*d/b*a*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+c
os(b*x+a)^2)*sin(b*x+a))+c^2*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+
a)^2)*sin(b*x+a)))
```

maxima [B] time = 0.60, size = 375, normalized size = 2.04

$$-\frac{3600\left(3\sin\left(bx+a\right)^{5}-5\sin\left(bx+a\right)^{3}\right)c^{2}-\frac{7200\left(3\sin(bx+a)^{5}-5\sin(bx+a)^{3}\right)acd}{b}+\frac{3600\left(3\sin(bx+a)^{5}-5\sin(bx+a)^{3}\right)a^{2}d^{2}}{b^{2}}+\frac{30}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/54000*(3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c^2 - 7200*(3*sin(b*x
+ a)^5 - 5*sin(b*x + a)^3)*a*c*d/b + 3600*(3*sin(b*x + a)^5 - 5*sin(b*x + a
)^3)*a^2*d^2/b^2 + 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b
*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x
+ 3*a) - 450*cos(b*x + a))*c*d/b - 30*(45*(b*x + a)*sin(5*b*x + 5*a) + 75*(
b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a)
 + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*a*d^2/b^2 + (270*(b*x + a)*cos(5
*b*x + 5*a) + 750*(b*x + a)*cos(3*b*x + 3*a) - 13500*(b*x + a)*cos(b*x + a)
 + 27*(25*(b*x + a)^2 - 2)*sin(5*b*x + 5*a) + 125*(9*(b*x + a)^2 - 2)*sin(3
*b*x + 3*a) - 6750*((b*x + a)^2 - 2)*sin(b*x + a))*d^2/b^2)/b
```

mupad [B] time = 0.85, size = 295, normalized size = 1.60

$$\frac{52\,d^2\,x\cos\left(a+b\,x\right)^5}{225\,b^2}-\frac{52\,d^2\cos\left(a+b\,x\right)^4\,\sin\left(a+b\,x\right)}{225\,b^3}-\frac{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^3\,\left(338\,d^2-225\,b^2\,c^2\right)}{675\,b^3}-\frac{2\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (52*d^2*x*cos(a + b*x)^5)/(225*b^2) - (52*d^2*cos(a + b*x)^4*sin(a + b*x))/
(225*b^3) - (cos(a + b*x)^2*sin(a + b*x)^3*(338*d^2 - 225*b^2*c^2))/(675*b^
3) - (2*sin(a + b*x)^5*(428*d^2 - 225*b^2*c^2))/(3375*b^3) + (2*d^2*x^2*sin
(a + b*x)^5)/(15*b) + (52*c*d*cos(a + b*x)^5)/(225*b^2) + (4*c*d*cos(a + b*
x)*sin(a + b*x)^4)/(15*b^2) + (4*c*d*x*sin(a + b*x)^5)/(15*b) + (d^2*x^2*co
s(a + b*x)^2*sin(a + b*x)^3)/(3*b) + (26*c*d*cos(a + b*x)^3*sin(a + b*x)^2)
/(45*b^2) + (4*d^2*x*cos(a + b*x)*sin(a + b*x)^4)/(15*b^2) + (26*d^2*x*cos(
a + b*x)^3*sin(a + b*x)^2)/(45*b^2) + (2*c*d*x*cos(a + b*x)^2*sin(a + b*x)^
3)/(3*b)
```

sympy [A] time = 6.23, size = 382, normalized size = 2.08

$$\begin{cases}\frac{2c^{2}\sin^{5}\left(a+bx\right)}{15b}+\frac{c^{2}\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{4cdx\sin^{5}\left(a+bx\right)}{15b}+\frac{2cdx\sin^{3}\left(a+bx\right)\cos^{2}\left(a+bx\right)}{3b}+\frac{2d^{2}x^{2}\sin^{5}\left(a+bx\right)}{15b}+\frac{d^{2}x^{2}\sin^{3}\left(a+bx\right)\cos^{2}\left(\right.}{3b}\\\left(c^{2}x+cdx^{2}+\frac{d^{2}x^{3}}{3}\right)\sin^{2}\left(a\right)\cos^{3}\left(a\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c**2*sin(a + b*x)**5/(15*b) + c**2*sin(a + b*x)**3*cos(a + b*x
)**2/(3*b) + 4*c*d*x*sin(a + b*x)**5/(15*b) + 2*c*d*x*sin(a + b*x)**3*cos(a
 + b*x)**2/(3*b) + 2*d**2*x**2*sin(a + b*x)**5/(15*b) + d**2*x**2*sin(a + b
*x)**3*cos(a + b*x)**2/(3*b) + 4*c*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2)
 + 26*c*d*sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*c*d*cos(a + b*x)**
5/(225*b**2) + 4*d**2*x*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 26*d**2*x*
sin(a + b*x)**2*cos(a + b*x)**3/(45*b**2) + 52*d**2*x*cos(a + b*x)**5/(225*
b**2) - 856*d**2*sin(a + b*x)**5/(3375*b**3) - 338*d**2*sin(a + b*x)**3*cos
(a + b*x)**2/(675*b**3) - 52*d**2*sin(a + b*x)*cos(a + b*x)**4/(225*b**3),
Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**2*cos(a)**3, True))
```

3.149 $\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

```
[Out] 1/8*d*cos(b*x+a)/b^2-1/144*d*cos(3*b*x+3*a)/b^2-1/400*d*cos(5*b*x+5*a)/b^2+
1/8*(d*x+c)*sin(b*x+a)/b-1/48*(d*x+c)*sin(3*b*x+3*a)/b-1/80*(d*x+c)*sin(5*b
*x+5*a)/b
```

Rubi [A] time = 0.09, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2638}

$$\frac{d\cos(a+bx)}{8b^2}-\frac{d\cos(3a+3bx)}{144b^2}-\frac{d\cos(5a+5bx)}{400b^2}+\frac{(c+dx)\sin(a+bx)}{8b}-\frac{(c+dx)\sin(3a+3bx)}{48b}-\frac{(c+dx)\sin(5a+5bx)}{80b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (d*Cos[a + b*x])/(8*b^2) - (d*Cos[3*a + 3*b*x])/(144*b^2) - (d*Cos[5*a + 5*
b*x])/(400*b^2) + ((c + d*x)*Sin[a + b*x])/(8*b) - ((c + d*x)*Sin[3*a + 3*b
*x])/(48*b) - ((c + d*x)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)\cos(a+bx) - \frac{1}{16}(c+dx)\cos(3a+3bx) - \frac{1}{16}(c+dx)\mathrm{co}\right.$$
$$= -\left(\frac{1}{16}\int (c+dx)\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)\cos(5a+5bx)\,dx$$
$$= \frac{(c+dx)\sin(a+bx)}{8b} - \frac{(c+dx)\sin(3a+3bx)}{48b} - \frac{(c+dx)\sin(5a+5bx}{80b}$$
$$= \frac{d\cos(a+bx)}{8b^2} - \frac{d\cos(3a+3bx)}{144b^2} - \frac{d\cos(5a+5bx)}{400b^2} + \frac{(c+dx)\sin(a}{8b}$$

Mathematica [A] time = 0.36, size = 110, normalized size = 1.01

$$-\frac{-450bc\sin(a+bx)+75bc\sin(3(a+bx))+45bc\sin(5(a+bx))-450bdx\sin(a+bx)+75bdx\sin(3(a+bx))}{3600b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -1/3600*(-450*d*Cos[a + b*x] + 25*d*Cos[3*(a + b*x)] + 9*d*Cos[5*(a + b*x)]
 - 450*b*c*Sin[a + b*x] - 450*b*d*x*Sin[a + b*x] + 75*b*c*Sin[3*(a + b*x)]
+ 75*b*d*x*Sin[3*(a + b*x)] + 45*b*c*Sin[5*(a + b*x)] + 45*b*d*x*Sin[5*(a +
 b*x)])/b^2
```

fricas [A] time = 0.76, size = 91, normalized size = 0.83

$$-\frac{9\,d\cos(bx+a)^5 - 5\,d\cos(bx+a)^3 - 30\,d\cos(bx+a) + 15\left(3\,(bdx+bc)\cos(bx+a)^4 - 2\,bdx - (bdx+bc)\,\mathrm{cc}\right.}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/225*(9*d*cos(b*x + a)^5 - 5*d*cos(b*x + a)^3 - 30*d*cos(b*x + a) + 15*(3
*(b*d*x + b*c)*cos(b*x + a)^4 - 2*b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 - 2*
b*c)*sin(b*x + a))/b^2
```

giac [A] time = 2.97, size = 106, normalized size = 0.97

$$-\frac{d\cos(5\,bx+5\,a)}{400\,b^2} - \frac{d\cos(3\,bx+3\,a)}{144\,b^2} + \frac{d\cos(bx+a)}{8\,b^2} - \frac{(bdx+bc)\sin(5\,bx+5\,a)}{80\,b^2} - \frac{(bdx+bc)\sin(3\,bx+3\,a)}{48\,b^2} +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] -1/400*d*cos(5*b*x + 5*a)/b^2 - 1/144*d*cos(3*b*x + 3*a)/b^2 + 1/8*d*cos(b*x + a)/b^2 - 1/80*(b*d*x + b*c)*sin(5*b*x + 5*a)/b^2 - 1/48*(b*d*x + b*c)*sin(3*b*x + 3*a)/b^2 + 1/8*(b*d*x + b*c)*sin(b*x + a)/b^2

maple [A] time = 0.02, size = 175, normalized size = 1.61

$$\frac{\frac{d\left(\frac{(bx+a)\left(2+\cos^2(bx+a)\right)\sin(bx+a)}{3}+\frac{\left(\cos^3(bx+a)\right)}{45}+\frac{2\cos(bx+a)}{15}-\frac{(bx+a)\left(\frac{8}{3}+\cos^4(bx+a)+\frac{4\left(\cos^2(bx+a)\right)}{3}\right)\sin(bx+a)}{5}-\frac{\left(\cos^5(bx+a)\right)}{25}\right)}{b}-\frac{da\left(-\frac{\left(\cos^4(bx+a)\right)\sin(bx+a)}{5}+\frac{(2+c}{}\right.}{b}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x)

[Out] 1/b*(1/b*d*(1/3*(b*x+a)*(2+cos(b*x+a)^2)*sin(b*x+a)+1/45*cos(b*x+a)^3+2/15*cos(b*x+a)-1/5*(b*x+a)*(8/3+cos(b*x+a)^4+4/3*cos(b*x+a)^2)*sin(b*x+a)-1/25*cos(b*x+a)^5)-1/b*d*a*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a))+c*(-1/5*cos(b*x+a)^4*sin(b*x+a)+1/15*(2+cos(b*x+a)^2)*sin(b*x+a)))

maxima [A] time = 0.95, size = 139, normalized size = 1.28

$$-\frac{240\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)c-\frac{240\left(3\sin(bx+a)^5-5\sin(bx+a)^3\right)ad}{b}+\frac{(45(bx+a)\sin(5bx+5a)+75(bx+a)\sin(3bx+3a)-45}{}}{3600b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")

[Out] -1/3600*(240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*c - 240*(3*sin(b*x + a)^5 - 5*sin(b*x + a)^3)*a*d/b + (45*(b*x + a)*sin(5*b*x + 5*a) + 75*(b*x + a)*sin(3*b*x + 3*a) - 450*(b*x + a)*sin(b*x + a) + 9*cos(5*b*x + 5*a) + 25*cos(3*b*x + 3*a) - 450*cos(b*x + a))*d/b)/b

mupad [B] time = 0.47, size = 119, normalized size = 1.09

$$\frac{26\,d\cos(a+b\,x)^5+65\,d\cos(a+b\,x)^3\sin(a+b\,x)^2+30\,d\,\cos(a+b\,x)\,\sin(a+b\,x)^4+30\,b\,c\sin(a+b\,x)^5+30}{225\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x),x)

```
[Out] (26*d*cos(a + b*x)^5 + 65*d*cos(a + b*x)^3*sin(a + b*x)^2 + 30*d*cos(a + b*
x)*sin(a + b*x)^4 + 30*b*c*sin(a + b*x)^5 + 30*b*d*x*sin(a + b*x)^5 + 75*b*
c*cos(a + b*x)^2*sin(a + b*x)^3 + 75*b*d*x*cos(a + b*x)^2*sin(a + b*x)^3)/(
225*b^2)
```

sympy [A] time = 3.10, size = 163, normalized size = 1.50

$$\begin{cases} \frac{2c\sin^5(a+bx)}{15b} + \frac{c\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2dx\sin^5(a+bx)}{15b} + \frac{dx\sin^3(a+bx)\cos^2(a+bx)}{3b} + \frac{2d\sin^4(a+bx)\cos(a+bx)}{15b^2} + \frac{13d\sin^2(a+bx)\cos \\ \left(cx + \frac{dx^2}{2}\right)\sin^2(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Piecewise((2*c*sin(a + b*x)**5/(15*b) + c*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*x*sin(a + b*x)**5/(15*b) + d*x*sin(a + b*x)**3*cos(a + b*x)**2/(
3*b) + 2*d*sin(a + b*x)**4*cos(a + b*x)/(15*b**2) + 13*d*sin(a + b*x)**2*co
s(a + b*x)**3/(45*b**2) + 26*d*cos(a + b*x)**5/(225*b**2), Ne(b, 0)), ((c*x
 + d*x**2/2)*sin(a)**2*cos(a)**3, True))
```

3.150 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=185

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d}-\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{8d}$$

```
[Out] -1/16*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d-1/16*Ci(3*b*c/d+3*b*x)*cos(3*a-3
*b*c/d)/d+1/8*Ci(b*c/d+b*x)*cos(a-b*c/d)/d+1/16*Si(5*b*c/d+5*b*x)*sin(5*a-5
*b*c/d)/d+1/16*Si(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d-1/8*Si(b*c/d+b*x)*sin(a
-b*c/d)/d
```

Rubi [A] time = 0.28, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {4406, 3303, 3299, 3302}

$$\frac{\cos\left(a-\frac{bc}{d}\right)\text{CosIntegral}\left(\frac{bc}{d}+bx\right)}{8d}-\frac{\cos\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d}-\frac{\cos\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/(8*d) - (Cos[3*a - (3*b*c)/d]
*CosIntegral[(3*b*c)/d + 3*b*x])/(16*d) - (Cos[5*a - (5*b*c)/d]*CosIntegral
[(5*b*c)/d + 5*b*x])/(16*d) - (Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])
/(8*d) + (Sin[3*a - (3*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d) + (Si
n[5*a - (5*b*c)/d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{c+dx}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)} - \frac{\cos(3a+3bx)}{16(c+dx)} - \frac{\cos(5a+5bx)}{16(c+dx)}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{c+dx}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{c+dx}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{c+dx}\,dx$$

$$= -\left(\frac{1}{16}\cos\left(5a-\frac{5bc}{d}\right)\int \frac{\cos\left(\frac{5bc}{d}+5bx\right)}{c+dx}\,dx\right) - \frac{1}{16}\cos\left(3a-\frac{3bc}{d}\right)\int \frac{\cos\left(\frac{3bc}{d}\right.}{c+}$$

$$= \frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d} - \frac{\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d} - \frac{\cos\left(5a-\frac{5bc}{d}\right)\mathrm{C}}{16d}$$

Mathematica [A] time = 0.51, size = 154, normalized size = 0.83

$$\frac{2\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right) - \cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3b(c+dx)}{d}\right) - \cos\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5b(c+dx)}{d}\right) - 2\sin\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x),x]
```

```
[Out] (2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] - Cos[3*a - (3*b*c)/d]*CosInte
gral[(3*b*(c + d*x))/d] - Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*(c + d*x))/
d] - 2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] + Sin[3*a - (3*b*c)/d]*Sin
Integral[(3*b*(c + d*x))/d] + Sin[5*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*
x))/d])/(16*d)
```

fricas [A] time = 0.61, size = 229, normalized size = 1.24

$$\frac{2\left(\mathrm{Ci}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ci}\left(-\frac{bdx+bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right) - \left(\mathrm{Ci}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ci}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right) - \left(\mathrm{Ci}\left(\frac{5\,(bdx+bc)}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/32*(2*(cos_integral((b*d*x + b*c)/d) + cos_integral(-(b*d*x + b*c)/d))*co
s(-(b*c - a*d)/d) - (cos_integral(3*(b*d*x + b*c)/d) + cos_integral(-3*(b*d
*x + b*c)/d))*cos(-3*(b*c - a*d)/d) - (cos_integral(5*(b*d*x + b*c)/d) + co
s_integral(-5*(b*d*x + b*c)/d))*cos(-5*(b*c - a*d)/d) + 2*sin(-5*(b*c - a*d
)/d)*sin_integral(5*(b*d*x + b*c)/d) + 2*sin(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*sin(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d))/
d
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 252, normalized size = 1.36

$$\frac{\frac{b\left(\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}\right)}{8}-\frac{b\left(\frac{5\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\sin\left(\frac{-5da+5cb}{d}\right)}{d}+\frac{5\,\mathrm{Ci}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}\right)}{80}-\frac{b\left(3\,\mathrm{S}\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] 1/b*(1/8*b*(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*d+b*c)/
d)*cos((-a*d+b*c)/d)/d)-1/80*b*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+
b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d)-1/48*b*(3*S
i(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*
c)/d)*cos(3*(-a*d+b*c)/d)/d))
```

maxima [C] time = 0.51, size = 408, normalized size = 2.21

$$-\frac{2\,b\left(E_1\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_1\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-b\left(E_1\left(\frac{3i\,bc+3i\,(bx+a)d-3i\,ad}{d}\right)+E_1\left(-\frac{3i\,bc+3i\,(bx+a)d-3\cdots}{d}\right.\right.\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/32*(2*b*(exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + exp_inte
gral_e(1, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*c - a*d)/d) - b*(exp
_integral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) + exp_integral_e(1,
 -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*cos(-3*(b*c - a*d)/d) - b*(exp_
integral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) + exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b*(-2*I*
exp_integral_e(1, (I*b*c + I*(b*x + a)*d - I*a*d)/d) + 2*I*exp_integral_e(1
, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b*(I*exp_integ
ral_e(1, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - I*exp_integral_e(1, -(3
*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) + b*(I*exp_in
tegral_e(1, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d) - I*exp_integral_e(1,
-(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))*sin(-5*(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x), x)
```

3.151 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=257

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{8d^2}-\frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\right.}{8d^2}$$

```
[Out] -1/8*cos(b*x+a)/d/(d*x+c)+1/16*cos(3*b*x+3*a)/d/(d*x+c)+1/16*cos(5*b*x+5*a)
/d/(d*x+c)-1/8*b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+3/16*b*cos(3*a-3*b*c/d)*Si(
3*b*c/d+3*b*x)/d^2+5/16*b*cos(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^2+5/16*b*Ci(
5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^2+3/16*b*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/
d)/d^2-1/8*b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A] time = 0.34, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{5b\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{16d^2}+\frac{3b\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{16d^2}-\frac{b\sin\left(a-\frac{bc}{d}\right)\text{CosIntegr}}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Cos[a + b*x]/(8*d*(c + d*x)) + Cos[3*a + 3*b*x]/(16*d*(c + d*x)) + Cos[5*a
 + 5*b*x]/(16*d*(c + d*x)) + (5*b*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a -
(5*b*c)/d])/(16*d^2) + (3*b*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c
)/d])/(16*d^2) - (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(8*d^2) -
(b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(8*d^2) + (3*b*Cos[3*a - (3
*b*c)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(16*d^2) + (5*b*Cos[5*a - (5*b*c)/
d]*SinIntegral[(5*b*c)/d + 5*b*x])/(16*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^2}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^2} - \frac{\cos(3a+3bx)}{16(c+dx)^2} - \frac{\cos(5a+5bx)}{16(c+dx)^2}\right)dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^2}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^2}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} - \frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{8d} + \frac{(3b)\int \frac{\mathrm{s}}{}}{}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{\left(5b\cos\left(5a-\frac{5bc}{d}\right)\right)\int \frac{\sin\left(\right.}{}}{16d}$$

$$= -\frac{\cos(a+bx)}{8d(c+dx)} + \frac{\cos(3a+3bx)}{16d(c+dx)} + \frac{\cos(5a+5bx)}{16d(c+dx)} + \frac{5b\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)\sin\left(5a\right.}{16d^2}$$

Mathematica [A] time = 2.11, size = 212, normalized size = 0.82

$$\frac{-2\left(b\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+b\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(b\left(\frac{c}{d}+x\right)\right)+\frac{d\cos(a+bx)}{c+dx}\right)+5b\sin\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5b(c+dx)}{d}\right)+3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] ((d*Cos[3*(a + b*x)])/(c + d*x) + (d*Cos[5*(a + b*x)])/(c + d*x) + 5*b*CosI
ntegral[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + 3*b*CosIntegral[(3*b*(c +
 d*x))/d]*Sin[3*a - (3*b*c)/d] - 2*((d*Cos[a + b*x])/(c + d*x) + b*CosInteg
ral[b*(c/d + x)]*Sin[a - (b*c)/d] + b*Cos[a - (b*c)/d]*SinIntegral[b*(c/d +
 x)]) + 3*b*Cos[3*a - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] + 5*b*Cos[5
*a - (5*b*c)/d]*SinIntegral[(5*b*(c + d*x))/d])/(16*d^2)
```

fricas [A] time = 0.51, size = 339, normalized size = 1.32

$$\frac{32\,d\cos\left(bx+a\right)^{5}-32\,d\cos\left(bx+a\right)^{3}+10\left(bdx+bc\right)\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{5\,(bdx+bc)}{d}\right)+6\left(bdx+bc\right)\cos\left(-\frac{3\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*d*cos(b*x + a)^5 - 32*d*cos(b*x + a)^3 + 10*(b*d*x + b*c)*cos(-5*(
b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 6*(b*d*x + b*c)*cos(-3*(b*c
 - a*d)/d)*sin_integral(3*(b*d*x + b*c)/d) - 4*(b*d*x + b*c)*cos(-(b*c - a*
d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b*d*x + b*c)*cos_integral((b*d*x
+ b*c)/d) + (b*d*x + b*c)*cos_integral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/
d) + 3*((b*d*x + b*c)*cos_integral(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_i
ntegral(-3*(b*d*x + b*c)/d))*sin(-3*(b*c - a*d)/d) + 5*((b*d*x + b*c)*cos_i
ntegral(5*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-5*(b*d*x + b*c)/d)
)*sin(-5*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 367, normalized size = 1.43

$$\frac{\frac{b^{2}\left(-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}\right)}{8}-\frac{b^{2}\left(-\frac{5\cos(5bx+5a)}{((bx+a)d-da+cb)d}-\frac{5\left(\frac{5\,\mathrm{Si}\left(5bx+5a+\frac{-5da+5cb}{d}\right)\cos\left(\frac{-5da+5cb}{d}\right)}{d}-\frac{5\,\mathrm{Ci}}{}\right)}{d}\right)}{80}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] 1/b*(1/8*b^2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-a*d+b*c)/d)*cos
((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)/d)-1/80*b^2*(-
5*cos(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d
)-1/48*b^2*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a
*d+b*c)/d)*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d
+b*c)/d)/d)/d))
```

maxima [C] time = 0.57, size = 439, normalized size = 1.71

$$-\frac{1073741824\,b^2\left(E_2\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_2\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^2\left(E_2\left(\frac{3i\,bc+3i\,(bx+a)d-3i}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^2*(exp_integral_e(2, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(2, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^2*(exp_integral_e(2, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^2*(-1073741824*I*exp_integral_e(2, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(2, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^2*(536870912*I*exp_inte
gral_e(2, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(2, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^2*(536870912*I*exp_integral_e(2, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(2, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**2, x)
```

3.152 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=338

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\mathrm{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\mathrm{Ci}\left(\frac{5bc}{d}+5bx\right)}{32d^3}+\frac{b^2\sin\left(a\right.}{}$$

```
[Out] 25/32*b^2*Ci(5*b*c/d+5*b*x)*cos(5*a-5*b*c/d)/d^3+9/32*b^2*Ci(3*b*c/d+3*b*x)
*cos(3*a-3*b*c/d)/d^3-1/16*b^2*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^3-1/16*cos(b*x+
a)/d/(d*x+c)^2+1/32*cos(3*b*x+3*a)/d/(d*x+c)^2+1/32*cos(5*b*x+5*a)/d/(d*x+c
)^2-25/32*b^2*Si(5*b*c/d+5*b*x)*sin(5*a-5*b*c/d)/d^3-9/32*b^2*Si(3*b*c/d+3*
b*x)*sin(3*a-3*b*c/d)/d^3+1/16*b^2*Si(b*c/d+b*x)*sin(a-b*c/d)/d^3+1/16*b*si
n(b*x+a)/d^2/(d*x+c)-3/32*b*sin(3*b*x+3*a)/d^2/(d*x+c)-5/32*b*sin(5*b*x+5*a
)/d^2/(d*x+c)
```

Rubi [A] time = 0.44, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{CosIntegral}\left(\frac{bc}{d}+bx\right)}{16d^3}+\frac{9b^2\cos\left(3a-\frac{3bc}{d}\right)\mathrm{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^3}+\frac{25b^2\cos\left(5a-\frac{5bc}{d}\right)\mathrm{CosI}}{32d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] -Cos[a + b*x]/(16*d*(c + d*x)^2) + Cos[3*a + 3*b*x]/(32*d*(c + d*x)^2) + Co
s[5*a + 5*b*x]/(32*d*(c + d*x)^2) - (b^2*Cos[a - (b*c)/d]*CosIntegral[(b*c)
/d + b*x])/(16*d^3) + (9*b^2*Cos[3*a - (3*b*c)/d]*CosIntegral[(3*b*c)/d + 3
*b*x])/(32*d^3) + (25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[(5*b*c)/d + 5*b*
x])/(32*d^3) + (b*Sin[a + b*x])/(16*d^2*(c + d*x)) - (3*b*Sin[3*a + 3*b*x])
/(32*d^2*(c + d*x)) - (5*b*Sin[5*a + 5*b*x])/(32*d^2*(c + d*x)) + (b^2*Sin[
a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/(16*d^3) - (9*b^2*Sin[3*a - (3*b*c
)/d]*SinIntegral[(3*b*c)/d + 3*b*x])/(32*d^3) - (25*b^2*Sin[5*a - (5*b*c)/d
]*SinIntegral[(5*b*c)/d + 5*b*x])/(32*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^3} - \frac{\cos(3a+3bx)}{16(c+dx)^3} - \frac{\cos(5a+5bx)}{16(c+dx)^3}\right) dx$$
$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^3}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^3}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^3}\,dx$$
$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx}{16d} + \frac{(3b)\int}{}$$
$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a}{32d^2(c}$$
$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} + \frac{b\sin(a+bx)}{16d^2(c+dx)} - \frac{3b\sin(3a}{32d^2(c}$$
$$= -\frac{\cos(a+bx)}{16d(c+dx)^2} + \frac{\cos(3a+3bx)}{32d(c+dx)^2} + \frac{\cos(5a+5bx)}{32d(c+dx)^2} - \frac{b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+b\right.}{16d^3}$$

Mathematica [A] time = 3.37, size = 283, normalized size = 0.84

$$\frac{-2b^2\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(b\left(\frac{c}{d}+x\right)\right)+9b^2\cos\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+25b^2\cos\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5b(c+dx)}{d}\right)+2b^2\sin\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^3,x]
```

```
[Out] ((d^2*Cos[3*(a + b*x)])/(c + d*x)^2 + (d^2*Cos[5*(a + b*x)])/(c + d*x)^2 -
2*b^2*Cos[a - (b*c)/d]*CosIntegral[b*(c/d + x)] + 9*b^2*Cos[3*a - (3*b*c)/d
]*CosIntegral[(3*b*(c + d*x))/d] + 25*b^2*Cos[5*a - (5*b*c)/d]*CosIntegral[
(5*b*(c + d*x))/d] + (2*d*(-(d*Cos[a + b*x]) + b*(c + d*x)*Sin[a + b*x]))/(
c + d*x)^2 - (3*b*d*Sin[3*(a + b*x)])/(c + d*x) - (5*b*d*Sin[5*(a + b*x)])/
(c + d*x) + 2*b^2*Sin[a - (b*c)/d]*SinIntegral[b*(c/d + x)] - 9*b^2*Sin[3*a
 - (3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d] - 25*b^2*Sin[5*a - (5*b*c)/d]*
SinIntegral[(5*b*(c + d*x))/d])/(32*d^3)
```

fricas [A] time = 0.54, size = 567, normalized size = 1.68

$$\frac{32\,d^2\cos(bx+a)^5 - 32\,d^2\cos(bx+a)^3 - 50\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{5\,(bc-ad)}{d}\right)\text{Si}\left(\frac{5\,(bdx+bc)}{d}\right) - 18\left(b^2d^2x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(32*d^2*cos(b*x + a)^5 - 32*d^2*cos(b*x + a)^3 - 50*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*sin(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) -
 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-3*(b*c - a*d)/d)*sin_integra
l(3*(b*d*x + b*c)/d) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-(b*c -
a*d)/d)*sin_integral((b*d*x + b*c)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*cos_integral((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*cos_integral(-(b*d*x + b*c)/d))*cos(-(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos_integral(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos_integral(-3*(b*d*x + b*c)/d))*cos(-3*(b*c - a*d)/d) +
 25*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(5*(b*d*x + b*c)/d)
+ (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-5*(b*d*x + b*c)/d))*c
os(-5*(b*c - a*d)/d) - 32*(5*(b*d^2*x + b*c*d)*cos(b*x + a)^4 - 3*(b*d^2*x
+ b*c*d)*cos(b*x + a)^2)*sin(b*x + a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 473, normalized size = 1.40

$$\frac{b^3\left(-\frac{\cos(bx+a)}{2((bx+a)d-da+cb)^2 d}-\frac{-\frac{\sin(bx+a)}{((bx+a)d-da+cb)d}+\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}+\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}\right)}{8}-\frac{b^3\left(-\frac{5\cos(5bx+5a)}{2((bx+a)d-da+cb)^2 d}-\frac{5\left(-\frac{5\sin(5bx+5a)}{((bx+a)d-da+cb)d}\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x)
```

```
[Out] 1/b*(1/8*b^3*(-1/2*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^2/d-1/2*(-sin(b*x+a)/((b*
x+a)*d-d*a+c*b)/d+(Si(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d+Ci(b*x+a+(-a*
d+b*c)/d)*cos((-a*d+b*c)/d)/d)/d)/d)-1/80*b^3*(-5/2*cos(5*b*x+5*a)/((b*x+a)
*d-d*a+c*b)^2/d-5/2*(-5*sin(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)/d+5*(5*Si(5*b*x+
5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d+5*Ci(5*b*x+5*a+5*(-a*d+b*c)/d)*co
s(5*(-a*d+b*c)/d)/d)/d)/d)-1/48*b^3*(-3/2*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b
```

```
)^2/d-3/2*(-3*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d+3*(3*Si(3*b*x+3*a+3*(-a*
d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d+3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*cos(3*(-a*d+
b*c)/d)/d)/d)/d))
```

maxima [C] time = 1.37, size = 474, normalized size = 1.40

$$-\frac{1073741824\,b^3\left(E_3\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_3\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^3\left(E_3\left(\frac{3i\,bc+3i\,(bx+a)d-3i}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^3*(exp_integral_e(3, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(3, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
*cos(-3*(b*c - a*d)/d) - 536870912*b^3*(exp_integral_e(3, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^3*(-1073741824*I*exp_integral_e(3, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(3, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^3*(536870912*I*exp_inte
gral_e(3, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(3, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^3*(536870912*I*exp_integral_e(3, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(3, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(a+bx\right)\cos^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**3,x)
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**3, x)
```

3.153 $\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=413

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{48d^4}+\frac{b^3\cos\left(a\right.}{}$$

```
[Out] -1/24*cos(b*x+a)/d/(d*x+c)^3+1/48*b^2*cos(b*x+a)/d^3/(d*x+c)+1/48*cos(3*b*x
+3*a)/d/(d*x+c)^3-3/32*b^2*cos(3*b*x+3*a)/d^3/(d*x+c)+1/48*cos(5*b*x+5*a)/d
/(d*x+c)^3-25/96*b^2*cos(5*b*x+5*a)/d^3/(d*x+c)+1/48*b^3*cos(a-b*c/d)*Si(b*
c/d+b*x)/d^4-9/32*b^3*cos(3*a-3*b*c/d)*Si(3*b*c/d+3*b*x)/d^4-125/96*b^3*cos
(5*a-5*b*c/d)*Si(5*b*c/d+5*b*x)/d^4-125/96*b^3*Ci(5*b*c/d+5*b*x)*sin(5*a-5*
b*c/d)/d^4-9/32*b^3*Ci(3*b*c/d+3*b*x)*sin(3*a-3*b*c/d)/d^4+1/48*b^3*Ci(b*c/
d+b*x)*sin(a-b*c/d)/d^4+1/48*b*sin(b*x+a)/d^2/(d*x+c)^2-1/32*b*sin(3*b*x+3*
a)/d^2/(d*x+c)^2-5/96*b*sin(5*b*x+5*a)/d^2/(d*x+c)^2
```

Rubi [A] time = 0.54, antiderivative size = 413, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{125b^3\sin\left(5a-\frac{5bc}{d}\right)\text{CosIntegral}\left(\frac{5bc}{d}+5bx\right)}{96d^4}-\frac{9b^3\sin\left(3a-\frac{3bc}{d}\right)\text{CosIntegral}\left(\frac{3bc}{d}+3bx\right)}{32d^4}+\frac{b^3\sin\left(a-\frac{bc}{d}\right)\text{Co}}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -Cos[a + b*x]/(24*d*(c + d*x)^3) + (b^2*Cos[a + b*x])/(48*d^3*(c + d*x)) +
Cos[3*a + 3*b*x]/(48*d*(c + d*x)^3) - (3*b^2*Cos[3*a + 3*b*x])/(32*d^3*(c +
 d*x)) + Cos[5*a + 5*b*x]/(48*d*(c + d*x)^3) - (25*b^2*Cos[5*a + 5*b*x])/(9
6*d^3*(c + d*x)) - (125*b^3*CosIntegral[(5*b*c)/d + 5*b*x]*Sin[5*a - (5*b*c
)/d])/(96*d^4) - (9*b^3*CosIntegral[(3*b*c)/d + 3*b*x]*Sin[3*a - (3*b*c)/d]
)/(32*d^4) + (b^3*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/(48*d^4) + (
b*Sin[a + b*x])/(48*d^2*(c + d*x)^2) - (b*Sin[3*a + 3*b*x])/(32*d^2*(c + d*
x)^2) - (5*b*Sin[5*a + 5*b*x])/(96*d^2*(c + d*x)^2) + (b^3*Cos[a - (b*c)/d]
*SinIntegral[(b*c)/d + b*x])/(48*d^4) - (9*b^3*Cos[3*a - (3*b*c)/d]*SinInte
gral[(3*b*c)/d + 3*b*x])/(32*d^4) - (125*b^3*Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*c)/d + 5*b*x])/(96*d^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
```

```
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^2(a+bx)}{(c+dx)^4}\,dx = \int \left(\frac{\cos(a+bx)}{8(c+dx)^4} - \frac{\cos(3a+3bx)}{16(c+dx)^4} - \frac{\cos(5a+5bx)}{16(c+dx)^4}\right) dx$$

$$= -\left(\frac{1}{16}\int \frac{\cos(3a+3bx)}{(c+dx)^4}\,dx\right) - \frac{1}{16}\int \frac{\cos(5a+5bx)}{(c+dx)^4}\,dx + \frac{1}{8}\int \frac{\cos(a+bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} - \frac{b\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx}{24d} + \frac{b\int \frac{\sin(}{(}}{}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} + \frac{\cos(5a+5bx)}{48d(c+dx)^3} + \frac{b\sin(a+bx)}{48d^2(c+dx)^2} - \frac{b\sin(3a}{32d^2(c}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos}{48}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos}{48}$$

$$= -\frac{\cos(a+bx)}{24d(c+dx)^3} + \frac{b^2\cos(a+bx)}{48d^3(c+dx)} + \frac{\cos(3a+3bx)}{48d(c+dx)^3} - \frac{3b^2\cos(3a+3bx)}{32d^3(c+dx)} + \frac{\cos}{48}$$

Mathematica [A] time = 3.40, size = 451, normalized size = 1.09

$$-\frac{27b^3(c+dx)^3\left(\sin\left(3a-\frac{3bc}{d}\right)\text{Ci}\left(\frac{3b(c+dx)}{d}\right)+\cos\left(3a-\frac{3bc}{d}\right)\text{Si}\left(\frac{3b(c+dx)}{d}\right)\right)+125b^3(c+dx)^3\left(\sin\left(5a-\frac{5bc}{d}\right)\text{Ci}\left(\frac{5}{}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^2)/(c + d*x)^4,x]
```

```
[Out] -1/96*(d*Cos[3*b*x]*((-2*d^2 + 9*b^2*(c + d*x)^2)*Cos[3*a] + 3*b*d*(c + d*x
)*Sin[3*a]) + d*Cos[5*b*x]*((-2*d^2 + 25*b^2*(c + d*x)^2)*Cos[5*a] + 5*b*d*
(c + d*x)*Sin[5*a]) + d*(3*b*d*(c + d*x)*Cos[3*a] - (-2*d^2 + 9*b^2*(c + d*
x)^2)*Sin[3*a])*Sin[3*b*x] + d*(5*b*d*(c + d*x)*Cos[5*a] - (-2*d^2 + 25*b^2
*(c + d*x)^2)*Sin[5*a])*Sin[5*b*x] - 2*(d*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)
^2)*Cos[a] + b*d*(c + d*x)*Sin[a]) + d*(b*d*(c + d*x)*Cos[a] - (-2*d^2 + b^
2*(c + d*x)^2)*Sin[a])*Sin[b*x] + b^3*(c + d*x)^3*(CosIntegral[b*(c/d + x)]
*Sin[a - (b*c)/d] + Cos[a - (b*c)/d]*SinIntegral[b*(c/d + x)])) + 27*b^3*(c
 + d*x)^3*(CosIntegral[(3*b*(c + d*x))/d]*Sin[3*a - (3*b*c)/d] + Cos[3*a -
(3*b*c)/d]*SinIntegral[(3*b*(c + d*x))/d]) + 125*b^3*(c + d*x)^3*(CosIntegr
al[(5*b*(c + d*x))/d]*Sin[5*a - (5*b*c)/d] + Cos[5*a - (5*b*c)/d]*SinIntegr
al[(5*b*(c + d*x))/d]))/(d^4*(c + d*x)^3)
```

fricas [B] time = 0.63, size = 811, normalized size = 1.96

$$-\frac{32\left(25\,b^2d^3x^2+50\,b^2cd^2x+25\,b^2c^2d-2\,d^3\right)\cos\left(bx+a\right)^5-32\left(29\,b^2d^3x^2+58\,b^2cd^2x+29\,b^2c^2d-2\,d^3\right)\cos\left(b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/192*(32*(25*b^2*d^3*x^2 + 50*b^2*c*d^2*x + 25*b^2*c^2*d - 2*d^3)*cos(b*x
 + a)^5 - 32*(29*b^2*d^3*x^2 + 58*b^2*c*d^2*x + 29*b^2*c^2*d - 2*d^3)*cos(b
*x + a)^3 + 250*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*c
os(-5*(b*c - a*d)/d)*sin_integral(5*(b*d*x + b*c)/d) + 54*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-3*(b*c - a*d)/d)*sin_integral
(3*(b*d*x + b*c)/d) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*cos(-(b*c - a*d)/d)*sin_integral((b*d*x + b*c)/d) + 192*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a) + 32*(5*(b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - 3*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*sin(b*x + a) - 2*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral((b*d*x + b
*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_inte
gral(-(b*d*x + b*c)/d))*sin(-(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^
2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(3*(b*d*x + b*c)/d) + (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-3*(b*d*x +
b*c)/d))*sin(-3*(b*c - a*d)/d) + 125*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*cos_integral(5*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-5*(b*d*x + b*c)/d))*sin
(-5*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 583, normalized size = 1.41

$$\frac{b^{4}\left(-\frac{\cos(bx+a)}{3((bx+a)d-da+cb)^{3}d}-\frac{-\frac{\sin(bx+a)}{2((bx+a)d-da+cb)^{2}d}+\frac{-\frac{\cos(bx+a)}{((bx+a)d-da+cb)d}-\frac{\frac{\mathrm{Si}\left(bx+a+\frac{-da+cb}{d}\right)\cos\left(\frac{-da+cb}{d}\right)}{d}-\frac{\mathrm{Ci}\left(bx+a+\frac{-da+cb}{d}\right)\sin\left(\frac{-da+cb}{d}\right)}{d}}{d}}{2d}}{3d}\right)}{8}-\frac{b^{4}\left(-\frac{5\cos(5bx+5a)}{3((bx+a)d-da+cb)^{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x)
```

```
[Out] 1/b*(1/8*b^4*(-1/3*cos(b*x+a)/((b*x+a)*d-d*a+c*b)^3/d-1/3*(-1/2*sin(b*x+a)/
((b*x+a)*d-d*a+c*b)^2/d+1/2*(-cos(b*x+a)/((b*x+a)*d-d*a+c*b)/d-(Si(b*x+a+(-
a*d+b*c)/d)*cos((-a*d+b*c)/d)/d-Ci(b*x+a+(-a*d+b*c)/d)*sin((-a*d+b*c)/d)/d)
/d)/d)/d)-1/80*b^4*(-5/3*cos(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^3/d-5/3*(-5/2*s
in(5*b*x+5*a)/((b*x+a)*d-d*a+c*b)^2/d+5/2*(-5*cos(5*b*x+5*a)/((b*x+a)*d-d*a
+c*b)/d-5*(5*Si(5*b*x+5*a+5*(-a*d+b*c)/d)*cos(5*(-a*d+b*c)/d)/d-5*Ci(5*b*x+
5*a+5*(-a*d+b*c)/d)*sin(5*(-a*d+b*c)/d)/d)/d)/d)/d)-1/48*b^4*(-cos(3*b*x+3*
a)/((b*x+a)*d-d*a+c*b)^3/d-(-3/2*sin(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)^2/d+3/2
*(-3*cos(3*b*x+3*a)/((b*x+a)*d-d*a+c*b)/d-3*(3*Si(3*b*x+3*a+3*(-a*d+b*c)/d)
*cos(3*(-a*d+b*c)/d)/d-3*Ci(3*b*x+3*a+3*(-a*d+b*c)/d)*sin(3*(-a*d+b*c)/d)/d
)/d)/d)/d))
```

maxima [C] time = 1.27, size = 524, normalized size = 1.27

$$-\frac{1073741824\,b^{4}\left(E_{4}\left(\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)+E_{4}\left(-\frac{i\,bc+i\,(bx+a)d-i\,ad}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)-536870912\,b^{4}\left(E_{4}\left(\frac{3i\,bc+3i\,(bx+a)d-3i}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/17179869184*(1073741824*b^4*(exp_integral_e(4, (I*b*c + I*(b*x + a)*d -
I*a*d)/d) + exp_integral_e(4, -(I*b*c + I*(b*x + a)*d - I*a*d)/d))*cos(-(b*
c - a*d)/d) - 536870912*b^4*(exp_integral_e(4, (3*I*b*c + 3*I*(b*x + a)*d -
 3*I*a*d)/d) + exp_integral_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))
```

```
*cos(-3*(b*c - a*d)/d) - 536870912*b^4*(exp_integral_e(4, (5*I*b*c + 5*I*(b
*x + a)*d - 5*I*a*d)/d) + exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5
*I*a*d)/d))*cos(-5*(b*c - a*d)/d) + b^4*(-1073741824*I*exp_integral_e(4, (I
*b*c + I*(b*x + a)*d - I*a*d)/d) + 1073741824*I*exp_integral_e(4, -(I*b*c +
 I*(b*x + a)*d - I*a*d)/d))*sin(-(b*c - a*d)/d) + b^4*(536870912*I*exp_inte
gral_e(4, (3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d) - 536870912*I*exp_integr
al_e(4, -(3*I*b*c + 3*I*(b*x + a)*d - 3*I*a*d)/d))*sin(-3*(b*c - a*d)/d) +
b^4*(536870912*I*exp_integral_e(4, (5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d)
 - 536870912*I*exp_integral_e(4, -(5*I*b*c + 5*I*(b*x + a)*d - 5*I*a*d)/d))
*sin(-5*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2}{\left(c+d\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^2)/(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\cos^3(a+bx)}{(c+dx)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(sin(a + b*x)**2*cos(a + b*x)**3/(c + d*x)**4, x)
```

3.154 $\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=285

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m\right.}{b}$$

```
[Out] -3*2^(-7-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I
*b*(d*x+c)/d)^m)-3*2^(-7-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*
(a-b*c/d))/((I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*exp(6*I*(a-b*c/d))*(d*x+c)
^m*GAMMA(1+m,-6*I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+2^(-7-m)*3^(-1-m)*(d*
x+c)^m*GAMMA(1+m,6*I*b*(d*x+c)/d)/b/exp(6*I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.32, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3308, 2181}

$$-\frac{3\,2^{-m-7}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-7}3^{-m-1}e^{6i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*2^(-7 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(
c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) - (3*2^(-7 - m)*(c + d*x)^m*Gamm
a[1 + m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x)
)/d)^m) + (2^(-7 - m)*3^(-1 - m)*E^((6*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[
1 + m, ((-6*I)*b*(c + d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-7 - m)*
3^(-1 - m)*(c + d*x)^m*Gamma[1 + m, ((6*I)*b*(c + d*x))/d])/(b*E^((6*I)*(a
- (b*c)/d))*((I*b*(c + d*x))/d)^m)
```

Rule 2181

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> -Simp[(F^(g*(e - (c*f)/d))*(c + d*x)^FracPart[m]*Gamma[m + 1, (-((f*g*Lo
g[F])/d))*(c + d*x)])/(d*(-((f*g*Log[F])/d))^(IntPart[m] + 1)*(-((f*g*Log[F
]*(c + d*x))/d))^FracPart[m]), x] /; FreeQ[{F, c, d, e, f, g, m}, x] && !I
ntegerQ[m]
```

Rule 3308

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
```

```
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^m \cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^m \sin(2a+2bx) - \frac{1}{32}(c+dx)^m \sin(6a+6bx)\right) dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^m \sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^m \sin(2a+2bx)$$

$$= -\left(\frac{1}{64} i \int e^{-i(6a+6bx)}(c+dx)^m\,dx\right) + \frac{1}{64} i \int e^{i(6a+6bx)}(c+dx)^m\,dx + \frac{3}{64}$$

$$= -\frac{3\,2^{-7-m} e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m \left(-\frac{ib(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b} - \frac{3\,2^{-7}}{}$$

Mathematica [A] time = 3.34, size = 255, normalized size = 0.89

$$\frac{2^{-m-7}3^{-m-1}e^{-\frac{6i(ad+bc)}{d}}(c+dx)^m \left(\frac{b^2(c+dx)^2}{d^2}\right)^{-m} \left(-3^{m+2}e^{4ia+\frac{8ibc}{d}}\left(-\frac{ib(c+dx)}{d}\right)^m \Gamma\left(m+1,\frac{2ib(c+dx)}{d}\right) - 3^{m+2}e^{4i\left(2a+\frac{bc}{d}\right)}\left(\frac{ib(c+d}{d}\right.\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (2^(-7 - m)*3^(-1 - m)*(c + d*x)^m*(-(3^(2 + m)*E^((4*I)*(2*a + (b*c)/d))*(
(I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-2*I)*b*(c + d*x))/d]) - 3^(2 + m)*E^((
4*I)*a + ((8*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((2*I)*b*(c +
 d*x))/d] + E^((12*I)*a)*((I*b*(c + d*x))/d)^m*Gamma[1 + m, ((-6*I)*b*(c +
d*x))/d] + E^(((12*I)*b*c)/d)*(((-I)*b*(c + d*x))/d)^m*Gamma[1 + m, ((6*I)*
b*(c + d*x))/d]))/(b*E^(((6*I)*(b*c + a*d))/d)*((b^2*(c + d*x)^2)/d^2)^m)
```

fricas [A] time = 0.50, size = 184, normalized size = 0.65

$$\frac{e^{\left(-\frac{dm\log\left(\frac{6ib}{d}\right)-6ibc+6iad}{d}\right)}\Gamma\left(m+1,\frac{6ibdx+6ibc}{d}\right) - 9e^{\left(-\frac{dm\log\left(\frac{2ib}{d}\right)-2ibc+2iad}{d}\right)}\Gamma\left(m+1,\frac{2ibdx+2ibc}{d}\right) - 9e^{\left(-\frac{dm\log\left(-\frac{2ib}{d}\right)+2ibc-2iad}{d}\right)}\Gamma\left(\right.}{384b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(e^(-(d*m*log(6*I*b/d) - 6*I*b*c + 6*I*a*d)/d)*gamma(m + 1, (6*I*b*d*
x + 6*I*b*c)/d) - 9*e^(-(d*m*log(2*I*b/d) - 2*I*b*c + 2*I*a*d)/d)*gamma(m +
 1, (2*I*b*d*x + 2*I*b*c)/d) - 9*e^(-(d*m*log(-2*I*b/d) + 2*I*b*c - 2*I*a*d
)/d)*gamma(m + 1, (-2*I*b*d*x - 2*I*b*c)/d) + e^(-(d*m*log(-6*I*b/d) + 6*I*
b*c - 6*I*a*d)/d)*gamma(m + 1, (-6*I*b*d*x - 6*I*b*c)/d))/b
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

maple [F] time = 0.25, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cos^3(bx+a)\right)\left(\sin^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cos(bx+a)^3 \sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m*cos(b*x + a)^3*sin(b*x + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3 \sin(a+b\,x)^3 (c+d\,x)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^m, x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.155 $\int (c+dx)^4 \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=233

$$-\frac{9d^4\cos(2a+2bx)}{128b^5}+\frac{d^4\cos(6a+6bx)}{10368b^5}-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos}{64b^3}$$

```
[Out] -9/128*d^4*cos(2*b*x+2*a)/b^5+9/64*d^2*(d*x+c)^2*cos(2*b*x+2*a)/b^3-3/64*(d
*x+c)^4*cos(2*b*x+2*a)/b+1/10368*d^4*cos(6*b*x+6*a)/b^5-1/576*d^2*(d*x+c)^2
*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^4*cos(6*b*x+6*a)/b-9/64*d^3*(d*x+c)*sin(2
*b*x+2*a)/b^4+3/32*d*(d*x+c)^3*sin(2*b*x+2*a)/b^2+1/1728*d^3*(d*x+c)*sin(6*
b*x+6*a)/b^4-1/288*d*(d*x+c)^3*sin(6*b*x+6*a)/b^2
```

Rubi [A] time = 0.27, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$-\frac{9d^3(c+dx)\sin(2a+2bx)}{64b^4}+\frac{d^3(c+dx)\sin(6a+6bx)}{1728b^4}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{d^2(c+dx)^2\cos(6a+6bx)}{576b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-9*d^4*Cos[2*a + 2*b*x])/(128*b^5) + (9*d^2*(c + d*x)^2*Cos[2*a + 2*b*x])/
(64*b^3) - (3*(c + d*x)^4*Cos[2*a + 2*b*x])/(64*b) + (d^4*Cos[6*a + 6*b*x])
/(10368*b^5) - (d^2*(c + d*x)^2*Cos[6*a + 6*b*x])/(576*b^3) + ((c + d*x)^4*
Cos[6*a + 6*b*x])/(192*b) - (9*d^3*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^4) + (
3*d*(c + d*x)^3*Sin[2*a + 2*b*x])/(32*b^2) + (d^3*(c + d*x)*Sin[6*a + 6*b*x
])/(1728*b^4) - (d*(c + d*x)^3*Sin[6*a + 6*b*x])/(288*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^4\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^4\sin(2a+2bx)-\frac{1}{32}(c+dx)^4\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^4\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^4\sin(2a+2bx)\,d$$

$$= -\frac{3(c+dx)^4\cos(2a+2bx)}{64b}+\frac{(c+dx)^4\cos(6a+6bx)}{192b}-\frac{d\int (c+dx)^3\,c}{4}$$

$$= -\frac{3(c+dx)^4\cos(2a+2bx)}{64b}+\frac{(c+dx)^4\cos(6a+6bx)}{192b}+\frac{3d(c+dx)^3\,\mathrm{si}}{32}$$

$$= \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)^2}{57}$$

$$= \frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)^2}{57}$$

$$= -\frac{9d^4\cos(2a+2bx)}{128b^5}+\frac{9d^2(c+dx)^2\cos(2a+2bx)}{64b^3}-\frac{3(c+dx)^4\cos(2a}{64b}$$

Mathematica [A] time = 1.54, size = 153, normalized size = 0.66

$$\frac{-12bd(c+dx)\sin(2(a+bx))\left(\cos(4(a+bx))\left(6b^2(c+dx)^2-d^2\right)-78b^2(c+dx)^2+121d^2\right)-243\cos(2(a+bx))\Big(}{10368b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-243*(3*d^4 - 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Cos[2*(a + b*x)]
+ (d^4 - 18*b^2*d^2*(c + d*x)^2 + 54*b^4*(c + d*x)^4)*Cos[6*(a + b*x)] - 12
*b*d*(c + d*x)*(121*d^2 - 78*b^2*(c + d*x)^2 + (-d^2 + 6*b^2*(c + d*x)^2)*C
os[4*(a + b*x)])*Sin[2*(a + b*x)])/(10368*b^5)
```

fricas [B] time = 0.50, size = 546, normalized size = 2.34

$$\frac{27\,b^4d^4x^4+108\,b^4cd^3x^3+2\left(54\,b^4d^4x^4+216\,b^4cd^3x^3+54\,b^4c^4-18\,b^2c^2d^2+d^4+18\left(18\,b^4c^2d^2-b^2d^4\right)x^2+36\Big(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 2*(54*b^4*d^4*x^4 + 216*b^4*c*d
^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2 - b^2*d^4)*
x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^6 - 3*(54*b^4*d^4*x^4 +
216*b^4*c*d^3*x^3 + 54*b^4*c^4 - 18*b^2*c^2*d^2 + d^4 + 18*(18*b^4*c^2*d^2
- b^2*d^4)*x^2 + 36*(6*b^4*c^3*d - b^2*c*d^3)*x)*cos(b*x + a)^4 + 18*(9*b^4
*c^2*d^2 - 5*b^2*d^4)*x^2 + 18*(9*b^2*d^4*x^2 + 18*b^2*c*d^3*x + 9*b^2*c^2*
d^2 - 5*d^4)*cos(b*x + a)^2 + 36*(3*b^4*c^3*d - 5*b^2*c*d^3)*x - 12*((6*b^3
*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b*c*d^3 + (18*b^3*c^2*d^2 - b*d
^4)*x)*cos(b*x + a)^5 - (6*b^3*d^4*x^3 + 18*b^3*c*d^3*x^2 + 6*b^3*c^3*d - b
*c*d^3 + (18*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^3 - 3*(3*b^3*d^4*x^3 + 9*
b^3*c*d^3*x^2 + 3*b^3*c^3*d - 5*b*c*d^3 + (9*b^3*c^2*d^2 - 5*b*d^4)*x)*cos(
b*x + a))*sin(b*x + a))/b^5
```

giac [A] time = 1.13, size = 359, normalized size = 1.54

$$\frac{\left(54\,b^4d^4x^4+216\,b^4cd^3x^3+324\,b^4c^2d^2x^2+216\,b^4c^3dx+54\,b^4c^4-18\,b^2d^4x^2-36\,b^2cd^3x-18\,b^2c^2d^2+d^4\right)\cos(}{10368\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/10368*(54*b^4*d^4*x^4 + 216*b^4*c*d^3*x^3 + 324*b^4*c^2*d^2*x^2 + 216*b^4
*c^3*d*x + 54*b^4*c^4 - 18*b^2*d^4*x^2 - 36*b^2*c*d^3*x - 18*b^2*c^2*d^2 +
d^4)*cos(6*b*x + 6*a)/b^5 - 3/128*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4
*c^2*d^2*x^2 + 8*b^4*c^3*d*x + 2*b^4*c^4 - 6*b^2*d^4*x^2 - 12*b^2*c*d^3*x -
 6*b^2*c^2*d^2 + 3*d^4)*cos(2*b*x + 2*a)/b^5 - 1/1728*(6*b^3*d^4*x^3 + 18*b
^3*c*d^3*x^2 + 18*b^3*c^2*d^2*x + 6*b^3*c^3*d - b*d^4*x - b*c*d^3)*sin(6*b*
x + 6*a)/b^5 + 3/64*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 6*b^3*c^2*d^2*x + 2*
b^3*c^3*d - 3*b*d^4*x - 3*b*c*d^3)*sin(2*b*x + 2*a)/b^5
```

maple [B] time = 0.13, size = 2061, normalized size = 8.85

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^4*d^4*(1/4*(b*x+a)^4*sin(b*x+a)^4-(b*x+a)^3*(-1/4*(sin(b*x+a)^3+3/
2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/12*(b*x+a)^2*sin(b*x+a)^4+1/6*(b*
x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/9*(b*x
+a)^2+1/216*sin(b*x+a)^4+5/36*sin(b*x+a)^2+1/4*(b*x+a)^2*cos(b*x+a)^2-1/2*(
b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/8*(b*x+a)^4-1/6*(b*x+a)^
4*sin(b*x+a)^6+2/3*(b*x+a)^3*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(
```

```
b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/18*(b*x+a)^2*sin(b*x+a)^6-1/9*(b*x+a)
*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+
5/16*a)-1/324*sin(b*x+a)^6)-4/b^4*a*d^4*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*
x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(
b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-
1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1
/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^
3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/
216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b^3*c*d^3
*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+
a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*x+a)^2-1/16*co
s(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6+1/2*(b*x+a)^2
*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+
5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8
*sin(b*x+a))*cos(b*x+a))+6/b^4*a^2*d^4*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x
+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x
+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b
*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16
*b*x+5/16*a)+1/108*sin(b*x+a)^6)-12/b^3*a*c*d^3*(1/4*(b*x+a)^2*sin(b*x+a)^4
-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+
1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)
^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*
x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+6/b^2*c^2*d^2*(1/4*(b*x+a)^2*sin(
b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x
+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*si
n(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a)
)*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)-4/b^4*a^3*d^4*(1/4*(b*x+a
)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*
a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x
+a))*cos(b*x+a))+12/b^3*a^2*c*d^3*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a
)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/3
6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-12/b^2*a*c^2*
d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)
-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)
^3+15/8*sin(b*x+a))*cos(b*x+a))+4/b*c^3*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(s
in(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+
a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+1/b^4
*a^4*d^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b^3*a^3*c*d^3
*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+6/b^2*a^2*c^2*d^2*(-1/6
*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-4/b*a*c^3*d*(-1/6*sin(b*x+a)^
2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^4*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*
cos(b*x+a)^4))
```

maxima [B] time = 0.39, size = 1033, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/10368*(864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^4 - 3456*(2*sin(b*x +
 a)^6 - 3*sin(b*x + a)^4)*a*c^3*d/b + 5184*(2*sin(b*x + a)^6 - 3*sin(b*x +
a)^4)*a^2*c^2*d^2/b^2 - 3456*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*c*d^
3/b^3 + 864*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^4*d^4/b^4 - 36*(6*(b*x
+ a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) +
27*sin(2*b*x + 2*a))*c^3*d/b + 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x
+ a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a*c^2*d^2/b
^2 - 108*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - si
n(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*c*d^3/b^3 + 36*(6*(b*x + a)*cos(6
*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b
*x + 2*a))*a^3*d^4/b^4 - 18*((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*
(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x
 + a)*sin(2*b*x + 2*a))*c^2*d^2/b^2 + 36*((18*(b*x + a)^2 - 1)*cos(6*b*x +
6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*
a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/b^3 - 18*((18*(b*x + a)^2 - 1)
*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*s
in(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^4/b^4 - 6*(6*(6*(b*
x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*co
s(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2
 - 1)*sin(2*b*x + 2*a))*c*d^3/b^3 + 6*(6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*
x + 6*a) - 162*(2*(b*x + a)^3 - 3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x +
a)^2 - 1)*sin(6*b*x + 6*a) + 243*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*a*d^
4/b^4 - ((54*(b*x + a)^4 - 18*(b*x + a)^2 + 1)*cos(6*b*x + 6*a) - 243*(2*(b
*x + a)^4 - 6*(b*x + a)^2 + 3)*cos(2*b*x + 2*a) - 6*(6*(b*x + a)^3 - b*x -
a)*sin(6*b*x + 6*a) + 486*(2*(b*x + a)^3 - 3*b*x - 3*a)*sin(2*b*x + 2*a))*d
^4/b^4)/b
```

mupad [B] time = 2.58, size = 576, normalized size = 2.47

$$-\frac{729\, d^4\, \cos\left(2\, a+2\, b\, x\right)-d^4\, \cos\left(6\, a+6\, b\, x\right)+486\, b^4\, c^4\, \cos\left(2\, a+2\, b\, x\right)-54\, b^4\, c^4\, \cos\left(6\, a+6\, b\, x\right)-972\, b^3\, c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] -(729*d^4*cos(2*a + 2*b*x) - d^4*cos(6*a + 6*b*x) + 486*b^4*c^4*cos(2*a + 2
*b*x) - 54*b^4*c^4*cos(6*a + 6*b*x) - 972*b^3*c^3*d*sin(2*a + 2*b*x) + 36*b
```

```
^3*c^3*d*sin(6*a + 6*b*x) - 1458*b^2*c^2*d^2*cos(2*a + 2*b*x) + 18*b^2*c^2*
d^2*cos(6*a + 6*b*x) - 1458*b^2*d^4*x^2*cos(2*a + 2*b*x) + 486*b^4*d^4*x^4*
cos(2*a + 2*b*x) + 18*b^2*d^4*x^2*cos(6*a + 6*b*x) - 54*b^4*d^4*x^4*cos(6*a
 + 6*b*x) - 972*b^3*d^4*x^3*sin(2*a + 2*b*x) + 36*b^3*d^4*x^3*sin(6*a + 6*b
*x) + 1458*b*c*d^3*sin(2*a + 2*b*x) - 6*b*c*d^3*sin(6*a + 6*b*x) + 1458*b*d
^4*x*sin(2*a + 2*b*x) - 6*b*d^4*x*sin(6*a + 6*b*x) + 2916*b^4*c^2*d^2*x^2*c
os(2*a + 2*b*x) - 324*b^4*c^2*d^2*x^2*cos(6*a + 6*b*x) - 2916*b^2*c*d^3*x*c
os(2*a + 2*b*x) + 1944*b^4*c^3*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d^3*x*cos(6*
a + 6*b*x) - 216*b^4*c^3*d*x*cos(6*a + 6*b*x) + 1944*b^4*c*d^3*x^3*cos(2*a
+ 2*b*x) - 216*b^4*c*d^3*x^3*cos(6*a + 6*b*x) - 2916*b^3*c^2*d^2*x*sin(2*a
+ 2*b*x) - 2916*b^3*c*d^3*x^2*sin(2*a + 2*b*x) + 108*b^3*c^2*d^2*x*sin(6*a
+ 6*b*x) + 108*b^3*c*d^3*x^2*sin(6*a + 6*b*x))/(10368*b^5)
```

sympy [A] time = 31.30, size = 1334, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**4*sin(a + b*x)**6/(12*b) + c**4*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c**3*d*x*sin(a + b*x)**6/(6*b) + c**3*d*x*sin(a + b*x)**4*cos(a
+ b*x)**2/(2*b) - c**3*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c**3*d*x
*cos(a + b*x)**6/(6*b) + c**2*d**2*x**2*sin(a + b*x)**6/(4*b) + 3*c**2*d**2
*x**2*sin(a + b*x)**4*cos(a + b*x)**2/(4*b) - 3*c**2*d**2*x**2*sin(a + b*x)
**2*cos(a + b*x)**4/(4*b) - c**2*d**2*x**2*cos(a + b*x)**6/(4*b) + c*d**3*x
**3*sin(a + b*x)**6/(6*b) + c*d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(2*
b) - c*d**3*x**3*sin(a + b*x)**2*cos(a + b*x)**4/(2*b) - c*d**3*x**3*cos(a
+ b*x)**6/(6*b) + d**4*x**4*sin(a + b*x)**6/(24*b) + d**4*x**4*sin(a + b*x)
**4*cos(a + b*x)**2/(8*b) - d**4*x**4*sin(a + b*x)**2*cos(a + b*x)**4/(8*b)
 - d**4*x**4*cos(a + b*x)**6/(24*b) + c**3*d*sin(a + b*x)**5*cos(a + b*x)/(
6*b**2) + 4*c**3*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c**3*d*sin(a
+ b*x)*cos(a + b*x)**5/(6*b**2) + c**2*d**2*x*sin(a + b*x)**5*cos(a + b*x)/
(2*b**2) + 4*c**2*d**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2) + c**2*d*
*2*x*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + c*d**3*x**2*sin(a + b*x)**5*co
s(a + b*x)/(2*b**2) + 4*c*d**3*x**2*sin(a + b*x)**3*cos(a + b*x)**3/(3*b**2
) + c*d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(2*b**2) + d**4*x**3*sin(a + b
*x)**5*cos(a + b*x)/(6*b**2) + 4*d**4*x**3*sin(a + b*x)**3*cos(a + b*x)**3/
(9*b**2) + d**4*x**3*sin(a + b*x)*cos(a + b*x)**5/(6*b**2) - 7*c**2*d**2*si
n(a + b*x)**6/(36*b**3) - c**2*d**2*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3
) + c**2*d**2*cos(a + b*x)**6/(12*b**3) - 5*c*d**3*x*sin(a + b*x)**6/(18*b*
*3) - c*d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(3*b**3) + c*d**3*x*sin(a +
b*x)**2*cos(a + b*x)**4/(3*b**3) + 5*c*d**3*x*cos(a + b*x)**6/(18*b**3) - 5
*d**4*x**2*sin(a + b*x)**6/(36*b**3) - d**4*x**2*sin(a + b*x)**4*cos(a + b*
x)**2/(6*b**3) + d**4*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(6*b**3) + 5*d**
```

```
4*x**2*cos(a + b*x)**6/(36*b**3) - 5*c*d**3*sin(a + b*x)**5*cos(a + b*x)/(1
8*b**4) - 31*c*d**3*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*c*d**3*si
n(a + b*x)*cos(a + b*x)**5/(18*b**4) - 5*d**4*x*sin(a + b*x)**5*cos(a + b*x
)/(18*b**4) - 31*d**4*x*sin(a + b*x)**3*cos(a + b*x)**3/(54*b**4) - 5*d**4*
x*sin(a + b*x)*cos(a + b*x)**5/(18*b**4) + 61*d**4*sin(a + b*x)**6/(648*b**
5) + 31*d**4*sin(a + b*x)**4*cos(a + b*x)**2/(216*b**5) - 5*d**4*cos(a + b*
x)**6/(108*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 +
c*d**3*x**4 + d**4*x**5/5)*sin(a)**3*cos(a)**3, True))
```

3.156 $\int (c+dx)^3 \cos^3(a+bx) \sin^3(a+bx)\,dx$

Optimal. Leaf size=181

$$-\frac{9d^3\sin(2a+2bx)}{256b^4}+\frac{d^3\sin(6a+6bx)}{6912b^4}+\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a}{128b^2}$$

```
[Out] 9/128*d^2*(d*x+c)*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^3*cos(2*b*x+2*a)/b-1/1152
*d^2*(d*x+c)*cos(6*b*x+6*a)/b^3+1/192*(d*x+c)^3*cos(6*b*x+6*a)/b-9/256*d^3*
sin(2*b*x+2*a)/b^4+9/128*d*(d*x+c)^2*sin(2*b*x+2*a)/b^2+1/6912*d^3*sin(6*b*
x+6*a)/b^4-1/384*d*(d*x+c)^2*sin(6*b*x+6*a)/b^2
```

Rubi [A] time = 0.22, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {4406, 3296, 2637}

$$\frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{d^2(c+dx)\cos(6a+6bx)}{1152b^3}+\frac{9d(c+dx)^2\sin(2a+2bx)}{128b^2}-\frac{d(c+dx)^2\sin(6a+6bx)}{384b^2}-\frac{9d^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (9*d^2*(c + d*x)*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^3*Cos[2*a + 2*b
*x])/(64*b) - (d^2*(c + d*x)*Cos[6*a + 6*b*x])/(1152*b^3) + ((c + d*x)^3*Co
s[6*a + 6*b*x])/(192*b) - (9*d^3*Sin[2*a + 2*b*x])/(256*b^4) + (9*d*(c + d*
x)^2*Sin[2*a + 2*b*x])/(128*b^2) + (d^3*Sin[6*a + 6*b*x])/(6912*b^4) - (d*(
c + d*x)^2*Sin[6*a + 6*b*x])/(384*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^3\sin(2a+2bx)-\frac{1}{32}(c+dx)^3\sin(6a+6bx)\right)dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^3\sin(6a+6bx)\,dx\right)+\frac{3}{32}\int (c+dx)^3\sin(2a+2bx)$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}-\frac{d\int (c+dx)^2}{}$$
$$= -\frac{3(c+dx)^3\cos(2a+2bx)}{64b}+\frac{(c+dx)^3\cos(6a+6bx)}{192b}+\frac{9d(c+dx)^2}{12}$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)}{11}$$
$$= \frac{9d^2(c+dx)\cos(2a+2bx)}{128b^3}-\frac{3(c+dx)^3\cos(2a+2bx)}{64b}-\frac{d^2(c+dx)}{11}$$

Mathematica [A] time = 2.31, size = 132, normalized size = 0.73

$$\frac{-324b(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+12b(c+dx)\cos(6(a+bx))\left(6b^2(c+dx)^2-d^2\right)-4d\sin(2(}{13824b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-324*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + 12*b*(c +
 d*x)*(-d^2 + 6*b^2*(c + d*x)^2)*Cos[6*(a + b*x)] - 4*d*(121*d^2 - 234*b^2*
(c + d*x)^2 + (-d^2 + 18*b^2*(c + d*x)^2)*Cos[4*(a + b*x)])*Sin[2*(a + b*x)
])/(13824*b^4)
```

fricas [B] time = 0.46, size = 349, normalized size = 1.93

$$\frac{9b^3d^3x^3+27b^3cd^2x^2+6\left(6b^3d^3x^3+18b^3cd^2x^2+6b^3c^3-bcd^2+\left(18b^3c^2d-bd^3\right)x\right)\cos(bx+a)^6-9\left(6b^3d^3x\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 6*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x
^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^6 - 9*(6*
```

```
b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 6*b^3*c^3 - b*c*d^2 + (18*b^3*c^2*d - b*d^
3)*x)*cos(b*x + a)^4 + 27*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 + 3*(9*b^3*c^2
*d - 5*b*d^3)*x - ((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*c
os(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*cos(
b*x + a)^3 - 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d - 5*d^3)*cos(b
*x + a))*sin(b*x + a))/b^4
```

giac [A] time = 0.38, size = 241, normalized size = 1.33

$$\frac{\left(6\,b^3d^3x^3+18\,b^3cd^2x^2+18\,b^3c^2dx+6\,b^3c^3-bd^3x-bcd^2\right)\cos\left(6\,bx+6\,a\right)}{1152\,b^4}-\frac{3\left(2\,b^3d^3x^3+6\,b^3cd^2x^2+6\,b^3c^2dx+\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/1152*(6*b^3*d^3*x^3 + 18*b^3*c*d^2*x^2 + 18*b^3*c^2*d*x + 6*b^3*c^3 - b*d
^3*x - b*c*d^2)*cos(6*b*x + 6*a)/b^4 - 3/128*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x
^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 3*b*d^3*x - 3*b*c*d^2)*cos(2*b*x + 2*a)/b^
4 - 1/6912*(18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*c^2*d - d^3)*sin(6*b*x
 + 6*a)/b^4 + 9/256*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin
(2*b*x + 2*a)/b^4
```

maple [B] time = 0.02, size = 1100, normalized size = 6.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^3*d^3*(1/4*(b*x+a)^3*sin(b*x+a)^4-3/4*(b*x+a)^2*(-1/4*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)-1/24*(b*x+a)*sin(b*x+a)^4-1/96*
(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/18*b*x-1/18*a+1/8*(b*x+a)*cos(b*
x+a)^2-1/16*cos(b*x+a)*sin(b*x+a)+1/12*(b*x+a)^3-1/6*(b*x+a)^3*sin(b*x+a)^6
+1/2*(b*x+a)^2*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*
x+a)+5/16*b*x+5/16*a)+1/36*(b*x+a)*sin(b*x+a)^6+1/216*(sin(b*x+a)^5+5/4*sin
(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-3/b^3*a*d^3*(1/4*(b*x+a)^2*sin(b*x+a
)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*
a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x
+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos
(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^2*c*d^2*(1/4*(b*x+a)^2*sin
(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*
x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*s
in(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a
))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b*x+a)^6)+3/b^3*a^2*d^3*(1/4*(b*x+
```

```
a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24
*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*
x+a))*cos(b*x+a))-6/b^2*a*c*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^
3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*
(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+3/b*c^2*d*(1/4*
(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x
-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*s
in(b*x+a))*cos(b*x+a))-1/b^3*a^3*d^3*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*c
os(b*x+a)^4)+3/b^2*a^2*c*d^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a
)^4)-3/b*a*c^2*d*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^3*(-1
/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

maxima [B] time = 0.35, size = 602, normalized size = 3.33

$$-\frac{576\left(2\,\sin\left(bx+a\right)^{6}-3\,\sin\left(bx+a\right)^{4}\right)c^{3}-\frac{1728\left(2\,\sin(bx+a)^{6}-3\,\sin(bx+a)^{4}\right)ac^{2}d}{b}+\frac{1728\left(2\,\sin(bx+a)^{6}-3\,\sin(bx+a)^{4}\right)a^{2}cd^{2}}{b^{2}}-\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/6912*(576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^3 - 1728*(2*sin(b*x +
a)^6 - 3*sin(b*x + a)^4)*a*c^2*d/b + 1728*(2*sin(b*x + a)^6 - 3*sin(b*x + a
)^4)*a^2*c*d^2/b^2 - 576*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*a^3*d^3/b^3
- 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*
b*x + 6*a) + 27*sin(2*b*x + 2*a))*c^2*d/b + 36*(6*(b*x + a)*cos(6*b*x + 6*a
) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))
*a*c*d^2/b^2 - 18*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x +
2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*a^2*d^3/b^3 - 6*((18*(b*x +
a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b
*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*c*d^2/b^2 + 6*((
18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x + a)^2 - 1)*cos(2*b*x + 2
*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*sin(2*b*x + 2*a))*a*d^3/
b^3 - (6*(6*(b*x + a)^3 - b*x - a)*cos(6*b*x + 6*a) - 162*(2*(b*x + a)^3 -
3*b*x - 3*a)*cos(2*b*x + 2*a) - (18*(b*x + a)^2 - 1)*sin(6*b*x + 6*a) + 243
*(2*(b*x + a)^2 - 1)*sin(2*b*x + 2*a))*d^3/b^3)/b
```

mupad [B] time = 1.23, size = 366, normalized size = 2.02

$$-\frac{243\,d^{3}\,\sin\left(2\,a+2\,b\,x\right)-d^{3}\,\sin\left(6\,a+6\,b\,x\right)+324\,b^{3}\,c^{3}\,\cos\left(2\,a+2\,b\,x\right)-36\,b^{3}\,c^{3}\,\cos\left(6\,a+6\,b\,x\right)-486\,b^{2}\,c^{2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] -(243*d^3*sin(2*a + 2*b*x) - d^3*sin(6*a + 6*b*x) + 324*b^3*c^3*cos(2*a + 2
*b*x) - 36*b^3*c^3*cos(6*a + 6*b*x) - 486*b^2*c^2*d*sin(2*a + 2*b*x) + 18*b
^2*c^2*d*sin(6*a + 6*b*x) + 324*b^3*d^3*x^3*cos(2*a + 2*b*x) - 36*b^3*d^3*x
^3*cos(6*a + 6*b*x) - 486*b^2*d^3*x^2*sin(2*a + 2*b*x) + 18*b^2*d^3*x^2*sin
(6*a + 6*b*x) - 486*b*c*d^2*cos(2*a + 2*b*x) + 6*b*c*d^2*cos(6*a + 6*b*x) -
 486*b*d^3*x*cos(2*a + 2*b*x) + 6*b*d^3*x*cos(6*a + 6*b*x) + 972*b^3*c^2*d*
x*cos(2*a + 2*b*x) - 108*b^3*c^2*d*x*cos(6*a + 6*b*x) - 972*b^2*c*d^2*x*sin
(2*a + 2*b*x) + 36*b^2*c*d^2*x*sin(6*a + 6*b*x) + 972*b^3*c*d^2*x^2*cos(2*a
 + 2*b*x) - 108*b^3*c*d^2*x^2*cos(6*a + 6*b*x))/(6912*b^4)
```

sympy [A] time = 18.63, size = 857, normalized size = 4.73

$$\begin{cases} \frac{c^3\sin^6(a+bx)}{12b} + \frac{c^3\sin^4(a+bx)\cos^2(a+bx)}{4b} + \frac{c^2dx\sin^6(a+bx)}{8b} + \frac{3c^2dx\sin^4(a+bx)\cos^2(a+bx)}{8b} - \frac{3c^2dx\sin^2(a+bx)\cos^4(a+bx)}{8b} - \frac{c^2dx\cos^6(a}{8b} \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\sin^3(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**3*sin(a + b*x)**6/(12*b) + c**3*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c**2*d*x*sin(a + b*x)**6/(8*b) + 3*c**2*d*x*sin(a + b*x)**4*cos(
a + b*x)**2/(8*b) - 3*c**2*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - c**2
*d*x*cos(a + b*x)**6/(8*b) + c*d**2*x**2*sin(a + b*x)**6/(8*b) + 3*c*d**2*x
**2*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - 3*c*d**2*x**2*sin(a + b*x)**2*c
os(a + b*x)**4/(8*b) - c*d**2*x**2*cos(a + b*x)**6/(8*b) + d**3*x**3*sin(a
+ b*x)**6/(24*b) + d**3*x**3*sin(a + b*x)**4*cos(a + b*x)**2/(8*b) - d**3*x
**3*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d**3*x**3*cos(a + b*x)**6/(24*b
) + c**2*d*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + c**2*d*sin(a + b*x)**3*c
os(a + b*x)**3/(3*b**2) + c**2*d*sin(a + b*x)*cos(a + b*x)**5/(8*b**2) + c*
d**2*x*sin(a + b*x)**5*cos(a + b*x)/(4*b**2) + 2*c*d**2*x*sin(a + b*x)**3*c
os(a + b*x)**3/(3*b**2) + c*d**2*x*sin(a + b*x)*cos(a + b*x)**5/(4*b**2) +
d**3*x**2*sin(a + b*x)**5*cos(a + b*x)/(8*b**2) + d**3*x**2*sin(a + b*x)**3
*cos(a + b*x)**3/(3*b**2) + d**3*x**2*sin(a + b*x)*cos(a + b*x)**5/(8*b**2)
 - 7*c*d**2*sin(a + b*x)**6/(72*b**3) - c*d**2*sin(a + b*x)**4*cos(a + b*x)
**2/(6*b**3) + c*d**2*cos(a + b*x)**6/(24*b**3) - 5*d**3*x*sin(a + b*x)**6/
(72*b**3) - d**3*x*sin(a + b*x)**4*cos(a + b*x)**2/(12*b**3) + d**3*x*sin(a
 + b*x)**2*cos(a + b*x)**4/(12*b**3) + 5*d**3*x*cos(a + b*x)**6/(72*b**3) -
 5*d**3*sin(a + b*x)**5*cos(a + b*x)/(72*b**4) - 31*d**3*sin(a + b*x)**3*co
s(a + b*x)**3/(216*b**4) - 5*d**3*sin(a + b*x)*cos(a + b*x)**5/(72*b**4), N
e(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*sin(a)**3
*cos(a)**3, True))
```

3.157 $\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=129

$$\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}+\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}-\frac{3(c+dx)^2\cos(2a+}{64b}$$

```
[Out] 3/128*d^2*cos(2*b*x+2*a)/b^3-3/64*(d*x+c)^2*cos(2*b*x+2*a)/b-1/3456*d^2*cos
(6*b*x+6*a)/b^3+1/192*(d*x+c)^2*cos(6*b*x+6*a)/b+3/64*d*(d*x+c)*sin(2*b*x+2
*a)/b^2-1/576*d*(d*x+c)*sin(6*b*x+6*a)/b^2
```

Rubi [A] time = 0.14, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {4406, 3296, 2638}

$$\frac{3d(c+dx)\sin(2a+2bx)}{64b^2}-\frac{d(c+dx)\sin(6a+6bx)}{576b^2}+\frac{3d^2\cos(2a+2bx)}{128b^3}-\frac{d^2\cos(6a+6bx)}{3456b^3}-\frac{3(c+dx)^2\cos(2a+}{64b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (3*d^2*Cos[2*a + 2*b*x])/(128*b^3) - (3*(c + d*x)^2*Cos[2*a + 2*b*x])/(64*b
) - (d^2*Cos[6*a + 6*b*x])/(3456*b^3) + ((c + d*x)^2*Cos[6*a + 6*b*x])/(192
*b) + (3*d*(c + d*x)*Sin[2*a + 2*b*x])/(64*b^2) - (d*(c + d*x)*Sin[6*a + 6*
b*x])/(576*b^2)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^2 \cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^2\sin(2a+2bx) - \frac{1}{32}(c+dx)^2\sin(6a+6bx)\right)dx$$
$$= -\left(\frac{1}{32}\int (c+dx)^2\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^2\sin(2a+2bx)\,d$$
$$= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b} - \frac{d\int (c+dx)\,\mathrm{c}}{9}$$
$$= -\frac{3(c+dx)^2\cos(2a+2bx)}{64b} + \frac{(c+dx)^2\cos(6a+6bx)}{192b} + \frac{3d(c+dx)\sin}{64b}$$
$$= \frac{3d^2\cos(2a+2bx)}{128b^3} - \frac{3(c+dx)^2\cos(2a+2bx)}{64b} - \frac{d^2\cos(6a+6bx)}{3456b^3} + -$$

Mathematica [A] time = 0.56, size = 91, normalized size = 0.71

$$\frac{-81\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+\cos(6(a+bx))\left(18b^2(c+dx)^2-d^2\right)-6bd(c+dx)(\sin(6(a+bx))-27\,\mathrm{si}}{3456b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-81*(-d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + (-d^2 + 18*b^2*(c + d*x)
^2)*Cos[6*(a + b*x)] - 6*b*d*(c + d*x)*(-27*Sin[2*(a + b*x)] + Sin[6*(a + b
*x)]))/(3456*b^3)
```

fricas [A] time = 0.47, size = 194, normalized size = 1.50

$$\frac{2\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2\right)\cos\left(bx+a\right)^6+9\,b^2d^2x^2+18\,b^2cdx-3\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/216*(2*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(b*x + a)^6
+ 9*b^2*d^2*x^2 + 18*b^2*c*d*x - 3*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*
c^2 - d^2)*cos(b*x + a)^4 + 9*d^2*cos(b*x + a)^2 - 6*(2*(b*d^2*x + b*c*d)*c
os(b*x + a)^5 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)^3 - 3*(b*d^2*x + b*c*d)*co
s(b*x + a))*sin(b*x + a))/b^3
```

giac [A] time = 0.41, size = 145, normalized size = 1.12

$$\frac{\left(18\,b^2d^2x^2+36\,b^2cdx+18\,b^2c^2-d^2\right)\cos\left(6\,bx+6\,a\right)}{3456\,b^3}-\frac{3\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(2\,bx+2\,a\right)}{128\,b^3}-\frac{\left(bd^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/3456*(18*b^2*d^2*x^2 + 36*b^2*c*d*x + 18*b^2*c^2 - d^2)*cos(6*b*x + 6*a)/
b^3 - 3/128*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(2*b*x + 2*a
)/b^3 - 1/576*(b*d^2*x + b*c*d)*sin(6*b*x + 6*a)/b^3 + 3/64*(b*d^2*x + b*c*
d)*sin(2*b*x + 2*a)/b^3
```

maple [B] time = 0.02, size = 498, normalized size = 3.86

$$\frac{\frac{d^2\left(\frac{(bx+a)^2\left(\sin^4(bx+a)\right)}{4}-\frac{(bx+a)\left(-\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{4}+\frac{3bx}{8}+\frac{3a}{8}\right)}{2}+\frac{(bx+a)^2}{24}-\frac{\left(\sin^4(bx+a)\right)}{72}-\frac{\left(\sin^2(bx+a)\right)}{24}-\frac{(bx+a)^2\left(\sin^6(bx+a)\right)}{6}+\frac{(bx+a)\left(-\frac{\left(\sin^5(bx+a)\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b^2}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b^2*d^2*(1/4*(b*x+a)^2*sin(b*x+a)^4-1/2*(b*x+a)*(-1/4*(sin(b*x+a)^3+
3/2*sin(b*x+a))*cos(b*x+a)+3/8*b*x+3/8*a)+1/24*(b*x+a)^2-1/72*sin(b*x+a)^4-
1/24*sin(b*x+a)^2-1/6*(b*x+a)^2*sin(b*x+a)^6+1/3*(b*x+a)*(-1/6*(sin(b*x+a)^
5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a)+5/16*b*x+5/16*a)+1/108*sin(b
*x+a)^6)-2/b^2*a*d^2*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b
*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)
^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))+2/b*c*d*(1/4*(b*x+a)*sin(b
*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos(b*x+a)-1/24*b*x-1/24*a-1/6*(
b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin(b*x+a)^3+15/8*sin(b*x+a))*co
s(b*x+a))+d^2/b^2*a^2*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)-2*
c*d/b*a*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)+c^2*(-1/6*sin(b*
x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4))
```

maxima [B] time = 0.41, size = 303, normalized size = 2.35

$$-\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c^2-\frac{576\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)acd}{b}+\frac{288\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)a^2d^2}{b^2}-\frac{6(6(\cdots}{\cdots}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/3456*(288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c^2 - 576*(2*sin(b*x + a
)^6 - 3*sin(b*x + a)^4)*a*c*d/b + 288*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)
*a^2*d^2/b^2 - 6*(6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2
*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*c*d/b + 6*(6*(b*x + a)*cos(6*
b*x + 6*a) - 54*(b*x + a)*cos(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*
x + 2*a))*a*d^2/b^2 - ((18*(b*x + a)^2 - 1)*cos(6*b*x + 6*a) - 81*(2*(b*x +
 a)^2 - 1)*cos(2*b*x + 2*a) - 6*(b*x + a)*sin(6*b*x + 6*a) + 162*(b*x + a)*
sin(2*b*x + 2*a))*d^2/b^2)/b
```

mupad [B] time = 0.81, size = 202, normalized size = 1.57

$$\frac{81\,d^2\,\cos\left(2\,a+2\,b\,x\right)-d^2\,\cos\left(6\,a+6\,b\,x\right)-162\,b^2\,c^2\,\cos\left(2\,a+2\,b\,x\right)+18\,b^2\,c^2\,\cos\left(6\,a+6\,b\,x\right)+162\,b\,c\,d\,\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] (81*d^2*cos(2*a + 2*b*x) - d^2*cos(6*a + 6*b*x) - 162*b^2*c^2*cos(2*a + 2*b
*x) + 18*b^2*c^2*cos(6*a + 6*b*x) + 162*b*c*d*sin(2*a + 2*b*x) - 6*b*c*d*si
n(6*a + 6*b*x) - 162*b^2*d^2*x^2*cos(2*a + 2*b*x) + 18*b^2*d^2*x^2*cos(6*a
+ 6*b*x) + 162*b*d^2*x*sin(2*a + 2*b*x) - 6*b*d^2*x*sin(6*a + 6*b*x) - 324*
b^2*c*d*x*cos(2*a + 2*b*x) + 36*b^2*c*d*x*cos(6*a + 6*b*x))/(3456*b^3)
```

sympy [A] time = 9.99, size = 461, normalized size = 3.57

$$\begin{cases} \frac{c^2 \sin^6(a+bx)}{12b} + \frac{c^2 \sin^4(a+bx)\cos^2(a+bx)}{4b} + \frac{cdx \sin^6(a+bx)}{12b} + \frac{cdx \sin^4(a+bx)\cos^2(a+bx)}{4b} - \frac{cdx \sin^2(a+bx)\cos^4(a+bx)}{4b} - \frac{cdx \cos^6(a+bx)}{12b} \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\sin^3(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c**2*sin(a + b*x)**6/(12*b) + c**2*sin(a + b*x)**4*cos(a + b*x)*
*2/(4*b) + c*d*x*sin(a + b*x)**6/(12*b) + c*d*x*sin(a + b*x)**4*cos(a + b*x
)**2/(4*b) - c*d*x*sin(a + b*x)**2*cos(a + b*x)**4/(4*b) - c*d*x*cos(a + b*
x)**6/(12*b) + d**2*x**2*sin(a + b*x)**6/(24*b) + d**2*x**2*sin(a + b*x)**4
*cos(a + b*x)**2/(8*b) - d**2*x**2*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) -
d**2*x**2*cos(a + b*x)**6/(24*b) + c*d*sin(a + b*x)**5*cos(a + b*x)/(12*b**
2) + 2*c*d*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + c*d*sin(a + b*x)*cos(
a + b*x)**5/(12*b**2) + d**2*x*sin(a + b*x)**5*cos(a + b*x)/(12*b**2) + 2*d
**2*x*sin(a + b*x)**3*cos(a + b*x)**3/(9*b**2) + d**2*x*sin(a + b*x)*cos(a
```

```
+ b*x)**5/(12*b**2) - 7*d**2*sin(a + b*x)**6/(216*b**3) - d**2*sin(a + b*x)
**4*cos(a + b*x)**2/(18*b**3) + d**2*cos(a + b*x)**6/(72*b**3), Ne(b, 0)),
((c**2*x + c*d*x**2 + d**2*x**3/3)*sin(a)**3*cos(a)**3, True))
```

3.158 $\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=77

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

```
[Out] -3/64*(d*x+c)*cos(2*b*x+2*a)/b+1/192*(d*x+c)*cos(6*b*x+6*a)/b+3/128*d*sin(2
*b*x+2*a)/b^2-1/1152*d*sin(6*b*x+6*a)/b^2
```

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4406, 3296, 2637}

$$\frac{3d\sin(2a+2bx)}{128b^2}-\frac{d\sin(6a+6bx)}{1152b^2}-\frac{3(c+dx)\cos(2a+2bx)}{64b}+\frac{(c+dx)\cos(6a+6bx)}{192b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)*Cos[6*a + 6*b*x])/(192*
b) + (3*d*Sin[2*a + 2*b*x])/(128*b^2) - (d*Sin[6*a + 6*b*x])/(1152*b^2)
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)\sin(2a+2bx) - \frac{1}{32}(c+dx)\sin(6a+6bx)\right) dx$$
$$= -\left(\frac{1}{32}\int (c+dx)\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)\sin(2a+2bx)\,dx$$
$$= -\frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b} - \frac{d\int \cos(6a+6\ldots}{192b}$$
$$= -\frac{3(c+dx)\cos(2a+2bx)}{64b} + \frac{(c+dx)\cos(6a+6bx)}{192b} + \frac{3d\sin(2a+2b\ldots}{128b^2}$$

Mathematica [A] time = 0.22, size = 63, normalized size = 0.82

$$\frac{-54b(c+dx)\cos(2(a+bx)) + 6b(c+dx)\cos(6(a+bx)) + d(27\sin(2(a+bx)) - \sin(6(a+bx)))}{1152b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*b*(c + d*x)*Cos[2*(a + b*x)] + 6*b*(c + d*x)*Cos[6*(a + b*x)] + d*(27*
Sin[2*(a + b*x)] - Sin[6*(a + b*x)]))/(1152*b^2)
```

fricas [A] time = 0.44, size = 87, normalized size = 1.13

$$\frac{12\,(bdx+bc)\cos(bx+a)^6 - 18\,(bdx+bc)\cos(bx+a)^4 + 3\,bdx - \left(2\,d\cos(bx+a)^5 - 2\,d\cos(bx+a)^3 - 3\,d\cos\ldots\right.}{72\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/72*(12*(b*d*x + b*c)*cos(b*x + a)^6 - 18*(b*d*x + b*c)*cos(b*x + a)^4 + 3
*b*d*x - (2*d*cos(b*x + a)^5 - 2*d*cos(b*x + a)^3 - 3*d*cos(b*x + a))*sin(b
*x + a))/b^2
```

giac [A] time = 0.24, size = 75, normalized size = 0.97

$$\frac{(bdx+bc)\cos(6\,bx+6\,a)}{192\,b^2} - \frac{3\,(bdx+bc)\cos(2\,bx+2\,a)}{64\,b^2} - \frac{d\sin(6\,bx+6\,a)}{1152\,b^2} + \frac{3\,d\sin(2\,bx+2\,a)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/192*(b*d*x + b*c)*cos(6*b*x + 6*a)/b^2 - 3/64*(b*d*x + b*c)*cos(2*b*x + 2
*a)/b^2 - 1/1152*d*sin(6*b*x + 6*a)/b^2 + 3/128*d*sin(2*b*x + 2*a)/b^2
```

maple [B] time = 0.02, size = 176, normalized size = 2.29

$$\frac{\frac{d\left(\frac{(bx+a)\left(\sin^4(bx+a)\right)}{4}+\frac{\left(\sin^3(bx+a)+\frac{3\sin(bx+a)}{2}\right)\cos(bx+a)}{16}-\frac{bx}{24}-\frac{a}{24}-\frac{(bx+a)\left(\sin^6(bx+a)\right)}{6}-\frac{\left(\sin^5(bx+a)+\frac{5\left(\sin^3(bx+a)\right)}{4}+\frac{15\sin(bx+a)}{8}\right)\cos(bx+a)}{36}\right)}{b}-\frac{da\left(-\frac{\left(\sin^2(bx+a)\right)(\cdots}{6}\cdots\right.}{\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 1/b*(1/b*d*(1/4*(b*x+a)*sin(b*x+a)^4+1/16*(sin(b*x+a)^3+3/2*sin(b*x+a))*cos
(b*x+a)-1/24*b*x-1/24*a-1/6*(b*x+a)*sin(b*x+a)^6-1/36*(sin(b*x+a)^5+5/4*sin
(b*x+a)^3+15/8*sin(b*x+a))*cos(b*x+a))-1/b*d*a*(-1/6*sin(b*x+a)^2*cos(b*x+a
)^4-1/12*cos(b*x+a)^4)+c*(-1/6*sin(b*x+a)^2*cos(b*x+a)^4-1/12*cos(b*x+a)^4)
)
```

maxima [A] time = 0.80, size = 119, normalized size = 1.55

$$-\frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)c-\frac{96\left(2\sin(bx+a)^6-3\sin(bx+a)^4\right)ad}{b}-\frac{(6(bx+a)\cos(6bx+6a)-54(bx+a)\cos(2bx+2a)-\sin(6\cdots}{b}}{1152b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1152*(96*(2*sin(b*x + a)^6 - 3*sin(b*x + a)^4)*c - 96*(2*sin(b*x + a)^6
- 3*sin(b*x + a)^4)*a*d/b - (6*(b*x + a)*cos(6*b*x + 6*a) - 54*(b*x + a)*co
s(2*b*x + 2*a) - sin(6*b*x + 6*a) + 27*sin(2*b*x + 2*a))*d/b)/b
```

mupad [B] time = 0.71, size = 84, normalized size = 1.09

$$\frac{\frac{27\,d\,\sin(2\,a+2\,b\,x)}{4}-\frac{d\,\sin(6\,a+6\,b\,x)}{4}-\frac{27\,b\,c\,\cos(2\,a+2\,b\,x)}{2}+\frac{3\,b\,c\,\cos(6\,a+6\,b\,x)}{2}-\frac{27\,b\,d\,x\,\cos(2\,a+2\,b\,x)}{2}+\frac{3\,b\,d\,x\,\cos(6\,a+6\,b\,x)}{2}}{288\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x),x)
```

```
[Out] ((27*d*sin(2*a + 2*b*x))/4 - (d*sin(6*a + 6*b*x))/4 - (27*b*c*cos(2*a + 2*b
*x))/2 + (3*b*c*cos(6*a + 6*b*x))/2 - (27*b*d*x*cos(2*a + 2*b*x))/2 + (3*b*
d*x*cos(6*a + 6*b*x))/2)/(288*b^2)
```

sympy [A] time = 5.33, size = 201, normalized size = 2.61

$$\begin{cases} \frac{c\sin^6(a+bx)}{12b} + \frac{c\sin^4(a+bx)\cos^2(a+bx)}{4b} + \frac{dx\sin^6(a+bx)}{24b} + \frac{dx\sin^4(a+bx)\cos^2(a+bx)}{8b} - \frac{dx\sin^2(a+bx)\cos^4(a+bx)}{8b} - \frac{dx\cos^6(a+bx)}{24b} + \frac{d}{} \\ \left(cx + \frac{dx^2}{2}\right)\sin^3(a)\cos^3(a) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Piecewise((c*sin(a + b*x)**6/(12*b) + c*sin(a + b*x)**4*cos(a + b*x)**2/(4*
b) + d*x*sin(a + b*x)**6/(24*b) + d*x*sin(a + b*x)**4*cos(a + b*x)**2/(8*b)
 - d*x*sin(a + b*x)**2*cos(a + b*x)**4/(8*b) - d*x*cos(a + b*x)**6/(24*b) +
 d*sin(a + b*x)**5*cos(a + b*x)/(24*b**2) + d*sin(a + b*x)**3*cos(a + b*x)*
*3/(9*b**2) + d*sin(a + b*x)*cos(a + b*x)**5/(24*b**2), Ne(b, 0)), ((c*x +
d*x**2/2)*sin(a)**3*cos(a)**3, True))
```

3.159 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=129

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{32d}-\frac{\cos\left(6a-\frac{6bc}{d}\right)\text{S}}{32d}$$

```
[Out] 3/32*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/32*cos(6*a-6*b*c/d)*Si(6*b*c/d+
6*b*x)/d-1/32*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d+3/32*Ci(2*b*c/d+2*b*x)*s
in(2*a-2*b*c/d)/d
```

Rubi [A] time = 0.25, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4406, 3303, 3299, 3302}

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{32d}+\frac{3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{32d}+\frac{3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -(CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(32*d) + (3*CosInteg
ral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(32*d) + (3*Cos[2*a - (2*b*c)/
d]*SinIntegral[(2*b*c)/d + 2*b*x])/(32*d) - (Cos[6*a - (6*b*c)/d]*SinIntegr
al[(6*b*c)/d + 6*b*x])/(32*d)
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{c+dx}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)} - \frac{\sin(6a+6bx)}{32(c+dx)}\right)dx$$
$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{c+dx}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{c+dx}\,dx$$
$$= -\left(\frac{1}{32}\cos\left(6a-\frac{6bc}{d}\right)\int \frac{\sin\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx\right) + \frac{1}{32}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin}{}$$
$$= -\frac{\text{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{32d} + \frac{3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{32d} + \frac{3\cos\left(2a\right.}{}$$

Mathematica [A] time = 0.31, size = 110, normalized size = 0.85

$$-\frac{\sin\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6b(c+dx)}{d}\right) - 3\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) - 3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \cos\left(6a-\frac{6bc}{d}\right)\text{Si}\left(\right.}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x),x]
```

```
[Out] -1/32*(CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 3*CosIntegral[
(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 3*Cos[2*a - (2*b*c)/d]*SinIntegra
l[(2*b*(c + d*x))/d] + Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])
/d
```

fricas [A] time = 0.44, size = 156, normalized size = 1.21

$$\frac{3\left(\text{Ci}\left(\frac{2(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{2(bdx+bc)}{d}\right)\right)\sin\left(-\frac{2(bc-ad)}{d}\right) - \left(\text{Ci}\left(\frac{6(bdx+bc)}{d}\right)+\text{Ci}\left(-\frac{6(bdx+bc)}{d}\right)\right)\sin\left(-\frac{6(bc-ad)}{d}\right) - 2\cos\left(\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/64*(3*(cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d)
)*sin(-2*(b*c - a*d)/d) - (cos_integral(6*(b*d*x + b*c)/d) + cos_integral(-
6*(b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d) - 2*cos(-6*(b*c - a*d)/d)*sin_int
egral(6*(b*d*x + b*c)/d) + 6*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x +
b*c)/d))/d
```

giac [C] time = 0.57, size = 6046, normalized size = 46.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] -1/64*(imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^
2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(
b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3*b*c/
d)^2*tan(b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*t
an(a)^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integr
al(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real
_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)*tan(
b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan
(3*b*c/d)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x
+ 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*real_part(cos
_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2
+ imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)
^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b
*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)
^2*tan(3*b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*t
an(3*b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(3
*b*c/d)^2 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*t
an(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/
d)*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(
```

```
2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + imag_part(cos_inte
gral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 2*sin_integral(6
*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*
x + b*c)/d)*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + 4*imag_part(cos_integral(6*b
*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 4*imag_part(co
s_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 +
 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3*b*c/d)*tan(b*c/d
)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2*ta
n(b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*
c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^
2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*t
an(3*a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*
c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 3*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + imag_part(cos_i
ntegral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 2*sin_int
egral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + 2*real_part(c
os_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) + 2*real_par
t(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2*tan(3*b*c/d) - 6*real
_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 - 6*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(3*b*c/d)^2 -
 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c/d)^
2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2*tan(3*b*c
/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2*tan(
b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2*ta
n(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/
d)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*ta
n(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)
^2*tan(3*b*c/d)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x + 6*
b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x -
6*b*c/d))*tan(3*a)*tan(a)^2*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x +
 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(
-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_i
ntegral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(
cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real
_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2*tan(b*c/d)^2 -
 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2*tan(b*
c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*t
```

an(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/ d)^2*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan (a)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)^2 - 3* imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)^2 + imag_part(c os_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(a)^2 - 2*sin_integral(6*(b*d* x + b*c)/d)*tan(3*a)^2*tan(a)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2 *tan(3*b*c/d) - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a) ^2*tan(3*b*c/d) + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(a)^2*tan(3 *b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^ 2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 - ima g_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d)^2 + 2*sin_in tegral(6*(b*d*x + b*c)/d)*tan(3*a)^2*tan(3*b*c/d)^2 - 6*sin_integral(2*(b*d *x + b*c)/d)*tan(3*a)^2*tan(3*b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b *c/d))*tan(a)^2*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d)) *tan(a)^2*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(3*b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*ta n(3*b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(3*b*c/d)^2 - 12*imag_part(co s_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d) + 12*imag_part(co s_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a)*tan(b*c/d) - 24*sin_integra l(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(a)*tan(b*c/d) - 12*imag_part(cos_integr al(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) + 12*imag_part(cos_in tegral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - 24*sin_integra l(2*(b*d*x + b*c)/d)*tan(a)*tan(3*b*c/d)^2*tan(b*c/d) - imag_part(cos_integ ral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 + 3*imag_part(cos_integral(2* b*x + 2*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/ d))*tan(3*a)^2*tan(b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2* tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2*tan(b*c/d)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(b*c/d)^2 - 3*imag_par t(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 3*imag_part(cos_in tegral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d)^2 - imag_part(cos_integral(-6 *b*x - 6*b*c/d))*tan(a)^2*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)* tan(a)^2*tan(b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2*tan(b*c/ d)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)*tan (b*c/d)^2 - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/ d)*tan(b*c/d)^2 + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(3*b*c/d)*t an(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)^2*tan(b *c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c /d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/ d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d)^ 2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*c/d)^2*tan(b*c/d)^2 + 6*sin_i ntegral(2*(b*d*x + b*c)/d)*tan(3*b*c/d)^2*tan(b*c/d)^2 - 6*real_part(cos_in tegral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(a) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(a) + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(a)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(a)^2 + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d) + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2*tan(3*b*c/d) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2*tan(3*b*c/d) - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2*tan(3*b*c/d) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(3*b*c/d)^2 - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(3*b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2*tan(b*c/d) - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2*tan(b*c/d) - 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2*tan(b*c/d) + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d) + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2*tan(b*c/d) + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(b*c/d)^2 + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d)^2 - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)*tan(b*c/d)^2 - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d)*tan(b*c/d)^2 - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*a)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*a)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*a)^2 + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(a)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(a)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(a)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(a)^2 + 4*imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a)*tan(3*b*c/d) - 4*imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a)*tan(3*b*c/d) + 8*sin_integral(6*(b*d*x + b*c)/d)*tan(3*a)*tan(3*b*c/d) - imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*b*c/d)^2 - 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(3*b*c/d)^2 + 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(3*b*c/d)^2 + imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d)^2 - 2*sin_integral(6*(b*d*x + b*c)/d)*tan(3*b*c/d)^2 - 6*sin_integral(2*(b*d*x + b*c)/d)*tan(3*b*c/d)^2 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(a)*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(a)*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(a)*tan(b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d))*tan(b*c/d)^2 + 3*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - 3*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - imag_part(cos_integral(-6*b*x - 6*b*c/d))*tan(b*c/d)^2 + 2*sin_integral(6*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 6*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 2*real_part(cos_integral(6*b*x + 6*b*c/d))*tan(3*a) + 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*a) -

```
 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(a) - 6*real_part(cos_integr
al(-2*b*x - 2*b*c/d))*tan(a) - 2*real_part(cos_integral(6*b*x + 6*b*c/d))*t
an(3*b*c/d) - 2*real_part(cos_integral(-6*b*x - 6*b*c/d))*tan(3*b*c/d) + 6*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 6*real_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*c/d) + imag_part(cos_integral(6*b*x + 6*b*c/d)
) - 3*imag_part(cos_integral(2*b*x + 2*b*c/d)) + 3*imag_part(cos_integral(-
2*b*x - 2*b*c/d)) - imag_part(cos_integral(-6*b*x - 6*b*c/d)) + 2*sin_integ
ral(6*(b*d*x + b*c)/d) - 6*sin_integral(2*(b*d*x + b*c)/d))/(d*tan(3*a)^2*t
an(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(a)^2*tan(3*b*c/d)^2
+ d*tan(3*a)^2*tan(a)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(3*b*c/d)^2*tan(b*c/
d)^2 + d*tan(a)^2*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a)^2*tan(a)^2 + d*t
an(3*a)^2*tan(3*b*c/d)^2 + d*tan(a)^2*tan(3*b*c/d)^2 + d*tan(3*a)^2*tan(b*c
/d)^2 + d*tan(a)^2*tan(b*c/d)^2 + d*tan(3*b*c/d)^2*tan(b*c/d)^2 + d*tan(3*a
)^2 + d*tan(a)^2 + d*tan(3*b*c/d)^2 + d*tan(b*c/d)^2 + d)
```

maple [A] time = 0.02, size = 178, normalized size = 1.38

$$\frac{-\frac{b\left(\frac{6\,\mathrm{Si}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\cos\left(\frac{-6da+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\sin\left(\frac{-6da+6cb}{d}\right)}{d}\right)}{192}+\frac{3b\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-}{}\right.}{d}}{64}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x)
```

```
[Out] 1/b*(-1/192*b*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d-6*Ci(6*
b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)+3/64*b*(2*Si(2*b*x+2*a+2*(-a
*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d
+b*c)/d)/d))
```

maxima [C] time = 0.72, size = 274, normalized size = 2.12

$$\frac{b\left(-3i\,E_1\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_1\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b\left(i\,E_1\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_1\left(-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/64*(b*(-3*I*exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d) +
3*I*exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(b*
c - a*d)/d) + b*(I*exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/
d) - I*exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos(-6*
(b*c - a*d)/d) - 3*b*(exp_integral_e(1, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*
d)/d) + exp_integral_e(1, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*sin(-2
```

```
*(b*c - a*d)/d) + b*(exp_integral_e(1, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d
)/d) + exp_integral_e(1, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*sin(-6*
(b*c - a*d)/d))/(b*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3\left(a+bx\right)\cos^3\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x), x)
```

3.160 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=179

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^2}+\frac{3b\sin\left(6a-\right.}{}$$

```
[Out] -3/16*b*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^2+3/16*b*Ci(2*b*c/d+2*b*x)*cos
(2*a-2*b*c/d)/d^2+3/16*b*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^2-3/16*b*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2-3/32*sin(2*b*x+2*a)/d/(d*x+c)+1/32*sin(6
*b*x+6*a)/d/(d*x+c)
```

Rubi [A] time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{3b\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^2}-\frac{3b\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^2}-\frac{3b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}\right.}{16d^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (3*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) - (3*b*C
os[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(16*d^2) - (3*Sin[2*a +
 2*b*x])/(32*d*(c + d*x)) + Sin[6*a + 6*b*x]/(32*d*(c + d*x)) - (3*b*Sin[2*
a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*d^2) + (3*b*Sin[6*a - (6
*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^2)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^2}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^2} - \frac{\sin(6a+6bx)}{32(c+dx)^2}\right)dx \\
&= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^2}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx \\
&= -\frac{3\sin(2a+2bx)}{32d(c+dx)} + \frac{\sin(6a+6bx)}{32d(c+dx)} + \frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{16d} - \frac{(3b)\int \frac{\cos(6a+6bx)}{c+dx}}{16d} \\
&= -\frac{3\sin(2a+2bx)}{32d(c+dx)} + \frac{\sin(6a+6bx)}{32d(c+dx)} - \frac{\left(3b\cos\left(6a-\frac{6bc}{d}\right)\right)\int \frac{\cos\left(\frac{6bc}{d}+6bx\right)}{c+dx}\,dx}{16d} + \\
&= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^2} - \frac{3b\cos\left(6a-\frac{6bc}{d}\right)\operatorname{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^2} - \frac{3\sin(2}{32d(}
\end{aligned}$$

Mathematica [A] time = 0.96, size = 189, normalized size = 1.06

$$\frac{6b(c+dx)\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2b(c+dx)}{d}\right) - 6b(c+dx)\cos\left(6a-\frac{6bc}{d}\right)\operatorname{Ci}\left(\frac{6b(c+dx)}{d}\right) - 6b(c+dx)\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] (6*b*(c + d*x)*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] - 6*b*(c
 + d*x)*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*(c + d*x))/d] - 3*d*Cos[2*b*x
]*Sin[2*a] + d*Cos[6*b*x]*Sin[6*a] - 3*d*Cos[2*a]*Sin[2*b*x] + d*Cos[6*a]*S
in[6*b*x] - 6*b*(c + d*x)*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/
d] + 6*b*(c + d*x)*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c + d*x))/d])/(32
*d^2*(c + d*x))
```

fricas [**A**] time = 0.48, size = 248, normalized size = 1.39

$$\frac{6\,(bdx+bc)\sin\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{6\,(bdx+bc)}{d}\right)-6\,(bdx+bc)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+3\left((bdx+bc)\,\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(6*(b*d*x + b*c)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d)
 - 6*(b*d*x + b*c)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) +
3*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integr
al(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) - 3*((b*d*x + b*c)*cos_integr
al(6*(b*d*x + b*c)/d) + (b*d*x + b*c)*cos_integral(-6*(b*d*x + b*c)/d))*cos
(-6*(b*c - a*d)/d) + 32*(d*cos(b*x + a)^5 - d*cos(b*x + a)^3)*sin(b*x + a))
/(d^3*x + c*d^2)
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**A**] time = 0.02, size = 256, normalized size = 1.43

$$\frac{-\frac{b^{2}\left(-\frac{6\sin(6bx+6a)}{((bx+a)d-da+cb)d}+\frac{\frac{36\,\mathrm{Si}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\sin\left(\frac{-6da+6cb}{d}\right)}{d}+\frac{36\,\mathrm{Ci}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\cos\left(\frac{-6da+6cb}{d}\right)}{d}}{d}\right)}{192}+\frac{3b^{2}\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)}{d}}{}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] 1/b*(-1/192*b^2*(-6*sin(6*b*x+6*a)/((b*x+a)*d-d*a+c*b)/d+6*(6*Si(6*b*x+6*a+
6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*
(-a*d+b*c)/d)/d)/d)+3/64*b^2*(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*
Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b
*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d))
```

maxima [C] time = 0.48, size = 301, normalized size = 1.68

$$\frac{b^2\left(-3i\,E_2\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_2\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^2\left(i\,E_2\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/64*(b^2*(-3*I*exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^2*(I*exp_integral_e(2, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(2, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^2*(exp_integral_e(2, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(2, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^2*(exp_integral_e(2, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(2, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b*c*d + (b*x + a)*d^2 - a*d^2)*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^3\,\sin(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^3(a+bx)\cos^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)**3*cos(a + b*x)**3/(c + d*x)**2, x)
```

3.161 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=235

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{16d^3}+\frac{9b^2\cos\left(\right.}{}$$

```
[Out] -3/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)+3/32*b*cos(6*b*x+6*a)/d^2/(d*x+c)-3/16*b
^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^3+9/16*b^2*cos(6*a-6*b*c/d)*Si(6*b*
c/d+6*b*x)/d^3+9/16*b^2*Ci(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^3-3/16*b^2*Ci(
2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-3/64*sin(2*b*x+2*a)/d/(d*x+c)^2+1/64*si
n(6*b*x+6*a)/d/(d*x+c)^2
```

Rubi [A] time = 0.35, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$\frac{9b^2\sin\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{16d^3}-\frac{3b^2\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{16d^3}-\frac{3b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\right.}{16d^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)) + (3*b*Cos[6*a + 6*b*x])/(32*d^2
*(c + d*x)) + (9*b^2*CosIntegral[(6*b*c)/d + 6*b*x]*Sin[6*a - (6*b*c)/d])/(
16*d^3) - (3*b^2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(16*d
^3) - (3*Sin[2*a + 2*b*x])/(64*d*(c + d*x)^2) + Sin[6*a + 6*b*x]/(64*d*(c +
 d*x)^2) - (3*b^2*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(16*
d^3) + (9*b^2*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(16*d^3)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^3} - \frac{\sin(6a+6bx)}{32(c+dx)^3}\right) dx$$

$$= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^3}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} + \frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{32d} - \frac{(3b)\int \frac{\cos(6a+6bx)}{(c+dx)^2}}{32d}$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} -$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} - \frac{3\sin(2a+2bx)}{64d(c+dx)^2} + \frac{\sin(6a+6bx)}{64d(c+dx)^2} +$$

$$= -\frac{3b\cos(2a+2bx)}{32d^2(c+dx)} + \frac{3b\cos(6a+6bx)}{32d^2(c+dx)} + \frac{9b^2\text{Ci}\left(\frac{6bc}{d}+6bx\right)\sin\left(6a-\frac{6bc}{d}\right)}{16d^3} -$$

Mathematica [A] time = 1.04, size = 239, normalized size = 1.02

$$\frac{6b^2(c+dx)^2\left(6\sin\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6b(c+dx)}{d}\right)-2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right)-2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right)+6\cos\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^3,x]
```

```
[Out] (-3*d*Cos[2*b*x]*(2*b*(c + d*x)*Cos[2*a] + d*Sin[2*a]) + d*Cos[6*b*x]*(6*b*
(c + d*x)*Cos[6*a] + d*Sin[6*a]) + 3*d*(-(d*Cos[2*a]) + 2*b*(c + d*x)*Sin[2
*a])*Sin[2*b*x] + d*(d*Cos[6*a] - 6*b*(c + d*x)*Sin[6*a])*Sin[6*b*x] + 6*b^
2*(c + d*x)^2*(6*CosIntegral[(6*b*(c + d*x))/d]*Sin[6*a - (6*b*c)/d] - 2*Co
sIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] - 2*Cos[2*a - (2*b*c)/d]*
SinIntegral[(2*b*(c + d*x))/d] + 6*Cos[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
 + d*x))/d]))/(64*d^3*(c + d*x)^2)
```

fricas [A] time = 0.54, size = 434, normalized size = 1.85

$$\frac{96\left(bd^2x+bcd\right)\cos\left(bx+a\right)^6-144\left(bd^2x+bcd\right)\cos\left(bx+a\right)^4+48\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+18\left(b^2d^2x^2+2\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/32*(96*(b*d^2*x + b*c*d)*cos(b*x + a)^6 - 144*(b*d^2*x + b*c*d)*cos(b*x +
 a)^4 + 48*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*cos(-6*(b*c - a*d)/d)*sin_integral(6*(b*d*x + b*c)/d) - 6*(b^2*
d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*
x + b*c)/d) + 16*(d^2*cos(b*x + a)^5 - d^2*cos(b*x + a)^3)*sin(b*x + a) - 3
*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(2*(b*d*x + b*c)/d) + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-2*(b*d*x + b*c)/d))*sin(
-2*(b*c - a*d)/d) + 9*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(6
*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_integral(-6*(
b*d*x + b*c)/d))*sin(-6*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 329, normalized size = 1.40

$$\frac{-\frac{b^3\left(-\frac{3\sin(6bx+6a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{18\cos(6bx+6a)}{((bx+a)d-da+cb)d}-\frac{18\left(\frac{6\,\mathrm{Si}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\cos\left(\frac{-6da+6cb}{d}\right)}{d}-\frac{6\,\mathrm{Ci}\left(6bx+6a+\frac{-6da+6cb}{d}\right)\sin\left(\frac{-6da+6cb}{d}\right)}{d}\right)}{d}}{d}\right)}{192}+\frac{3b^3\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2 d}\right.}{}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x)
```

```
[Out] 1/b*(-1/192*b^3*(-3*sin(6*b*x+6*a)/((b*x+a)*d-d*a+c*b)^2/d+3*(-6*cos(6*b*x+
6*a)/((b*x+a)*d-d*a+c*b)/d-6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*
c)/d)/d-6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d+b*c)/d)/d)/d)/d)+3/64*b^
3*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*
a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x
+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d))
```

maxima [C] time = 0.69, size = 336, normalized size = 1.43

$$\frac{b^3\left(-3i\,E_3\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_3\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^3\left(i\,E_3\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/64*(b^3*(-3*I*exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^3*(I*exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^3*(exp_integral_e(3, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(3, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^3*(exp_integral_e(3, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(3, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d
^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3}{\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.162 $\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=287

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{Ci}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{8d^4}-\frac{9b^3\sin\left(\right.}{}$$

```
[Out] 9/8*b^3*Ci(6*b*c/d+6*b*x)*cos(6*a-6*b*c/d)/d^4-1/8*b^3*Ci(2*b*c/d+2*b*x)*co
s(2*a-2*b*c/d)/d^4-1/32*b*cos(2*b*x+2*a)/d^2/(d*x+c)^2+1/32*b*cos(6*b*x+6*a
)/d^2/(d*x+c)^2-9/8*b^3*Si(6*b*c/d+6*b*x)*sin(6*a-6*b*c/d)/d^4+1/8*b^3*Si(2
*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^4-1/32*sin(2*b*x+2*a)/d/(d*x+c)^3+1/16*b^2
*sin(2*b*x+2*a)/d^3/(d*x+c)+1/96*sin(6*b*x+6*a)/d/(d*x+c)^3-3/16*b^2*sin(6*
b*x+6*a)/d^3/(d*x+c)
```

Rubi [A] time = 0.42, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4406, 3297, 3303, 3299, 3302}

$$-\frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{8d^4}+\frac{9b^3\cos\left(6a-\frac{6bc}{d}\right)\text{CosIntegral}\left(\frac{6bc}{d}+6bx\right)}{8d^4}+\frac{b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\right.}{8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] -(b*Cos[2*a + 2*b*x])/(32*d^2*(c + d*x)^2) + (b*Cos[6*a + 6*b*x])/(32*d^2*(
c + d*x)^2) - (b^3*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/(8*
d^4) + (9*b^3*Cos[6*a - (6*b*c)/d]*CosIntegral[(6*b*c)/d + 6*b*x])/(8*d^4)
- Sin[2*a + 2*b*x]/(32*d*(c + d*x)^3) + (b^2*Sin[2*a + 2*b*x])/(16*d^3*(c +
 d*x)) + Sin[6*a + 6*b*x]/(96*d*(c + d*x)^3) - (3*b^2*Sin[6*a + 6*b*x])/(16
*d^3*(c + d*x)) + (b^3*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])
/(8*d^4) - (9*b^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*c)/d + 6*b*x])/(8*d
^4)
```

Rule 3297

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[((
c + d*x)^(m + 1)*Sin[e + f*x])/(d*(m + 1)), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^3(a+bx)\sin^3(a+bx)}{(c+dx)^4}\,dx &= \int \left(\frac{3\sin(2a+2bx)}{32(c+dx)^4} - \frac{\sin(6a+6bx)}{32(c+dx)^4}\right) dx \\
&= -\left(\frac{1}{32}\int \frac{\sin(6a+6bx)}{(c+dx)^4}\,dx\right) + \frac{3}{32}\int \frac{\sin(2a+2bx)}{(c+dx)^4}\,dx \\
&= -\frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^3}\,dx}{16d} - \frac{b\int \frac{\cos(6a+6bx)}{(c+dx)^3}\,dx}{16d} \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{\sin(6a+6bx)}{96d(c+dx)^3} - \cdots \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} + \cdots \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{\sin(2a+2bx)}{32d(c+dx)^3} + \frac{b^2\sin(2a+2bx)}{16d^3(c+dx)} + \cdots \\
&= -\frac{b\cos(2a+2bx)}{32d^2(c+dx)^2} + \frac{b\cos(6a+6bx)}{32d^2(c+dx)^2} - \frac{b^3\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{8d^4} + \cdots
\end{aligned}
$$

Mathematica [A] time = 4.94, size = 554, normalized size = 1.93

$$
\frac{12b^3c^3\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right) - 108b^3c^3\sin\left(6a-\frac{6bc}{d}\right)\operatorname{Si}\left(\frac{6b(c+dx)}{d}\right) + 36b^3c^2dx\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b(c+dx)}{d}\right) - \cdots}{\cdots}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[a + b*x]^3*Sin[a + b*x]^3)/(c + d*x)^4,x]
```

```
[Out] (-3*b*c*d^2*Cos[2*(a + b*x)] - 3*b*d^3*x*Cos[2*(a + b*x)] + 3*b*c*d^2*Cos[6
*(a + b*x)] + 3*b*d^3*x*Cos[6*(a + b*x)] - 12*b^3*(c + d*x)^3*Cos[2*a - (2*
b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + 108*b^3*(c + d*x)^3*Cos[6*a - (6*b
*c)/d]*CosIntegral[(6*b*(c + d*x))/d] + 6*b^2*c^2*d*Sin[2*(a + b*x)] - 3*d^
3*Sin[2*(a + b*x)] + 12*b^2*c*d^2*x*Sin[2*(a + b*x)] + 6*b^2*d^3*x^2*Sin[2*
(a + b*x)] - 18*b^2*c^2*d*Sin[6*(a + b*x)] + d^3*Sin[6*(a + b*x)] - 36*b^2*
c*d^2*x*Sin[6*(a + b*x)] - 18*b^2*d^3*x^2*Sin[6*(a + b*x)] + 12*b^3*c^3*Sin
[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c^2*d*x*Sin[2*a -
 (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 36*b^3*c*d^2*x^2*Sin[2*a - (2*
b*c)/d]*SinIntegral[(2*b*(c + d*x))/d] + 12*b^3*d^3*x^3*Sin[2*a - (2*b*c)/d
]*SinIntegral[(2*b*(c + d*x))/d] - 108*b^3*c^3*Sin[6*a - (6*b*c)/d]*SinInte
gral[(6*b*(c + d*x))/d] - 324*b^3*c^2*d*x*Sin[6*a - (6*b*c)/d]*SinIntegral[
```

```
(6*b*(c + d*x))/d] - 324*b^3*c*d^2*x^2*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*
b*(c + d*x))/d] - 108*b^3*d^3*x^3*Sin[6*a - (6*b*c)/d]*SinIntegral[(6*b*(c
+ d*x))/d])/(96*d^4*(c + d*x)^3)
```

fricas [B] time = 0.56, size = 638, normalized size = 2.22

$$\frac{48\left(bd^{3}x+bcd^{2}\right)\cos\left(bx+a\right)^{6}-72\left(bd^{3}x+bcd^{2}\right)\cos\left(bx+a\right)^{4}+24\left(bd^{3}x+bcd^{2}\right)\cos\left(bx+a\right)^{2}-54\left(b^{3}d^{3}x^{3}+3\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/48*(48*(b*d^3*x + b*c*d^2)*cos(b*x + a)^6 - 72*(b*d^3*x + b*c*d^2)*cos(b*
x + a)^4 + 24*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2 - 54*(b^3*d^3*x^3 + 3*b^3*
c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*sin(-6*(b*c - a*d)/d)*sin_integral(6*(
b*d*x + b*c)/d) + 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^
3)*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 3*((b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d
) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(
-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d) + 27*((b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(6*(b*d*x + b*c)/d) + (b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-6*(b*d*x + b
*c)/d))*cos(-6*(b*c - a*d)/d) - 16*((18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b
^2*c^2*d - d^3)*cos(b*x + a)^5 - (18*b^2*d^3*x^2 + 36*b^2*c*d^2*x + 18*b^2*
c^2*d - d^3)*cos(b*x + a)^3 + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*c
os(b*x + a))*sin(b*x + a))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.02, size = 404, normalized size = 1.41

$$\frac{\frac{3b^4\left(-\frac{2\sin(2bx+2a)}{3((bx+a)d-da+cb)^3 d}+\frac{-\frac{2\cos(2bx+2a)}{3((bx+a)d-da+cb)^2 d}-\frac{2\left(-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}\right)}{3d}}{d}\right)}{64} - \cdots}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x)
```

```
[Out] 1/b*(3/64*b^4*(-2/3*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^3/d+2/3*(-cos(2*b*x+
2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*
Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b
*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/192*b^4*(-2*sin(6*b*x+6*a)/((b*x+a
)*d-d*a+c*b)^3/d+2*(-3*cos(6*b*x+6*a)/((b*x+a)*d-d*a+c*b)^2/d-3*(-6*sin(6*b
*x+6*a)/((b*x+a)*d-d*a+c*b)/d+6*(6*Si(6*b*x+6*a+6*(-a*d+b*c)/d)*sin(6*(-a*d
+b*c)/d)/d+6*Ci(6*b*x+6*a+6*(-a*d+b*c)/d)*cos(6*(-a*d+b*c)/d)/d)/d)/d)/d))
```

maxima [C] time = 0.86, size = 386, normalized size = 1.34

$$\frac{b^4\left(-3i\,E_4\left(\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)+3i\,E_4\left(-\frac{2i\,bc+2i\,(bx+a)d-2i\,ad}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+b^4\left(i\,E_4\left(\frac{6i\,bc+6i\,(bx+a)d-6i\,ad}{d}\right)-i\,E_4 \cdots\right.}{64\left(b^3c^3d-3\,ab^2c^2d^2+3 \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^3*sin(b*x+a)^3/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*(b^4*(-3*I*exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d)
+ 3*I*exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*cos(-2*(
b*c - a*d)/d) + b^4*(I*exp_integral_e(4, (6*I*b*c + 6*I*(b*x + a)*d - 6*I*a
*d)/d) - I*exp_integral_e(4, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))*cos
(-6*(b*c - a*d)/d) - 3*b^4*(exp_integral_e(4, (2*I*b*c + 2*I*(b*x + a)*d -
2*I*a*d)/d) + exp_integral_e(4, -(2*I*b*c + 2*I*(b*x + a)*d - 2*I*a*d)/d))*
sin(-2*(b*c - a*d)/d) + b^4*(exp_integral_e(4, (6*I*b*c + 6*I*(b*x + a)*d -
 6*I*a*d)/d) + exp_integral_e(4, -(6*I*b*c + 6*I*(b*x + a)*d - 6*I*a*d)/d))
*sin(-6*(b*c - a*d)/d))/((b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*
x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*b)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (a+b\,x)^3\,\sin (a+b\,x)^3}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4,x)
```

```
[Out] int((cos(a + b*x)^3*sin(a + b*x)^3)/(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**3*sin(b*x+a)**3/(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.163 $\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=152

$$\operatorname{Int}(\cot(a+bx)(c+dx)^m,x)+\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx}{}$$

```
[Out] 2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b*
(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*
c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*cot(b*x+a),x)
```

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Cot[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx &= \int (c+dx)^m \cot(a+bx)\,dx-\int (c+dx)^m \cos(a+bx)\sin(a+bx)\,dx\\ &= \int (c+dx)^m \cot(a+bx)\,dx-\int \frac{1}{2}(c+dx)^m \sin(2a+2bx)\,dx\\ &= -\left(\frac{1}{2}\int (c+dx)^m \sin(2a+2bx)\,dx\right)+\int (c+dx)^m \cot(a+bx)\,dx\\ &= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx+\int (c\\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e^{-}}{}\end{aligned}$$

Mathematica [A] time = 7.77, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]^2*Cot[a + b*x], x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos(bx+a)^2\cot(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)^2\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cos^2(bx+a)\right)\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
[Out] int((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)^2\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)^2*cot(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\mathrm{cot}\left(a+b\,x\right)\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m,x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + d x\right)^{m} \cos^{2}{\left (a + b x \right)} \cot{\left (a + b x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)**m*cos(a + b*x)**2*cot(a + b*x), x)
```

3.164 $\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=307

$$-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\sin^2(a+bx)}{4b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}+\frac{3d^3(c+dx)\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c+d$$

```
[Out] -3/2*c*d^3*x/b^3-3/4*d^4*x^2/b^3+1/4*(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^
4*ln(1-exp(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+
3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,e
xp(2*I*(b*x+a)))/b^4-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5+3/2*d^3*(d*x+c
)*cos(b*x+a)*sin(b*x+a)/b^4-d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2-3/4*d^4*s
in(b*x+a)^2/b^5+3/2*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3-1/2*(d*x+c)^4*sin(b*x+a)
^2/b
```

Rubi [A] time = 0.34, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4408, 4404, 3311, 32, 3310, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{3d^4\text{Po}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*c*d^3*x)/(2*b^3) - (3*d^4*x^2)/(4*b^3) + (c + d*x)^4/(4*b) - ((I/5)*(c
+ d*x)^5)/d + ((c + d*x)^4*Log[1 - E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c +
d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3,
 E^((2*I)*(a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a +
b*x))])/b^4 - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) + (3*d^3*(c +
 d*x)*Cos[a + b*x]*Sin[a + b*x])/(2*b^4) - (d*(c + d*x)^3*Cos[a + b*x]*Sin[
a + b*x])/b^2 - (3*d^4*Sin[a + b*x]^2)/(4*b^5) + (3*d^2*(c + d*x)^2*Sin[a +
 b*x]^2)/(2*b^3) - ((c + d*x)^4*Sin[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
```

```
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^4 \cot(a+bx)\,dx - \int (c+dx)^4 \cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1-e^{2i(a+bx)}}\,dx + \frac{(2}{}$$
$$= -\frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)^3\cos(a+bx)\sin}{b^2}$$
$$= \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{2id(c+dx)^3\mathrm{Li}}{b^2}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx}\right.}{b}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx}\right.}{b}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx}\right.}{b}$$
$$= -\frac{3cd^3x}{2b^3} - \frac{3d^4x^2}{4b^3} + \frac{(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1-e^{2i(a+bx}\right.}{b}$$

Mathematica [B] time = 6.52, size = 2828, normalized size = 9.21

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)
*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E
^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
 b*x))]))/E^((2*I)*a)))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a)
 + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))
*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(
a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^(
(2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^
((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 -
(d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))
*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 +
```

```
E^((-I)*(a + b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*
(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog
[4, -E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)
*a) - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I
)*b^2*x^2*PolyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*
x))] + (6*I)*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) + (c^4
*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b
*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(160*b^5) - ((I/160)*Sin
[2*a + 2*b*x])/b^5)*(80*b^5*c^4*x*Cos[a + 2*b*x] + 160*b^5*c^3*d*x^2*Cos[a
+ 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] + 80*b^5*c*d^3*x^4*Cos[a + 2*
b*x] + 16*b^5*d^4*x^5*Cos[a + 2*b*x] + 80*b^5*c^4*x*Cos[3*a + 2*b*x] + 160*
b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + 160*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + 80*
b^5*c*d^3*x^4*Cos[3*a + 2*b*x] + 16*b^5*d^4*x^5*Cos[3*a + 2*b*x] + (10*I)*b
^4*c^4*Cos[3*a + 4*b*x] - 20*b^3*c^3*d*Cos[3*a + 4*b*x] - (30*I)*b^2*c^2*d^
2*Cos[3*a + 4*b*x] + 30*b*c*d^3*Cos[3*a + 4*b*x] + (15*I)*d^4*Cos[3*a + 4*b
*x] + (40*I)*b^4*c^3*d*x*Cos[3*a + 4*b*x] - 60*b^3*c^2*d^2*x*Cos[3*a + 4*b*
x] - (60*I)*b^2*c*d^3*x*Cos[3*a + 4*b*x] + 30*b*d^4*x*Cos[3*a + 4*b*x] + (6
0*I)*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*x] - 60*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] -
 (30*I)*b^2*d^4*x^2*Cos[3*a + 4*b*x] + (40*I)*b^4*c*d^3*x^3*Cos[3*a + 4*b*x
] - 20*b^3*d^4*x^3*Cos[3*a + 4*b*x] + (10*I)*b^4*d^4*x^4*Cos[3*a + 4*b*x] -
 (10*I)*b^4*c^4*Cos[5*a + 4*b*x] + 20*b^3*c^3*d*Cos[5*a + 4*b*x] + (30*I)*b
^2*c^2*d^2*Cos[5*a + 4*b*x] - 30*b*c*d^3*Cos[5*a + 4*b*x] - (15*I)*d^4*Cos[
5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Cos[5*a + 4*b*x] + 60*b^3*c^2*d^2*x*Cos[5
*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Cos[5*a + 4*b*x] - 30*b*d^4*x*Cos[5*a + 4*
b*x] - (60*I)*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Cos[5*a +
 4*b*x] + (30*I)*b^2*d^4*x^2*Cos[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Cos[5*
a + 4*b*x] + 20*b^3*d^4*x^3*Cos[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Cos[5*a +
 4*b*x] + 20*b^4*c^4*Sin[a] - (40*I)*b^3*c^3*d*Sin[a] - 60*b^2*c^2*d^2*Sin[
a] + (60*I)*b*c*d^3*Sin[a] + 30*d^4*Sin[a] + 80*b^4*c^3*d*x*Sin[a] - (120*I
)*b^3*c^2*d^2*x*Sin[a] - 120*b^2*c*d^3*x*Sin[a] + (60*I)*b*d^4*x*Sin[a] + 1
20*b^4*c^2*d^2*x^2*Sin[a] - (120*I)*b^3*c*d^3*x^2*Sin[a] - 60*b^2*d^4*x^2*S
in[a] + 80*b^4*c*d^3*x^3*Sin[a] - (40*I)*b^3*d^4*x^3*Sin[a] + 20*b^4*d^4*x^
4*Sin[a] + (80*I)*b^5*c^4*x*Sin[a + 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[a +
2*b*x] + (160*I)*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[
a + 2*b*x] + (16*I)*b^5*d^4*x^5*Sin[a + 2*b*x] + (80*I)*b^5*c^4*x*Sin[3*a +
 2*b*x] + (160*I)*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] + (160*I)*b^5*c^2*d^2*x^3*
Sin[3*a + 2*b*x] + (80*I)*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] + (16*I)*b^5*d^4*x
^5*Sin[3*a + 2*b*x] - 10*b^4*c^4*Sin[3*a + 4*b*x] - (20*I)*b^3*c^3*d*Sin[3*
a + 4*b*x] + 30*b^2*c^2*d^2*Sin[3*a + 4*b*x] + (30*I)*b*c*d^3*Sin[3*a + 4*b
*x] - 15*d^4*Sin[3*a + 4*b*x] - 40*b^4*c^3*d*x*Sin[3*a + 4*b*x] - (60*I)*b^
3*c^2*d^2*x*Sin[3*a + 4*b*x] + 60*b^2*c*d^3*x*Sin[3*a + 4*b*x] + (30*I)*b*d
^4*x*Sin[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] - (60*I)*b^3*c*
d^3*x^2*Sin[3*a + 4*b*x] + 30*b^2*d^4*x^2*Sin[3*a + 4*b*x] - 40*b^4*c*d^3*x
^3*Sin[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Sin[3*a + 4*b*x] - 10*b^4*d^4*x^4*
Sin[3*a + 4*b*x] + 10*b^4*c^4*Sin[5*a + 4*b*x] + (20*I)*b^3*c^3*d*Sin[5*a +
```

```
 4*b*x] - 30*b^2*c^2*d^2*Sin[5*a + 4*b*x] - (30*I)*b*c*d^3*Sin[5*a + 4*b*x]
 + 15*d^4*Sin[5*a + 4*b*x] + 40*b^4*c^3*d*x*Sin[5*a + 4*b*x] + (60*I)*b^3*c
^2*d^2*x*Sin[5*a + 4*b*x] - 60*b^2*c*d^3*x*Sin[5*a + 4*b*x] - (30*I)*b*d^4*
x*Sin[5*a + 4*b*x] + 60*b^4*c^2*d^2*x^2*Sin[5*a + 4*b*x] + (60*I)*b^3*c*d^3
*x^2*Sin[5*a + 4*b*x] - 30*b^2*d^4*x^2*Sin[5*a + 4*b*x] + 40*b^4*c*d^3*x^3*
Sin[5*a + 4*b*x] + (20*I)*b^3*d^4*x^3*Sin[5*a + 4*b*x] + 10*b^4*d^4*x^4*Sin
[5*a + 4*b*x]) - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I
*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTa
n[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2
*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
 + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[
a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.66, size = 1453, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 48*d^4*polylog(5, cos(b*x + a) + I*si
n(b*x + a)) + 48*d^4*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 48*d^4*pol
ylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 48*d^4*polylog(5, -cos(b*x + a) -
 I*sin(b*x + a)) + 3*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - (2*b^4*d^4*x^4 + 8*b^4
*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4
)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 2*(2*b^3*d^4*x^3
+ 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*
cos(b*x + a)*sin(b*x + a) + 2*(2*b^4*c^3*d - 3*b^2*c*d^3)*x - (-8*I*b^3*d^4
*x^3 - 24*I*b^3*c*d^3*x^2 - 24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*dilog(cos(b
*x + a) + I*sin(b*x + a)) - (8*I*b^3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^
3*c^2*d^2*x + 8*I*b^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (8*I*b^
3*d^4*x^3 + 24*I*b^3*c*d^3*x^2 + 24*I*b^3*c^2*d^2*x + 8*I*b^3*c^3*d)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) - (-8*I*b^3*d^4*x^3 - 24*I*b^3*c*d^3*x^2 -
24*I*b^3*c^2*d^2*x - 8*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3
*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*log(cos(b*x + a) - I*si
n(b*x + a) + 1) - 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4
*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/
2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x
^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
+ 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^
3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) - I*si
```

```
n(b*x + a) + 1) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, cos(b*x + a) + I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, cos(b*x + a) - I
*sin(b*x + a)) - (-48*I*b*d^4*x - 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) +
I*sin(b*x + a)) - (48*I*b*d^4*x + 48*I*b*c*d^3)*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3,
 cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2
*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*d^4*x^2 + 2*b^2*c
*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 24*(b^2*
d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a)))/b^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \cos (bx + a)^2 \cot (bx + a) \ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)^2*cot(b*x + a), x)
```

maple [B] time = 0.54, size = 1326, normalized size = 4.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+12/b^3*
c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))
-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^
2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2
*I*c^2*d^2*x^3-2*I*c^3*d*x^2-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*p
olylog(5,exp(I*(b*x+a)))/b^5+1/8*(2*b^4*d^4*x^4+8*b^4*c*d^3*x^3+12*b^4*c^2*
d^2*x^2+8*b^4*c^3*d*x+2*b^4*c^4-6*b^2*d^4*x^2-12*b^2*c*d^3*x-6*b^2*c^2*d^2+
3*d^4)/b^5*cos(2*b*x+2*a)+I*c^4*x-2/b*c^4*ln(exp(I*(b*x+a)))+1/b*c^4*ln(exp
(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-1/5*I*d^4*x^5+4/b*c^3*d*ln(exp(
I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*
x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2+24/b^3*c*d^3*polylog(3,-exp(I
*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*
(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polyl
og(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+2*I/b^4*d^4*
a^4*x-4*I/b^2*c^3*d*a^2+8*I/b^3*c^2*d^2*a^3-6*I/b^4*c*d^3*a^4-4*I/b^2*d^4*p
olylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b
^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)
```

```
))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I
*(b*x+a)))+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a
))-1)+8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)
)-1)-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-
1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-8*I/b^
3*c*d^3*a^3*x+12*I/b^2*c^2*d^2*a^2*x-8*I/b*c^3*d*a*x-12*I/b^2*c*d^3*polylog
(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-12*I/
b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x
+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+a)))
*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/4/b^4*d*(2*b^2*d^3*x^3+6*b^2*c*
d^2*x^2+6*b^2*c^2*d*x+2*b^2*c^3-3*d^3*x-3*c*d^2)*sin(2*b*x+2*a)
```

maxima [B] time = 1.06, size = 1635, normalized size = 5.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/40*(20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^4 - 80*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c^3*d/b + 120*(sin(b*x + a)^2 - log(sin(b*x + a)^2)
)*a^2*c^2*d^2/b^2 - 80*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*c*d^3/b^3
 + 20*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^4*d^4/b^4 - (-8*I*(b*x + a)^
5*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^4 - 960*d^4*polylog(5, -e^(I
*b*x + I*a)) - 960*d^4*polylog(5, e^(I*b*x + I*a)) + (-80*I*b^2*c^2*d^2 + 1
60*I*a*b*c*d^3 - 80*I*a^2*d^4)*(b*x + a)^3 + (-80*I*b^3*c^3*d + 240*I*a*b^2
*c^2*d^2 - 240*I*a^2*b*c*d^3 + 80*I*a^3*d^4)*(b*x + a)^2 + (40*I*(b*x + a)^
4*d^4 + (160*I*b*c*d^3 - 160*I*a*d^4)*(b*x + a)^3 + (240*I*b^2*c^2*d^2 - 48
0*I*a*b*c*d^3 + 240*I*a^2*d^4)*(b*x + a)^2 + (160*I*b^3*c^3*d - 480*I*a*b^2
*c^2*d^2 + 480*I*a^2*b*c*d^3 - 160*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x +
a), cos(b*x + a) + 1) + (-40*I*(b*x + a)^4*d^4 + (-160*I*b*c*d^3 + 160*I*a*
d^4)*(b*x + a)^3 + (-240*I*b^2*c^2*d^2 + 480*I*a*b*c*d^3 - 240*I*a^2*d^4)*(
b*x + a)^2 + (-160*I*b^3*c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 +
160*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 5*(2*(
b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 3*(2*a^2 - 1)*d^4 + 8*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(2*b^2*c^2*d^2 - 4*a*b*c*d^3 + (2*a^2 - 1)*d^4
)*(b*x + a)^2 + 4*(2*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 3*(2*a^2 - 1)*b*c*d^3 -
(2*a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (-160*I*b^3*c^3*d + 480*I*
a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 160*I*a^3*d^4 +
 (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2*d^2 + 960*I*a
*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (-160*I*b^3*
c^3*d + 480*I*a*b^2*c^2*d^2 - 480*I*a^2*b*c*d^3 - 160*I*(b*x + a)^3*d^4 + 1
60*I*a^3*d^4 + (-480*I*b*c*d^3 + 480*I*a*d^4)*(b*x + a)^2 + (-480*I*b^2*c^2
*d^2 + 960*I*a*b*c*d^3 - 480*I*a^2*d^4)*(b*x + a))*dilog(e^(I*b*x + I*a)) +
 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2
```

```
*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*
b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b
*x + a) + 1) + 20*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 - 960*I*a
*d^4)*polylog(4, -e^(I*b*x + I*a)) + (960*I*b*c*d^3 + 960*I*(b*x + a)*d^4 -
 960*I*a*d^4)*polylog(4, e^(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^
(I*b*x + I*a)) + 480*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4
 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, e^(I*b*x + I*a)) - 10*(2*b^3*c
^3*d - 6*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(2*a^2 - 1)*b*c*d^3 - (2*a^3
 - 3*a)*d^4 + 6*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(2*b^2*c^2*d^2 - 4*a*b*c*
d^3 + (2*a^2 - 1)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/b^4)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\cot(a+b\,x)\,\left(c+d\,x\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4,x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4\cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*cos(a + b*x)**2*cot(a + b*x), x)
```

3.165 $\int (c+dx)^3 \cos^2(a+bx) \cot(a+bx)\, dx$

Optimal. Leaf size=246

$$\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}-\frac{3id(c+d}{}$$

```
[Out] -3/8*d^3*x/b^3+1/4*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1-exp(2*I*(b*
x+a)))/b-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*
polylog(3,exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4+3
/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2+
3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

Rubi [A] time = 0.28, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4408, 4404, 3311, 32, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\,\text{s}}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) - ((I/4)*(c + d*x)^4)/d + ((c + d*x)
^3*Log[1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^
((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])
/(2*b^3) + (((3*I)/4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^3*Cos
[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x
])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin[a
+ b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^3\cot(a+bx)\,dx - \int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1-e^{2i(a+bx)}}\,dx + \frac{(3d}{}$$
$$= -\frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)\sin}{4b^2}$$
$$= \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c+dx)^2\mathrm{Li}_2}{2b^2}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c}{}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c}{}$$
$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{3id(c}{}$$

Mathematica [B] time = 6.41, size = 1918, normalized size = 7.80

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] -1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I
)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 +
E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a
+ b*x))]))/E^((2*I)*a)))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) +
 (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(
b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a
+ b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((
-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(4*b^4)
+ (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a
]))/(b*(Cos[a]^2 + Sin[a]^2)) + Csc[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)
*Sin[2*a + 2*b*x])/b^4)*(32*b^4*c^3*x*Cos[a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos
[a + 2*b*x] + 32*b^4*c*d^2*x^3*Cos[a + 2*b*x] + 8*b^4*d^3*x^4*Cos[a + 2*b*x
] + 32*b^4*c^3*x*Cos[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Cos[3*a + 2*b*x] + 32*
```

```
b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + 8*b^4*d^3*x^4*Cos[3*a + 2*b*x] + (4*I)*b^3
*c^3*Cos[3*a + 4*b*x] - 6*b^2*c^2*d*Cos[3*a + 4*b*x] - (6*I)*b*c*d^2*Cos[3*
a + 4*b*x] + 3*d^3*Cos[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Cos[3*a + 4*b*x] -
 12*b^2*c*d^2*x*Cos[3*a + 4*b*x] - (6*I)*b*d^3*x*Cos[3*a + 4*b*x] + (12*I)*
b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - 6*b^2*d^3*x^2*Cos[3*a + 4*b*x] + (4*I)*b^3
*d^3*x^3*Cos[3*a + 4*b*x] - (4*I)*b^3*c^3*Cos[5*a + 4*b*x] + 6*b^2*c^2*d*Co
s[5*a + 4*b*x] + (6*I)*b*c*d^2*Cos[5*a + 4*b*x] - 3*d^3*Cos[5*a + 4*b*x] -
(12*I)*b^3*c^2*d*x*Cos[5*a + 4*b*x] + 12*b^2*c*d^2*x*Cos[5*a + 4*b*x] + (6*
I)*b*d^3*x*Cos[5*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Cos[5*a + 4*b*x] + 6*b^2
*d^3*x^2*Cos[5*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Cos[5*a + 4*b*x] + 8*b^3*c^3*
Sin[a] - (12*I)*b^2*c^2*d*Sin[a] - 12*b*c*d^2*Sin[a] + (6*I)*d^3*Sin[a] + 2
4*b^3*c^2*d*x*Sin[a] - (24*I)*b^2*c*d^2*x*Sin[a] - 12*b*d^3*x*Sin[a] + 24*b
^3*c*d^2*x^2*Sin[a] - (12*I)*b^2*d^3*x^2*Sin[a] + 8*b^3*d^3*x^3*Sin[a] + (3
2*I)*b^4*c^3*x*Sin[a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[a + 2*b*x] + (32*I
)*b^4*c*d^2*x^3*Sin[a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[a + 2*b*x] + (32*I)*
b^4*c^3*x*Sin[3*a + 2*b*x] + (48*I)*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + (32*I)
*b^4*c*d^2*x^3*Sin[3*a + 2*b*x] + (8*I)*b^4*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^
3*c^3*Sin[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Sin[3*a + 4*b*x] + 6*b*c*d^2*Sin[3
*a + 4*b*x] + (3*I)*d^3*Sin[3*a + 4*b*x] - 12*b^3*c^2*d*x*Sin[3*a + 4*b*x]
- (12*I)*b^2*c*d^2*x*Sin[3*a + 4*b*x] + 6*b*d^3*x*Sin[3*a + 4*b*x] - 12*b^3
*c*d^2*x^2*Sin[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*Sin[3*a + 4*b*x] - 4*b^3*d^
3*x^3*Sin[3*a + 4*b*x] + 4*b^3*c^3*Sin[5*a + 4*b*x] + (6*I)*b^2*c^2*d*Sin[5
*a + 4*b*x] - 6*b*c*d^2*Sin[5*a + 4*b*x] - (3*I)*d^3*Sin[5*a + 4*b*x] + 12*
b^3*c^2*d*x*Sin[5*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Sin[5*a + 4*b*x] - 6*b*d^
3*x*Sin[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + (6*I)*b^2*d^3*x^
2*Sin[5*a + 4*b*x] + 4*b^3*d^3*x^3*Sin[5*a + 4*b*x]) - (3*c^2*d*Csc[a]*Sec[
a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Lo
g[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + Ar
cTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[
Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1
+ Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.59, size = 984, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, cos(b*x + a) +
I*sin(b*x + a)) + 24*I*d^3*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 24*I
*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) - 24*I*d^3*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
```

```
c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*
dilog(cos(b*x + a) + I*sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x
 + 12*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (12*I*b^2*d^3*x^2
 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a))
 - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) -
 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
- a^3*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 4*(b^3*d^3*x
^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*
d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*
x + a) - I*sin(b*x + a) + 1) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*s
in(b*x + a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x
+ a)) - 24*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/
b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos(bx+a)^2 \cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)^2*cot(b*x + a), x)
```

maple [B] time = 0.46, size = 899, normalized size = 3.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2
*polylog(2,-exp(I*(b*x+a)))*x-6*I/b*c^2*d*a*x+6*I*d^3*polylog(4,exp(I*(b*x+
a)))/b^4+I*c^3*x+1/8/b^3*(2*b^2*d^3*x^3+6*b^2*c*d^2*x^2+6*b^2*c^2*d*x+2*b^2
*c^3-3*d^3*x-3*c*d^2)*cos(2*b*x+2*a)-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+2/b
^4*d^3*a^3*ln(exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*
c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3
*d^3*polylog(3,-exp(I*(b*x+a)))*x-3/2*I/b^4*a^4*d^3+6*I/b^4*d^3*polylog(4,-
exp(I*(b*x+a)))-I*c*d^2*x^3-3/2*I*c^2*d*x^2-1/4*I*d^3*x^4+1/b*c^3*ln(exp(I*
```

```
(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)-2/b*c^3*ln(exp(I*(b*x+a)))+3/b^3*c
*d^2*a^2*ln(exp(I*(b*x+a))-1)-6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-3*I/b^2*c^
2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/
b^2*c^2*d*a^2-2*I/b^3*a^3*d^3*x+4*I/b^3*c*d^2*a^3-3*I/b^2*d^3*polylog(2,exp
(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(ex
p(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(
b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a
)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-
1)+6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*
d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-3/16*d*(2*b^2
*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^4*sin(2*b*x+2*a)
```

maxima [B] time = 0.80, size = 967, normalized size = 3.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/16*(8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^3 - 24*(sin(b*x + a)^2 -
log(sin(b*x + a)^2))*a*c^2*d/b + 24*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*
a^2*c*d^2/b^2 - 8*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a^3*d^3/b^3 - (-4*
I*(b*x + a)^4*d^3 + (-16*I*b*c*d^2 + 16*I*a*d^3)*(b*x + a)^3 + 96*I*d^3*pol
ylog(4, -e^(I*b*x + I*a)) + 96*I*d^3*polylog(4, e^(I*b*x + I*a)) + (-24*I*b
^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d
^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a)^2 + (48*I*b^2*c^2*d - 96*I*a*b*c
*d^2 + 48*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (
-16*I*(b*x + a)^3*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x + a)^2 + (-48*I*b
^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) + 2*(2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b
*x + a))*cos(2*b*x + 2*a) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x +
 a)^2*d^3 - 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*dilog(-e
^(I*b*x + I*a)) + (-48*I*b^2*c^2*d + 96*I*a*b*c*d^2 - 48*I*(b*x + a)^2*d^3
- 48*I*a^2*d^3 + (-96*I*b*c*d^2 + 96*I*a*d^3)*(b*x + a))*dilog(e^(I*b*x + I
*a)) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*cos(b*x + a) + 1) + 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(
b*x + a)^2 - 2*cos(b*x + a) + 1) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*pol
ylog(3, -e^(I*b*x + I*a)) + 96*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3,
 e^(I*b*x + I*a)) - 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a
^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/b^3)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^2\cot(a+b\,x)\,\left(c+d\,x\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)**2*cot(a + b*x), x)
```

3.166 $\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=181

$$\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\sin^2(a+bx)}{4b^3}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{(c+dx)^2\log\left(1-\right.}{b}$$

```
[Out] 1/2*c*d*x/b+1/4*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1-exp(2*I*(b*x+a))
)/b-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,exp(2*I*(
b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2/b
^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.23, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4408, 4404, 3310, 3717, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[
1 - E^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 + (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^2
*Sin[a + b*x]^2)/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)^2\cot(a+bx)\,dx - \int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1-e^{2i(a+bx)}}\,dx + \frac{d}{}$$
$$= -\frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)\sin}{2b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\mathrm{Li}}{b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\mathrm{Li}}{b^2}$$
$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id(c+dx)\mathrm{Li}}{b^2}$$

Mathematica [B] time = 2.90, size = 564, normalized size = 3.12

$$\frac{48b^3cdx^2\cot(a) - 48b^3cdx^2e^{i\tan^{-1}(\tan(a))}\cot(a)\sqrt{\sec^2(a)} + 48b^2c^2\log(\sin(a+bx)) - 6b^2c^2\csc(a)\sin(a+2bx) +}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]^2*Cot[a + b*x],x]

[Out] ((48*I)*b^2*c*d*Pi*x + (16*I)*b^3*d^2*x^3 - (96*I)*b^2*c*d*x*ArcTan[Tan[a]]
 + 48*b^3*c*d*x^2*Cot[a] - 6*b*c*d*Cos[a + 2*b*x]*Csc[a] - 6*b*d^2*x*Cos[a
+ 2*b*x]*Csc[a] + 6*b*c*d*Cos[3*a + 2*b*x]*Csc[a] + 6*b*d^2*x*Cos[3*a + 2*b
*x]*Csc[a] + 48*b*c*d*Pi*Log[1 + E^((-2*I)*b*x)] + 48*b^2*d^2*x^2*Log[1 - E
^((-I)*(a + b*x))] + 48*b^2*d^2*x^2*Log[1 + E^((-I)*(a + b*x))] + 96*b^2*c*
d*x*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + 96*b*c*d*ArcTan[Tan[a]]*Log
[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] - 48*b*c*d*Pi*Log[Cos[b*x]] + 48*b^2
*c^2*Log[Sin[a + b*x]] - 96*b*c*d*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a
]]]] + (96*I)*b*d^2*x*PolyLog[2, -E^((-I)*(a + b*x))] + (96*I)*b*d^2*x*Poly
Log[2, E^((-I)*(a + b*x))] - (48*I)*b*c*d*PolyLog[2, E^((2*I)*(b*x + ArcTan
[Tan[a]]))] + 96*d^2*PolyLog[3, -E^((-I)*(a + b*x))] + 96*d^2*PolyLog[3, E^
((-I)*(a + b*x))] - 48*b^3*c*d*E^(I*ArcTan[Tan[a]])*x^2*Cot[a]*Sqrt[Sec[a]^
2] - 6*b^2*c^2*Csc[a]*Sin[a + 2*b*x] + 3*d^2*Csc[a]*Sin[a + 2*b*x] - 12*b^2
*c*d*x*Csc[a]*Sin[a + 2*b*x] - 6*b^2*d^2*x^2*Csc[a]*Sin[a + 2*b*x] + 6*b^2*
c^2*Csc[a]*Sin[3*a + 2*b*x] - 3*d^2*Csc[a]*Sin[3*a + 2*b*x] + 12*b^2*c*d*x*
Csc[a]*Sin[3*a + 2*b*x] + 6*b^2*d^2*x^2*Csc[a]*Sin[3*a + 2*b*x])/(48*b^3)
```

fricas [C] time = 0.56, size = 594, normalized size = 3.28

$$-\frac{b^2d^2x^2 + 2\,b^2cdx - \left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx+a\right)^2 - 4\,d^2\mathrm{polylog}\left(3, \cos\left(bx+a\right) + i\,\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 - 4*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 4
*d^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x
+ a) + I*sin(b*x + a)) - 4*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)
*dilog(cos(b*x + a) + I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(cos
(b*x + a) - I*sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-cos(b*x + a)
 + I*sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-cos(b*x + a) - I*sin
(b*x + a)) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*s
in(b*x + a) + 1) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a)
 - I*sin(b*x + a) + 1) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x
 + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-
1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x +
a) + 1))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\cos\left(bx+a\right)^2\cot\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cos(b*x + a)^2*cot(b*x + a), x)
```

maple [B] time = 0.49, size = 535, normalized size = 2.96

$$\frac{d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^3}-\frac{2d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^2}{b}+\frac{4id}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b*d^2
*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d^2*ln(exp
(I*(b*x+a))+1)*x^2+4/3*I/b^3*d^2*a^3-I*c*d*x^2-4*I/b*c*d*a*x+2*d^2*polylog(
3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+I*c^2*x-2/b*c^2*
ln(exp(I*(b*x+a)))+1/b*c^2*ln(exp(I*(b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1
)-1/3*I*d^2*x^3-1/4*d*(d*x+c)*sin(2*b*x+2*a)/b^2+2/b*c*d*ln(1-exp(I*(b*x+a)
))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x+4/b^2*
c*d*a*ln(exp(I*(b*x+a)))-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*a^2*x
-2*I/b^2*c*d*a^2-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polyl
og(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*p
olylog(2,exp(I*(b*x+a)))+1/8*(2*b^2*d^2*x^2+4*b^2*c*d*x+2*b^2*c^2-d^2)/b^3*
cos(2*b*x+2*a)
```

maxima [B] time = 0.46, size = 522, normalized size = 2.88

$$-\frac{12\left(\sin\left(bx+a\right)^2-\log\left(\sin\left(bx+a\right)^2\right)\right)c^2-\frac{24\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2-\log\left(\sin(bx+a)^2\right)\right)a^2d^2}{b^2}-\frac{-8i\,(\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*c^2 - 24*(sin(b*x + a)^2 -
 log(sin(b*x + a)^2))*a*c*d/b + 12*(sin(b*x + a)^2 - log(sin(b*x + a)^2))*a
^2*d^2/b^2 - (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)^2
 + 48*d^2*polylog(3, -e^(I*b*x + I*a)) + 48*d^2*polylog(3, e^(I*b*x + I*a))
 + (24*I*(b*x + a)^2*d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin
(b*x + a), cos(b*x + a) + 1) + (-24*I*(b*x + a)^2*d^2 + (-48*I*b*c*d + 48*I
*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 3*(2*(b*x + a
)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (-48*I*b*c*
d - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(-e^(I*b*x + I*a)) + (-48*I*b*c*d
 - 48*I*(b*x + a)*d^2 + 48*I*a*d^2)*dilog(e^(I*b*x + I*a)) + 12*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*cos(b*x + a) + 1) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*lo
g(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 6*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(a+b\,x\right)^2\,\cot\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)**2*cot(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)**2*cot(a + b*x), x)
```

3.167 $\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$

Optimal. Leaf size=114

$$-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/4*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b-1/2*I*d*polylo
g(2,exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin(b
*x+a)^2/b
```

Rubi [A] time = 0.13, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4408, 4404, 2635, 8, 3717, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}+\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}-\frac{i(c$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x)/(4*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 - E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Sin
[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx = \int (c+dx)\cot(a+bx)\,dx - \int (c+dx)\cos(a+bx)\sin(a+bx)\,dx$$
$$= -\frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx + \frac{d\int \mathrm{s}}{}$$
$$= -\frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} -$$
$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+b}{4b^2}$$
$$= \frac{dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2} - \frac{d\cos}{}$$

Mathematica [A] time = 0.35, size = 131, normalized size = 1.15

$$\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\mathrm{Li}_2\left(e^{2i(a+bx)}\right)\right)\right)}{b^2} - \frac{d\sin(2(a+bx))}{8b^2} - \frac{ad\log(\sin(a+bx))}{b^2} - \frac{c\sin^2(a}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]^2*Cot[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (c*Log[Sin[a + b*x]])/b - (a*d*Log[Sin[a + b
*x]])/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2
 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 - (c*Sin[a + b*x]^2)/(2*b) - (d*S
in[2*(a + b*x)])/(8*b^2)
```

fricas [B] time = 0.59, size = 292, normalized size = 2.56

$$-\frac{bdx - 2\,(bdx+bc)\cos\,(bx+a)^2 + d\cos\,(bx+a)\sin\,(bx+a) + 2i\,d\mathrm{Li}_2\,(\cos\,(bx+a) + i\,\sin\,(bx+a)) - 2i\,d\mathrm{Li}_2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) - 2*I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - 2*I*d*dilog(-cos(b*x + a) + I*sin(b*x + a)) + 2*I*d*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - 2*(b*d*x + b*c)*log(cos(b*x + a) + I*sin(
b*x + a) + 1) - 2*(b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 2*
(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*(b*c - a*
```

```
d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 2*(b*d*x + a*d)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*d*x + a*d)*log(-cos(b*x + a) - I
*sin(b*x + a) + 1))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cos(bx+a)^2\cot(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cos(b*x + a)^2*cot(b*x + a), x)
```

maple [B] time = 0.41, size = 249, normalized size = 2.18

$$icx-\frac{id\,x^2}{2}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{id\operatorname{polylog}\left(2,-e^{i(bx+a)}\right)}{b^2}-\frac{id\operatorname{polylog}\left(2,e^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x)
```

```
[Out] I*c*x-2*I/b*d*a*x+1/b*c*ln(exp(I*(b*x+a))-1)+1/b*c*ln(exp(I*(b*x+a))+1)-2/b
*c*ln(exp(I*(b*x+a)))-1/2*I*d*x^2-I/b^2*d*a^2-I*d*polylog(2,exp(I*(b*x+a)))
/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*d*
ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-1/b^2*d*a*ln(exp(I*(b
*x+a))-1)+2/b^2*d*a*ln(exp(I*(b*x+a)))+1/4*(d*x+c)*cos(2*b*x+2*a)/b-1/8*d*s
in(2*b*x+2*a)/b^2
```

maxima [B] time = 0.79, size = 222, normalized size = 1.95

$$\frac{-4i\,b^2dx^2-8i\,b^2cx-8i\,bdx\arctan\left(\sin(bx+a),-\cos(bx+a)+1\right)+8i\,bc\arctan\left(\sin(bx+a),\cos(bx+a)-1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)^2*cot(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x - 8*I*b*d*x*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) + 8*I*b*c*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (8*I*b*d*x
+ 8*I*b*c)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + 2*(b*d*x + b*c)*cos(2*
b*x + 2*a) - 8*I*d*dilog(-e^(I*b*x + I*a)) - 8*I*d*dilog(e^(I*b*x + I*a)) +
 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ 4*(b*d*x + b*c)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
 - d*sin(2*b*x + 2*a))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)^2\cot(a+b\,x)\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x),x)
[Out] int(cos(a + b*x)^2*cot(a + b*x)*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos^2(a+bx)\cot(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)**2*cot(b*x+a),x)
[Out] Integral((c + d*x)*cos(a + b*x)**2*cot(a + b*x), x)
```

3.168 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=82

$$\text{Int}\left(\frac{\cot(a+bx)}{c+dx},x\right)-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+Unintegrable(cot(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
```

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Cot[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx &= \int \frac{\cot(a+bx)}{c+dx}\,dx-\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx\\ &= \int \frac{\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx\\ &= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right)+\int \frac{\cot(a+bx)}{c+dx}\,dx\\ &= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right)-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2b\right.}{c+dx}\\ &= -\frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\int \frac{\cot(a+bx}{c+dx}\end{aligned}$$

Mathematica [A] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x),x]
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(bx+a\right)^{2}\cot\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)^{2}\cot\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^{2}\left(bx+a\right)\right)\cot\left(bx+a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-i\,E_{1}\left(\frac{2i\,bdx+2i\,bc}{d}\right)+i\,E_{1}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+4\,d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^{2}+\sin(bx+a)^{2}+2\,\cos(bx+a)+1\right)}\,dx-4\,d}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*((-I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + I*exp_integral_e(1,
-(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*d*integrate(sin(b*x +
a)/((d*x + c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)
*cos(b*x + a) + c), x) - 4*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)
^2 + (d*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) - (
exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d*x
+ 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/d
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x), x)
```

3.169 $\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$\text{Int}\left(\frac{\cot(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] -b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(cot(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Cot[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx &= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx\\
&= \int \frac{\cot(a+bx)}{(c+dx)^2}\,dx-\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right)+\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin}{}}{d}\\
&= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+\right.}{d^2}
\end{aligned}$$

Mathematica [A] time = 2.49, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Cos[a + b*x]^2*Cot[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos(bx+a)^2\cot(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)^2*cot(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(bx+a)^2\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)^2*cot(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{\left(\cos^2(bx+a)\right)\cot(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
[Out] int(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(-i\,E_2\left(\frac{2i\,bdx+2i\,bc}{d}\right)+i\,E_2\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+4\left(d^2x+cd\right)\int\frac{\sin(bx+a)}{(dx+c)^2\left(\cos(bx+a)^2+\sin(bx+a)^2+2\,\cos(bx+a)+1\right)}\,d}{4\left(d^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)^2*cot(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*((-I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + I*exp_integral_e(2,
-(2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 4*(d^2*x + c*d)*integrat
e(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^
2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x +
 c^2)*cos(b*x + a)), x) - 4*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 +
 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x +
c^2)*sin(b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) -
 (exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d*
x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)^2\cot(a+b\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)^2*cot(a + b*x))/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)**2*cot(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)**2*cot(a + b*x)/(c + d*x)**2, x)
```

3.170 $\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=154

$$\text{Int}\left(\cot(a+bx)\csc(a+bx)(c+dx)^m,x\right)+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m \ldots}{}$$

```
[Out] CannotIntegrate((d*x+c)^m*cot(b*x+a)*csc(b*x+a),x)+1/2*I*exp(I*(a-b*c/d))*(
d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)-1/2*I*(d*x+c)^m*G
AMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Cot[a + b*x]*Csc[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx &= -\int (c+dx)^m \cos(a+bx)\,dx+\int (c+dx)^m \cot(a+bx)\csc(a+bx)\,dx\\ &= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^{m}\ldots\\ &= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)\ldots}{}\end{aligned}$$

Mathematica [A] time = 9.89, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Cos[a + b*x]*Cot[a + b*x]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\cos(bx+a)\cot(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\cot(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\left(\cot^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
[Out] int((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\cos(bx+a)\cot(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cos(b*x + a)*cot(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^m, x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \cos(a + bx) \cot^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cos(a + b*x)*cot(a + b*x)**2, x)
```

3.171 $\int (c+dx)^4 \cos(a+bx) \cot^2(a+bx) \, dx$

Optimal. Leaf size=299

$$-\frac{24id^4\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^5}+\frac{24id^4\text{Li}_4\left(e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\sin(a+bx)}{b^5}-\frac{24d^3(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{Li}_3\left(e^{i(a+\right.}}{b^4}$$

```
[Out] -8*d*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b^2+24*d^3*(d*x+c)*cos(b*x+a)/b^4-4*
d*(d*x+c)^3*cos(b*x+a)/b^2-(d*x+c)^4*csc(b*x+a)/b+12*I*d^2*(d*x+c)^2*polylo
g(2,-exp(I*(b*x+a)))/b^3-12*I*d^2*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^3-2
4*d^3*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^4+24*d^3*(d*x+c)*polylog(3,exp(I
*(b*x+a)))/b^4-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I*d^4*polylog(4,e
xp(I*(b*x+a)))/b^5-24*d^4*sin(b*x+a)/b^5+12*d^2*(d*x+c)^2*sin(b*x+a)/b^3-(d
*x+c)^4*sin(b*x+a)/b
```

Rubi [A] time = 0.29, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {4408, 3296, 2637, 4410, 4183, 2531, 6609, 2282, 6589}

$$-\frac{24d^3(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{24d^3(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^4}+\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-8*d*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b^2 + (24*d^3*(c + d*x)*Cos[a +
 b*x])/b^4 - (4*d*(c + d*x)^3*Cos[a + b*x])/b^2 - ((c + d*x)^4*Csc[a + b*x]
)/b + ((12*I)*d^2*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((12*I)*d
^2*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (24*d^3*(c + d*x)*PolyLog
[3, -E^(I*(a + b*x))])/b^4 + (24*d^3*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])
/b^4 - ((24*I)*d^4*PolyLog[4, -E^(I*(a + b*x))])/b^5 + ((24*I)*d^4*PolyLog[
4, E^(I*(a + b*x))])/b^5 - (24*d^4*Sin[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*
Sin[a + b*x])/b^3 - ((c + d*x)^4*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^4\cos(a+bx)\,dx + \int (c+dx)^4\cot(a+bx)\csc(a+bx)\,dx$$

$$= -\frac{(c+dx)^4\csc(a+bx)}{b} - \frac{(c+dx)^4\sin(a+bx)}{b} + \frac{(4d)\int (c+dx)^3\csc(}{b}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{4d(c+dx)^3\cos(a+bx)}{b^2} - \frac{(c+dx)^4}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+d}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+d}{}$$

$$= -\frac{8d(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^3(c+dx)\cos(a+bx)}{b^4} - \frac{4d(c+d}{}$$

Mathematica [B] time = 1.71, size = 798, normalized size = 2.67

$$\frac{\csc(a+bx)\left(-3c^4b^4-3d^4x^4b^4-12cd^3x^3b^4-18c^2d^2x^2b^4-12c^3dxb^4+c^4\cos(2(a+bx))b^4+d^4x^4\cos(2(a+bx)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^4*c^4 + 12*b^2*c^2*d^2 - 24*d^4 - 12*b^4*c^3*d*x + 24*b
^2*c*d^3*x - 18*b^4*c^2*d^2*x^2 + 12*b^2*d^4*x^2 - 12*b^4*c*d^3*x^3 - 3*b^4
*d^4*x^4 + b^4*c^4*Cos[2*(a + b*x)] - 12*b^2*c^2*d^2*Cos[2*(a + b*x)] + 24*
d^4*Cos[2*(a + b*x)] + 4*b^4*c^3*d*x*Cos[2*(a + b*x)] - 24*b^2*c*d^3*x*Cos[
2*(a + b*x)] + 6*b^4*c^2*d^2*x^2*Cos[2*(a + b*x)] - 12*b^2*d^4*x^2*Cos[2*(a
 + b*x)] + 4*b^4*c*d^3*x^3*Cos[2*(a + b*x)] + b^4*d^4*x^4*Cos[2*(a + b*x)]
```

```
- 16*b^3*c^3*d*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48*b^3
*c^2*d^2*x*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 48*b^3*c*d
^3*x^2*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 16*b^3*d^4*x^3
*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + (24*I)*b^2*d^2*(c +
d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] - (24*I)*b^2
*d^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - 4
8*b*c*d^3*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] - 48*b*d^
4*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*c*d^3*Po
lyLog[3, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] + 48*b*d^4*x*PolyLog[3
, Cos[a + b*x] + I*Sin[a + b*x]]*Sin[a + b*x] - (48*I)*d^4*PolyLog[4, -Cos[
a + b*x] - I*Sin[a + b*x]]*Sin[a + b*x] + (48*I)*d^4*PolyLog[4, Cos[a + b*x
] + I*Sin[a + b*x]]*Sin[a + b*x] - 4*b^3*c^3*d*Sin[2*(a + b*x)] + 24*b*c*d^
3*Sin[2*(a + b*x)] - 12*b^3*c^2*d^2*x*Sin[2*(a + b*x)] + 24*b*d^4*x*Sin[2*(
a + b*x)] - 12*b^3*c*d^3*x^2*Sin[2*(a + b*x)] - 4*b^3*d^4*x^3*Sin[2*(a + b*
x)]))/(2*b^5)
```

fricas [C] time = 0.63, size = 1233, normalized size = 4.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 12*b^2*c^2*d^2 - 12*I*d^4*p
olylog(4, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4,
 cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 12*I*d^4*polylog(4, -cos(b*x
 + a) + I*sin(b*x + a))*sin(b*x + a) + 12*I*d^4*polylog(4, -cos(b*x + a) -
I*sin(b*x + a))*sin(b*x + a) + 24*d^4 + 12*(b^4*c^2*d^2 - b^2*d^4)*x^2 - (b
^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 - 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c
^2*d^2 - 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d - 6*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4
*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d - 6*b*c*d^3 + 3*(b^3*c^2*d^2 -
2*b*d^4)*x)*cos(b*x + a)*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*
x - 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6
*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(cos(b*x + a) - I
*sin(b*x + a))*sin(b*x + a) - (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^
2*c^2*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b^2*d^
4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*dilog(-cos(b*x + a) - I*sin(b*x
 + a))*sin(b*x + a) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x +
b^3*c^3*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 2*(b^3*d^4
*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c
*d^3 - a^3*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x +
 a) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*log(-1/2*co
s(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b^
3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*
```

```
log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 2*(b^3*d^4*x^3 + 3*b
^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 12*(b*d^4*x + b*c*d
^3)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - 12*(b*d^4*x +
b*c*d^3)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 12*(b*d^4
*x + b*c*d^3)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 12*
(b*d^4*x + b*c*d^3)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a)
 + 8*(b^4*c^3*d - 3*b^2*c*d^3)*x)/(b^5*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^4 \cos(bx + a) \cot(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cos(b*x + a)*cot(b*x + a)^2, x)
```

maple [B] time = 0.16, size = 1056, normalized size = 3.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 8/b^5*d^4*a^3*arctanh(exp(I*(b*x+a)))-24/b^4*d^3*c*polylog(3,-exp(I*(b*x+a)
))+24/b^4*d^3*c*polylog(3,exp(I*(b*x+a)))-8/b^2*d*c^3*arctanh(exp(I*(b*x+a)
))-24/b^4*d^4*polylog(3,-exp(I*(b*x+a)))*x+24/b^4*d^4*polylog(3,exp(I*(b*x+
a)))*x-24*I*d^4*polylog(4,-exp(I*(b*x+a)))/b^5+24*I/b^3*d^3*c*polylog(2,-ex
p(I*(b*x+a)))*x-24*I/b^3*d^3*c*polylog(2,exp(I*(b*x+a)))*x+24*I*d^4*polylog
(4,exp(I*(b*x+a)))/b^5+1/2*I*(d^4*x^4*b^4+4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2
+4*b^4*c^3*d*x+4*I*b^3*d^4*x^3+b^4*c^4-12*b^2*d^4*x^2+12*I*b^3*c*d^3*x^2-24
*b^2*c*d^3*x+12*I*b^3*c^2*d^2*x-12*c^2*d^2*b^2+4*I*b^3*c^3*d-24*I*b*d^4*x+2
4*d^4-24*I*b*c*d^3)/b^5*exp(I*(b*x+a))-2*I*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x
^2+4*c^3*d*x+c^4)*exp(I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)-1/2*I*(d^4*x^4*b^4+
4*b^4*c*d^3*x^3+6*b^4*c^2*d^2*x^2+4*b^4*c^3*d*x-4*I*b^3*d^4*x^3+b^4*c^4-12*
b^2*d^4*x^2-12*I*b^3*c*d^3*x^2-24*b^2*c*d^3*x-12*I*b^3*c^2*d^2*x-12*c^2*d^2
*b^2-4*I*b^3*c^3*d+24*I*b*d^4*x+24*d^4+24*I*b*c*d^3)/b^5*exp(-I*(b*x+a))+12
/b^2*d^2*c^2*ln(1-exp(I*(b*x+a)))*x+12/b^3*d^2*c^2*ln(1-exp(I*(b*x+a)))*a-1
2/b^2*d^2*c^2*ln(exp(I*(b*x+a))+1)*x-12/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)*a+
12/b^2*d^3*c*ln(1-exp(I*(b*x+a)))*x^2-12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a^2
-12/b^2*d^3*c*ln(exp(I*(b*x+a))+1)*x^2+12/b^4*d^3*c*ln(exp(I*(b*x+a))+1)*a^
2+4/b^2*d^4*ln(1-exp(I*(b*x+a)))*x^3+4/b^5*d^4*ln(1-exp(I*(b*x+a)))*a^3-4/b
^2*d^4*ln(exp(I*(b*x+a))+1)*x^3-4/b^5*d^4*ln(exp(I*(b*x+a))+1)*a^3-24/b^4*d
```

```
^3*c*a^2*arctanh(exp(I*(b*x+a)))+24/b^3*d^2*c^2*a*arctanh(exp(I*(b*x+a)))+1
2*I/b^3*d^4*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^3*d^4*polylog(2,exp(I*(b*
x+a)))*x^2+12*I/b^3*d^2*c^2*polylog(2,-exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*pol
ylog(2,exp(I*(b*x+a)))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)\,\cot\left(a+b\,x\right)^2\,\left(c+d\,x\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4,x)
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cos(b*x+a)*cot(b*x+a)**2,x)
[Out] Integral((c + d*x)**4*cos(a + b*x)*cot(a + b*x)**2, x)
```

3.172 $\int (c+dx)^3 \cos(a+bx) \cot^2(a+bx)\, dx$

Optimal. Leaf size=216

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\cos(a+bx)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+$$

```
[Out] -6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)
^2*cos(b*x+a)/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(
b*x+a)))/b^3-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,
-exp(I*(b*x+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*sin(
b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

Rubi [A] time = 0.22, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4408, 3296, 2638, 4410, 4183, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}}{b^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 + (6*d^3*Cos[a + b*x])/b^4
- (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I
)*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 +
 (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*Sin[a + b*x])/b
^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\,dx + \int (c+dx)^3\cot(a+bx)\csc(a+bx)\,dx$$

$$= -\frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b} + \frac{(3d)\int (c+dx)^2\csc(}{b}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{(c+dx)^3 c}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} - \frac{(c+dx)^3 c}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a}{b^2}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a}{b^2}$$

Mathematica [B] time = 1.42, size = 539, normalized size = 2.50

$$\frac{\csc(a+bx)\left(b^3c^3\cos(2(a+bx)) + 3b^3c^2dx\cos(2(a+bx)) + 3b^3cd^2x^2\cos(2(a+bx)) + b^3d^3x^3\cos(2(a+bx)) - \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (Csc[a + b*x]*(-3*b^3*c^3 + 6*b*c*d^2 - 9*b^3*c^2*d*x + 6*b*d^3*x - 9*b^3*c
*d^2*x^2 - 3*b^3*d^3*x^3 + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a +
b*x)] + 3*b^3*c^2*d*x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3
*c*d^2*x^2*Cos[2*(a + b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] + 6*b^2*c^2*d*Lo
g[1 - E^(I*(a + b*x))]*Sin[a + b*x] + 12*b^2*c*d^2*x*Log[1 - E^(I*(a + b*x)
)]*Sin[a + b*x] + 6*b^2*d^3*x^2*Log[1 - E^(I*(a + b*x))]*Sin[a + b*x] - 6*b
^2*c^2*d*Log[1 + E^(I*(a + b*x))]*Sin[a + b*x] - 12*b^2*c*d^2*x*Log[1 + E^(
I*(a + b*x))]*Sin[a + b*x] - 6*b^2*d^3*x^2*Log[1 + E^(I*(a + b*x))]*Sin[a +
 b*x] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))]*Sin[a + b*x] -
(12*I)*b*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))]*Sin[a + b*x] - 12*d^3*Po
lyLog[3, -E^(I*(a + b*x))]*Sin[a + b*x] + 12*d^3*PolyLog[3, E^(I*(a + b*x))
]*Sin[a + b*x] - 3*b^2*c^2*d*Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*
b^2*c*d^2*x*Sin[2*(a + b*x)] - 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

fricas [C] time = 0.56, size = 797, normalized size = 3.69

$$-\frac{4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 4\,b^3c^3 - 6\,d^3\mathrm{polylog}\left(3, \cos\left(bx+a\right) + i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right) - 6\,d^3\mathrm{polylog}\left(3, \mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 4*b^3*c^3 - 6*d^3*polylog(3, cos(b
*x + a) + I*sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*
sin(b*x + a))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) + 6*d^3*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x
+ a) - 12*b*c*d^2 - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
+ 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*
x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*
d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I
*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x
 - 6*I*b*c*d^2)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b
*d^3*x + 6*I*b*c*d^2)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*log(cos(
b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3
*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a
^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x +
 a) + 1)*sin(b*x + a) + 12*(b^3*c^2*d - b*d^3)*x)/(b^4*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cos (bx+a) \cot (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cos(b*x + a)*cot(b*x + a)^2, x)
```

maple [B] time = 0.13, size = 649, normalized size = 3.00

$$\frac{i\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3\right)\mathrm{e}^{i(bx+a)}}{2b^4}-\frac{i\left(d^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*
b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
```

```
-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6
*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a
))+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+6/b^2*d^2*c*ln(1-exp(I*(b*x+a))
)*x+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6
/b^3*d^2*c*ln(exp(I*(b*x+a))+1)*a-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^4+6*d^
3*polylog(3,exp(I*(b*x+a)))/b^4-6/b^2*d*c^2*arctanh(exp(I*(b*x+a)))-6/b^4*d
^3*a^2*arctanh(exp(I*(b*x+a)))+12/b^3*d^2*c*a*arctanh(exp(I*(b*x+a)))-6*I/b
^3*d^3*polylog(2,exp(I*(b*x+a)))*x-6*I/b^3*d^2*c*polylog(2,exp(I*(b*x+a)))-
3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^2+6*I/b
^3*d^2*c*polylog(2,-exp(I*(b*x+a)))+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*x^2-3/b^
4*d^3*ln(1-exp(I*(b*x+a)))*a^2-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(
I*(b*x+a))/b/(exp(2*I*(b*x+a))-1)
```

maxima [B] time = 1.96, size = 11018, normalized size = 51.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/sin(b*x + a) + sin(b*x + a)) - 6*a*c^2*d*(1/sin(b*x + a) + s
in(b*x + a))/b + 6*a^2*c*d^2*(1/sin(b*x + a) + sin(b*x + a))/b^2 - 2*a^3*d^
3*(1/sin(b*x + a) + sin(b*x + a))/b^3 - 3*(((b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a)
+ a - sin(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2
*(4*(b*x + a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + c
os(b*x + a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)
^2 + (8*(b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*
a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) -
 sin(b*x + a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a
)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x
 + a)*cos(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) +
 cos(b*x + a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2
 + cos(b*x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2
)*sin(2*b*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3
*a) + 2*(3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a
)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +
a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c
os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x
+ a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*
a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)
```

```
^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
 + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x +
 a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) +
 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)
^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin
(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*
(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(
2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*
a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*si
n(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x
+ 2*a) + a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2
*a)^2 + (b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x +
 a)*sin(b*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2
*b*x + 2*a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x +
a))*sin(2*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x
+ a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(
2*b*x + 2*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a
)*cos(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)
^2 + b*x + a)*sin(b*x + a) - cos(b*x + a))*c^2*d/(((cos(2*b*x + 2*a)^2 + si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x +
a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + c
os(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x
+ a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*
a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)
^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
 + 3*a) + sin(b*x + a)^2)*b) + 6*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x +
 2*a) + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin
(2*b*x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x
+ a)*cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x +
a))*cos(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x +
 3*a)^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(
b*x + a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(
2*b*x + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x
+ a))*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b
*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos
(b*x + a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*
x + a)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b
*x + 2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(
3*(b*x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x
```

```
 + a) + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1)
+ ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + s
in(b*x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) +
a - sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (
b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b
*x + a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*
a) - (b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2
*b*x + 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 +
13*(b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)
*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2
*a) + a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x
 + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x
+ a)*sin(b*x + a) - cos(b*x + a))*a*c*d^2/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
 + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x
+ a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*c
os(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + co
s(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(
b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
 + sin(b*x + a)^2)*b^2) - 3*(((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
 + 1)*cos(3*b*x + 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*
x + 2*a))*sin(3*b*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*
cos(b*x + a)*sin(2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*c
os(2*b*x + 2*a) + 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)
^2 - ((b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x +
 a)*cos(2*b*x + 2*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x
```

```
 + 2*a) + 1)*cos(3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))
*sin(2*b*x + 2*a) - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x +
a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x
+ a)*sin(b*x + a) - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^
2 + sin(b*x + a)^2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a
)^2 + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x +
2*a) + sin(2*b*x + 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*
x + a)*sin(b*x + a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a)
 + cos(b*x + a))*cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2
)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*si
n(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*
x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x +
 a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a
) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(
b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((c
os(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x
+ 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*
x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2
*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*c
os(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x
+ a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x +
 a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + s
in(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - s
in(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x +
 a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x +
a)^2 + b*x + 2*(((b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) -
(b*x + a)*cos(b*x + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x
+ 2*a) - sin(b*x + a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b
*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(
b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) +
 a)*sin(3*b*x + 3*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)
*sin(b*x + a)^2)*sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*
sin(b*x + a) - cos(b*x + a))*a^2*d^3/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b
*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 -
2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*
sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*
b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x
 + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x +
```

```
 a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + si
n(b*x + a)^2)*b^3) - 2*(-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*(I*a - 1)*d^3
 - 3*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a)^2 + (I*(b*x + a)^3*d^3 - 6*I*b*c
*d^2 - 6*(-I*a - 1)*d^3 + (3*I*b*c*d^2 - 3*(I*a + 1)*d^3)*(b*x + a)^2 - (6*
b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a))*cos(3*b*x + 3*a)^2 + (6*I*(b*x + a)^3
*d^3 - 12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (18*I*b*c*d^2 - 18*
I*a*d^3)*(b*x + a)^2)*cos(b*x + a)^2 + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 -
6*(I*a + 1)*d^3 + (-3*I*b*c*d^2 - 3*(-I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^
2 - (6*a - 6*I)*d^3)*(b*x + a))*sin(3*b*x + 3*a)^2 - 12*((b*x + a)^3*d^3 -
2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*co
s(b*x + a)*sin(b*x + a) + (-6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 + 12*I*(b*x
+ a)*d^3 - 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2)*sin(b*x +
 a)^2 - (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (
12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d
^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a
)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a
)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (
-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(b*x + a)
 - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a)
- (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I
*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*si
n(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*
x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (
12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d
^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a
)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a
)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (
-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(b*x + a)
 - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (6*I*(b*x + a)^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a)
- (6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I
*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*si
n(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*
x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + ((-7*I*(b*x + a)^3*d^3 +
 18*I*b*c*d^2 - 6*(3*I*a + 1)*d^3 + (-21*I*b*c*d^2 - 3*(-7*I*a - 1)*d^3)*(b
*x + a)^2 + (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*cos(b*x + a) + (7*(b*
x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d
^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*sin(b*x + a))*
cos(3*b*x + 3*a) + (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 - 3*
(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b
```

```
*x + a))*cos(2*b*x + 2*a) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d
^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a) - 12
*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((-
12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + 12*(b*c*d^2
+ (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 1
2*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d
^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-1
2*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x
+ 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) + (12*I*b*c*d^2
 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) - 12*(b*
c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(-e^(I*b*x + I*a)) + ((12
*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x +
 a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^
3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
 - 12*I*a*d^3)*cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x
+ a))*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*
cos(b*x + a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b
*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3)*cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d
^3)*sin(b*x + a))*sin(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*s
in(b*x + a))*dilog(e^(I*b*x + I*a)) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*
d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*
b*x + 2*a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*
sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d
^3)*(b*x + a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d
^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c
*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2
 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3
)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b
*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (6*I*
b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*
d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) + (-3*I*(b*x + a)^2*
d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2
 - a*d^3)*(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(2*b*x + 2*a) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a
))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*
I*a*d^3)*(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2
*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*
b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*
a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^
3)*(b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d^3 +
 (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(b*x + a) + 3*((b*x + a)^2*d^3 +
```

```
2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - (3*I*(b*x +
 a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (12*d^3*cos(b*x + a) + 12*I
*d^3*sin(b*x + a) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3
)*cos(3*b*x + 3*a) - 12*(d^3*cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b*x +
 2*a) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*s
in(3*b*x + 3*a) - (12*I*d^3*cos(b*x + a) - 12*d^3*sin(b*x + a))*sin(2*b*x +
 2*a))*polylog(3, -e^(I*b*x + I*a)) + (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b
*x + a) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*cos(3*b*
x + 3*a) - 12*(d^3*cos(b*x + a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) + (1
2*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*sin(3*b*x +
3*a) + (-12*I*d^3*cos(b*x + a) + 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*pol
ylog(3, e^(I*b*x + I*a)) - ((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + (12*a - 12*I)
*d^3 + (6*b*c*d^2 - (6*a - 6*I)*d^3)*(b*x + a)^2 - (-12*I*b*c*d^2 - 12*(-I*
a - 1)*d^3)*(b*x + a))*cos(3*b*x + 3*a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 +
 (18*a - 6*I)*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c*
d^2 - 6*(I*a + 3)*d^3)*(b*x + a))*cos(b*x + a) - (7*I*(b*x + a)^3*d^3 - 18*
I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x +
a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^3)*(b*x + a))*sin(b*x + a))*sin(3*b*x +
3*a) - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3*a +
 3*I)*d^3)*(b*x + a)^2 + 6*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a))*sin(2*b*
x + 2*a))/(2*b^3*cos(b*x + a) + 2*I*b^3*sin(b*x + a) + (2*b^3*cos(2*b*x + 2
*a) + 2*I*b^3*sin(2*b*x + 2*a) - 2*b^3)*cos(3*b*x + 3*a) - 2*(b^3*cos(b*x +
 a) + I*b^3*sin(b*x + a))*cos(2*b*x + 2*a) - (-2*I*b^3*cos(2*b*x + 2*a) + 2
*b^3*sin(2*b*x + 2*a) + 2*I*b^3)*sin(3*b*x + 3*a) - (2*I*b^3*cos(b*x + a) -
 2*b^3*sin(b*x + a))*sin(2*b*x + 2*a)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*cos(a + b*x)*cot(a + b*x)**2, x)
```

3.173 $\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx) \, dx$

Optimal. Leaf size=139

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\sin(a+bx)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{(c+$$

```
[Out] -4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-2*d*(d*x+c)*cos(b*x+a)/b^2-(d*x+c)
^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,ex
p(I*(b*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

Rubi [A] time = 0.15, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4408, 3296, 2637, 4410, 4183, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\cos(a+bx)}{b^2}-\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{2d^2 s}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] (-4*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (2*d*(c + d*x)*Cos[a + b*x]
)/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -E^(I*(a + b*x
))])/b^3 - ((2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 + (2*d^2*Sin[a + b*x
])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\,dx + \int (c+dx)^2\cot(a+bx)\csc(a+bx)\,dx$$

$$= -\frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} + \frac{(2d)\int (c+dx)\csc(a}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\operatorname{cs}}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\operatorname{cs}}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d(c+dx)\cos(a+bx)}{b^2} - \frac{(c+dx)^2\operatorname{cs}}{b}$$

Mathematica [B] time = 3.92, size = 310, normalized size = 2.23

$$-\frac{2\cos(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(c+dx)\right)+2\sin(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*(8*b*c*d*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + 2*b^2*(c + d*x)^2*Csc
[a] - 4*d^2*(2*ArcTan[Tan[a]]*ArcTanh[Cos[a] - Sin[a]*Tan[(b*x)/2]] + (((b*
x + ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(
b*x + ArcTan[Tan[a]]))]) + I*PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - I*
PolyLog[2, E^(I*(b*x + ArcTan[Tan[a]]))])*Sec[a])/Sqrt[Sec[a]^2]) + 2*Cos[b
*x]*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a]) - b^2*(c +
 d*x)^2*Csc[a/2]*Csc[(a + b*x)/2]*Sin[(b*x)/2] + b^2*(c + d*x)^2*Sec[a/2]*S
ec[(a + b*x)/2]*Sin[(b*x)/2] + 2*((-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d
*(c + d*x)*Sin[a])*Sin[b*x])/b^3
```

fricas [B] time = 0.50, size = 448, normalized size = 3.22

$$-\frac{2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-i\,d^2\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + I*d^2*dilog(cos(b*x + a) + I*si
n(b*x + a))*sin(b*x + a) - I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b
*x + a) + I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - I*d^2*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)*s
in(b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(
b*x + a) + (b*d^2*x + b*c*d)*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) - (b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) +
1)*sin(b*x + a) - (b*d^2*x + a*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)
*sin(b*x + a) - 2*d^2)/(b^3*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\cos\left(bx+a\right)\cot\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*cos(b*x + a)*cot(b*x + a)^2, x)

maple [B] time = 0.13, size = 332, normalized size = 2.39

$$\frac{i\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)e^{i(bx+a)}}{2b^3}-\frac{i\left(d^2x^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)e}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x)

[Out] 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(b*x+a))-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*exp(-I*(b*x+a))-2*I*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I*(b*x+a))-1)-4/b^2*d*c*arctanh(exp(I*(b*x+a)))-2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x-2/b^3*d^2*ln(exp(I*(b*x+a))+1)*a+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2/b^2*d^2*ln(1-exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3+4/b^3*d^2*a*arctanh(exp(I*(b*x+a)))

maxima [B] time = 1.66, size = 3284, normalized size = 23.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")

[Out] (b^2*d^2*x^2*(-I*cos(a) + sin(a)) + b^2*c^2*(-I*cos(a) + sin(a)) - b*c*d*(2*cos(a) + 2*I*sin(a)) - 2*d^2*(-I*cos(a) + sin(a)) - (2*b^2*c*d*(I*cos(a) - sin(a)) + b*d^2*(2*cos(a) + 2*I*sin(a)))*x - ((4*b*d^2*x*(-I*cos(a) + sin(a)) + 4*b*c*d*(-I*cos(a) + sin(a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*cos(2*b*x + 3*a) - 4*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) - ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a) - 4*(b*d^2*x + b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) + 4*(b*d^2*x*(I*cos(a) - sin(a)) + b*c*d*(I*cos(a) - sin(a)))*cos(b*x + a) + (b*d^2*x*(4*cos(a) + 4*I*sin(a)) + b*c*d*(4*cos(a) + 4*I*sin(a)) - 4*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (4*I*b*d^2*x + 4*I*b*c*d)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + (4*(b*d^2*x + b*c*d)*cos(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - (b*d^2*x*(4*cos(a) + 4*I*sin(a)) + b*c*d*(4*cos(a) + 4*I*sin(a)))*sin(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (4*b*c*d*(-I*cos(a) + sin(a))*cos(b*x + a) + b*c*d*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*b*c*d*(I*cos(a) - sin(a)) - 4*I*b*c*d*cos(2*b*x + 3*a) + 4*b*c*d*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) - (-4*I*b*

```
c*d*cos(b*x + a) + 4*b*c*d*sin(b*x + a))*cos(2*b*x + 3*a) - (b*c*d*(4*cos(a
) + 4*I*sin(a)) - 4*b*c*d*cos(2*b*x + 3*a) - 4*I*b*c*d*sin(2*b*x + 3*a))*si
n(3*b*x + 3*a) - 4*(b*c*d*cos(b*x + a) + I*b*c*d*sin(b*x + a))*sin(2*b*x +
3*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (4*b*d^2*x*(I*cos(a) - sin(
a))*cos(b*x + a) - b*d^2*x*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + 4*(b*d^2*
x*(-I*cos(a) + sin(a)) + I*b*d^2*x*cos(2*b*x + 3*a) - b*d^2*x*sin(2*b*x + 3
*a))*cos(3*b*x + 3*a) + 4*(-I*b*d^2*x*cos(b*x + a) + b*d^2*x*sin(b*x + a))*
cos(2*b*x + 3*a) + (b*d^2*x*(4*cos(a) + 4*I*sin(a)) - 4*b*d^2*x*cos(2*b*x +
 3*a) - 4*I*b*d^2*x*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + (4*b*d^2*x*cos(b*x
 + a) + 4*I*b*d^2*x*sin(b*x + a))*sin(2*b*x + 3*a))*arctan2(sin(b*x + a), -
cos(b*x + a) + 1) + ((I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*
b^2*c*d - 2*b*d^2)*x)*cos(3*b*x + 3*a) + (-I*b^2*d^2*x^2 - I*b^2*c^2 + 2*b*
c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*cos(b*x + a) - (b^2*d^2*x^2 + b
^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(3*b*x + 3*a) + (b
^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*sin(b*x
 + a))*cos(3*b*x + 4*a) + ((-6*I*b^2*d^2*x^2 - 12*I*b^2*c*d*x - 6*I*b^2*c^2
 + 4*I*d^2)*cos(b*x + 2*a) + 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2
*d^2)*sin(b*x + 2*a))*cos(3*b*x + 3*a) + (I*b^2*d^2*x^2 + I*b^2*c^2 + 2*b*c
*d - 2*I*d^2 + (2*I*b^2*c*d + 2*b*d^2)*x)*cos(2*b*x + 3*a) + ((6*I*b^2*d^2*
x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2 - 4*I*d^2)*cos(b*x + a) - 2*(3*b^2*d^2*x
^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*sin(b*x + a))*cos(b*x + 2*a) - (4*d^2
*(-I*cos(a) + sin(a))*cos(b*x + a) + d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x +
a) + (4*d^2*(I*cos(a) - sin(a)) - 4*I*d^2*cos(2*b*x + 3*a) + 4*d^2*sin(2*b*
x + 3*a))*cos(3*b*x + 3*a) - (-4*I*d^2*cos(b*x + a) + 4*d^2*sin(b*x + a))*c
os(2*b*x + 3*a) - (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4
*I*d^2*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - 4*(d^2*cos(b*x + a) + I*d^2*sin
(b*x + a))*sin(2*b*x + 3*a))*dilog(-e^(I*b*x + I*a)) - (4*d^2*(I*cos(a) - s
in(a))*cos(b*x + a) - d^2*(4*cos(a) + 4*I*sin(a))*sin(b*x + a) + (4*d^2*(-I
*cos(a) + sin(a)) + 4*I*d^2*cos(2*b*x + 3*a) - 4*d^2*sin(2*b*x + 3*a))*cos(
3*b*x + 3*a) - (4*I*d^2*cos(b*x + a) - 4*d^2*sin(b*x + a))*cos(2*b*x + 3*a)
 + (d^2*(4*cos(a) + 4*I*sin(a)) - 4*d^2*cos(2*b*x + 3*a) - 4*I*d^2*sin(2*b*
x + 3*a))*sin(3*b*x + 3*a) + 4*(d^2*cos(b*x + a) + I*d^2*sin(b*x + a))*sin(
2*b*x + 3*a))*dilog(e^(I*b*x + I*a)) + ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) +
b*c*d*(2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) + (-2*
I*b*d^2*x - 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b
*c*d)*cos(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*
a) - (b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(
b*x + a) - (2*b*d^2*x*(-I*cos(a) + sin(a)) + 2*b*c*d*(-I*cos(a) + sin(a)) -
 (-2*I*b*d^2*x - 2*I*b*c*d)*cos(2*b*x + 3*a) - 2*(b*d^2*x + b*c*d)*sin(2*b*
x + 3*a))*sin(3*b*x + 3*a) + ((2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a) - 2*(b
*d^2*x + b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) - 2*(b*d^2*x*(I*cos(a) - sin
(a)) + b*c*d*(I*cos(a) - sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) - ((b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*
(2*cos(a) + 2*I*sin(a)) - 2*(b*d^2*x + b*c*d)*cos(2*b*x + 3*a) - (2*I*b*d^2
*x + 2*I*b*c*d)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + (2*(b*d^2*x + b*c*d)*c
```

```
os(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*sin(b*x + a))*cos(2*b*x + 3*a) - (
b*d^2*x*(2*cos(a) + 2*I*sin(a)) + b*c*d*(2*cos(a) + 2*I*sin(a)))*cos(b*x +
a) + (2*b*d^2*x*(I*cos(a) - sin(a)) + 2*b*c*d*(I*cos(a) - sin(a)) - (2*I*b*
d^2*x + 2*I*b*c*d)*cos(2*b*x + 3*a) + 2*(b*d^2*x + b*c*d)*sin(2*b*x + 3*a))
*sin(3*b*x + 3*a) - ((-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a) + 2*(b*d^2*x +
 b*c*d)*sin(b*x + a))*sin(2*b*x + 3*a) + 2*(b*d^2*x*(-I*cos(a) + sin(a)) +
b*c*d*(-I*cos(a) + sin(a)))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*cos(b*x + a) + 1) - ((b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*d - 2*d^2 + 2*
(b^2*c*d + I*b*d^2)*x)*cos(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 + 2*I*b*c*
d - 2*d^2 + 2*(b^2*c*d + I*b*d^2)*x)*cos(b*x + a) - (-I*b^2*d^2*x^2 - I*b^2
*c^2 + 2*b*c*d + 2*I*d^2 + (-2*I*b^2*c*d + 2*b*d^2)*x)*sin(3*b*x + 3*a) - (
I*b^2*d^2*x^2 + I*b^2*c^2 - 2*b*c*d - 2*I*d^2 + (2*I*b^2*c*d - 2*b*d^2)*x)*
sin(b*x + a))*sin(3*b*x + 4*a) + (2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^
2 - 2*d^2)*cos(b*x + 2*a) + (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x + 6*I*b^2*c^2
 - 4*I*d^2)*sin(b*x + 2*a))*sin(3*b*x + 3*a) - (b^2*d^2*x^2 + b^2*c^2 - 2*I
*b*c*d - 2*d^2 + 2*(b^2*c*d - I*b*d^2)*x)*sin(2*b*x + 3*a) - (2*(3*b^2*d^2*
x^2 + 6*b^2*c*d*x + 3*b^2*c^2 - 2*d^2)*cos(b*x + a) - (-6*I*b^2*d^2*x^2 - 1
2*I*b^2*c*d*x - 6*I*b^2*c^2 + 4*I*d^2)*sin(b*x + a))*sin(b*x + 2*a))/(b^3*(
2*cos(a) + 2*I*sin(a))*cos(b*x + a) + 2*b^3*(I*cos(a) - sin(a))*sin(b*x + a
) - (b^3*(2*cos(a) + 2*I*sin(a)) - 2*b^3*cos(2*b*x + 3*a) - 2*I*b^3*sin(2*b
*x + 3*a))*cos(3*b*x + 3*a) - 2*(b^3*cos(b*x + a) + I*b^3*sin(b*x + a))*cos
(2*b*x + 3*a) + (2*b^3*(-I*cos(a) + sin(a)) + 2*I*b^3*cos(2*b*x + 3*a) - 2*
b^3*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) - (2*I*b^3*cos(b*x + a) - 2*b^3*sin(
b*x + a))*sin(2*b*x + 3*a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(a+b\,x)\,\cot(a+b\,x)^2\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cos(a+bx) \cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*cos(a + b*x)*cot(a + b*x)**2, x)
```

3.174 $\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$

Optimal. Leaf size=58

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

```
[Out] -d*arctanh(cos(b*x+a))/b^2-d*cos(b*x+a)/b^2-(d*x+c)*csc(b*x+a)/b-(d*x+c)*si
n(b*x+a)/b
```

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4408, 3296, 2638, 4410, 3770}

$$-\frac{d\cos(a+bx)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{(c+dx)\csc(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Cos[a + b*x]])/b^2) - (d*Cos[a + b*x])/b^2 - ((c + d*x)*Csc[a
+ b*x])/b - ((c + d*x)*Sin[a + b*x])/b
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4410

```
Int[Cot[(a_.) + (b_.)*(x_)]^(p_.)*Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Simp[((c + d*x)^m*Csc[a + b*x]^n)/(b*n), x]
+ Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Csc[a + b*x]^n, x], x] /; FreeQ[{
a, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\cot(a+bx)\csc(a+bx)\,dx$$
$$= -\frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+dx)\sin(a+bx)}{b} + \frac{d\int \csc(a+bx)\,dx}{b} + \frac{d}{\ }$$
$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{(c+}{\ }$$

Mathematica [A] time = 0.68, size = 104, normalized size = 1.79

$$-\frac{2bc\sin(a+bx) + 2bc\csc(a+bx) + 2bdx\sin(a+bx) + 2d\cos(a+bx) + bdx\tan\left(\frac{1}{2}(a+bx)\right) + bdx\cot\left(\frac{1}{2}(a+\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cos[a + b*x]*Cot[a + b*x]^2,x]
```

```
[Out] -1/2*(2*d*Cos[a + b*x] + b*d*x*Cot[(a + b*x)/2] + 2*b*c*Csc[a + b*x] + 2*d*
Log[Cos[(a + b*x)/2]] - 2*d*Log[Sin[(a + b*x)/2]] + 2*b*c*Sin[a + b*x] + 2*
b*d*x*Sin[a + b*x] + b*d*x*Tan[(a + b*x)/2])/b^2
```

fricas [A] time = 0.46, size = 95, normalized size = 1.64

$$-\frac{4\,bdx - 2\,(bdx + bc)\cos(bx+a)^2 + 2\,d\cos(bx+a)\sin(bx+a) + d\log\left(\frac{1}{2}\cos(bx+a) + \frac{1}{2}\right)\sin(bx+a) - d\,\mathrm{l}}{2\,b^2\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
 a) + d*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - d*log(-1/2*cos(b*x + a)
+ 1/2)*sin(b*x + a) + 4*b*c)/(b^2*sin(b*x + a))
```

giac [B] time = 5.16, size = 1967, normalized size = 33.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
6*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 -
 d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1
/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(t
an(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^3 + d*log(4*(tan(1/2*b*x)^4 + 2
*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 +
 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)
^4*tan(1/2*a)^3 + 6*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - d*log(4*(tan(1/2*b*
x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)
^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*ta
n(1/2*b*x)^3*tan(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(
1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(
1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*
b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*d*t
an(1/2*b*x)^4*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 - 2*d*tan(1/
2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan(1/2*b*x)^4 - 8*b*d*x*tan(1/2*b*x)^3*tan(1
/2*a) - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a)
+ tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a)
+ 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a) + d*log(4*(tan(1/2*b*x)^
4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x
)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2
*b*x)^4*tan(1/2*a) - 12*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 8*b*d*x*tan(1/2
*b*x)*tan(1/2*a)^3 + b*d*x*tan(1/2*a)^4 - d*log(4*(tan(1/2*b*x)^4*tan(1/2*a
)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b
*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan
(1/2*a)^4 + d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2
*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2
*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4 -
 8*b*c*tan(1/2*b*x)^3*tan(1/2*a) + 2*d*tan(1/2*b*x)^4*tan(1/2*a) - 12*b*c*t
an(1/2*b*x)^2*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 8*b*c*tan(1
/2*b*x)*tan(1/2*a)^3 + 12*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + b*c*tan(1/2*a)^4
+ 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b*d*x*tan(1/2*b*x)^2 + d*log(4*(tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2
*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))
*tan(1/2*b*x)^3 - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + t
an(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + t
an(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3 + 8*b*d*x*tan(1/2*b*x)*tan(
```

```
1/2*a) + 6*b*d*x*tan(1/2*a)^2 + d*log(4*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*ta
n(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 - 2*
tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^3 - d*log(4*(ta
n(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 +
tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan(1/2*a)^2 +
1))*tan(1/2*a)^3 + 6*b*c*tan(1/2*b*x)^2 - 2*d*tan(1/2*b*x)^3 + 8*b*c*tan(1/
2*b*x)*tan(1/2*a) - 12*d*tan(1/2*b*x)^2*tan(1/2*a) + 6*b*c*tan(1/2*a)^2 - 1
2*d*tan(1/2*b*x)*tan(1/2*a)^2 - 2*d*tan(1/2*a)^3 + b*d*x + d*log(4*(tan(1/2
*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2
*a)^2 + tan(1/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))
*tan(1/2*b*x) - d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan
(1/2*b*x)^2*tan(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan
(1/2*a)^2)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x) + d*log(4*(tan(1/2*b*x)^4*tan(1
/2*a)^2 - 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan(1/2*a)^2 + tan(1
/2*b*x)^2 - 2*tan(1/2*b*x)*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a) -
 d*log(4*(tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^3*tan(1/2*a) + tan(1/2*b*x)^2*tan
(1/2*a)^2 + tan(1/2*b*x)^2 + 2*tan(1/2*b*x)*tan(1/2*a) + tan(1/2*a)^2)/(tan
(1/2*a)^2 + 1))*tan(1/2*a) + b*c + 2*d*tan(1/2*b*x) + 2*d*tan(1/2*a))/(b^2*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^2*tan(1/2
*b*x)^4*tan(1/2*a) + b^2*tan(1/2*b*x)*tan(1/2*a)^4 - b^2*tan(1/2*b*x)^3 - b
^2*tan(1/2*a)^3 - b^2*tan(1/2*b*x) - b^2*tan(1/2*a))
```

maple [C] time = 0.14, size = 124, normalized size = 2.14

$$\frac{i\left(bdx+cb+id\right)e^{i(bx+a)}}{2b^2}-\frac{i\left(bdx+cb-id\right)e^{-i(bx+a)}}{2b^2}-\frac{2ie^{i(bx+a)}\left(dx+c\right)}{b\left(e^{2i(bx+a)}-1\right)}-\frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2}+\frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x)
```

```
[Out] 1/2*I*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))-1/2*I*(b*d*x+c*b-I*d)/b^2*exp(-I*(
b*x+a))-2*I*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))-1)-d/b^2*ln(exp(I*(b
*x+a))+1)+d/b^2*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.39, size = 2110, normalized size = 36.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/sin(b*x + a) + sin(b*x + a)) - 2*a*d*(1/sin(b*x + a) + sin(b*x
 + a))/b - (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(3*b*x +
 3*a)^3 + (b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*sin(3*b
```

```
*x + 3*a)^3 - 6*(b*x + a)*sin(b*x + a)^3 - 2*(4*(b*x + a)*cos(b*x + a)*sin(
2*b*x + 2*a) - (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a) +
 3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a)^2 - ((b*x + a)*s
in(b*x + a) + cos(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*cos(2*b*x + 2
*a)*sin(b*x + a) + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) + 1)*cos(
3*b*x + 3*a) - 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)
 - 8*(b*x + a)*sin(b*x + a))*sin(3*b*x + 3*a)^2 - ((b*x + a)*sin(b*x + a) +
 cos(b*x + a))*sin(2*b*x + 2*a)^2 + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a)
 - (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^
2 + 2)*cos(2*b*x + 2*a) + cos(2*b*x + 2*a)^2 + cos(b*x + a)^2 + (13*(b*x +
a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) + sin(2*b*x
+ 2*a)^2 + sin(b*x + a)^2 + 1)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*sin(b*x +
a)^3 + (3*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) + cos(b*x + a))*
cos(2*b*x + 2*a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x
+ 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*
x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)
+ 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a
)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2
- 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos
(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*
b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x +
 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(c
os(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b
*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)
^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 - 2*(cos(2*b*x + 2*a)^2*cos(b*x + a
) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos
(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x
 + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x +
 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(
3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + ((b*x - (b*x + a)*cos(2*b*x + 2*a) + a - sin(2*b*x + 2*a))*
cos(3*b*x + 3*a)^2 + (b*x + a)*cos(2*b*x + 2*a)^2 + (b*x + a)*cos(b*x + a)^
2 + (b*x + a)*sin(2*b*x + 2*a)^2 + 13*(b*x + a)*sin(b*x + a)^2 + b*x + 2*((
(b*x + a)*cos(b*x + a) + sin(b*x + a))*cos(2*b*x + 2*a) - (b*x + a)*cos(b*x
 + a) - ((b*x + a)*sin(b*x + a) - cos(b*x + a))*sin(2*b*x + 2*a) - sin(b*x
+ a))*cos(3*b*x + 3*a) - ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
 a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*sin(3*b*x + 3
*a) - 6*((b*x + a)*cos(b*x + a)^3 + (b*x + a)*cos(b*x + a)*sin(b*x + a)^2)*
sin(2*b*x + 2*a) - (6*(b*x + a)*cos(b*x + a)^2 + b*x + a)*sin(b*x + a) - co
s(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*
a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x +
2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 - 2*cos(2*b*x + 2*a) + 1)
```

```
*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2
- 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 - 2*
cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) - 2*(cos(b*x
 + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 - 2*(cos(2*b*x
+ 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) - 2*cos(2*b*x + 2*a
)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b))/b
```

mupad [B] time = 2.31, size = 162, normalized size = 2.79

$$e^{a\,1i+b\,x\,1i}\left(\frac{(b\,c+d\,1i)\,1i}{2\,b^2}+\frac{d\,x\,1i}{2\,b}\right)+e^{-a\,1i-b\,x\,1i}\left(\frac{(-b\,c+d\,1i)\,1i}{2\,b^2}-\frac{d\,x\,1i}{2\,b}\right)-\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}\,1i+1i\right)}{b^2}+\frac{d\,\ln\left(d\,2i-d\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)*cot(a + b*x)^2*(c + d*x),x)
```

```
[Out] exp(a*1i + b*x*1i)*(((d*1i + b*c)*1i)/(2*b^2) + (d*x*1i)/(2*b)) + exp(- a*1
i - b*x*1i)*(((d*1i - b*c)*1i)/(2*b^2) - (d*x*1i)/(2*b)) - (d*log(exp(a*1i
+ b*x*1i)*1i + 1i))/b^2 + (d*log(d*2i - d*exp(a*1i)*exp(b*x*1i)*2i))/b^2 +
(2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i)*1i - 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cos(a+bx)\cot^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cos(b*x+a)*cot(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*cos(a + b*x)*cot(a + b*x)**2, x)
```

3.175 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=75

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{c+dx},x\right)-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c),x)-Ci(b*c/d+b*x)*cos(a-b*c/d)
/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Cot[a + b*x]*Csc[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{c+dx}\,dx\\
&= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\text{co}}{}\\
&= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\cot(a+bx)\csc(a+}{c+dx}
\end{aligned}$$

Mathematica [A] time = 3.72, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^{2}}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\left(\cot^{2}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x +
I*b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d)
```

```
+ (b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*
x + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*
b*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) +
(b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
+ I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*
c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*
cos(2*b*x + 2*a)^2 - 4*d*cos(b*x + a)*sin(2*b*x + 2*a) + (b*c*(exp_integral
_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-
(b*c - a*d)/d) - b*c*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_inte
gral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d) + (b*d*(exp_integral_e
(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*cos(-(b
*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - I*exp_integr
al_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x)*sin(2*b*x + 2*a)^2 +
 (b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x
 + I*b*c)/d))*cos(-(b*c - a*d)/d) - b*d*(I*exp_integral_e(1, (I*b*d*x + I*b
*c)/d) - I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d)/d))*x
- (2*b*c*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*c*(-2*I*exp_integral_e(1, (I*b*d*x
 + I*b*c)/d) + 2*I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c - a*d
)/d) + (2*b*d*(exp_integral_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -
(I*b*d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + b*d*(-2*I*exp_integral_e(1, (I*
b*d*x + I*b*c)/d) + 2*I*exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin(-(b*c
- a*d)/d))*x - 4*d*sin(b*x + a))*cos(2*b*x + 2*a) - 2*(b*d^3*x + b*c*d^2 +
(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*
a)^2 - 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^
2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(b*x + a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x
 + b*c^2)*cos(b*x + a)), x) - 2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*co
s(2*b*x + 2*a)^2 + (b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a)^2 - 2*(b*d^3*x + b*
c*d^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(b*d^2*x^2 + 2*b*c*d*x + b*
c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)^2 + (b*d^2*x^2 + 2*b*c*d
*x + b*c^2)*sin(b*x + a)^2 - 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(b*x + a)
), x) - 4*d*sin(b*x + a))/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x +
2*a)^2 + (b*d^2*x + b*c*d)*sin(2*b*x + 2*a)^2 - 2*(b*d^2*x + b*c*d)*cos(2*b
*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+b\,x)\,\cot(a+b\,x)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x), x)
```

3.176 $\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=93

$$\text{Int}\left(\frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2},x\right)+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\cos(a+bx)}{d(c+dx)}$$

```
[Out] CannotIntegrate(cot(b*x+a)*csc(b*x+a)/(d*x+c)^2,x)+cos(b*x+a)/d/(d*x+c)+b*c
os(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a-b*c/d)/d^2
```

Rubi [A] time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][(C
ot[a + b*x]*Csc[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\cot(a+bx)\csc(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\text{cot}
\end{aligned}$$

Mathematica [A] time = 4.04, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2,x]
[Out] Integrate[(Cos[a + b*x]*Cot[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^{2}}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(cos(b*x + a)*cot(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\cot\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(cos(b*x + a)*cot(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(bx+a\right)\left(\cot^{2}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
[Out] int(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(a+bx)\cot(a+bx)^2}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2,x)
```

```
[Out] int((cos(a + b*x)*cot(a + b*x)^2)/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(a+bx)\cot^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(b*x+a)*cot(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cos(a + b*x)*cot(a + b*x)**2/(c + d*x)**2, x)
```

3.177 $\int (c+dx)^m \cot^3(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\cot^3(a+bx)(c+dx)^m,x\right)$$

[Out] Unintegrable((d*x+c)^m*cot(b*x+a)^3,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Cot[a + b*x]^3,x]

[Out] Defer[Int][(c + d*x)^m*Cot[a + b*x]^3, x]

Rubi steps

$$\int (c+dx)^m \cot^3(a+bx)\,dx = \int (c+dx)^m \cot^3(a+bx)\,dx$$

Mathematica [A] time = 11.48, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Cot[a + b*x]^3,x]

[Out] Integrate[(c + d*x)^m*Cot[a + b*x]^3, x]

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \cot(bx+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="fricas")

[Out] integral((d*x + c)^m*cot(b*x + a)^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cot^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cot(b*x+a)^3,x)
[Out] int((d*x+c)^m*cot(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cot(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*cot(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \cot(a+b\,x)^3\,(c+d\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^m,x)
[Out] int(cot(a + b*x)^3*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cot(b*x+a)**3,x)
[Out] Integral((c + d*x)**m*cot(a + b*x)**3, x)
```

3.178 $\int (c+dx)^4 \cot^3(a+bx)\,dx$

Optimal. Leaf size=302

$$\frac{3d^4\mathrm{Li}_3\left(e^{2i(a+bx)}\right)}{b^5}+\frac{3d^4\mathrm{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{6id^3(c+dx)\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\mathrm{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\mathrm{Li}_3\left(e^{2\ldots}\right.}{b^3}$$

```
[Out] -2*I*d*(d*x+c)^3/b^2-1/2*(d*x+c)^4/b+1/5*I*(d*x+c)^5/d-2*d*(d*x+c)^3*cot(b*
x+a)/b^2-1/2*(d*x+c)^4*cot(b*x+a)^2/b+6*d^2*(d*x+c)^2*ln(1-exp(2*I*(b*x+a))
)/b^3-(d*x+c)^4*ln(1-exp(2*I*(b*x+a)))/b-6*I*d^3*(d*x+c)*polylog(2,exp(2*I*
(b*x+a)))/b^4+2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*d^4*polylog
(3,exp(2*I*(b*x+a)))/b^5-3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x+a)))/b^3-3*
I*d^3*(d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,exp(2*I*(b*
x+a)))/b^5
```

Rubi [A] time = 0.46, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3717, 2190, 2531, 2282, 6589, 32, 6609}

$$-\frac{3d^2(c+dx)^2\mathrm{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}-\frac{6id^3(c+dx)\mathrm{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\mathrm{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}+\frac{2i\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] ((-2*I)*d*(c + d*x)^3)/b^2 - (c + d*x)^4/(2*b) + ((I/5)*(c + d*x)^5)/d - (2
*d*(c + d*x)^3*Cot[a + b*x])/b^2 - ((c + d*x)^4*Cot[a + b*x]^2)/(2*b) + (6*
d^2*(c + d*x)^2*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^4*Log[1 - E^
((2*I)*(a + b*x))])/b - ((6*I)*d^3*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 + (3*d^4
*PolyLog[3, E^((2*I)*(a + b*x))])/b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, E^((2
*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*(a + b*x))]
)/b^4 + (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \cot^3(a+bx)\,dx = -\frac{(c+dx)^4\cot^2(a+bx)}{2b} + \frac{(2d)\int (c+dx)^3\cot^2(a+bx)\,dx}{b} - \int (c+dx)^4\cot(a+b$$

$$= \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+}{1-e^{2i(a+}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2}{2b}$$

$$= -\frac{2id(c+dx)^3}{b^2} - \frac{(c+dx)^4}{2b} + \frac{i(c+dx)^5}{5d} - \frac{2d(c+dx)^3\cot(a+bx)}{b^2} - \frac{(c+dx)^4\cot^2}{2b}$$

Mathematica [B] time = 7.12, size = 1534, normalized size = 5.08

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Cot[a + b*x]^3,x]
```

```
[Out] -1/5*(x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Cot[a
]) - ((c + d*x)^4*Csc[a + b*x]^2)/(2*b) + (c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x
^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x
))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1
 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I
)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)
*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^3 - (d^4*
E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((
```

```
-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))
] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))
*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E
^((2*I)*a)))/b^5 + (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3
*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2
*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*Po
lyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] -
 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b
^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b
*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 + (d^4*E^(I*a)
*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 -
 E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 + E^((-I)*(a
+ b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*(a + b*x))]
- (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, -E^((-I)
*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (20*(-
1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*Po
lyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*x))] + (6*I)
*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) - (c^4*Csc[a]*(-(b
*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 +
 Sin[a]^2)) + (6*c^2*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[
a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (2*Csc[a]*Csc[a + b*x]*
(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin
[b*x]))/b^2 + (2*c^3*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*
x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[T
an[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*Ar
cTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x +
ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^
2 + Sin[a]^2)]) - (6*c*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((
I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcT
an[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] +
2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*
x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^4*Sqrt[Sec[a]^2*(Cos
[a]^2 + Sin[a]^2)])
```

<u>fricas</u> [C] time = 0.58, size = 1747, normalized size = 5.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^4*d^4*x^4 + 16*b^4*c*d^3*x^3 + 24*b^4*c^2*d^2*x^2 + 16*b^4*c^3*d*x
 + 4*b^4*c^4 + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*
I*b*c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^
3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*
```

```
b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (4*I*b^3*d^4*x^3
 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d - 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 -
b*d^4)*x + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d + 12*I*b*
c*d^3 - 12*I*(b^3*c^2*d^2 - b*d^4)*x)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2
*a) - I*sin(2*b*x + 2*a)) + 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^
2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*
d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*co
s(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) +
 2*(b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2 - 4*(a^3 - 3*a)*b*c*d
^3 + (a^4 - 6*a^2)*d^4 - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 - 1)*b^2*c^2*d^2
 - 4*(a^3 - 3*a)*b*c*d^3 + (a^4 - 6*a^2)*d^4)*cos(2*b*x + 2*a))*log(-1/2*co
s(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(
a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*
(b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*si
n(2*b*x + 2*a) + 1) + 2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 - 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x - (b^4*d^4*x^4 + 4*b^4*c*d
^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 - 3*a)*b*c*d^3 - (a^4 -
 6*a^2)*d^4 + 6*(b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(b^4*c^3*d - 3*b^2*c*d^3)*x
)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 3*(d^
4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a))
 + 3*(d^4*cos(2*b*x + 2*a) - d^4)*polylog(5, cos(2*b*x + 2*a) - I*sin(2*b*x
 + 2*a)) + (6*I*b*d^4*x + 6*I*b*c*d^3 + (-6*I*b*d^4*x - 6*I*b*c*d^3)*cos(2*
b*x + 2*a))*polylog(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (-6*I*b*d^4
*x - 6*I*b*c*d^3 + (6*I*b*d^4*x + 6*I*b*c*d^3)*cos(2*b*x + 2*a))*polylog(4,
 cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x +
b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4)*cos(2
*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + 6*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 - d^4 - (b^2*d^4*x^2 + 2*b^2*c*d^3*x + b
^2*c^2*d^2 - d^4)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b
*x + 2*a)) + 8*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + b^3*c^3*d
)*sin(2*b*x + 2*a))/(b^5*cos(2*b*x + 2*a) - b^5)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \cot(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*cot(b*x + a)^3, x)
```

maple [B] time = 0.20, size = 1868, normalized size = 6.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cot(b*x+a)^3,x)
```

```
[Out] -1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-12/b^3
*c^2*d^2*polylog(3,-exp(I*(b*x+a)))-12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a))
)+1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))-12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x
^2-12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2+1/5*I*d^4*x^5+I*c*d^3*x^4-24*I
/b^3*d^3*c*a*x+12*d^4*polylog(3,-exp(I*(b*x+a)))/b^5+12*d^4*polylog(3,exp(I
*(b*x+a)))/b^5+24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,exp(I
*(b*x+a)))/b^5+8*I/b*a*c^3*d*x-12*I/b^2*a^2*c^2*d^2*x+8*I/b^3*c*d^3*a^3*x+1
2*I/b^2*polylog(2,exp(I*(b*x+a)))*c^2*d^2*x+12*I/b^2*c*d^3*polylog(2,exp(I*
(b*x+a)))*x^2+6*I/b^4*c*d^3*a^4-24*I/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))+4*
I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-8*I/b^3*a^3*c^2*d^2+4*I/b^2*d^4*polyl
og(2,exp(I*(b*x+a)))*x^3-24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-2*I/b^4*d
^4*a^4*x+4*I/b^2*a^2*c^3*d-I*c^4*x+2/b*c^4*ln(exp(I*(b*x+a)))-1/b*c^4*ln(ex
p(I*(b*x+a))+1)-1/b*c^4*ln(exp(I*(b*x+a))-1)+6/b^5*d^4*a^2*ln(exp(I*(b*x+a)
)-1)-12/b^5*d^4*a^2*ln(exp(I*(b*x+a)))+6/b^3*d^2*c^2*ln(exp(I*(b*x+a))-1)+6
/b^3*d^2*c^2*ln(exp(I*(b*x+a))+1)-12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-6/b^5*d
^4*a^2*ln(1-exp(I*(b*x+a)))+6/b^3*d^4*ln(1-exp(I*(b*x+a)))*x^2+6/b^3*d^4*ln
(exp(I*(b*x+a))+1)*x^2-4*I/b^2*d^4*x^3+8*I/b^5*d^4*a^3-24*I/b^4*d^4*polylog
(4,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3
*d*polylog(2,-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+2*
(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*
exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+
b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I
*(b*x+a))+2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))+6*I*c*d^3*x^2+6*I*c^2*d^2*
x+2*I*c^3*d)/b^2/(exp(2*I*(b*x+a))-1)^2-4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x-4/
b*c^3*d*ln(1-exp(I*(b*x+a)))*x-4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a-6/b*c^2*d
^2*ln(exp(I*(b*x+a))+1)*x^2-24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x+6/b^3
*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))-6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2-24/b
^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x-8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+4/b^4
*c*d^3*a^3*ln(exp(I*(b*x+a))-1)-8/b^4*c*d^3*a^3*ln(exp(I*(b*x+a)))-6/b^3*c^
2*d^2*a^2*ln(exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+4/b^2*
c^3*d*a*ln(exp(I*(b*x+a))-1)-1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4-1/b*d^4*ln(ex
p(I*(b*x+a))+1)*x^4-4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3-4/b*c*d^3*ln(1-exp(I
*(b*x+a)))*x^3-4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3+2*I*c^2*d^2*x^3+2*I*c^3
*d*x^2+12/b^3*d^3*c*ln(exp(I*(b*x+a))+1)*x+12/b^3*d^3*c*ln(1-exp(I*(b*x+a))
)*x+12/b^4*d^3*c*ln(1-exp(I*(b*x+a)))*a+24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))+1
2*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*polylog(2,-ex
p(I*(b*x+a)))*x-8/5*I/b^5*d^4*a^5-12/b^4*d^3*c*a*ln(exp(I*(b*x+a))-1)-12*I/
```

```
b^4*d^3*c*polylog(2,-exp(I*(b*x+a)))-12*I/b^4*d^3*c*polylog(2,exp(I*(b*x+a)
))+12*I/b^4*d^4*a^2*x-12*I/b^2*d^3*c*x^2-12*I/b^4*d^3*c*a^2-12*I/b^4*d^4*po
lylog(2,-exp(I*(b*x+a)))*x-12*I/b^4*d^4*polylog(2,exp(I*(b*x+a)))*x
```

maxima [B] time = 4.85, size = 7111, normalized size = 23.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 4*a*c^3*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/sin(b*x + a)^2 + log(sin(
b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3
 + a^4*d^4*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^4 - 2*(2*(b*x + a)^5*
d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + 10*
(b*c*d^3 - a*d^4)*(b*x + a)^4 + 20*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b
*x + a)^3 + 20*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
 + a)^2 - (10*(b*x + a)^4*d^4 - 60*b^2*c^2*d^2 + 120*a*b*c*d^3 - 60*a^2*d^4
 + 40*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2
- 1)*d^4)*(b*x + a)^2 + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d
^3 - (a^3 - 3*a)*d^4)*(b*x + a) + 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*
a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 -
2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 20*((
b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*
d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^
2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)
*(b*x + a))*cos(2*b*x + 2*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*c^2*d^2 + 1
20*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x + a)^3 + (
60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x + a)^2 + (
40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d^3 + (-40*I
*a^3 + 120*I*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (-20*I*(b*x + a)^4*d^4 +
 120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4 + (-80*I*b*c*d^3 + 80*
I*a*d^4)*(b*x + a)^3 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2
+ 120*I)*d^4)*(b*x + a)^2 + (-80*I*b^3*c^3*d + 240*I*a*b^2*c^2*d^2 + (-240*
I*a^2 + 240*I)*b*c*d^3 + (80*I*a^3 - 240*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (60*b^2*c^2*d^2 - 120*a*b*c*
d^3 + 60*a^2*d^4 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(4*b*x + 4*a
) - 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(2*b*x + 2*a) - (-60*I*b^2
*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4)*sin(4*b*x + 4*a) - (120*I*b^2*c^
2*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) - 1) + (10*(b*x + a)^4*d^4 + 40*(b*c*d^3 - a*d^4)*(b*x +
 a)^3 + 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 40*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a
```

```
) + 10*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2
- 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) - 20*
((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a
^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-10*I*(b*
x + a)^4*d^4 + (-40*I*b*c*d^3 + 40*I*a*d^4)*(b*x + a)^3 + (-60*I*b^2*c^2*d^
2 + 120*I*a*b*c*d^3 + (-60*I*a^2 + 60*I)*d^4)*(b*x + a)^2 + (-40*I*b^3*c^3*
d + 120*I*a*b^2*c^2*d^2 + (-120*I*a^2 + 120*I)*b*c*d^3 + (40*I*a^3 - 120*I*
a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (20*I*(b*x + a)^4*d^4 + (80*I*b*c*d^3
 - 80*I*a*d^4)*(b*x + a)^3 + (120*I*b^2*c^2*d^2 - 240*I*a*b*c*d^3 + (120*I*
a^2 - 120*I)*d^4)*(b*x + a)^2 + (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + (24
0*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4)*(b*x + a))*sin(2*b*x
+ 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 2*((b*x + a)^5*d^4 + 5*(
b*c*d^3 - a*d^4)*(b*x + a)^4 + 10*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 2)*d^
4)*(b*x + a)^3 + 10*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 2)*b*c*d^3 - (a
^3 - 6*a)*d^4)*(b*x + a)^2 - 60*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a))*cos(4*b*x + 4*a) - (4*(b*x + a)^5*d^4 + 40*b^3*c^3*d - 120*a*b^2*c^2*
d^2 + 120*a^2*b*c*d^3 - 40*a^3*d^4 + (20*b*c*d^3 - (20*a - 20*I)*d^4)*(b*x
+ a)^4 + (40*b^2*c^2*d^2 - (80*a - 80*I)*b*c*d^3 + 40*(a^2 - 2*I*a - 1)*d^4
)*(b*x + a)^3 + (40*b^3*c^3*d - (120*a - 120*I)*b^2*c^2*d^2 + 120*(a^2 - 2*
I*a - 1)*b*c*d^3 - (40*a^3 - 120*I*a^2 - 120*a)*d^4)*(b*x + a)^2 + (80*I*b^
3*c^3*d - 120*(2*I*a + 1)*b^2*c^2*d^2 + (240*I*a^2 + 240*a)*b*c*d^3 + (-80*
I*a^3 - 120*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (40*b^3*c^3*d - 120*a*b
^2*c^2*d^2 + 40*(b*x + a)^3*d^4 + 120*(a^2 - 1)*b*c*d^3 - 40*(a^3 - 3*a)*d^
4 + 120*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 120*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a
^2 - 1)*d^4)*(b*x + a) + 40*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
+ 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a)
- 80*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 -
 (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-40*I*b^3*c^3*d + 12
0*I*a*b^2*c^2*d^2 - 40*I*(b*x + a)^3*d^4 + (-120*I*a^2 + 120*I)*b*c*d^3 + (
40*I*a^3 - 120*I*a)*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-12
0*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 120*I)*d^4)*(b*x + a))*si
n(4*b*x + 4*a) - (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*d^2 + 80*I*(b*x + a)^3*d
^4 + (240*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240*I*a)*d^4 + (240*I*b*c*d
^3 - 240*I*a*d^4)*(b*x + a)^2 + (240*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + (240
*I*a^2 - 240*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) +
 (40*b^3*c^3*d - 120*a*b^2*c^2*d^2 + 40*(b*x + a)^3*d^4 + 120*(a^2 - 1)*b*c
*d^3 - 40*(a^3 - 3*a)*d^4 + 120*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 120*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a) + 40*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + (b*x + a)^3*d^4 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^
3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x
 + a))*cos(4*b*x + 4*a) - 80*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + (b*x + a)^3*d^4
```

```
 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a)
 - (-40*I*b^3*c^3*d + 120*I*a*b^2*c^2*d^2 - 40*I*(b*x + a)^3*d^4 + (-120*I*
a^2 + 120*I)*b*c*d^3 + (40*I*a^3 - 120*I*a)*d^4 + (-120*I*b*c*d^3 + 120*I*a
*d^4)*(b*x + a)^2 + (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 + (-120*I*a^2 + 1
20*I)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (80*I*b^3*c^3*d - 240*I*a*b^2*c^2*
d^2 + 80*I*(b*x + a)^3*d^4 + (240*I*a^2 - 240*I)*b*c*d^3 + (-80*I*a^3 + 240
*I*a)*d^4 + (240*I*b*c*d^3 - 240*I*a*d^4)*(b*x + a)^2 + (240*I*b^2*c^2*d^2
 - 480*I*a*b*c*d^3 + (240*I*a^2 - 240*I)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*d
ilog(e^(I*b*x + I*a)) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b
*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b
^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*
b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 6
0*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*c^2*d^2 - 60*I*a*b
*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x + a)^3 + (-30*I*b
^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*x + a)^2 + (-20*I*
b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d^3 + (20*I*a^3 - 6
0*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (10*I*(b*x + a)^4*d^4 - 60*I*b^2*
c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3 - 40*I*a*d^4)*(b*x
 + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2 - 60*I)*d^4)*(b*x
 + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*a^2 - 120*I)*b*c*d
^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 5*((b*x + a)^
4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x
 + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^
3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a
))*sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 -
6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3
+ (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)
*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*
c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x
 + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*
x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d
^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a) + (-5*I*(b*x + a)^4*d^4 + 30*I*b^2*
c^2*d^2 - 60*I*a*b*c*d^3 + 30*I*a^2*d^4 + (-20*I*b*c*d^3 + 20*I*a*d^4)*(b*x
 + a)^3 + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 + (-30*I*a^2 + 30*I)*d^4)*(b*
x + a)^2 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^2 + 60*I)*b*c*d
^3 + (20*I*a^3 - 60*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (10*I*(b*x + a)
^4*d^4 - 60*I*b^2*c^2*d^2 + 120*I*a*b*c*d^3 - 60*I*a^2*d^4 + (40*I*b*c*d^3
 - 40*I*a*d^4)*(b*x + a)^3 + (60*I*b^2*c^2*d^2 - 120*I*a*b*c*d^3 + (60*I*a^2
 - 60*I)*d^4)*(b*x + a)^2 + (40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + (120*I*
a^2 - 120*I)*b*c*d^3 + (-40*I*a^3 + 120*I*a)*d^4)*(b*x + a))*cos(2*b*x + 2*
a) + 5*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c
*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(
b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 -
```

```
3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 10*((b*x + a)^4*d^4 - 6*b^2*c^2*d^2
 + 12*a*b*c*d^3 - 6*a^2*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*
d^2 - 2*a*b*c*d^3 + (a^2 - 1)*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*(a^2 - 1)*b*c*d^3 - (a^3 - 3*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*
log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (240*I*d^4*cos(
4*b*x + 4*a) - 480*I*d^4*cos(2*b*x + 2*a) - 240*d^4*sin(4*b*x + 4*a) + 480*
d^4*sin(2*b*x + 2*a) + 240*I*d^4)*polylog(5, -e^(I*b*x + I*a)) - (240*I*d^4
*cos(4*b*x + 4*a) - 480*I*d^4*cos(2*b*x + 2*a) - 240*d^4*sin(4*b*x + 4*a) +
 480*d^4*sin(2*b*x + 2*a) + 240*I*d^4)*polylog(5, e^(I*b*x + I*a)) - (240*b
*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 + 240*(b*c*d^3 + (b*x + a)*d^4 - a*d
^4)*cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*
a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4 - 240*I*a*d^4)*sin(4*b*x + 4*a) +
 (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 + 480*I*a*d^4)*sin(2*b*x + 2*a))*pol
ylog(4, -e^(I*b*x + I*a)) - (240*b*c*d^3 + 240*(b*x + a)*d^4 - 240*a*d^4 +
240*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) - 480*(b*c*d^3 + (b*
x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (240*I*b*c*d^3 + 240*I*(b*x + a)*d^4
 - 240*I*a*d^4)*sin(4*b*x + 4*a) + (-480*I*b*c*d^3 - 480*I*(b*x + a)*d^4 +
480*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) - (-120*I*b^2*c^
2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4
+ (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*I*b^2*c^2*d^2 + 240*I*a*
b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^
3 + 240*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (240*I*b^2*c^2*d^2 - 480*I*a
*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240*I)*d^4 + (480*I*b*c*d^3
 - 480*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 120*(b^2*c^2*d^2 - 2*a*b*c*d^
3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*
b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d
^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x
+ I*a)) - (-120*I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (
-120*I*a^2 + 120*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a) + (-120*
I*b^2*c^2*d^2 + 240*I*a*b*c*d^3 - 120*I*(b*x + a)^2*d^4 + (-120*I*a^2 + 120
*I)*d^4 + (-240*I*b*c*d^3 + 240*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (240
*I*b^2*c^2*d^2 - 480*I*a*b*c*d^3 + 240*I*(b*x + a)^2*d^4 + (240*I*a^2 - 240
*I)*d^4 + (480*I*b*c*d^3 - 480*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 120*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 240*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + (a^2 - 1)*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a
))*polylog(3, e^(I*b*x + I*a)) - (-2*I*(b*x + a)^5*d^4 + (-10*I*b*c*d^3 + 1
0*I*a*d^4)*(b*x + a)^4 + (-20*I*b^2*c^2*d^2 + 40*I*a*b*c*d^3 + (-20*I*a^2 +
 40*I)*d^4)*(b*x + a)^3 + (-20*I*b^3*c^3*d + 60*I*a*b^2*c^2*d^2 + (-60*I*a^
2 + 120*I)*b*c*d^3 + (20*I*a^3 - 120*I*a)*d^4)*(b*x + a)^2 + (120*I*b^2*c^2
*d^2 - 240*I*a*b*c*d^3 + 120*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) - (4*I*
(b*x + a)^5*d^4 + 40*I*b^3*c^3*d - 120*I*a*b^2*c^2*d^2 + 120*I*a^2*b*c*d^3
- 40*I*a^3*d^4 + (20*I*b*c*d^3 - 20*(I*a + 1)*d^4)*(b*x + a)^4 + (40*I*b^2*
c^2*d^2 - 80*(I*a + 1)*b*c*d^3 + (40*I*a^2 + 80*a - 40*I)*d^4)*(b*x + a)^3
+ (40*I*b^3*c^3*d - 120*(I*a + 1)*b^2*c^2*d^2 + (120*I*a^2 + 240*a - 120*I)
```

```
*b*c*d^3 + (-40*I*a^3 - 120*a^2 + 120*I*a)*d^4)*(b*x + a)^2 - (80*b^3*c^3*d
 - (240*a - 120*I)*b^2*c^2*d^2 + 240*(a^2 - I*a)*b*c*d^3 - 40*(2*a^3 - 3*I*
a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-10*I*b^4*cos(4*b*x + 4*a) + 20*I*b
^4*cos(2*b*x + 2*a) + 10*b^4*sin(4*b*x + 4*a) - 20*b^4*sin(2*b*x + 2*a) - 1
0*I*b^4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(a+b\,x)^3\,(c+d\,x)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \cot^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**4*cot(a + b*x)**3, x)
```

3.179 $\int (c+dx)^3 \cot^3(a+bx)\,dx$

Optimal. Leaf size=256

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{Li}}{2b^2}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-3/2*d*(d*x+c)^2*co
t(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+c)*ln(1-exp(2*I*(b*x+a
)))/b^3-(d*x+c)^3*ln(1-exp(2*I*(b*x+a)))/b-3/2*I*d^3*polylog(2,exp(2*I*(b*x
+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)
*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.37, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3720, 3717, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{PolyLog}}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) + ((I/4)*(c + d*x)^4)/d
- (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d*x)^3*Cot[a + b*x]^2)/(2*
b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b^3 - ((c + d*x)^3*Log[
1 - E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))
])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3
*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*Po
lyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
```

```
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \cot^3(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(3d)\int (c+dx)^2\cot^2(a+bx)\,dx}{2b} - \int (c+dx)^3\cot(a+bx$$

$$= \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+d}{1-e^{2i(a+bx}}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} + \frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(}{2b}$$

Mathematica [B] time = 6.89, size = 994, normalized size = 3.88

$$-\frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\sin(a)-bx\cos(a))c^3}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{3d\csc(a)\sec(a)\left(b^2e^{i\tan^{-1}(\tan(a))}x^2 + \frac{\left(ibx\left(2\tan^{-}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Cot[a + b*x]^3,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cot[a]) - ((c + d*x)^3*
Csc[a + b*x]^2)/(2*b) + (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 -
 E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x
*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((
2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*Pol
yLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(2*b^3) + (d^3*E^(I*a)*Csc[a]*(
(b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a
 + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] -
(6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*
PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2
*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2
*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))
/E^((2*I)*a)))/(4*b^4) - (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] +
 Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*(-(b
*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2
 + Sin[a]^2)) + (3*Csc[a]*Csc[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x]
 + d^3*x^2*Sin[b*x]))/(2*b^2) + (3*c^2*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan
[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Lo
g[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sq
rt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan
[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x
)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + P
i*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyL
og[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^
4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.55, size = 1135, normalized size = 4.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(8*b^3*d^3*x^3 + 24*b^3*c*d^2*x^2 + 24*b^3*c^2*d*x + 8*b^3*c^3 + (-6*I*
b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3 + (6*I*b^2*d^3*x^2
 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*cos(2*b*x + 2*a))*dilog(cos(
2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x +
6*I*b^2*c^2*d - 6*I*d^3 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^
2*d + 6*I*d^3)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a
)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (
```

```
b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x
+ 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*
c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3 - (b^3*c^3 - 3*
a*b^2*c^2*d + 3*(a^2 - 1)*b*c*d^2 - (a^3 - 3*a)*d^3)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 4*(b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^
2*d - b*d^3)*x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c
*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-co
s(2*b*x + 2*a) + I*sin(2*b*x + 2*a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*
x - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 -
 3*a)*d^3 + 3*(b^3*c^2*d - b*d^3)*x)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a) + 1) + (-3*I*d^3*cos(2*b*x + 2*a) + 3*I*d^3)*polylog
(4, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) + (3*I*d^3*cos(2*b*x + 2*a) - 3*
I*d^3)*polylog(4, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*polylog(3, cos(2*b*x + 2*a) +
I*sin(2*b*x + 2*a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(2*b*x
+ 2*a))*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 12*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + b^2*c^2*d)*sin(2*b*x + 2*a))/(b^4*cos(2*b*x + 2*a) - b^4
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \cot (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*cot(b*x + a)^3, x)
```

maple [B] time = 0.14, size = 1194, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cot(b*x+a)^3,x)
```

```
[Out] 1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3*I/b^2*a^2*c^2*d+3*I/b^2*c^2*d*polylog(2
,exp(I*(b*x+a)))+3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4*I/b^3*a^3*c*d^
2+2*I/b^3*d^3*a^3*x+1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)-2/b^4*d^3*a^3*ln(exp
(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))-6/b^3*c*d^2*polylog(3,e
xp(I*(b*x+a)))-6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(3,-e
xp(I*(b*x+a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-e
xp(I*(b*x+a)))/b^4-1/b*c^3*ln(exp(I*(b*x+a))-1)-1/b*c^3*ln(exp(I*(b*x+a))+1
```

```
)+2/b*c^3*ln(exp(I*(b*x+a)))+6*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x+(2*
b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2
*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c
^2*d*exp(2*I*(b*x+a))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*
c^2*d)/b^2/(exp(2*I*(b*x+a))-1)^2+3/2*I*c^2*d*x^2+3/b^3*d^2*c*ln(exp(I*(b*x
+a))-1)+3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3/b
^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln
(1-exp(I*(b*x+a)))*a-3/b^4*d^3*a*ln(exp(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*
(b*x+a)))-3*I/b^2*d^3*x^2-3*I/b^4*d^3*a^2-3*I/b^4*d^3*polylog(2,-exp(I*(b*x
+a)))+3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-e
xp(I*(b*x+a)))-3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*a^2*ln(exp(
I*(b*x+a)))-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x-3/b*c^2*d*ln(1-exp(I*(b*x+a)))
*x-3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a+3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))-
3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b^2
*c^2*d*a*ln(exp(I*(b*x+a))-1)-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-1/b*d^3*ln(1
-exp(I*(b*x+a)))*x^3-1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b
*x+a))+1)*x^3-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+6*I/b^2*polylog(2,exp(I*(
b*x+a)))*c*d^2*x-6*I/b^3*d^3*a*x-6*I/b^4*d^3*polylog(4,exp(I*(b*x+a)))+3/2*
I/b^4*d^3*a^4
```

maxima [B] time = 1.74, size = 3952, normalized size = 15.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 3*a*c^2*d*(1/sin(b*x +
 a)^2 + log(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*
x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2))/b^3 - 2*(
(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + 4*(b*c*d^2 - a
*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (4*
(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*((b*x + a)^3*d
^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3*
d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4*I*(b*x + a)
^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^
2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*si
n(4*b*x + 4*a) + (-8*I*(b*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-24*I
*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 + (-
24*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 - a*d^3)*cos(4*b*x +
 4*a) - 24*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3
```

)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 - 24*I*a*d^3)*sin(2*b*x + 2*a))*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^3*d^3 + 12*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 8*((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12*I
*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 12*I
)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (8*I*(b*x + a)^3*d^3 + (24*I*b*c*d^2 -
 24*I*a*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + (24*I*a^2 - 2
4*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a)
+ 1) + ((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d -
2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) - (2*(b*x + a)^4*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^
2*d^3 + (8*b*c*d^2 - (8*a - 8*I)*d^3)*(b*x + a)^3 + (12*b^2*c^2*d - (24*a -
 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 + (24*I*b^2*c^2*d -
24*(2*I*a + 1)*b*c*d^2 + (24*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a)
 + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 2
4*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d
^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 24*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I
*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b
*x + a))*sin(4*b*x + 4*a) - (24*I*b^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x +
a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 + (48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*s
in(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 1
2*(b*x + a)^2*d^3 + 12*(a^2 - 1)*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*
(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a
d^3)(b*x + a))*cos(4*b*x + 4*a) - 24*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)
^2*d^3 + (a^2 - 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
(-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 12*
I)*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (24*I*b
^2*c^2*d - 48*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d^3 + (24*I*a^2 - 24*I)*d^3 +
(48*I*b*c*d^2 - 48*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I
*a)) - (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*
I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*
d^3)*(b*x + a) + (-2*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*
c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a
^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*
c*d^2 + 12*I*a*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^
2*d - 24*I*a*b*c*d^2 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
 2*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a)
- 4*((b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^
2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a)

```
)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-2*I*(b*x +
a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x + a) + (-2
*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(
b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 6*I)*d^3)*(b*x
+ a))*cos(4*b*x + 4*a) + (4*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2
 + (12*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x + a)^3*d^3
- 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 4*((b*x + a)^3*d^3
 - 3*b*c*d^2 + 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2
*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3
*cos(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) +
 24*d^3)*polylog(4, -e^(I*b*x + I*a)) - (24*d^3*cos(4*b*x + 4*a) - 48*d^3*c
os(2*b*x + 2*a) + 24*I*d^3*sin(4*b*x + 4*a) - 48*I*d^3*sin(2*b*x + 2*a) + 2
4*d^3)*polylog(4, e^(I*b*x + I*a)) - (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 +
24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(4*b*x +
4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 48*I*a*d^3)*cos(2*b*x + 2*a) +
24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 48*(b*c*d^2 + (b*x
+ a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*
d^3 + 24*I*a*d^3)*cos(4*b*x + 4*a) + (48*I*b*c*d^2 + 48*I*(b*x + a)*d^3 - 4
8*I*a*d^3)*cos(2*b*x + 2*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*
x + 4*a) - 48*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3
, e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^3 + (-4*I*b*c*d^2 + 4*I*a*d^3)*(b*x
+ a)^3 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a
)^2 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (2*I*(b*x +
 a)^4*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (8*I*b*c*d^2 -
 8*(I*a + 1)*d^3)*(b*x + a)^3 + (12*I*b^2*c^2*d - 24*(I*a + 1)*b*c*d^2 + (1
2*I*a^2 + 24*a - 12*I)*d^3)*(b*x + a)^2 - (24*b^2*c^2*d - (48*a - 24*I)*b*c
*d^2 + 24*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-4*I*b^3*cos(4*b*x
 + 4*a) + 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) - 8*b^3*sin(2*b
*x + 2*a) - 4*I*b^3))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot(a+bx)^3 (c+dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \cot^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*cot(a + b*x)**3, x)
```

3.180 $\int (c+dx)^2 \cot^3(a+bx)\,dx$

Optimal. Leaf size=168

$$-\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\cot(a+bx)}{b^2}-\frac{(c+dx)^2\log\left(1-e^{2i(a+\cdots}\right.}{b}$$

```
[Out] -c*d*x/b-1/2*d^2*x^2/b+1/3*I*(d*x+c)^3/d-d*(d*x+c)*cot(b*x+a)/b^2-1/2*(d*x+
c)^2*cot(b*x+a)^2/b-(d*x+c)^2*ln(1-exp(2*I*(b*x+a)))/b+d^2*ln(sin(b*x+a))/b
^3+I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,exp(2*I*(b
*x+a)))/b^3
```

Rubi [A] time = 0.27, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3475, 3717, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\cot(a+bx)}{b^2}+\frac{d^2\log(\sin(a+bx))}{b^3}-\frac{(c+dx)^{\cdots}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) + ((I/3)*(c + d*x)^3)/d - (d*(c + d*x)*Cot[a
 + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b) - ((c + d*x)^2*Log[1 - E^
((2*I)*(a + b*x))])/b + (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b
^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \cot^3(a+bx)\,dx = -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{d\int (c+dx)\cot^2(a+bx)\,dx}{b} - \int (c+dx)^2\cot(a+bx)\,d$$

$$= \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx}{1-e^{2i(a+bx)}}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} + \frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} - \frac{(c+}{}$$

Mathematica [B] time = 6.68, size = 540, normalized size = 3.21

$$\frac{d^2\csc(a)(\sin(a)\log(\sin(a)\cos(bx)+\cos(a)\sin(bx)) - bx\cos(a))}{b^3\left(\sin^2(a)+\cos^2(a)\right)} + \frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Cot[a + b*x]^3,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cot[a]) - ((c + d*x)^2*Csc[a + b*x]^2)/
(2*b) + (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-
2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2
*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-
I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 +
 E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a
 + b*x))]))/E^((2*I)*a)))/(6*b^3) - (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*
x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[
a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(C
os[a]^2 + Sin[a]^2)) + (Csc[a]*Csc[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x])
)/b^2 + (c*d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2
*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Lo
g[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a
]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan
```

[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])

fricas [C] time = 0.47, size = 655, normalized size = 3.90

$$\frac{4\,b^2d^2x^2+8\,b^2cdx+4\,b^2c^2+\left(-2i\,bd^2x-2i\,bcd+\left(2i\,bd^2x+2i\,bcd\right)\cos\left(2\,bx+2\,a\right)\right)\mathrm{Li}_2\left(\cos\left(2\,bx+2\,a\right)+i\,\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d +
(2*I*b*d^2*x + 2*I*b*c*d)*cos(2*b*x + 2*a))*dilog(cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(2
*b*x + 2*a))*dilog(cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 2*(b^2*c^2 - 2*
a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(2*b*x +
 2*a))*log(-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b^2*c
^2 - 2*a*b*c*d + (a^2 - 1)*d^2 - (b^2*c^2 - 2*a*b*c*d + (a^2 - 1)*d^2)*cos(
2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2
*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*
d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(2*b*x + 2*a) + I*sin(
2*b*x + 2*a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(2*b*x + 2*a))*log(-cos(
2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) - (d^2*cos(2*b*x + 2*a) - d^2)*polyl
og(3, cos(2*b*x + 2*a) + I*sin(2*b*x + 2*a)) - (d^2*cos(2*b*x + 2*a) - d^2)
*polylog(3, cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a)) + 4*(b*d^2*x + b*c*d)*si
n(2*b*x + 2*a))/(b^3*cos(2*b*x + 2*a) - b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \cot (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*cot(b*x + a)^3, x)
```

maple [B] time = 0.11, size = 635, normalized size = 3.78

$$-\frac{d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^3}+\frac{2d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^3}-\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^2}{b}+\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}-\frac{d^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x^2}{b}-\frac{4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cot(b*x+a)^3,x)
```

```
[Out] -4/3*I/b^3*a^3*d^2-1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/b^3*d^2*a^2*ln(exp(
I*(b*x+a)))-1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2+1/b^3*d^2*ln(1-exp(I*(b*x+a)))
*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2+4*I/b*a*c*d*x+1/3*I*d^2*x^3+I*c*d*x^2
-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,exp(I*(b*x+a)))/b^3+2
/b*c^2*ln(exp(I*(b*x+a)))-1/b*c^2*ln(exp(I*(b*x+a))-1)-1/b*c^2*ln(exp(I*(b*
x+a))+1)-I*c^2*x-2/b*c*d*ln(1-exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1-exp(I*(b*x+a
)))*a-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-4/b^2*c*d*a*ln(exp(I*(b*x+a)))+2/b^2*c
*d*a*ln(exp(I*(b*x+a))-1)+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(
b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+
a))+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1
)-2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*I/b^2*polyl
og(2,exp(I*(b*x+a)))*d^2*x+2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-2*I/b^2*a^
2*d^2*x+2*I/b^2*a^2*c*d+2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*
d*polylog(2,-exp(I*(b*x+a)))
```

maxima [B] time = 0.71, size = 1966, normalized size = 11.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2)) - 2*a*c*d*(1/sin(b*x + a
)^2 + log(sin(b*x + a)^2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)
^2))/b^2 - 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)^2 + 12*b*c*d
- 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) - 6*d^2 + 6*
((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(4*b*x + 4*a) - 12
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*cos(2*b*x + 2*a) + (
6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*sin(4*
b*x + 4*a) + (-12*I*(b*x + a)^2*d^2 + (-24*I*b*c*d + 24*I*a*d^2)*(b*x + a)
+ 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*
d^2*cos(4*b*x + 4*a) - 12*d^2*cos(2*b*x + 2*a) + 6*I*d^2*sin(4*b*x + 4*a) -
 12*I*d^2*sin(2*b*x + 2*a) + 6*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1)
 + (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 6*((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 12*((b*x + a)^2*d^2 + 2*(b
*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^2 + (-12*I*
b*c*d + 12*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2*d^2 + (
24*I*b*c*d - 24*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
 -cos(b*x + a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*x + a)^2 -
6*(b*x + a)*d^2)*cos(4*b*x + 4*a) - (4*(b*x + a)^3*d^2 + (12*b*c*d - (12*a
- 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 + (24*I*b*c*d - 12*(2*I*a +
1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d
^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 24*(b*c*d + (b*x
```

```
 + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 1
2*I*a*d^2)*sin(4*b*x + 4*a) - (24*I*b*c*d + 24*I*(b*x + a)*d^2 - 24*I*a*d^2
)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (12*b*c*d + 12*(b*x + a)*d^2
- 12*a*d^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 24*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)
*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) - (24*I*b*c*d + 24*I*(b*x + a)*d^2 - 24
*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2*d^2
+ (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2 + (-
6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (6*I*(b*x +
a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*cos(2*b*x + 2*a)
+ 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x + 4*a)
- 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x + 2*a))
*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*(b*x + a
)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*
d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (6*I
*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a) - 6*I*d^2)*cos(2*b*x
 + 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x
 + 4*a) - 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x
 + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-12*I
*d^2*cos(4*b*x + 4*a) + 24*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(4*b*x + 4*a)
 - 24*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (-12*
I*d^2*cos(4*b*x + 4*a) + 24*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(4*b*x + 4*a
) - 24*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, e^(I*b*x + I*a)) - (-2*I
*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*x + a)*d^
2)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^3*d^2 + (12*I*b*c*d - 12*(I*a + 1)*d^2
)*(b*x + a)^2 + 12*I*b*c*d - 12*I*a*d^2 - (24*b*c*d - (24*a - 12*I)*d^2)*(b
*x + a))*sin(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) + 12*I*b^2*cos(2*b*x
+ 2*a) + 6*b^2*sin(4*b*x + 4*a) - 12*b^2*sin(2*b*x + 2*a) - 6*I*b^2))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+bx)^3 (c+dx)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \cot^3(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cot(b*x+a)**3,x)
[Out] Integral((c + d*x)**2*cot(a + b*x)**3, x)
```

3.181 $\int (c+dx)\cot^3(a+bx)\,dx$

Optimal. Leaf size=109

$$\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] -1/2*d*x/b+1/2*I*(d*x+c)^2/d-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+c)*cot(b*x+a)^2/
b-(d*x+c)*ln(1-exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.13, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3473, 8, 3717, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\log\left(1-e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -(d*x)/(2*b) + ((I/2)*(c + d*x)^2)/d - (d*Cot[a + b*x])/(2*b^2) - ((c + d*x
)*Cot[a + b*x]^2)/(2*b) - ((c + d*x)*Log[1 - E^((2*I)*(a + b*x))])/b + ((I/
2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\cot^3(a+bx)\,dx = -\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{d\int \cot^2(a+bx)\,dx}{2b}-\int (c+dx)\cot(a+bx)\,dx$$

$$= \frac{i(c+dx)^2}{2d}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+2i\int \frac{e^{2i(a+bx)}(c+dx)}{1-e^{2i(a+bx)}}\,dx-\frac{d}{}$$

$$= -\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{(c+dx)\log\left(1-e^{2i(a+\right.}}{b}$$

$$= -\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{(c+dx)\log\left(1-e^{2i(a+\right.}}{b}$$

$$= -\frac{dx}{2b}+\frac{i(c+dx)^2}{2d}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}-\frac{(c+dx)\log\left(1-e^{2i(a+\right.}}{b}$$

Mathematica [B] time = 6.17, size = 240, normalized size = 2.20

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{i\tan^{-1}(\tan(a))}+\frac{\tan(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx+\tan^{-1}(\tan(a))\right)}\right)+ibx\left(2\tan^{-1}(\tan(a))-\pi\right)-2\left(\tan^{-1}(\tan(a))+bx\right)\log\left(1-e^{2i\left(\tan^{-1}(\tan(a))+\right.}\right.\right.}{\sqrt{\tan^2(a)+1}}\right.}{2b^2\sqrt{\sec^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Cot[a + b*x]^3,x]
```

```
[Out] -1/2*(d*x^2*Cot[a]) - (d*x*Csc[a + b*x]^2)/(2*b) - (c*(Cot[a + b*x]^2 + 2*L
og[Cos[a + b*x]] + 2*Log[Tan[a + b*x]]))/(2*b) + (d*Csc[a]*Csc[a + b*x]*Sin
[b*x])/(2*b^2) + (d*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(
-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[
a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTa
n[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + Arc
Tan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2
 + Sin[a]^2)])
```

fricas [B] time = 0.46, size = 339, normalized size = 3.11

$$\frac{4\,bdx+4\,bc+(i\,d\cos(2\,bx+2\,a)-i\,d)\mathrm{Li}_2\left(\cos(2\,bx+2\,a)+i\,\sin(2\,bx+2\,a)\right)+(-i\,d\cos(2\,bx+2\,a)+i\,d)\mathrm{Li}_2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*b*d*x + 4*b*c + (I*d*cos(2*b*x + 2*a) - I*d)*dilog(cos(2*b*x + 2*a)
+ I*sin(2*b*x + 2*a)) + (-I*d*cos(2*b*x + 2*a) + I*d)*dilog(cos(2*b*x + 2*a
) - I*sin(2*b*x + 2*a)) + 2*(b*c - a*d - (b*c - a*d)*cos(2*b*x + 2*a))*log(
-1/2*cos(2*b*x + 2*a) + 1/2*I*sin(2*b*x + 2*a) + 1/2) + 2*(b*c - a*d - (b*c
 - a*d)*cos(2*b*x + 2*a))*log(-1/2*cos(2*b*x + 2*a) - 1/2*I*sin(2*b*x + 2*a
) + 1/2) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x + 2*a))*log(-cos(2*b*x
+ 2*a) + I*sin(2*b*x + 2*a) + 1) + 2*(b*d*x + a*d - (b*d*x + a*d)*cos(2*b*x
 + 2*a))*log(-cos(2*b*x + 2*a) - I*sin(2*b*x + 2*a) + 1) + 2*d*sin(2*b*x +
2*a))/(b^2*cos(2*b*x + 2*a) - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\cot(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*cot(b*x + a)^3, x)
```

maple [B] time = 0.09, size = 281, normalized size = 2.58

$$\frac{id\operatorname{polylog}\left(2,-\mathrm{e}^{i(bx+a)}\right)}{b^{2}}-icx+\frac{2bdx\,\mathrm{e}^{2i(bx+a)}+2bc\,\mathrm{e}^{2i(bx+a)}-id\,\mathrm{e}^{2i(bx+a)}+id}{b^{2}\left(\mathrm{e}^{2i(bx+a)}-1\right)^{2}}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cot(b*x+a)^3,x)
```

```
[Out] I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*c*x+(2*b*d*x*exp(2*I*(b*x+a))+2*b*c*ex
p(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+I*d)/b^2/(exp(2*I*(b*x+a))-1)^2-1/b*c*l
n(exp(I*(b*x+a))-1)-1/b*c*ln(exp(I*(b*x+a))+1)+2/b*c*ln(exp(I*(b*x+a)))+I/b
^2*d*polylog(2,exp(I*(b*x+a)))+1/2*I*d*x^2+I/b^2*d*a^2-1/b*d*ln(exp(I*(b*x+
a))+1)*x+2*I/b*d*a*x-1/b*d*ln(1-exp(I*(b*x+a)))*x-1/b^2*d*ln(1-exp(I*(b*x+a
)))*a+1/b^2*d*a*ln(exp(I*(b*x+a))-1)-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.53, size = 839, normalized size = 7.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] (b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a
) - 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*
a) + (-4*I*b*d*x - 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
 + a) + 1) - (2*b*c*cos(4*b*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin
(4*b*x + 4*a) - 4*I*b*c*sin(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos
(b*x + a) - 1) + (2*b*d*x*cos(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I
*b*d*x*sin(4*b*x + 4*a) - 4*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin
(b*x + a), -cos(b*x + a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) -
(2*b^2*d*x^2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*cos(2*b*x + 2*a) + (2
*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d
*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(4*b*x + 4*a) -
4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*
d)*dilog(e^(I*b*x + I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b*
x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*x
 + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(4*b
*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(4*b*
x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
```

```
a)^2 - 2*cos(b*x + a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2*c*x)*sin(4*b*x + 4*a)
 - (2*I*b^2*d*x^2 - 4*b*c - 4*(-I*b^2*c + b*d)*x + 2*I*d)*sin(2*b*x + 2*a)
+ 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*
b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(a+bx)^3 (c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3*(c + d*x),x)
```

```
[Out] int(cot(a + b*x)^3*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\cot^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cot(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*cot(a + b*x)**3, x)
```

3.182 $\int \frac{\cot^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x),x]
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx = \int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 8.08, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x),x]
[Out] Integrate[Cot[a + b*x]^3/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(cot(b*x + a)^3/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(cot(b*x + a)^3/(d*x + c), x)
```

maple [A] time = 2.65, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c),x)
[Out] int(cot(b*x+a)^3/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/(c + d*x),x)
[Out] int(cot(a + b*x)^3/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cot(a + b*x)**3/(c + d*x), x)
```

3.183 $\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\cot^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(cot(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Cot[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 9.29, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Integrate[Cot[a + b*x]^3/(c + d*x)^2, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\cot\left(bx+a\right)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(cot(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(cot(b*x + a)^3/(d*x + c)^2, x)
```

maple [A] time = 4.19, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(cot(b*x+a)^3/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\cot(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(cot(a + b*x)^3/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cot(a + b*x)**3/(c + d*x)**2, x)
```

3.184 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)}{4096b^{7/2}}$$

[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/32*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3

Rubi [A] time = 0.80, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d}{}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]

[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^2)

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)\int(c+}{}$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+dx}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c}}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c}}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c}}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c}}{}$$

Mathematica [A] time = 12.15, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\,\cos(2(a+bx))-256b^3c^2\sqrt{c+dx}\,\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx))-25}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
```

```
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

fricas [A] time = 0.57, size = 376, normalized size = 0.92

$$-\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{4\,(bc-ad)}{d}\right)C\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

giac [C] time = 3.32, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(512*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^3 + 24*c*d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*
b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x
 + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*
(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2
```

```
*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d -
 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2
+ 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/
d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d
- 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*
b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*
I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^
(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 -
72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b + 4
*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^
2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(
3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 7
2*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I
*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d
- 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^
3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*
sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d
^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d
^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b
*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*
x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x
 + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192*(-I*
sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4
```

```
*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(d*x
+ c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b
+ 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*
c - 4*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.04, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\cdots\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.56, size = 547, normalized size = 1.34

$$\frac{\left(1280\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+10240\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-15\sqrt{dx+c}\,b^{2}\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 1024
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) -
 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b
 - b*c + a*d)/d) + ((480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^
(1/4)*cos(-2*(b*c - a*d)/d) + (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (
(30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) +
(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*e
rf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(4
80*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*
d)/d) - (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2
*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx) (c+dx)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.185 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2}{\ }\right.}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.56, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx + \frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{(3d)\int \sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} + \frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b} - \frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b} - \frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\,\mathrm{co}}{}$$

Mathematica [A] time = 2.98, size = 393, normalized size = 1.12

$$\frac{-3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

fricas [A] time = 0.50, size = 294, normalized size = 0.84

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 2.65, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(64*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
)*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(
3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 -
16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16*(I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
```

```
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2
*I*a*d)/d)/b^2)/d^2) + 16*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c -
I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sq
rt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8
*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*
x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d
*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.04, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+\cdots}{\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

maxima [C] time = 0.51, size = 503, normalized size = 1.43

$$-\frac{\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1024\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\,\sqrt{dx+c}\,b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-768\,\sqrt{dx+\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 102
4*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 768*sqrt(d*x + c)*b^2*sin(2*((d
*x + c)*b - b*c + a*d)/d) - (-(48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) -
 (-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (
6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*
sqrt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((48*I - 48)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I + 48)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.186 $\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\cdots}{\sqrt{\cdots}}\right)}{64b^{3/2}}$$

```
[Out] 1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(
3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x+
c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int \sqrt{c+dx}\sin(4a+4bx)\,dx+\frac{1}{4}\int \sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{d\int \frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right.}{6}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Su}}{}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a\right.}{}$$

Mathematica [A] time = 0.59, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\right.}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

fricas [A] time = 0.51, size = 233, normalized size = 0.78

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+8\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

giac [C] time = 2.82, size = 818, normalized size = 2.74

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+8\left(i\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

maple [A] time = 0.04, size = 286, normalized size = 0.96

$$\frac{-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.54, size = 425, normalized size = 1.42

$$-\frac{\left(\frac{32\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{128\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left((8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(8i+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 128*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(8*I +
 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I
 - 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \sin (a + bx) \cos^3 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

3.187 $\int \sqrt{c+dx} \cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=299

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\ldots}{\ldots}\right)}{64b^{3/2}}$$

```
[Out] 1/128*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^
(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/128*FresnelS(2*b^(1/2)*2^(1/2)/Pi
^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(
3/2)+1/16*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2
))*d^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/P
i^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-1/8*cos(2*b*x+2*a)*(d*x+
c)^(1/2)/b-1/32*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{3/2}}+\frac{\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(4a\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -(Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(8*b) - (Sqrt[c + d*x]*Cos[4*a + 4*b*x])/
(32*b) + (Sqrt[d]*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[
2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(3/2)) + (Sqrt[d]*Sqrt[Pi]*Cos[2*a - (
2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(16*b^(3/
2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]]*Sin[4*a - (4*b*c)/d])/(64*b^(3/2)) - (Sqrt[d]*Sqrt[Pi]*FresnelS[(2*Sq
rt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(16*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin(a+bx)\,dx = \int\left(\frac{1}{4}\sqrt{c+dx}\sin(2a+2bx)+\frac{1}{8}\sqrt{c+dx}\sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int\sqrt{c+dx}\sin(4a+4bx)\,dx+\frac{1}{4}\int\sqrt{c+dx}\sin(2a+2bx)\,dx$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{d\int\frac{\cos(4a+4bx)}{\sqrt{c+dx}}dx}{64b}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\left(d\cos\left(4a-\frac{4bc}{d}\right)\right.}{6}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\cos\left(4a-\frac{4bc}{d}\right)\mathrm{Su}}{}$$
$$= -\frac{\sqrt{c+dx}\cos(2a+2bx)}{8b}-\frac{\sqrt{c+dx}\cos(4a+4bx)}{32b}+\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(4a\right.}{}$$

Mathematica [A] time = 0.16, size = 264, normalized size = 0.88

$$\frac{\sqrt{2\pi}\cos\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)+8\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-\sqrt{2\pi}\sin\left(4a-\frac{4bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\sqrt{\right.}{128b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-16*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 4*Sqrt[b/d]*Sqrt[c + d*x]*C
os[4*(a + b*x)] + Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[2/Pi]*Sqrt[c + d*x]] + 8*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b/
d]*Sqrt[c + d*x])/Sqrt[Pi]] - Sqrt[2*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sq
rt[c + d*x]]*Sin[4*a - (4*b*c)/d] - 8*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c
 + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(128*b*Sqrt[b/d])
```

fricas [A] time = 0.55, size = 233, normalized size = 0.78

$$\frac{\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-\sqrt{2}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right)+8\,\pi d\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/128*(sqrt(2)*pi*d*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos(2*sqrt
(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_sin
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) + 8*pi*d*sqr
t(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi*d)
)) - 8*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(
-2*(b*c - a*d)/d) - 4*(8*b*cos(b*x + a)^4 - 3*b)*sqrt(d*x + c))/b^2
```

giac [C] time = 0.98, size = 818, normalized size = 2.74

$$-\frac{-\frac{i\sqrt{2}\sqrt{\pi}(8bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{4ibc-4iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+\frac{i\sqrt{2}\sqrt{\pi}(8bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-4ibc+4iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+8\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/256*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a
*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*(I*sqrt(2)*sqrt(pi)*d*e
rf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*
c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*
d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4
*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*
d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt
(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 8*I*sqrt(pi)*(4*b*c + I*d)*d*
erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I
*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*
d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*
c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*
d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*
I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x
+ c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b +
 4*I*b*c - 4*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 286, normalized size = 0.96

$$-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\cos\left(\frac{4(dx+c)b}{d}+\frac{4da-4cb}{d}\right)}{32b}+ \cdots \over d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/32/b*d*Pi^(
1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c
)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/
2)*b/d))-1/64/b*d*(d*x+c)^(1/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+1/256/b*d*
2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2
)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(
1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.51, size = 425, normalized size = 1.42

$$-\frac{\left(\frac{32\sqrt{dx+c}\,b^2\cos\left(\frac{4((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{128\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left((8i-8)\cdot 4^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{2(bc-ad)}{d}\right)+(8i+ \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/1024*(32*sqrt(d*x + c)*b^2*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 128*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((8*I - 8)*4^(1/4)*s
qrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (8*I + 8)*4^(1/4)
*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(2*I*b/d)) + ((2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-4*(b*
c - a*d)/d) + (2*I + 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*
d)/d))*erf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(2*I + 2)*sqrt(2)*sqrt(pi)*b*(b
^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (2*I - 2)*sqrt(2)*sqrt(pi)*b*(b^2/d^2
)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(8*I +
 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (8*I
 - 8)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \sin(a + bx) \cos^3(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)*cos(a + b*x)**3, x)
```

3.188 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=351

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(4a-\frac{4bc}{d}\right)S\left(\frac{2\ldots}{\ldots}\right)}{512b^{5/2}}$$

```
[Out] -1/8*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(3/2)*cos(4*b*x+4*a)/b-3/1
024*d^(3/2)*cos(4*a-4*b*c/d)*FresnelS(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)-3/1024*d^(3/2)*FresnelC(2*b^(1/2)*2^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1/2)/b^(5
/2)-3/64*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/
Pi^(1/2))*Pi^(1/2)/b^(5/2)-3/64*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+3/32*d*sin(2*b*x+2*a)*(d*
x+c)^(1/2)/b^2+3/256*d*sin(4*b*x+4*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.57, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\sin\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{512b^{5/2}}-\frac{3\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{64b^{5/2}}-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] -((c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(8*b) - ((c + d*x)^(3/2)*Cos[4*a + 4*b*
x])/(32*b) - (3*d^(3/2)*Sqrt[Pi/2]*Cos[4*a - (4*b*c)/d]*FresnelS[(2*Sqrt[b]
*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*Co
s[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/
(64*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/2]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c
+ d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(512*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi]*
FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d]
)/(64*b^(5/2)) + (3*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(32*b^2) + (3*d*Sqrt[
c + d*x]*Sin[4*a + 4*b*x])/(256*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{3/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{3/2}\sin(4a+4bx)\right)dx$$

$$= \frac{1}{8}\int (c+dx)^{3/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{3/2}\sin(2a+2bx)\,dx$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{(3d)\int \sqrt{c+}}{}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}+\frac{3d\sqrt{c+dx}}{3}$$

$$= -\frac{(c+dx)^{3/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{3/2}\cos(4a+4bx)}{32b}-\frac{3d^{3/2}\sqrt{\frac{\pi}{2}}\ \mathrm{co}}{}$$

Mathematica [A] time = 1.20, size = 393, normalized size = 1.12

$$\frac{-3\sqrt{2\pi}\,d\sin\left(4a-\frac{4bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)-48\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right)-3\sqrt{2\pi}\,d\cos\left(4a-\frac{4bc}{d}\right)S}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-128*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 128*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] - 32*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[4*(a + b*
x)] - 32*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 3*d*Sqrt[2*Pi]*Co
s[4*a - (4*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]] - 48*d*Sq
rt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]
- 3*d*Sqrt[2*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (
4*b*c)/d] - 48*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Si
n[2*a - (2*b*c)/d] + 96*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 12*Sqr
t[b/d]*d*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(1024*b^2*Sqrt[b/d])
```

fricas [A] time = 0.51, size = 294, normalized size = 0.84

$$-\frac{3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+3\sqrt{2}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4\,(bc-ad)}{d}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/1024*(3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_sin(
2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 3*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*f
resnel_cos(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d) +
48*pi*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d))) + 48*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt
(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 16*(16*(b^2*d*x + b^2*c)*cos(b*x + a)^4
 - 6*b^2*d*x - 6*b^2*c - 3*(2*b*d*cos(b*x + a)^3 + 3*b*d*cos(b*x + a))*sin(
b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 3.87, size = 1503, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/2048*(64*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(
-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/
sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1))
)*c^2 + d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d*erf(-s
qrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4
*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(
3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*(d*x +
c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 -
16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d - 3*sqrt
(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2 + 16*(I
*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(
I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqr
```

```
t(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x + c)*b*c
*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)
/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sq
rt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(
b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*s
qrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2
*I*a*d)/d)/b^2)/d^2) + 16*(-I*sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)
*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d
)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c -
I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sq
rt(pi)*(4*b*c + I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8
*I*sqrt(pi)*(4*b*c - I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)*b) + 4*sqrt(d*x + c)*d*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16
*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*
x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d
*e^((-4*I*(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 376, normalized size = 1.07

$$\frac{-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/16/b*d*(1/4
/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d)^
(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d
)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1
/64/b*d*(d*x+c)^(3/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+3/64/b*d*(1/8/b*d*(d
*x+c)^(1/2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d
)^(1/2)*(cos(4*(a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)
^(1/2)*b/d)+sin(4*(a*d-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d))))
```

maxima [C] time = 0.53, size = 503, normalized size = 1.43

$$-\frac{\left(\frac{256\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)}{d}+\frac{1024\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\,\sqrt{dx+c}\,b^2\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)-768\,\sqrt{dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/8192*(256*(d*x + c)^(3/2)*b^3*cos(4*((d*x + c)*b - b*c + a*d)/d)/d + 102
4*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(4*((d*x + c)*b - b*c + a*d)/d) - 768*sqrt(d*x + c)*b^2*sin(2*((d
*x + c)*b - b*c + a*d)/d) - (-(48*I + 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (48*I - 48)*4^(1/4)*sqrt(2)*sqrt(pi)*b*
d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) -
 (-(6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) + (
6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*
sqrt(d*x + c)*sqrt(I*b/d)) - ((6*I - 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4
)*cos(-4*(b*c - a*d)/d) - (6*I + 6)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*si
n(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) - ((48*I - 48)*4^(1/
4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) - (48*I + 48)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sq
rt(d*x + c)*sqrt(-2*I*b/d)))*d/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.189 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=407

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(4a-\frac{4bc}{d}\right)S}{4096b^{7/2}}$$

```
[Out] -1/8*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b-1/32*(d*x+c)^(5/2)*cos(4*b*x+4*a)/b+5/3
2*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2+5/256*d*(d*x+c)^(3/2)*sin(4*b*x+4*a)/b
^2-15/8192*d^(5/2)*cos(4*a-4*b*c/d)*FresnelC(2*b^(1/2)*2^(1/2)/Pi^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/8192*d^(5/2)*FresnelS(2*b^(
1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(4*a-4*b*c/d)*2^(1/2)*Pi^(1
/2)/b^(7/2)-15/256*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+15/256*d^(5/2)*FresnelS(2*b^(1/2)*(d*x
+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+15/128*d^2*co
s(2*b*x+2*a)*(d*x+c)^(1/2)/b^3+15/2048*d^2*cos(4*b*x+4*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 0.67, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\cos\left(4a-\frac{4bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4096b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{256b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (15*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(128*b^3) - ((c + d*x)^(5/2)*Cos[2*
a + 2*b*x])/(8*b) + (15*d^2*Sqrt[c + d*x]*Cos[4*a + 4*b*x])/(2048*b^3) - ((
c + d*x)^(5/2)*Cos[4*a + 4*b*x])/(32*b) - (15*d^(5/2)*Sqrt[Pi/2]*Cos[4*a -
(4*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(4096*b^
(7/2)) - (15*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt
[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(256*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*Fres
nelS[(2*Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[4*a - (4*b*c)/d])/(4
096*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqr
t[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(256*b^(7/2)) + (5*d*(c + d*x)^(3/2)*
Sin[2*a + 2*b*x])/(32*b^2) + (5*d*(c + d*x)^(3/2)*Sin[4*a + 4*b*x])/(256*b^
2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin(a+bx)\,dx = \int \left(\frac{1}{4}(c+dx)^{5/2}\sin(2a+2bx)+\frac{1}{8}(c+dx)^{5/2}\sin(4a+4bx)\right)dx$$
$$= \frac{1}{8}\int (c+dx)^{5/2}\sin(4a+4bx)\,dx+\frac{1}{4}\int (c+dx)^{5/2}\sin(2a+2bx)\,dx$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{(5d)\int (c+ \cdots}{}$$
$$= -\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}-\frac{(c+dx)^{5/2}\cos(4a+4bx)}{32b}+\frac{5d(c+dx)^{3 \cdots}}{\cdots}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+ \cdots}}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+ \cdots}}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+ \cdots}}{}$$
$$= \frac{15d^2\sqrt{c+dx}\,\cos(2a+2bx)}{128b^3}-\frac{(c+dx)^{5/2}\cos(2a+2bx)}{8b}+\frac{15d^2\sqrt{c+ \cdots}}{}$$

Mathematica [A] time = 8.39, size = 550, normalized size = 1.35

$$\frac{-1024b^3c^2\sqrt{c+dx}\,\cos(2(a+bx))-256b^3c^2\sqrt{c+dx}\,\cos(4(a+bx))-1024b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx))-256b \cdots}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x],x]
```

```
[Out] (-1024*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 960*b*d^2*Sqrt[c + d*x]*Cos
[2*(a + b*x)] - 2048*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 1024*b^3*d^
2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 256*b^3*c^2*Sqrt[c + d*x]*Cos[4*(a +
 b*x)] + 60*b*d^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 512*b^3*c*d*x*Sqrt[c + d
*x]*Cos[4*(a + b*x)] - 256*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[4*(a + b*x)] - 15*
Sqrt[b/d]*d^3*Sqrt[2*Pi]*Cos[4*a - (4*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[2/P
i]*Sqrt[c + d*x]] - 480*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresnel
C[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] + 15*Sqrt[b/d]*d^3*Sqrt[2*Pi]*Fresn
elS[2*Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*Sin[4*a - (4*b*c)/d] + 480*Sqrt[b
```

```
/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a - (
2*b*c)/d] + 1280*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 1280*b^2*d^2*x*Sq
rt[c + d*x]*Sin[2*(a + b*x)] + 160*b^2*c*d*Sqrt[c + d*x]*Sin[4*(a + b*x)] +
 160*b^2*d^2*x*Sqrt[c + d*x]*Sin[4*(a + b*x)])/(8192*b^4)
```

fricas [A] time = 0.53, size = 376, normalized size = 0.92

$$-\frac{15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{4(bc-ad)}{d}\right)C\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-15\sqrt{2}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{4(bc-ad)}{d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/8192*(15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-4*(b*c - a*d)/d)*fresnel_cos
(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 15*sqrt(2)*pi*d^3*sqrt(b/(pi*d))
*fresnel_sin(2*sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-4*(b*c - a*d)/d)
+ 480*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x +
c)*sqrt(b/(pi*d))) - 480*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) - 4*(192*b^3*d^2*x^2 + 384*b^3*c*d*x
+ 192*b^3*c^2 + 360*b*d^2*cos(b*x + a)^2 - 8*(64*b^3*d^2*x^2 + 128*b^3*c*d*
x + 64*b^3*c^2 - 15*b*d^2)*cos(b*x + a)^4 - 225*b*d^2 + 160*(2*(b^2*d^2*x +
 b^2*c*d)*cos(b*x + a)^3 + 3*(b^2*d^2*x + b^2*c*d)*cos(b*x + a))*sin(b*x +
a))*sqrt(d*x + c))/b^4
```

giac [C] time = 3.10, size = 2418, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16384*(512*(I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)) - I*sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*
d/sqrt(b^2*d^2) + 1)) + 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)) - 4*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d
^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1
)))*c^3 + 24*c*d^2*((I*sqrt(2)*sqrt(pi)*(64*b^2*c^2 + 16*I*b*c*d - 3*d^2)*d
*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*
b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 4*I*(8*I*(d*x
 + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-4*I*
(d*x + c)*b + 4*I*b*c - 4*I*a*d)/d)/b^2)/d^2 + (-I*sqrt(2)*sqrt(pi)*(64*b^2
```

```
*c^2 - 16*I*b*c*d - 3*d^2)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^2) - 4*I*(8*I*(d*x + c)^(3/2)*b*d - 16*I*sqrt(d*x + c)*b*c*d -
 3*sqrt(d*x + c)*d^2)*e^((4*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b^2)/d^2
+ 16*(I*sqrt(pi)*(16*b^2*c^2 + 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*
b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d - 8*I*sqrt(d*x +
 c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/
d)/b^2)/d^2 + 16*(-I*sqrt(pi)*(16*b^2*c^2 - 8*I*b*c*d - 3*d^2)*d*erf(-sqrt(
b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(4*I*(d*x + c)^(3/2)*b*d
- 8*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*
b*c + 2*I*a*d)/d)/b^2)/d^2) + d^3*((-I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 + 192*
I*b^2*c^2*d - 72*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^
(3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d + 40*(d*x + c)^(3/2)*b*d^2 -
72*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-4*I*(d*x + c)*b + 4
*I*b*c - 4*I*a*d)/d)/b^3)/d^3 + (I*sqrt(2)*sqrt(pi)*(512*b^3*c^3 - 192*I*b^
2*c^2*d - 72*b*c*d^2 + 15*I*d^3)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sq
rt(b^2*d^2) + 1)*b^3) - 4*I*(64*I*(d*x + c)^(5/2)*b^2*d - 192*I*(d*x + c)^(
3/2)*b^2*c*d + 192*I*sqrt(d*x + c)*b^2*c^2*d - 40*(d*x + c)^(3/2)*b*d^2 + 7
2*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x + c)*d^3)*e^((4*I*(d*x + c)*b - 4*I
*b*c + 4*I*a*d)/d)/b^3)/d^3 + 32*(-I*sqrt(pi)*(64*b^3*c^3 + 48*I*b^2*c^2*d
- 36*b*c*d^2 - 15*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2
) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^
3) - 2*I*(16*I*(d*x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*
sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqrt(d*x + c)*b*c*d
^2 - 15*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 32*(I*sqrt(pi)*(64*b^3*c^3 - 48*I*b^2*c^2*d - 36*b*c*d^2 + 15*I*d
^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b
*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(16*I*(d*
x + c)^(5/2)*b^2*d - 48*I*(d*x + c)^(3/2)*b^2*c*d + 48*I*sqrt(d*x + c)*b^2*
c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sqrt(d*x + c)*b*c*d^2 - 15*I*sqrt(d*x
 + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 192*(-I*
sqrt(2)*sqrt(pi)*(8*b*c + I*d)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((4*I*b*c - 4*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b) + I*sqrt(2)*sqrt(pi)*(8*b*c - I*d)*d*erf(-sqrt(2)*sqrt(b*d)*
sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-4*I*b*c + 4*I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 8*I*sqrt(pi)*(4*b*c + I*d)*d*erf(-sq
rt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d
)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 8*I*sqrt(pi)*(4*b*c - I*d)*d*er
f(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 4*sqrt(d*x + c)*d*e^((4
```

```
*I*(d*x + c)*b - 4*I*b*c + 4*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((2*I*(d*x
+ c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 16*sqrt(d*x + c)*d*e^((-2*I*(d*x + c)*b
+ 2*I*b*c - 2*I*a*d)/d)/b + 4*sqrt(d*x + c)*d*e^((-4*I*(d*x + c)*b + 4*I*b*
c - 4*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 470, normalized size = 1.15

$$-\frac{d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{8b}+\frac{5d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x)
```

```
[Out] 2/d*(-1/16/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+5/16/b*d*(1/4
/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d*x+
c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(cos
(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2*(a
*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1/64/b*d*(
d*x+c)^(5/2)*cos(4/d*(d*x+c)*b+4*(a*d-b*c)/d)+5/64/b*d*(1/8/b*d*(d*x+c)^(3/
2)*sin(4/d*(d*x+c)*b+4*(a*d-b*c)/d)-3/8/b*d*(-1/8/b*d*(d*x+c)^(1/2)*cos(4/d
*(d*x+c)*b+4*(a*d-b*c)/d)+1/32/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(cos(4*(a*d
-b*c)/d)*FresnelC(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(4*(
a*d-b*c)/d)*FresnelS(2*2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.52, size = 547, normalized size = 1.34

$$\frac{\left(1280\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{4\,((dx+c)b-bc+ad)}{d}\right)+10240\,(dx+c)^{\frac{3}{2}}b^3\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-32\left(\frac{64\,(dx+c)^{\frac{5}{2}}b^4}{d}-15\sqrt{dx+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/65536*(1280*(d*x + c)^(3/2)*b^3*sin(4*((d*x + c)*b - b*c + a*d)/d) + 1024
0*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 32*(64*(d*x + c)
^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(4*((d*x + c)*b - b*c + a*d)/d) -
 512*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)*b
 - b*c + a*d)/d) + ((480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^
(1/4)*cos(-2*(b*c - a*d)/d) + (480*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + (
(30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) +
(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*e
rf(2*sqrt(d*x + c)*sqrt(I*b/d)) + (-(30*I + 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*cos(-4*(b*c - a*d)/d) - (30*I - 30)*sqrt(2)*sqrt(pi)*b*d^2*(b^2
/d^2)^(1/4)*sin(-4*(b*c - a*d)/d))*erf(2*sqrt(d*x + c)*sqrt(-I*b/d)) + (-(4
80*I + 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*
d)/d) - (480*I - 480)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2
*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx) (c+dx)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.190 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)C}{32b^{7/2}}$$

```
[Out] 5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^
2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b-1
/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3/16
000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)*10^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(
7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC(b^(1/2)
*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/
b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)
-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 1.14, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
```

```
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) - \frac{1}{16}(c\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos(5a+$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{5/2}\sin}{80b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx}{}$$

Mathematica [C] time = 22.42, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d]
)/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x)
)/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]
*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c
)/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2
```

```
*b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^
(3/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^
2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]
*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] +
 (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) -
(c^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*
a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d])
 + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c
*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]
*(Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2
*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3
*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]
*d*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*S
in[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c
)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[S
qrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b
/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt
[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*
a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*
Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*
b*x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a
 - (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[
b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c
^2 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2
*b*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(
3200*Sqrt[5]*b^5)
```

fricas [A] time = 0.58, size = 548, normalized size = 0.89

$$-\frac{81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+625\,\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
```

```
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 18.66, size = 3677, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")

[Out] 1/864000*(1800*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*
erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*
d^2*(9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqr
t(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3
/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c
)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(
2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
```

+ c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^
2)/d^2 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d
- d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
- 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^
2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(
pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
x + c)(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqr
t(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I
*a*d)/d)/b^2)/d^2) - d^3*(27*(sqrt(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2
*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^3) + 10*(20*I*(d*x + c)^(5/2)*b^2*d - 60*I*(d*x + c)^(3/2)*b
^2*c*d + 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(
d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c -
5*I*a*d)/d)/b^3)/d^3 + 125*(sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d -
18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d
+ 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x +
c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*
d)/d)/b^3)/d^3 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*
c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b
^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*s
qrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^
2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3
- 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d
^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*
I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)
*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(sqrt(
6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x +
c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2
d + 10(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c
)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(sqrt(10)*
sqrt(pi)*(200*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*s

```
qrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c
+ 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 10*(-20*I*(d*x +
 c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c
)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) - 180*(9*sqrt(
10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c + I*
d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)
*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*
d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*s
qrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d
*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x +
c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x +
c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b
+ 5*I*b*c - 5*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.07, size = 716, normalized size = 1.16

$$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fres}\cdots\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{8b}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*(d*x+c)*b+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.57, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
 a*d)/d)/d + ((162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*
(b*c - a*d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(
-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((1250*I + 1250)*9^(1
/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (1250*I - 1250)*
9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(3*I*b/d)) + (-(202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((202500*I - 202500)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c
 - a*d)/d) + (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-(162*I - 162)*25^(1/4
)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(
1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-5*I*b/d)) + 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sq
rt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x
 + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(3*((d*x + c)*b - b*c
```

```
+ a*d)/d) - 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x
+ c)*b^3)*sin(((d*x + c)*b - b*c + a*d)/d))*d^2/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+bx)^3 \sin(a+bx)^2 (c+dx)^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.191 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=534

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)C\left(\right.}{800b^{5/2}}$$

```
[Out] 1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*
x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8000
*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-
5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(
b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/
576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*
a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/32*d
^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d
)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1/96*d*cos(3
*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.84, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}\,d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}\,d^{3/2}\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
```

```
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \frac{1}{16}(c+\right.$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos(5a+5b$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin(5}{80b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,\cos}{800b}$$

$$= \frac{3d\sqrt{c+dx}\,\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\,\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,\cos}{800b}$$

Mathematica [C] time = 12.00, size = 1043, normalized size = 1.95

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\,\mathrm{co}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
 + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2
/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*S
qrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[
2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) -
```

```
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d
] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt
[b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sq
rt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b
*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*
x)] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*
a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt
[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) -
(d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*
d*Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d
] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x
)] + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

fricas [A] time = 0.56, size = 446, normalized size = 0.84

$$\frac{27\,\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)C\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+125\,\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)C\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 7.72, size = 2293, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/144000*(300*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9
*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*
d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*
c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2
)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(2)*sqr
t(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b
*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(
-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(
(3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(pi)*(1
00*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x
+ c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/
d)/b^2)/d^2) - 20*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*s
qrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sq
rt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
```

```
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10
)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*
I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d
)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90
0*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d
*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x +
c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.06, size = 583, normalized size = 1.09

$$\frac{\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+c)}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.58, size = 754, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 + 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 1080*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a*
d)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)
/d - 54000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d + (
(54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(13500
*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I + 1
3500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + ((13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.192 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{}}{}\right.}{8b^{3/2}}$$

```
[Out] 1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(
3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^
(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/8
*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1/80*sin(5*
b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.69, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\cos(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\cos(5a+5bx)$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5}{80b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5}{80b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5}{80b}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\right.}{48b^{3/2}}$$

Mathematica [C] time = 6.94, size = 435, normalized size = 0.95

$$-\frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) - \sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3
*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[
3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(
Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x
]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5
*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]
*b*Sqrt[b/d])
```

fricas [A] time = 0.54, size = 365, normalized size = 0.80

$$\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5(bc-ad)}{d}\right)S\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3(bc-ad)}{d}\right)S\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)-450}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

giac [C] time = 2.15, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/14400*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*
b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqr
t(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi
)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 30*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
```

```
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 90*I*sqr
t(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)
*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)
*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b)/d
```

maple [A] time = 0.06, size = 444, normalized size = 0.97

$$\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.52, size = 674, normalized size = 1.47

$$-\frac{\sqrt{2}\left(\frac{360\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{600\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(-\frac{(18i+18)\cdot 2}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 + 600*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
^2 + (-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)
/d)/d + (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d
)/d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I + 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)
) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (90
0*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
 + c)*sqrt(I*b/d)) + (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c
 - a*d)/d)/d + (900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) +
 ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
 - (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/
d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^2\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

3.193 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=459

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{10}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}}$$

```
[Out] 1/800*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(3/2)+1/800*FresnelC(b^(1/2)*10^(1/2)/Pi^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*d^(1/2)*10^(1/2)*Pi^(1/2)/b^(
3/2)+1/288*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)
/d^(1/2))*d^(1/2)*6^(1/2)*Pi^(1/2)/b^(3/2)+1/288*FresnelC(b^(1/2)*6^(1/2)/P
i^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*d^(1/2)*6^(1/2)*Pi^(1/2)/b^
(3/2)-1/16*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/16*FresnelC(b^(1/2)*2^(1/2)/Pi^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/8
*sin(b*x+a)*(d*x+c)^(1/2)/b-1/48*sin(3*b*x+3*a)*(d*x+c)^(1/2)/b-1/80*sin(5*
b*x+5*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.67, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{10}}\sqrt{d}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{80b^{3/2}}+\frac{\sqrt{\frac{\pi}{6}}\sqrt{d}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}\sin\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] -(Sqrt[d]*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(8*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fre
snelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (Sqrt[d]*
Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x
])/Sqrt[d]])/(80*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*Sqrt[10/
Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(80*b^(3/2)) + (Sqrt[d]*S
qrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3
*b*c)/d])/(48*b^(3/2)) - (Sqrt[d]*Sqrt[Pi/2]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*S
qrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(8*b^(3/2)) + (Sqrt[c + d*x]*Sin[a
 + b*x])/(8*b) - (Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(48*b) - (Sqrt[c + d*x]*S
in[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}\sqrt{c+dx}\cos(a+bx) - \frac{1}{16}\sqrt{c+dx}\cos(3a+3bx) - \frac{1}{16}\sqrt{c+dx}\right.$$

$$= -\left(\frac{1}{16}\int \sqrt{c+dx}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int \sqrt{c+dx}\cos(5a+5bx)\,d$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5b}{80b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5b}{80b}$$

$$= \frac{\sqrt{c+dx}\sin(a+bx)}{8b} - \frac{\sqrt{c+dx}\sin(3a+3bx)}{48b} - \frac{\sqrt{c+dx}\sin(5a+5b}{80b}$$

$$= -\frac{\sqrt{d}\sqrt{\frac{\pi}{2}}\cos\left(a-\frac{bc}{d}\right)S\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{3/2}} + \frac{\sqrt{d}\sqrt{\frac{\pi}{6}}\cos\left(3a-\frac{3bc}{d}\right)S\left(\frac{\sqrt{b}}{}\right.}{48b^{3/2}}$$

Mathematica [C] time = 6.87, size = 435, normalized size = 0.95

$$-\frac{-\sqrt{2\pi}\sin\left(3a-\frac{3bc}{d}\right)C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) - \sqrt{2\pi}\cos\left(3a-\frac{3bc}{d}\right)S\left(\sqrt{\frac{b}{d}}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}\right) + 2\sqrt{3}\sqrt{\frac{b}{d}}\sqrt{c+dx}\sin(3}{96\sqrt{3}\,b\sqrt{\frac{b}{d}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/Sq
rt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/d]
)/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) - (-(Sqrt[2*Pi]*Cos[3
*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d] + 2*Sqrt[
3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)])/(96*Sqrt[3]*b*Sqrt[b/d]) - (-(
Sqrt[2*Pi]*Cos[5*a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x
]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5
*b*c)/d] + 2*Sqrt[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)])/(160*Sqrt[5]
*b*Sqrt[b/d])
```

fricas [A] time = 0.54, size = 365, normalized size = 0.80

$$\frac{9\sqrt{10}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5\,(bc-ad)}{d}\right)S\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+25\sqrt{6}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3\,(bc-ad)}{d}\right)S\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-45}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/7200*(9*sqrt(10)*pi*d*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_sin(sq
rt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 25*sqrt(6)*pi*d*sqrt(b/(pi*d))*cos(-
3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 450*sq
rt(2)*pi*d*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqrt(2)*sqrt(d*x
+ c)*sqrt(b/(pi*d))) - 450*sqrt(2)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(2)*
sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 25*sqrt(6)*pi*d*sqrt(b/
(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-3*(b*c - a*d
)/d) + 9*sqrt(10)*pi*d*sqrt(b/(pi*d))*fresnel_cos(sqrt(10)*sqrt(d*x + c)*sq
rt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(3*b*cos(b*x + a)^4 - b*cos(b*x +
 a)^2 - 2*b)*sqrt(d*x + c)*sin(b*x + a))/b^2
```

giac [C] time = 4.48, size = 1258, normalized size = 2.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/14400*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*
sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(
b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf
(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*
b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqr
t(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(
b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)
*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(
d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*
sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
+ 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi
)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b
^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
 + 1)*b) - 30*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
```

```
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 90*I*sqr
t(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d*x
 + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x + c)
*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I*(
d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x + c)
*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 444, normalized size = 0.97

$$\frac{\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{16b\sqrt{\frac{b}{d}}}-\frac{d\sqrt{dx+c}\,\sin\left(\frac{3(dx+c)b}{d}+\frac{3da-3cb}{d}\right)}{48b}+\frac{d\sqrt{\ldots}}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(1/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/32/b*d*2^(1/2)
*Pi^(1/2)/(b/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/
2)*(d*x+c)^(1/2)*b/d)+sin((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2
)*(d*x+c)^(1/2)*b/d))-1/96/b*d*(d*x+c)^(1/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/
d)+1/576/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)
/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))-1/160
/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+1/1600/b*d*2^(1/2)*Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1
/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(
1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))
```

maxima [C] time = 0.51, size = 674, normalized size = 1.47

$$-\frac{\sqrt{2}\left(\frac{360\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{5((dx+c)b-bc+ad)}{d}\right)}{d^2}+\frac{600\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{3((dx+c)b-bc+ad)}{d}\right)}{d^2}-\frac{3600\sqrt{2}\sqrt{dx+c}\,b^3\sin\left(\frac{(dx+c)b-bc+ad}{d}\right)}{d^2}+\left(-\frac{(18i+18}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/57600*sqrt(2)*(360*sqrt(2)*sqrt(d*x + c)*b^3*sin(5*((d*x + c)*b - b*c +
a*d)/d)/d^2 + 600*sqrt(2)*sqrt(d*x + c)*b^3*sin(3*((d*x + c)*b - b*c + a*d)
/d)/d^2 - 3600*sqrt(2)*sqrt(d*x + c)*b^3*sin(((d*x + c)*b - b*c + a*d)/d)/d
^2 + (-(18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)
/d)/d + (18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d
)/d)/d)*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + (-(50*I + 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d + (50*I - 50)*9^(1/4)*sqrt(pi)*b
^2*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(3*I*b/d)
) + ((900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d)/d - (90
0*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)/d)*erf(sqrt(d*x
 + c)*sqrt(I*b/d)) + (-(900*I - 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c
 - a*d)/d)/d + (900*I + 900)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/
d)/d)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + ((50*I - 50)*9^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d)/d - (50*I + 50)*9^(1/4)*sqrt(pi)*b^2*(
b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d)/d)*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) +
 ((18*I - 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d)/d
 - (18*I + 18)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d)/
d)*erf(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^2\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\sin^2(a+bx)\cos^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
[Out] Integral(sqrt(c + d*x)*sin(a + b*x)**2*cos(a + b*x)**3, x)
```

3.194 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=534

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\sqrt{\frac{6}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\cos\left(5a-\frac{5bc}{d}\right)C\left(\right.}{800b^{5/2}}$$

```
[Out] 1/8*(d*x+c)^(3/2)*sin(b*x+a)/b-1/48*(d*x+c)^(3/2)*sin(3*b*x+3*a)/b-1/80*(d*
x+c)^(3/2)*sin(5*b*x+5*a)/b+3/8000*d^(3/2)*cos(5*a-5*b*c/d)*FresnelC(b^(1/2
)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(5/2)-3/8000
*d^(3/2)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-
5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(5/2)+1/576*d^(3/2)*cos(3*a-3*b*c/d)*FresnelC(
b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)-1/
576*d^(3/2)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*
a-3*b*c/d)*6^(1/2)*Pi^(1/2)/b^(5/2)-3/32*d^(3/2)*cos(a-b*c/d)*FresnelC(b^(1
/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/32*d
^(3/2)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d
)*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*cos(b*x+a)*(d*x+c)^(1/2)/b^2-1/96*d*cos(3
*b*x+3*a)*(d*x+c)^(1/2)/b^2-3/800*d*cos(5*b*x+5*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.86, antiderivative size = 534, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{2}}d^{3/2}\cos\left(a-\frac{bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{5/2}}+\frac{\sqrt{\frac{\pi}{6}}d^{3/2}\cos\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{3\sqrt{\frac{\pi}{10}}d^{3/2}\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x]*Cos[a + b*x])/(16*b^2) - (d*Sqrt[c + d*x]*Cos[3*a + 3*b*
x])/(96*b^2) - (3*d*Sqrt[c + d*x]*Cos[5*a + 5*b*x])/(800*b^2) - (3*d^(3/2)*
Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqr
t[d]])/(16*b^(5/2)) + (d^(3/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*FresnelC[(Sq
rt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi
/10]*Cos[5*a - (5*b*c)/d]*FresnelC[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt
[d]])/(800*b^(5/2)) - (3*d^(3/2)*Sqrt[Pi/10]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*
Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(800*b^(5/2)) - (d^(3/2)*Sqrt
[Pi/6]*FresnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[3*a - (3*b*
c)/d])/(96*b^(5/2)) + (3*d^(3/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sq
rt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(16*b^(5/2)) + ((c + d*x)^(3/2)*Sin
```

```
[a + b*x])/(8*b) - ((c + d*x)^(3/2)*Sin[3*a + 3*b*x])/(48*b) - ((c + d*x)^(
3/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{3/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{3/2}\cos(3a+3bx) - \frac{1}{16}(c$$

$$= -\left(\frac{1}{16}\int (c+dx)^{3/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{3/2}\cos(5a+$$

$$= \frac{(c+dx)^{3/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{3/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{3/2}\sin}{80b}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,c}{800}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,c}{800}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,c}{800}$$

$$= \frac{3d\sqrt{c+dx}\cos(a+bx)}{16b^2} - \frac{d\sqrt{c+dx}\cos(3a+3bx)}{96b^2} - \frac{3d\sqrt{c+dx}\,c}{800}$$

Mathematica [C] time = 11.71, size = 1043, normalized size = 1.95

$$-\frac{ice^{-\frac{i(bc+ad)}{d}}\sqrt{c+dx}\left(\frac{e^{2ia}\Gamma\left(\frac{3}{2},-\frac{ib(c+dx)}{d}\right)}{\sqrt{-\frac{ib(c+dx)}{d}}} - \frac{e^{\frac{2ibc}{d}}\Gamma\left(\frac{3}{2},\frac{ib(c+dx)}{d}\right)}{\sqrt{\frac{ib(c+dx)}{d}}}\right)}{16b} + \frac{d\left(\sqrt{\frac{b}{d}}\sqrt{2\pi}\,C\left(\sqrt{\frac{b}{d}}\sqrt{\frac{2}{\pi}}\sqrt{c+dx}\right)\left(2bc\sin\left(a-\frac{bc}{d}\right)-3d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d])/
Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x))/
d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (d*(Sqrt[b/d]*Sqr
t[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c)/d]
 + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2
/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2*b*S
qrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(32*b^3) - (c*(-(Sqrt[
2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]) -
```

```
Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*a - (3*b*c)/d
] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*Sqrt[3]*b*Sqrt
[b/d]) - (d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*
x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d]) + Sqrt[b/d]*Sq
rt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[3*a - (3*b
*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]*(Cos[3*(a + b*
x)] + 2*b*x*Sin[3*(a + b*x)])))/(192*Sqrt[3]*b^3) - (c*(-(Sqrt[2*Pi]*Cos[5*
a - (5*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*
FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt
[5]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) -
(d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*
d*Cos[5*a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi
]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d
] + 3*d*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x
)] + 10*b*x*Sin[5*(a + b*x)])))/(1600*Sqrt[5]*b^3)
```

fricas [A] time = 0.59, size = 446, normalized size = 0.84

$$\frac{27\sqrt{10}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{5(bc-ad)}{d}\right)C\left(\sqrt{10}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+125\sqrt{6}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{3(bc-ad)}{d}\right)C\left(\sqrt{6}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/72000*(27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_co
s(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 125*sqrt(6)*pi*d^2*sqrt(b/(pi*d)
)*cos(-3*(b*c - a*d)/d)*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d))) -
 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_cos(sqrt(2)
*sqrt(d*x + c)*sqrt(b/(pi*d))) + 6750*sqrt(2)*pi*d^2*sqrt(b/(pi*d))*fresnel
_sin(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) - 125*sqrt(6
)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))*s
in(-3*(b*c - a*d)/d) - 27*sqrt(10)*pi*d^2*sqrt(b/(pi*d))*fresnel_sin(sqrt(1
0)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) - 480*(9*b*d*cos(b*x
 + a)^5 - 5*b*d*cos(b*x + a)^3 - 30*b*d*cos(b*x + a) + 10*(3*(b^2*d*x + b^2
*c)*cos(b*x + a)^4 - 2*b^2*d*x - 2*b^2*c - (b^2*d*x + b^2*c)*cos(b*x + a)^2
)*sin(b*x + a))*sqrt(d*x + c))/b^3
```

giac [C] time = 7.04, size = 2293, normalized size = 4.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/144000*(300*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x +
 c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*
d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqr
t(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d
)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt
(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*s
qrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)
/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqr
t(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3
*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*e
rf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^2 + d^2*(9
*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*
sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)
/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3/2)*b*
d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c)*b +
5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 + 4*I*b*
c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2)
+ 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2
)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(2)*sqr
t(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x
 + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/
sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b
*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^2)/d^2
 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2
)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)
/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/2)*b*d
 - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b - I*b*c
 + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d - d^2
)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*(
-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^2)*e^(
(3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(pi)*(1
00*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c
)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b
*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqrt(d*x
+ c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/
d)/b^2)/d^2) - 20*(9*sqrt(10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*s
qrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/
d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I
*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)
*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sq
rt(2)*sqrt(pi)*(2*b*c + I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*
```

```
b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*
d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b
*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqr
t(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*
erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((
-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10
)*sqrt(pi)*(10*b*c - I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt
(b^2*d^2) + 1)*b) - 90*I*sqrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*
I*a*d)/d)/b - 150*I*sqrt(d*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d
)/d)/b + 900*I*sqrt(d*x + c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 90
0*I*sqrt(d*x + c)*d*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d
*x + c)*d*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x +
c)*d*e^((-5*I*(d*x + c)*b + 5*I*b*c - 5*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 583, normalized size = 1.09

$$\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{da-cb}{d}\right)\mathrm{S}\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)\right)}{4b\sqrt{\frac{b}{d}}}\right)}{8b}-\frac{d(dx+c)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(3/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-3/16/b*d*(-1/2/b
*d*(d*x+c)^(1/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+1/4/b*d*2^(1/2)*Pi^(1/2)/(b
/d)^(1/2)*(cos((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d)-sin((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1
/2)*b/d)))-1/96/b*d*(d*x+c)^(3/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/32/b*d
*(-1/6/b*d*(d*x+c)^(1/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/36/b*d*2^(1/2)*
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(cos(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*
3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/
Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))-1/160/b*d*(d*x+c)^(3/2)*s
in(5/d*(d*x+c)*b+5*(a*d-b*c)/d)+3/160/b*d*(-1/10/b*d*(d*x+c)^(1/2)*cos(5/d*
(d*x+c)*b+5*(a*d-b*c)/d)+1/100/b*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(co
s(5*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2
)*b/d)-sin(5*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d))))
```

maxima [C] time = 0.53, size = 754, normalized size = 1.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/576000*sqrt(2)*(3600*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(5*((d*x + c)*b - b*
c + a*d)/d)/d^2 + 6000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(3*((d*x + c)*b - b*c
 + a*d)/d)/d^2 - 36000*sqrt(2)*(d*x + c)^(3/2)*b^4*sin(((d*x + c)*b - b*c +
 a*d)/d)/d^2 + 1080*sqrt(2)*sqrt(d*x + c)*b^3*cos(5*((d*x + c)*b - b*c + a*
d)/d)/d + 3000*sqrt(2)*sqrt(d*x + c)*b^3*cos(3*((d*x + c)*b - b*c + a*d)/d)
/d - 54000*sqrt(2)*sqrt(d*x + c)*b^3*cos(((d*x + c)*b - b*c + a*d)/d)/d + (
(54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (
54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*cos(-3*(b*c - a*d)/d) + (250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b^2/d^
2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(3*I*b/d)) + (-(13500
*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (13500*I + 1
3500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*s
qrt(I*b/d)) + ((13500*I + 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-(b*c - a
*d)/d) + (13500*I - 13500)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d)
)*erf(sqrt(d*x + c)*sqrt(-I*b/d)) + (-(250*I + 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (250*I - 250)*9^(1/4)*sqrt(pi)*b^2*(b
^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-
(54*I + 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) - (
54*I - 54)*25^(1/4)*sqrt(pi)*b^2*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf
(sqrt(d*x + c)*sqrt(-5*I*b/d)))*d^2/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (a+b x)^3 \sin (a+b x)^2 (c+d x)^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.195 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx$

Optimal. Leaf size=615

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{10}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)C\left(\frac{\sqrt{b}\,\sqrt{\frac{6}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}\sin\left(a-\frac{bc}{d}\right)C\left(\right.}{32b^{7/2}}$$

```
[Out] 5/16*d*(d*x+c)^(3/2)*cos(b*x+a)/b^2-5/288*d*(d*x+c)^(3/2)*cos(3*b*x+3*a)/b^
2-1/160*d*(d*x+c)^(3/2)*cos(5*b*x+5*a)/b^2+1/8*(d*x+c)^(5/2)*sin(b*x+a)/b-1
/48*(d*x+c)^(5/2)*sin(3*b*x+3*a)/b-1/80*(d*x+c)^(5/2)*sin(5*b*x+5*a)/b-3/16
000*d^(5/2)*cos(5*a-5*b*c/d)*FresnelS(b^(1/2)*10^(1/2)/Pi^(1/2)*(d*x+c)^(1/
2)/d^(1/2))*10^(1/2)*Pi^(1/2)/b^(7/2)-3/16000*d^(5/2)*FresnelC(b^(1/2)*10^(
1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(5*a-5*b*c/d)*10^(1/2)*Pi^(1/2)/b^(
7/2)-5/3456*d^(5/2)*cos(3*a-3*b*c/d)*FresnelS(b^(1/2)*6^(1/2)/Pi^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*6^(1/2)*Pi^(1/2)/b^(7/2)-5/3456*d^(5/2)*FresnelC(b^(1/2)
*6^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(3*a-3*b*c/d)*6^(1/2)*Pi^(1/2)/
b^(7/2)+15/64*d^(5/2)*cos(a-b*c/d)*FresnelS(b^(1/2)*2^(1/2)/Pi^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^(5/2)*FresnelC(b^(1/2)*2^
(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(a-b*c/d)*2^(1/2)*Pi^(1/2)/b^(7/2)
-15/32*d^2*sin(b*x+a)*(d*x+c)^(1/2)/b^3+5/576*d^2*sin(3*b*x+3*a)*(d*x+c)^(1
/2)/b^3+3/1600*d^2*sin(5*b*x+5*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A] time = 1.02, antiderivative size = 615, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{3\sqrt{\frac{\pi}{10}}\,d^{5/2}\sin\left(5a-\frac{5bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{10}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{1600b^{7/2}}-\frac{5\sqrt{\frac{\pi}{6}}\,d^{5/2}\sin\left(3a-\frac{3bc}{d}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2)*Cos[a + b*x])/(16*b^2) - (5*d*(c + d*x)^(3/2)*Cos[3*a
+ 3*b*x])/(288*b^2) - (d*(c + d*x)^(3/2)*Cos[5*a + 5*b*x])/(160*b^2) + (15*
d^(5/2)*Sqrt[Pi/2]*Cos[a - (b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[2/Pi]*Sqrt[c + d
*x])/Sqrt[d]])/(32*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/6]*Cos[3*a - (3*b*c)/d]*Fr
esnelS[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (3*d^(5
/2)*Sqrt[Pi/10]*Cos[5*a - (5*b*c)/d]*FresnelS[(Sqrt[b]*Sqrt[10/Pi]*Sqrt[c +
 d*x])/Sqrt[d]])/(1600*b^(7/2)) - (3*d^(5/2)*Sqrt[Pi/10]*FresnelC[(Sqrt[b]*
Sqrt[10/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[5*a - (5*b*c)/d])/(1600*b^(7/2)) -
(5*d^(5/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[b]*Sqrt[6/Pi]*Sqrt[c + d*x])/Sqrt[d]]*
```

```
Sin[3*a - (3*b*c)/d])/(576*b^(7/2)) + (15*d^(5/2)*Sqrt[Pi/2]*FresnelC[(Sqrt
[b]*Sqrt[2/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[a - (b*c)/d])/(32*b^(7/2)) - (15
*d^2*Sqrt[c + d*x]*Sin[a + b*x])/(32*b^3) + ((c + d*x)^(5/2)*Sin[a + b*x])/
(8*b) + (5*d^2*Sqrt[c + d*x]*Sin[3*a + 3*b*x])/(576*b^3) - ((c + d*x)^(5/2)
*Sin[3*a + 3*b*x])/(48*b) + (3*d^2*Sqrt[c + d*x]*Sin[5*a + 5*b*x])/(1600*b^
3) - ((c + d*x)^(5/2)*Sin[5*a + 5*b*x])/(80*b)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^2(a+bx)\,dx = \int \left(\frac{1}{8}(c+dx)^{5/2}\cos(a+bx) - \frac{1}{16}(c+dx)^{5/2}\cos(3a+3bx) - \frac{1}{16}(c+$$

$$= -\left(\frac{1}{16}\int (c+dx)^{5/2}\cos(3a+3bx)\,dx\right) - \frac{1}{16}\int (c+dx)^{5/2}\cos(5a+5b$$

$$= \frac{(c+dx)^{5/2}\sin(a+bx)}{8b} - \frac{(c+dx)^{5/2}\sin(3a+3bx)}{48b} - \frac{(c+dx)^{5/2}\sin(5}{80b}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^3}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^3}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^3}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^3}{}$$

$$= \frac{5d(c+dx)^{3/2}\cos(a+bx)}{16b^2} - \frac{5d(c+dx)^{3/2}\cos(3a+3bx)}{288b^2} - \frac{d(c+dx)^3}{}$$

Mathematica [C] time = 21.75, size = 1795, normalized size = 2.92

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^2,x]
```

```
[Out] ((-1/16*I)*c^2*Sqrt[c + d*x]*((E^((2*I)*a)*Gamma[3/2, ((-I)*b*(c + d*x))/d]
)/Sqrt[((-I)*b*(c + d*x))/d] - (E^(((2*I)*b*c)/d)*Gamma[3/2, (I*b*(c + d*x)
)/d])/Sqrt[(I*b*(c + d*x))/d]))/(b*E^((I*(b*c + a*d))/d)) + (c*d*(Sqrt[b/d]
*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[a - (b*c
)/d] + 2*b*c*Sin[a - (b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sq
rt[2/Pi]*Sqrt[c + d*x]]*(2*b*c*Cos[a - (b*c)/d] + 3*d*Sin[a - (b*c)/d]) + 2
```

```
*b*Sqrt[c + d*x]*(3*Cos[a + b*x] + 2*b*x*Sin[a + b*x])))/(16*b^3) + ((b/d)^
(3/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*((4*b^
2*c^2 - 15*d^2)*Cos[a - (b*c)/d] + 12*b*c*d*Sin[a - (b*c)/d])) - Sqrt[2*Pi]
*FresnelC[Sqrt[b/d]*Sqrt[2/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[a - (b*c)/d] +
 (4*b^2*c^2 - 15*d^2)*Sin[a - (b*c)/d]) + 2*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2*b
*(c - 5*d*x)*Cos[a + b*x] + d*(-15 + 4*b^2*x^2)*Sin[a + b*x])))/(64*b^5) -
(c^2*(-(Sqrt[2*Pi]*Cos[3*a - (3*b*c)/d]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[
c + d*x]]) - Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*Sin[3*
a - (3*b*c)/d] + 2*Sqrt[3]*Sqrt[b/d]*Sqrt[c + d*x]*Sin[3*(a + b*x)]))/(96*S
qrt[3]*b*Sqrt[b/d]) - (c*d*(Sqrt[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/
Pi]*Sqrt[c + d*x]]*(-(d*Cos[3*a - (3*b*c)/d]) + 2*b*c*Sin[3*a - (3*b*c)/d])
 + Sqrt[b/d]*Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(2*b*c
*Cos[3*a - (3*b*c)/d] + d*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*b*Sqrt[c + d*x]
*(Cos[3*(a + b*x)] + 2*b*x*Sin[3*(a + b*x)])))/(96*Sqrt[3]*b^3) - ((b/d)^(3
/2)*d^2*(-(Sqrt[2*Pi]*FresnelS[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*((12*b^2
*c^2 - 5*d^2)*Cos[3*a - (3*b*c)/d] + 12*b*c*d*Sin[3*a - (3*b*c)/d])) - Sqrt
[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[6/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[3*a - (3
*b*c)/d] + (12*b^2*c^2 - 5*d^2)*Sin[3*a - (3*b*c)/d]) + 2*Sqrt[3]*Sqrt[b/d]
*d*Sqrt[c + d*x]*(-2*b*(c - 5*d*x)*Cos[3*(a + b*x)] + d*(-5 + 12*b^2*x^2)*S
in[3*(a + b*x)])))/(1152*Sqrt[3]*b^5) - (c^2*(-(Sqrt[2*Pi]*Cos[5*a - (5*b*c
)/d]*FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]) - Sqrt[2*Pi]*FresnelC[S
qrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*Sin[5*a - (5*b*c)/d] + 2*Sqrt[5]*Sqrt[b
/d]*Sqrt[c + d*x]*Sin[5*(a + b*x)]))/(160*Sqrt[5]*b*Sqrt[b/d]) - (c*d*(Sqrt
[b/d]*Sqrt[2*Pi]*FresnelC[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-3*d*Cos[5*
a - (5*b*c)/d] + 10*b*c*Sin[5*a - (5*b*c)/d]) + Sqrt[b/d]*Sqrt[2*Pi]*Fresne
lS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(10*b*c*Cos[5*a - (5*b*c)/d] + 3*d*
Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*b*Sqrt[c + d*x]*(3*Cos[5*(a + b*x)] + 10*
b*x*Sin[5*(a + b*x)])))/(800*Sqrt[5]*b^3) - ((b/d)^(3/2)*d^2*(-(Sqrt[2*Pi]*
FresnelS[Sqrt[b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*((20*b^2*c^2 - 3*d^2)*Cos[5*a
 - (5*b*c)/d] + 12*b*c*d*Sin[5*a - (5*b*c)/d])) - Sqrt[2*Pi]*FresnelC[Sqrt[
b/d]*Sqrt[10/Pi]*Sqrt[c + d*x]]*(-12*b*c*d*Cos[5*a - (5*b*c)/d] + (20*b^2*c
^2 - 3*d^2)*Sin[5*a - (5*b*c)/d]) + 2*Sqrt[5]*Sqrt[b/d]*d*Sqrt[c + d*x]*(-2
*b*(c - 5*d*x)*Cos[5*(a + b*x)] + d*(-3 + 20*b^2*x^2)*Sin[5*(a + b*x)])))/(
3200*Sqrt[5]*b^5)
```

fricas [A] time = 0.57, size = 548, normalized size = 0.89

$$-\frac{81\sqrt{10}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{5\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{10}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right)+625\sqrt{6}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{3\,(bc-ad)}{d}\right)\mathrm{S}\left(\sqrt{6}\,\sqrt{dx+c}\,\sqrt{\frac{b}{\pi d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/432000*(81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*cos(-5*(b*c - a*d)/d)*fresnel_
sin(sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d))) + 625*sqrt(6)*pi*d^3*sqrt(b/(pi*
```

```
d))*cos(-3*(b*c - a*d)/d)*fresnel_sin(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*d)))
 - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*cos(-(b*c - a*d)/d)*fresnel_sin(sqr
t(2)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 101250*sqrt(2)*pi*d^3*sqrt(b/(pi*d))*f
resnel_cos(sqrt(2)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-(b*c - a*d)/d) + 625*
sqrt(6)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(sqrt(6)*sqrt(d*x + c)*sqrt(b/(pi*
d)))*sin(-3*(b*c - a*d)/d) + 81*sqrt(10)*pi*d^3*sqrt(b/(pi*d))*fresnel_cos(
sqrt(10)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-5*(b*c - a*d)/d) + 480*(90*(b^2
*d^2*x + b^2*c*d)*cos(b*x + a)^5 - 50*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3
- 300*(b^2*d^2*x + b^2*c*d)*cos(b*x + a) - (120*b^3*d^2*x^2 + 240*b^3*c*d*x
 + 120*b^3*c^2 - 9*(20*b^3*d^2*x^2 + 40*b^3*c*d*x + 20*b^3*c^2 - 3*b*d^2)*c
os(b*x + a)^4 - 428*b*d^2 + (60*b^3*d^2*x^2 + 120*b^3*c*d*x + 60*b^3*c^2 +
11*b*d^2)*cos(b*x + a)^2)*sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 12.74, size = 3677, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/864000*(1800*(3*sqrt(10)*sqrt(pi)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x
+ c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b
*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*
d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*sqrt
(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqr
t(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 30*sqrt(2)*sqrt(pi)*d*erf(-1/2*sqrt(2)*
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d
)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 5*sqrt(6)*sqrt(pi)*d*erf(-1/2*sq
rt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c +
3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 3*sqrt(10)*sqrt(pi)*d*
erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^(
(-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c^3 + 18*c*
d^2*(9*(sqrt(10)*sqrt(pi)*(100*b^2*c^2 + 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqr
t(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(10*I*(d*x + c)^(3
/2)*b*d - 20*I*sqrt(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((-5*I*(d*x + c
)*b + 5*I*b*c - 5*I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 +
4*I*b*c*d - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1
)*b^2) - 6*(2*I*(d*x + c)^(3/2)*b*d - 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x +
c)*d^2)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*d)/d)/b^2)/d^2 - 2250*(sqrt(
2)*sqrt(pi)*(4*b^2*c^2 + 4*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sq
rt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(
I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x
```

```
+ c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^
2)/d^2 - 2250*(sqrt(2)*sqrt(pi)*(4*b^2*c^2 - 4*I*b*c*d - 3*d^2)*d*erf(-1/2*
sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c +
I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) + 2*(2*I*(d*x + c)^(3/
2)*b*d - 4*I*sqrt(d*x + c)*b*c*d - 3*sqrt(d*x + c)*d^2)*e^((I*(d*x + c)*b -
 I*b*c + I*a*d)/d)/b^2)/d^2 + 125*(sqrt(6)*sqrt(pi)*(12*b^2*c^2 - 4*I*b*c*d
 - d^2)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2)
 - 6*(-2*I*(d*x + c)^(3/2)*b*d + 4*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)*d^
2)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^2)/d^2 + 9*(sqrt(10)*sqrt(
pi)*(100*b^2*c^2 - 20*I*b*c*d - 3*d^2)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d
*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)
*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 10*(-10*I*(d*x + c)^(3/2)*b*d + 20*I*sqr
t(d*x + c)*b*c*d + 3*sqrt(d*x + c)*d^2)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I
*a*d)/d)/b^2)/d^2) - d^3*(27*(sqrt(10)*sqrt(pi)*(200*b^3*c^3 + 60*I*b^2*c^2
*d - 18*b*c*d^2 - 3*I*d^3)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*b
*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b
^2*d^2) + 1)*b^3) + 10*(20*I*(d*x + c)^(5/2)*b^2*d - 60*I*(d*x + c)^(3/2)*b
^2*c*d + 60*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(
d*x + c)*b*c*d^2 - 3*I*sqrt(d*x + c)*d^3)*e^((-5*I*(d*x + c)*b + 5*I*b*c -
5*I*a*d)/d)/b^3)/d^3 + 125*(sqrt(6)*sqrt(pi)*(72*b^3*c^3 + 36*I*b^2*c^2*d -
 18*b*c*d^2 - 5*I*d^3)*d*erf(-1/2*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sq
rt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^
2) + 1)*b^3) + 6*(12*I*(d*x + c)^(5/2)*b^2*d - 36*I*(d*x + c)^(3/2)*b^2*c*d
 + 36*I*sqrt(d*x + c)*b^2*c^2*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x +
c)*b*c*d^2 - 5*I*sqrt(d*x + c)*d^3)*e^((-3*I*(d*x + c)*b + 3*I*b*c - 3*I*a*
d)/d)/b^3)/d^3 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 + 12*I*b^2*c^2*d - 18*b*
c*d^2 - 15*I*d^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b
^3) - 2*(-4*I*(d*x + c)^(5/2)*b^2*d + 12*I*(d*x + c)^(3/2)*b^2*c*d - 12*I*s
qrt(d*x + c)*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^
2 + 15*I*sqrt(d*x + c)*d^3)*e^((-I*(d*x + c)*b + I*b*c - I*a*d)/d)/b^3)/d^3
 - 6750*(sqrt(2)*sqrt(pi)*(8*b^3*c^3 - 12*I*b^2*c^2*d - 18*b*c*d^2 + 15*I*d
^3)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d
)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*(4*
I*(d*x + c)^(5/2)*b^2*d - 12*I*(d*x + c)^(3/2)*b^2*c*d + 12*I*sqrt(d*x + c)
*b^2*c^2*d - 10*(d*x + c)^(3/2)*b*d^2 + 18*sqrt(d*x + c)*b*c*d^2 - 15*I*sqr
t(d*x + c)*d^3)*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b^3)/d^3 + 125*(sqrt(
6)*sqrt(pi)*(72*b^3*c^3 - 36*I*b^2*c^2*d - 18*b*c*d^2 + 5*I*d^3)*d*erf(-1/2
*sqrt(6)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c
 + 3*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 6*(-12*I*(d*x +
 c)^(5/2)*b^2*d + 36*I*(d*x + c)^(3/2)*b^2*c*d - 36*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c
)*d^3)*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b^3)/d^3 + 27*(sqrt(10)*
sqrt(pi)*(200*b^3*c^3 - 60*I*b^2*c^2*d - 18*b*c*d^2 + 3*I*d^3)*d*erf(-1/2*s
```

```
qrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-5*I*b*c
+ 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) + 10*(-20*I*(d*x +
 c)^(5/2)*b^2*d + 60*I*(d*x + c)^(3/2)*b^2*c*d - 60*I*sqrt(d*x + c)*b^2*c^2
*d + 10*(d*x + c)^(3/2)*b*d^2 - 18*sqrt(d*x + c)*b*c*d^2 + 3*I*sqrt(d*x + c
)*d^3)*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b^3)/d^3) - 180*(9*sqrt(
10)*sqrt(pi)*(10*b*c + I*d)*d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((5*I*b*c - 5*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c + I*d)*d*erf(-1/2*sqrt(6)*sqr
t(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((3*I*b*c - 3*I*a*d)/d)
/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)*sqrt(pi)*(2*b*c + I*
d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*
e^((I*b*c - I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - 450*sqrt(2)
*sqrt(pi)*(2*b*c - I*d)*d*erf(-1/2*sqrt(2)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/
sqrt(b^2*d^2) + 1)/d)*e^((-I*b*c + I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^
2) + 1)*b) + 25*sqrt(6)*sqrt(pi)*(6*b*c - I*d)*d*erf(-1/2*sqrt(6)*sqrt(b*d)
*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-3*I*b*c + 3*I*a*d)/d)/(sq
rt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*sqrt(10)*sqrt(pi)*(10*b*c - I*d)*
d*erf(-1/2*sqrt(10)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-5*I*b*c + 5*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 90*I*s
qrt(d*x + c)*d*e^((5*I*(d*x + c)*b - 5*I*b*c + 5*I*a*d)/d)/b - 150*I*sqrt(d
*x + c)*d*e^((3*I*(d*x + c)*b - 3*I*b*c + 3*I*a*d)/d)/b + 900*I*sqrt(d*x +
c)*d*e^((I*(d*x + c)*b - I*b*c + I*a*d)/d)/b - 900*I*sqrt(d*x + c)*d*e^((-I
*(d*x + c)*b + I*b*c - I*a*d)/d)/b + 150*I*sqrt(d*x + c)*d*e^((-3*I*(d*x +
c)*b + 3*I*b*c - 3*I*a*d)/d)/b + 90*I*sqrt(d*x + c)*d*e^((-5*I*(d*x + c)*b
+ 5*I*b*c - 5*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 716, normalized size = 1.16

$$\frac{\frac{d(dx+c)^{\frac{5}{2}}\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{8b}-\frac{5d\left(-\frac{d(dx+c)^{\frac{3}{2}}\cos\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}+\frac{3d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{(dx+c)b}{d}+\frac{da-cb}{d}\right)}{2b}-\frac{d\sqrt{2}\,\sqrt{\pi}\left(\cos\left(\frac{da-cb}{d}\right)S\left(\frac{\sqrt{2}\,\sqrt{dx+c}\,b}{\sqrt{\pi}\,\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{da-cb}{d}\right)\mathrm{Fresne}\right.}{4b\sqrt{\frac{b}{d}}}\right.}{2b}\right.}{8b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x)
```

```
[Out] 2/d*(1/16/b*d*(d*x+c)^(5/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-5/16/b*d*(-1/2/b
*d*(d*x+c)^(3/2)*cos(1/d*(d*x+c)*b+(a*d-b*c)/d)+3/2/b*d*(1/2/b*d*(d*x+c)^(1
/2)*sin(1/d*(d*x+c)*b+(a*d-b*c)/d)-1/4/b*d*2^(1/2)*Pi^(1/2)/(b/d)^(1/2)*(co
s((a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin
((a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))-1
/96/b*d*(d*x+c)^(5/2)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+5/96/b*d*(-1/6/b*d*(
d*x+c)^(3/2)*cos(3/d*(d*x+c)*b+3*(a*d-b*c)/d)+1/2/b*d*(1/6/b*d*(d*x+c)^(1/2
)*sin(3/d*(d*x+c)*b+3*(a*d-b*c)/d)-1/36/b*d*2^(1/2)*Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(cos(3*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^(1/2)*(d*
x+c)^(1/2)*b/d)+sin(3*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)/(b/d)^
(1/2)*(d*x+c)^(1/2)*b/d))))-1/160/b*d*(d*x+c)^(5/2)*sin(5/d*(d*x+c)*b+5*(a*
d-b*c)/d)+1/32/b*d*(-1/10/b*d*(d*x+c)^(3/2)*cos(5/d*(d*x+c)*b+5*(a*d-b*c)/d
)+3/10/b*d*(1/10/b*d*(d*x+c)^(1/2)*sin(5/d*(d*x+c)*b+5*(a*d-b*c)/d)-1/100/b
*d*2^(1/2)*Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(cos(5*(a*d-b*c)/d)*FresnelS(2^(1/2
)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(5*(a*d-b*c)/d)*Fresne
lC(2^(1/2)/Pi^(1/2)*5^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.58, size = 820, normalized size = 1.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/3456000*sqrt(2)*(10800*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(5*((d*x + c)*b -
b*c + a*d)/d)/d + 30000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(3*((d*x + c)*b - b*
c + a*d)/d)/d - 540000*sqrt(2)*(d*x + c)^(3/2)*b^4*cos(((d*x + c)*b - b*c +
 a*d)/d)/d + ((162*I + 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*
(b*c - a*d)/d) - (162*I - 162)*25^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(
-5*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(5*I*b/d)) + ((1250*I + 1250)*9^(1
/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c - a*d)/d) - (1250*I - 1250)*
9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*(b*c - a*d)/d))*erf(sqrt(d*x
+ c)*sqrt(3*I*b/d)) + (-(202500*I + 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*
cos(-(b*c - a*d)/d) + (202500*I - 202500)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*si
n(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(I*b/d)) + ((202500*I - 202500)*sq
rt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-(b*c - a*d)/d) - (202500*I + 202500)*sqrt
(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-I*b
/d)) + (-(1250*I - 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-3*(b*c
 - a*d)/d) + (1250*I + 1250)*9^(1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-3*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-3*I*b/d)) + (-(162*I - 162)*25^(1/4
)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*cos(-5*(b*c - a*d)/d) + (162*I + 162)*25^(
1/4)*sqrt(pi)*b^2*d*(b^2/d^2)^(1/4)*sin(-5*(b*c - a*d)/d))*erf(sqrt(d*x + c
)*sqrt(-5*I*b/d)) + 1080*(20*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 3*sqrt(2)*sq
rt(d*x + c)*b^3)*sin(5*((d*x + c)*b - b*c + a*d)/d) + 3000*(12*sqrt(2)*(d*x
 + c)^(5/2)*b^5/d^2 - 5*sqrt(2)*sqrt(d*x + c)*b^3)*sin(3*((d*x + c)*b - b*c
```

```
+ a*d)/d) - 54000*(4*sqrt(2)*(d*x + c)^(5/2)*b^5/d^2 - 15*sqrt(2)*sqrt(d*x
+ c)*b^3)*sin(((d*x + c)*b - b*c + a*d)/d))*d^2/b^6
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.196 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)S\left(\right.}{18432b^{7/2}}$$

```
[Out] -3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+1
5/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x+6
*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*FresnelS(
2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*
Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+45/102
4*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(d*x+c)^(1
/2)/b^3
```

Rubi [A] time = 0.90, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\,\text{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{5/2}\sin(2a+2$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} - \frac{(5d)\int(c}{}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{15d(c+}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c}}{}$$

Mathematica [A] time = 5.07, size = 550, normalized size = 1.35

$$\frac{-2592b^3c^2\sqrt{c+dx}\cos(2(a+bx)) + 288b^3c^2\sqrt{c+dx}\cos(6(a+bx)) - 2592b^3d^2x^2\sqrt{c+dx}\cos(2(a+bx)) + 28}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

fricas [A] time = 0.59, size = 445, normalized size = 1.09

$$\frac{5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 13.77, size = 2417, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/110592*(576*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 + 36*c*d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d
*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*
b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*
```

```
x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^
2 - 8*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*s
qrt(pi)*(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*
c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2
)/d^2 + 9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*
I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2
*c^2*d - 36*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)
*b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d*x + c)*b + 6*I*b*c
 - 6*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2
*d - 36*b*c*d^2 + 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*
b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqr
t(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c +
 6*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(pi)*(192*b^3*c^3 + 144*I*b^2*c^2*d - 10
8*b*c*d^2 - 45*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 2*I*(48*I*(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*s
qrt(d*x + c)*b^2*c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 108*sqrt(d*x + c)*b*c*d
^2 - 45*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 27*(I*sqrt(pi)*(192*b^3*c^3 - 144*I*b^2*c^2*d - 108*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*
I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*
(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*sqrt(d*x + c)
*b^2*c^2*d - 60*(d*x + c)^(3/2)*b*d^2 + 108*sqrt(d*x + c)*b*c*d^2 - 45*I*sq
rt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 14
4*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi)*(12*b*c + 3*I*d)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c -
2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*b*c
```

```
- 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt(d*x
 + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*
e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((-2*I
*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*x +
c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.05, size = 477, normalized size = 1.17

$$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.53, size = 557, normalized size = 1.37

$$-\frac{\left(1920\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)-51840\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-96\left(\frac{48\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-5\sqrt{dx+c}\,b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/884736*(1920*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 51
840*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 96*(48*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d)
+ 2592*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) + ((10*I - 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-6*(b*c - a*d)/d) + (10*I + 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (
-(2430*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c
 - a*d)/d) - (2430*I + 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((2430*I + 2430)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (24
30*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(10*I + 10)*36^(1/4)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (10*I - 10)*36^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-6*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.197 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\sqrt{\ldots}}{\ldots}\right)}{1536b^{5/2}}$$

```
[Out] -3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+1
/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2)*3
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2)/b^
(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*b*x+2*a
)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.63, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a\ldots\right)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right) dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2a+2b$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int\sqrt{c+}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\ c}{}$$

Mathematica [A] time = 2.83, size = 391, normalized size = 1.11

$$\frac{\sqrt{3\pi}\,d\sin\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 81\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\,d\cos\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

fricas [A] time = 0.56, size = 326, normalized size = 0.93

$$\frac{\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-81}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

giac [C] time = 12.68, size = 1502, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9216*(48*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d*erf(-sq
rt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(
3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d*x + c)*
b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*
b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)
*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(pi)*
(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
```

```
t(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*c*d + 9*
sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d^2 +
9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d
*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*
d)/d)/b^2)/d^2) + 8*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*
d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6
*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi
)*(12*b*c + 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I
*sqrt(pi)*(12*b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) - 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 5
4*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*
d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.04, size = 383, normalized size = 1.09

$$\frac{-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)S\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{64b}+\frac{d(dx}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.51, size = 513, normalized size = 1.46

$$\left(\frac{384\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{3456\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)}{d}-96\,\sqrt{dx+c}\,b^2\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)+2592\,\sqrt{dx+\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*(384*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 345
6*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 2592*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) - (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
((162*I + 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) - (162*I - 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (-(162*I - 162)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (162*I + 162)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)) - ((2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d
/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.198 $\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}\cdots}{\cdots}\right)}{384b^{3/2}}$$

```
[Out] -1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/2)/
b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.46, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(\cdots\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\sin(2a+2bx)\,d$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6a+6bx)}{\sqrt{c+dx}}}{384b}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\left(d\cos\left(6a-\frac{6}{}\right.\right.}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\cos\left(6a-\frac{6bc}{d}\right)}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos(6}{}$$

Mathematica [A] time = 1.25, size = 264, normalized size = 0.88

$$\frac{-\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) + 27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\sin\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{115}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

fricas [A] time = 0.51, size = 242, normalized size = 0.81

$$-\frac{\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)C\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,S\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right) - 27\,\pi}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

giac [C] time = 7.83, size = 818, normalized size = 2.74

$$-\frac{\frac{i\sqrt{3}\sqrt{\pi}(12bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{6ibc-6iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{3}\sqrt{\pi}(12bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-6ibc+6iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+12\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2304*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 12*(-I*sqrt(3)*sqrt(pi)
*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*
I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(
pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(
pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c
 - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 9*I*sqrt(pi)*(12*b*c + 3*
I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*
c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*
b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt
(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c
)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((
-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*
x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d
```

maple [A] time = 0.04, size = 293, normalized size = 0.98

$$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{6(dx+c)b}{d}+\frac{6da-6cb}{d}\right)}{192b}-}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.52, size = 435, normalized size = 1.45

$$\left(\frac{96\sqrt{dx+c}\,b^2\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{864\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left((2i-2)\cdot 36^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{6(bc-ad)}{d}\right)+(2i+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18432*(96*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 864*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((2*I - 2)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I + 2)*36^(1/
4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) + (-(54*I - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) - (54*I + 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((54*I + 5
4)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (54*I
 - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(
b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))
*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.199 $\int \sqrt{c+dx}\,\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=299

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{128b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2\sqrt{b}}{\cdots}\right.}{384b^{3/2}}$$

```
[Out] -1/1152*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/
d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+1/1152*FresnelS(2*b^(1/2)*3^(1/2)
/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*d^(1/2)*3^(1/2)*Pi^(1/2)/
b^(3/2)+3/128*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/2)/Pi^
(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/128*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*d^(1/2)*Pi^(1/2)/b^(3/2)-3/64*cos(2*b*x+2*a)
*(d*x+c)^(1/2)/b+1/192*cos(6*b*x+6*a)*(d*x+c)^(1/2)/b
```

Rubi [A] time = 0.45, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{384b^{3/2}}+\frac{3\sqrt{\pi}\sqrt{d}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{128b^{3/2}}+\sqrt{\frac{\pi}{3}}\sqrt{d}\sin\left(\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(64*b) + (Sqrt[c + d*x]*Cos[6*a + 6*b*x
])/(192*b) - (Sqrt[d]*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*S
qrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(384*b^(3/2)) + (3*Sqrt[d]*Sqrt[Pi]*Cos[
2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(1
28*b^(3/2)) + (Sqrt[d]*Sqrt[Pi/3]*FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d
*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(384*b^(3/2)) - (3*Sqrt[d]*Sqrt[Pi]*Fre
snelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(
128*b^(3/2))
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int \sqrt{c+dx}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}\sqrt{c+dx}\sin(2a+2bx) - \frac{1}{32}\sqrt{c+dx}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int \sqrt{c+dx}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int \sqrt{c+dx}\sin(2a+2bx)\,d$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{d\int \frac{\cos(6a+6bx)}{\sqrt{c+dx}}}{384b}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\left(d\cos\left(6a-\frac{6}{d}\right.\right.}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\cos\left(6a-\frac{6bc}{d}\right)}{}$$

$$= -\frac{3\sqrt{c+dx}\cos(2a+2bx)}{64b} + \frac{\sqrt{c+dx}\cos(6a+6bx)}{192b} - \frac{\sqrt{d}\sqrt{\frac{\pi}{3}}\cos\left(6\right.}{}$$

Mathematica [A] time = 0.52, size = 264, normalized size = 0.88

$$\frac{-\sqrt{3\pi}\cos\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) + 27\sqrt{\pi}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\sin\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{\frac{b}{d}}\right.}{115}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-54*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 6*Sqrt[b/d]*Sqrt[c + d*x]*C
os[6*(a + b*x)] - Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt
[3/Pi]*Sqrt[c + d*x]] + 27*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b
/d]*Sqrt[c + d*x])/Sqrt[Pi]] + Sqrt[3*Pi]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*S
qrt[c + d*x]]*Sin[6*a - (6*b*c)/d] - 27*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt
[c + d*x])/Sqrt[Pi]]*Sin[2*a - (2*b*c)/d])/(1152*b*Sqrt[b/d])
```

fricas [A] time = 0.56, size = 242, normalized size = 0.81

$$-\frac{\sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right) - \sqrt{3}\,\pi d\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right) - 27\,\pi a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/1152*(sqrt(3)*pi*d*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(2*sq
rt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - sqrt(3)*pi*d*sqrt(b/(pi*d))*fresnel_s
in(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 27*pi*d*
sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(pi
*d))) + 27*pi*d*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*sqrt(b/(pi*d)))*
sin(-2*(b*c - a*d)/d) - 48*(4*b*cos(b*x + a)^6 - 6*b*cos(b*x + a)^4 + b)*sq
rt(d*x + c))/b^2
```

giac [C] time = 3.74, size = 818, normalized size = 2.74

$$-\frac{\frac{i\sqrt{3}\sqrt{\pi}(12bc+id)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{6ibc-6iad}{d}\right)}}{\sqrt{bd}\left(\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}-\frac{i\sqrt{3}\sqrt{\pi}(12bc-id)d\,\mathrm{erf}\left(-\frac{\sqrt{3}\sqrt{bd}\sqrt{dx+c}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)}{d}\right)e^{\left(\frac{-6ibc+6iad}{d}\right)}}{\sqrt{bd}\left(-\frac{ibd}{\sqrt{b^2d^2}}+1\right)b}+12\left(-\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/2304*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*
x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I
*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(
3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I
*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) + 12*(-I*sqrt(3)*sqrt(pi)
*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*
I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(
pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^
((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(
pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c
 - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-
sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*
d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)))*c - 9*I*sqrt(pi)*(12*b*c + 3*
I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*
c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*
b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e
^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt
(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c
)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((
-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*
x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)/d
```

maple [A] time = 0.00, size = 293, normalized size = 0.98

$$\frac{-\frac{3d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{3d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)-\sin\left(\frac{2da-2cb}{d}\right)\mathrm{S}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{128b\sqrt{\frac{b}{d}}}+\frac{d\sqrt{dx+c}\,\cos\left(\frac{6(dx+c)b}{d}+\frac{6da-6cb}{d}\right)}{192b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+3/256/b*d*Pi
^(1/2)/(b/d)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(
1/2)*b/d))+1/384/b*d*(d*x+c)^(1/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4608/
b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/
2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*Fresn
elS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.51, size = 435, normalized size = 1.45

$$\left(\frac{96\sqrt{dx+c}\,b^2\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{864\sqrt{dx+c}\,b^2\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}+\left((2i-2)\cdot 36^{\frac{1}{4}}\sqrt{2}\sqrt{\pi}\,b\left(\frac{b^2}{d^2}\right)^{\frac{1}{4}}\cos\left(-\frac{6(bc-ad)}{d}\right)+(2i+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(1/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18432*(96*sqrt(d*x + c)*b^2*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 864*sq
rt(d*x + c)*b^2*cos(2*((d*x + c)*b - b*c + a*d)/d)/d + ((2*I - 2)*36^(1/4)*
sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I + 2)*36^(1/
4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x +
 c)*sqrt(6*I*b/d)) + (-(54*I - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/
4)*cos(-2*(b*c - a*d)/d) - (54*I + 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)
^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((54*I + 5
4)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (54*I
 - 54)*4^(1/4)*sqrt(2)*sqrt(pi)*b*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*er
f(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*(
b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*
b*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))
*d/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\,\sin(a+b\,x)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(1/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.200 $\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=351

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{\pi}}\right)}{512b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{b}\ \ldots\right.}{1536b^{5/2}}$$

```
[Out] -3/64*(d*x+c)^(3/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(3/2)*cos(6*b*x+6*a)/b+1
/4608*d^(3/2)*cos(6*a-6*b*c/d)*FresnelS(2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/4608*d^(3/2)*FresnelC(2*b^(1/2)*3
^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*Pi^(1/2)/b^
(5/2)-9/512*d^(3/2)*cos(2*a-2*b*c/d)*FresnelS(2*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2)/Pi^(1/2))*Pi^(1/2)/b^(5/2)-9/512*d^(3/2)*FresnelC(2*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(5/2)+9/256*d*sin(2*b*x+2*a
)*(d*x+c)^(1/2)/b^2-1/768*d*sin(6*b*x+6*a)*(d*x+c)^(1/2)/b^2
```

Rubi [A] time = 0.56, antiderivative size = 351, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}\sin\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{1536b^{5/2}}-\frac{9\sqrt{\pi}\,d^{3/2}\sin\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\sqrt{c+dx}}{\sqrt{\pi}\sqrt{d}}\right)}{512b^{5/2}}+\sqrt{\frac{\pi}{3}}\,d^{3/2}\cos\left(6a\ \ldots\right.$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-3*(c + d*x)^(3/2)*Cos[2*a + 2*b*x])/(64*b) + ((c + d*x)^(3/2)*Cos[6*a + 6
*b*x])/(192*b) + (d^(3/2)*Sqrt[Pi/3]*Cos[6*a - (6*b*c)/d]*FresnelS[(2*Sqrt[
b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[Pi]
*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])
])/(512*b^(5/2)) + (d^(3/2)*Sqrt[Pi/3]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[
c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d])/(1536*b^(5/2)) - (9*d^(3/2)*Sqrt[P
i]*FresnelC[(2*Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)
/d])/(512*b^(5/2)) + (9*d*Sqrt[c + d*x]*Sin[2*a + 2*b*x])/(256*b^2) - (d*Sq
rt[c + d*x]*Sin[6*a + 6*b*x])/(768*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{3/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{3/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{3/2}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{3/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{3/2}\sin(2a+2b$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} - \frac{d\int\sqrt{c+}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{9d\sqrt{c+d}}{}$$

$$= -\frac{3(c+dx)^{3/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{3/2}\cos(6a+6bx)}{192b} + \frac{d^{3/2}\sqrt{\frac{\pi}{3}}\,c}{}$$

Mathematica [A] time = 0.20, size = 391, normalized size = 1.11

$$\frac{\sqrt{3\pi}\,d\sin\left(6a-\frac{6bc}{d}\right)C\left(2\sqrt{\frac{b}{d}}\sqrt{\frac{3}{\pi}}\sqrt{c+dx}\right) - 81\sqrt{\pi}\,d\sin\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{\frac{b}{d}}\sqrt{c+dx}}{\sqrt{\pi}}\right) + \sqrt{3\pi}\,d\cos\left(6a-\frac{6bc}{d}\right)S\left(2\sqrt{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-216*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 216*b*Sqrt[b/d]*d*x*Sq
rt[c + d*x]*Cos[2*(a + b*x)] + 24*b*c*Sqrt[b/d]*Sqrt[c + d*x]*Cos[6*(a + b*
x)] + 24*b*Sqrt[b/d]*d*x*Sqrt[c + d*x]*Cos[6*(a + b*x)] + d*Sqrt[3*Pi]*Cos[
6*a - (6*b*c)/d]*FresnelS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]] - 81*d*Sqrt
[Pi]*Cos[2*a - (2*b*c)/d]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] +
d*Sqrt[3*Pi]*FresnelC[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*
c)/d] - 81*d*Sqrt[Pi]*FresnelC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*
a - (2*b*c)/d] + 162*Sqrt[b/d]*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] - 6*Sqrt[b/
d]*d*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(4608*b^2*Sqrt[b/d])
```

fricas [A] time = 0.58, size = 326, normalized size = 0.93

$$\frac{\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\cos\left(-\frac{6\,(bc-ad)}{d}\right)\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)+\sqrt{3}\,\pi d^2\sqrt{\frac{b}{\pi d}}\,\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6\,(bc-ad)}{d}\right)-81}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4608*(sqrt(3)*pi*d^2*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_sin(2*s
qrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) + sqrt(3)*pi*d^2*sqrt(b/(pi*d))*fresne
l_cos(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) - 81*pi
*d^2*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_sin(2*sqrt(d*x + c)*sqrt(
b/(pi*d))) - 81*pi*d^2*sqrt(b/(pi*d))*fresnel_cos(2*sqrt(d*x + c)*sqrt(b/(p
i*d)))*sin(-2*(b*c - a*d)/d) + 96*(8*(b^2*d*x + b^2*c)*cos(b*x + a)^6 - 12*
(b^2*d*x + b^2*c)*cos(b*x + a)^4 + 2*b^2*d*x + 2*b^2*c - (2*b*d*cos(b*x + a
)^5 - 2*b*d*cos(b*x + a)^3 - 3*b*d*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c
))/b^3
```

giac [C] time = 13.12, size = 1502, normalized size = 4.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/9216*(48*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*
b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(
b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d
/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d
^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^
2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)
))*c^2 + d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d*erf(-sq
rt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*
I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*x + c)^(
3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d*x + c)*
b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 - 8*I*
b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2
) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(d*x + c)
*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*sqrt(pi)*
(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqr
```

```
t(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2
) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*c*d + 9*
sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2)/d^2 +
9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x
+ c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-
I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d
*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*
d)/d)/b^2)/d^2) + 8*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(
b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(
sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*
d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6
*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi
)*(12*b*c + 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)
/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I
*sqrt(pi)*(12*b*c - 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*
d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
1)*b) - 6*sqrt(d*x + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 5
4*sqrt(d*x + c)*d*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d
*x + c)*d*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*
d*e^((-6*I*(d*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c)/d
```

maple [A] time = 0.00, size = 383, normalized size = 1.09

$$\frac{-\frac{3d(dx+c)^{\frac{3}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{9d\left(\frac{d\sqrt{dx+c}\,\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)S\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)+\sin\left(\frac{2da-2cb}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{dx+c}\,b}{\sqrt{\pi}\sqrt{\frac{b}{d}}\,d}\right)\right)}{8b\sqrt{\frac{b}{d}}}\right)}{64b}+\frac{d(dx}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(3/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+9/128/b*d*(1
/4/b*d*(d*x+c)^(1/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-1/8/b*d*Pi^(1/2)/(b/d
)^(1/2)*(cos(2*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b
/d)+sin(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
+1/384/b*d*(d*x+c)^(3/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/128/b*d*(1/12/b
*d*(d*x+c)^(1/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/144/b*d*2^(1/2)*Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(cos(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)
/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)+sin(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2
)*6^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)))
```

maxima [C] time = 0.54, size = 513, normalized size = 1.46

$$\left(\frac{384\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{6((dx+c)b-bc+ad)}{d}\right)}{d}-\frac{3456\,(dx+c)^{\frac{3}{2}}b^3\cos\left(\frac{2((dx+c)b-bc+ad)}{d}\right)}{d}-96\sqrt{dx+c}\,b^2\sin\left(\frac{6((dx+c)b-bc+ad)}{d}\right)+2592\sqrt{dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/73728*(384*(d*x + c)^(3/2)*b^3*cos(6*((d*x + c)*b - b*c + a*d)/d)/d - 345
6*(d*x + c)^(3/2)*b^3*cos(2*((d*x + c)*b - b*c + a*d)/d)/d - 96*sqrt(d*x +
c)*b^2*sin(6*((d*x + c)*b - b*c + a*d)/d) + 2592*sqrt(d*x + c)*b^2*sin(2*((
d*x + c)*b - b*c + a*d)/d) - (-(2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2
/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) + (2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d
*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) -
((162*I + 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a
*d)/d) - (162*I - 162)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*
(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) - (-(162*I - 162)*4^(1/4)*
sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (162*I + 162)*
4^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/d^2)^(1/4)*sin(-2*(b*c - a*d)/d))*erf(sqr
t(d*x + c)*sqrt(-2*I*b/d)) - ((2*I - 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*(b^2/
d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (2*I + 2)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-6*I*b/d)))*d
/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(a+b\,x)^3\sin(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.201 $\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx$

Optimal. Leaf size=407

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{\frac{3}{\pi}}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)C\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}\,\sqrt{\pi}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin\left(6a-\frac{6bc}{d}\right)S\left(\frac{2}{}\right.}{18432b^{7/2}}$$

```
[Out] -3/64*(d*x+c)^(5/2)*cos(2*b*x+2*a)/b+1/192*(d*x+c)^(5/2)*cos(6*b*x+6*a)/b+1
5/256*d*(d*x+c)^(3/2)*sin(2*b*x+2*a)/b^2-5/2304*d*(d*x+c)^(3/2)*sin(6*b*x+6
*a)/b^2+5/55296*d^(5/2)*cos(6*a-6*b*c/d)*FresnelC(2*b^(1/2)*3^(1/2)/Pi^(1/2
)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/55296*d^(5/2)*FresnelS(
2*b^(1/2)*3^(1/2)/Pi^(1/2)*(d*x+c)^(1/2)/d^(1/2))*sin(6*a-6*b*c/d)*3^(1/2)*
Pi^(1/2)/b^(7/2)-45/2048*d^(5/2)*cos(2*a-2*b*c/d)*FresnelC(2*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2)/Pi^(1/2))*Pi^(1/2)/b^(7/2)+45/2048*d^(5/2)*FresnelS(2*b^(1/
2)*(d*x+c)^(1/2)/d^(1/2)/Pi^(1/2))*sin(2*a-2*b*c/d)*Pi^(1/2)/b^(7/2)+45/102
4*d^2*cos(2*b*x+2*a)*(d*x+c)^(1/2)/b^3-5/9216*d^2*cos(6*b*x+6*a)*(d*x+c)^(1
/2)/b^3
```

Rubi [A] time = 0.67, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4406, 3296, 3306, 3305, 3351, 3304, 3352}

$$\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\cos\left(6a-\frac{6bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{\frac{3}{\pi}}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{18432b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}\cos\left(2a-\frac{2bc}{d}\right)\text{FresnelC}\left(\frac{2\sqrt{b}\,\sqrt{c+dx}}{\sqrt{\pi}\,\sqrt{d}}\right)}{2048b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (45*d^2*Sqrt[c + d*x]*Cos[2*a + 2*b*x])/(1024*b^3) - (3*(c + d*x)^(5/2)*Cos
[2*a + 2*b*x])/(64*b) - (5*d^2*Sqrt[c + d*x]*Cos[6*a + 6*b*x])/(9216*b^3) +
 ((c + d*x)^(5/2)*Cos[6*a + 6*b*x])/(192*b) + (5*d^(5/2)*Sqrt[Pi/3]*Cos[6*a
 - (6*b*c)/d]*FresnelC[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]])/(1843
2*b^(7/2)) - (45*d^(5/2)*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*FresnelC[(2*Sqrt[b]*
Sqrt[c + d*x])/(Sqrt[d]*Sqrt[Pi])])/(2048*b^(7/2)) - (5*d^(5/2)*Sqrt[Pi/3]*
FresnelS[(2*Sqrt[b]*Sqrt[3/Pi]*Sqrt[c + d*x])/Sqrt[d]]*Sin[6*a - (6*b*c)/d]
)/(18432*b^(7/2)) + (45*d^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[Pi])]*Sin[2*a - (2*b*c)/d])/(2048*b^(7/2)) + (15*d*(c + d*x)
^(3/2)*Sin[2*a + 2*b*x])/(256*b^2) - (5*d*(c + d*x)^(3/2)*Sin[6*a + 6*b*x])
/(2304*b^2)
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3304

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3305

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[(f*x^2)/d], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3306

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[(c*f)/d + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3351

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 3352

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]*Fresne
lC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)])/(f*Rt[d, 2]), x] /; FreeQ[{d, e, f}, x]
```

Rule 4406

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rubi steps

$$\int (c+dx)^{5/2}\cos^3(a+bx)\sin^3(a+bx)\,dx = \int \left(\frac{3}{32}(c+dx)^{5/2}\sin(2a+2bx) - \frac{1}{32}(c+dx)^{5/2}\sin(6a+6bx)\right)dx$$

$$= -\left(\frac{1}{32}\int (c+dx)^{5/2}\sin(6a+6bx)\,dx\right) + \frac{3}{32}\int (c+dx)^{5/2}\sin(2a+2b$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} - \frac{(5d)\int (c\ }{}$$

$$= -\frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} + \frac{(c+dx)^{5/2}\cos(6a+6bx)}{192b} + \frac{15d(c+}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+}}{}$$

$$= \frac{45d^2\sqrt{c+dx}\,\cos(2a+2bx)}{1024b^3} - \frac{3(c+dx)^{5/2}\cos(2a+2bx)}{64b} - \frac{5d^2\sqrt{c+}}{}$$

<u>**Mathematica**</u> [A] time = 2.74, size = 550, normalized size = 1.35

$$\frac{-2592b^3c^2\sqrt{c+dx}\,\cos(2(a+bx)) + 288b^3c^2\sqrt{c+dx}\,\cos(6(a+bx)) - 2592b^3d^2x^2\sqrt{c+dx}\,\cos(2(a+bx)) + 288}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cos[a + b*x]^3*Sin[a + b*x]^3,x]
```

```
[Out] (-2592*b^3*c^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 2430*b*d^2*Sqrt[c + d*x]*Co
s[2*(a + b*x)] - 5184*b^3*c*d*x*Sqrt[c + d*x]*Cos[2*(a + b*x)] - 2592*b^3*d
^2*x^2*Sqrt[c + d*x]*Cos[2*(a + b*x)] + 288*b^3*c^2*Sqrt[c + d*x]*Cos[6*(a
+ b*x)] - 30*b*d^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 576*b^3*c*d*x*Sqrt[c +
d*x]*Cos[6*(a + b*x)] + 288*b^3*d^2*x^2*Sqrt[c + d*x]*Cos[6*(a + b*x)] + 5*
Sqrt[b/d]*d^3*Sqrt[3*Pi]*Cos[6*a - (6*b*c)/d]*FresnelC[2*Sqrt[b/d]*Sqrt[3/P
i]*Sqrt[c + d*x]] - 1215*Sqrt[b/d]*d^3*Sqrt[Pi]*Cos[2*a - (2*b*c)/d]*Fresne
lC[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]] - 5*Sqrt[b/d]*d^3*Sqrt[3*Pi]*Fresn
```

```
elS[2*Sqrt[b/d]*Sqrt[3/Pi]*Sqrt[c + d*x]]*Sin[6*a - (6*b*c)/d] + 1215*Sqrt[
b/d]*d^3*Sqrt[Pi]*FresnelS[(2*Sqrt[b/d]*Sqrt[c + d*x])/Sqrt[Pi]]*Sin[2*a -
(2*b*c)/d] + 3240*b^2*c*d*Sqrt[c + d*x]*Sin[2*(a + b*x)] + 3240*b^2*d^2*x*S
qrt[c + d*x]*Sin[2*(a + b*x)] - 120*b^2*c*d*Sqrt[c + d*x]*Sin[6*(a + b*x)]
- 120*b^2*d^2*x*Sqrt[c + d*x]*Sin[6*(a + b*x)])/(55296*b^4)
```

fricas [A] time = 0.61, size = 445, normalized size = 1.09

$$5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\cos\left(-\frac{6(bc-ad)}{d}\right)\mathrm{C}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)-5\sqrt{3}\,\pi d^3\sqrt{\frac{b}{\pi d}}\,\mathrm{S}\left(2\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{\pi d}}\right)\sin\left(-\frac{6(bc-ad)}{d}\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/55296*(5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*cos(-6*(b*c - a*d)/d)*fresnel_cos(
2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d))) - 5*sqrt(3)*pi*d^3*sqrt(b/(pi*d))*f
resnel_sin(2*sqrt(3)*sqrt(d*x + c)*sqrt(b/(pi*d)))*sin(-6*(b*c - a*d)/d) -
1215*pi*d^3*sqrt(b/(pi*d))*cos(-2*(b*c - a*d)/d)*fresnel_cos(2*sqrt(d*x + c
)*sqrt(b/(pi*d))) + 1215*pi*d^3*sqrt(b/(pi*d))*fresnel_sin(2*sqrt(d*x + c)*
sqrt(b/(pi*d)))*sin(-2*(b*c - a*d)/d) + 96*(24*b^3*d^2*x^2 + 2*(48*b^3*d^2*
x^2 + 96*b^3*c*d*x + 48*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^6 + 48*b^3*c*d*x +
24*b^3*c^2 + 45*b*d^2*cos(b*x + a)^2 - 3*(48*b^3*d^2*x^2 + 96*b^3*c*d*x + 4
8*b^3*c^2 - 5*b*d^2)*cos(b*x + a)^4 - 25*b*d^2 - 20*(2*(b^2*d^2*x + b^2*c*d
)*cos(b*x + a)^5 - 2*(b^2*d^2*x + b^2*c*d)*cos(b*x + a)^3 - 3*(b^2*d^2*x +
b^2*c*d)*cos(b*x + a))*sin(b*x + a))*sqrt(d*x + c))/b^4
```

giac [C] time = 16.88, size = 2417, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] -1/110592*(576*(-I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*
(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sq
rt(b^2*d^2) + 1)) + I*sqrt(3)*sqrt(pi)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x +
c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*
b*d/sqrt(b^2*d^2) + 1)) + 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)) - 9*I*sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2
*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) +
 1)))*c^3 + 36*c*d^2*((-I*sqrt(3)*sqrt(pi)*(48*b^2*c^2 + 8*I*b*c*d - d^2)*d
*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*
b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 6*I*(-4*I*(d*
```

x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d - sqrt(d*x + c)*d^2)*e^((-6*I*(d
*x + c)*b + 6*I*b*c - 6*I*a*d)/d)/b^2)/d^2 + (I*sqrt(3)*sqrt(pi)*(48*b^2*c^
2 - 8*I*b*c*d - d^2)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^
2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2)
+ 1)*b^2) - 6*I*(-4*I*(d*x + c)^(3/2)*b*d + 8*I*sqrt(d*x + c)*b*c*d + sqrt(
d*x + c)*d^2)*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b^2)/d^2 + 9*(I*s
qrt(pi)*(48*b^2*c^2 + 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I
*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt
(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*I*sqrt(d*x + c)*b*
c*d + 9*sqrt(d*x + c)*d^2)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b^2
)/d^2 + 9*(-I*sqrt(pi)*(48*b^2*c^2 - 24*I*b*c*d - 9*d^2)*d*erf(-sqrt(b*d)*s
qrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*I*b*c + 2*I*a*d)/d)/(sqrt
(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^2) - 2*I*(12*I*(d*x + c)^(3/2)*b*d - 24*
I*sqrt(d*x + c)*b*c*d - 9*sqrt(d*x + c)*d^2)*e^((2*I*(d*x + c)*b - 2*I*b*c
+ 2*I*a*d)/d)/b^2)/d^2) + d^3*((I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 + 144*I*b^2
*c^2*d - 36*b*c*d^2 - 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(I*b*
d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^
2*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)
*b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d - 20*(d*x + c)^(3/2)*b*d^2 + 36*sq
rt(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((-6*I*(d*x + c)*b + 6*I*b*c
 - 6*I*a*d)/d)/b^3)/d^3 + (-I*sqrt(3)*sqrt(pi)*(576*b^3*c^3 - 144*I*b^2*c^2
*d - 36*b*c*d^2 + 5*I*d^3)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/s
qrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2
*d^2) + 1)*b^3) - 6*I*(-48*I*(d*x + c)^(5/2)*b^2*d + 144*I*(d*x + c)^(3/2)*
b^2*c*d - 144*I*sqrt(d*x + c)*b^2*c^2*d + 20*(d*x + c)^(3/2)*b*d^2 - 36*sqr
t(d*x + c)*b*c*d^2 + 5*I*sqrt(d*x + c)*d^3)*e^((6*I*(d*x + c)*b - 6*I*b*c +
 6*I*a*d)/d)/b^3)/d^3 + 27*(-I*sqrt(pi)*(192*b^3*c^3 + 144*I*b^2*c^2*d - 10
8*b*c*d^2 - 45*I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) +
 1)/d)*e^((2*I*b*c - 2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b^3)
- 2*I*(48*I*(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*s
qrt(d*x + c)*b^2*c^2*d + 60*(d*x + c)^(3/2)*b*d^2 - 108*sqrt(d*x + c)*b*c*d
^2 - 45*I*sqrt(d*x + c)*d^3)*e^((-2*I*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b
^3)/d^3 + 27*(I*sqrt(pi)*(192*b^3*c^3 - 144*I*b^2*c^2*d - 108*b*c*d^2 + 45*
I*d^3)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-2*
I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b^3) - 2*I*(48*I*
(d*x + c)^(5/2)*b^2*d - 144*I*(d*x + c)^(3/2)*b^2*c*d + 144*I*sqrt(d*x + c)
*b^2*c^2*d - 60*(d*x + c)^(3/2)*b*d^2 + 108*sqrt(d*x + c)*b*c*d^2 - 45*I*sq
rt(d*x + c)*d^3)*e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b^3)/d^3) + 14
4*(I*sqrt(3)*sqrt(pi)*(12*b*c + I*d)*d*erf(-sqrt(3)*sqrt(b*d)*sqrt(d*x + c)
*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((6*I*b*c - 6*I*a*d)/d)/(sqrt(b*d)*(I*b*d/s
qrt(b^2*d^2) + 1)*b) - I*sqrt(3)*sqrt(pi)*(12*b*c - I*d)*d*erf(-sqrt(3)*sqr
t(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-6*I*b*c + 6*I*a*d)/
d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 9*I*sqrt(pi)*(12*b*c + 3*I*d)
*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((2*I*b*c -
2*I*a*d)/d)/(sqrt(b*d)*(I*b*d/sqrt(b^2*d^2) + 1)*b) + 9*I*sqrt(pi)*(12*b*c

```
- 3*I*d)*d*erf(-sqrt(b*d)*sqrt(d*x + c)*(-I*b*d/sqrt(b^2*d^2) + 1)/d)*e^((-
2*I*b*c + 2*I*a*d)/d)/(sqrt(b*d)*(-I*b*d/sqrt(b^2*d^2) + 1)*b) - 6*sqrt(d*x
 + c)*d*e^((6*I*(d*x + c)*b - 6*I*b*c + 6*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*
e^((2*I*(d*x + c)*b - 2*I*b*c + 2*I*a*d)/d)/b + 54*sqrt(d*x + c)*d*e^((-2*I
*(d*x + c)*b + 2*I*b*c - 2*I*a*d)/d)/b - 6*sqrt(d*x + c)*d*e^((-6*I*(d*x +
c)*b + 6*I*b*c - 6*I*a*d)/d)/b)*c^2)/d
```

maple [A] time = 0.00, size = 477, normalized size = 1.17

$$-\frac{3d(dx+c)^{\frac{5}{2}}\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{64b}+\frac{15d\left(\frac{d(dx+c)^{\frac{3}{2}}\sin\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}-\frac{3d\left(-\frac{d\sqrt{dx+c}\,\cos\left(\frac{2(dx+c)b}{d}+\frac{2da-2cb}{d}\right)}{4b}+\frac{d\sqrt{\pi}\left(\cos\left(\frac{2da-2cb}{d}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{dx+c}}{\sqrt{\pi}\sqrt{\frac{b}{d}}}\right.\right.}{8b\sqrt{\frac{b}{d}}}\right.}{4b}\right.}{64b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x)
```

```
[Out] 2/d*(-3/128/b*d*(d*x+c)^(5/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+15/128/b*d*(
1/4/b*d*(d*x+c)^(3/2)*sin(2/d*(d*x+c)*b+2*(a*d-b*c)/d)-3/4/b*d*(-1/4/b*d*(d
*x+c)^(1/2)*cos(2/d*(d*x+c)*b+2*(a*d-b*c)/d)+1/8/b*d*Pi^(1/2)/(b/d)^(1/2)*(
cos(2*(a*d-b*c)/d)*FresnelC(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d)-sin(2
*(a*d-b*c)/d)*FresnelS(2/Pi^(1/2)/(b/d)^(1/2)*(d*x+c)^(1/2)*b/d))))+1/384/b
*d*(d*x+c)^(5/2)*cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-5/384/b*d*(1/12/b*d*(d*x+
c)^(3/2)*sin(6/d*(d*x+c)*b+6*(a*d-b*c)/d)-1/4/b*d*(-1/12/b*d*(d*x+c)^(1/2)*
cos(6/d*(d*x+c)*b+6*(a*d-b*c)/d)+1/144/b*d*2^(1/2)*Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(cos(6*(a*d-b*c)/d)*FresnelC(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(1/2)*(d*x
+c)^(1/2)*b/d)-sin(6*(a*d-b*c)/d)*FresnelS(2^(1/2)/Pi^(1/2)*6^(1/2)/(b/d)^(
1/2)*(d*x+c)^(1/2)*b/d)))))
```

maxima [C] time = 0.56, size = 557, normalized size = 1.37

$$-\frac{\left(1920\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{6\,((dx+c)b-bc+ad)}{d}\right)-51840\,(dx+c)^{\frac{3}{2}}b^{3}\sin\left(\frac{2\,((dx+c)b-bc+ad)}{d}\right)-96\left(\frac{48\,(dx+c)^{\frac{5}{2}}b^{4}}{d}-5\sqrt{dx+c}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cos(b*x+a)^3*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/884736*(1920*(d*x + c)^(3/2)*b^3*sin(6*((d*x + c)*b - b*c + a*d)/d) - 51
840*(d*x + c)^(3/2)*b^3*sin(2*((d*x + c)*b - b*c + a*d)/d) - 96*(48*(d*x +
c)^(5/2)*b^4/d - 5*sqrt(d*x + c)*b^2*d)*cos(6*((d*x + c)*b - b*c + a*d)/d)
+ 2592*(16*(d*x + c)^(5/2)*b^4/d - 15*sqrt(d*x + c)*b^2*d)*cos(2*((d*x + c)
*b - b*c + a*d)/d) + ((10*I - 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)
^(1/4)*cos(-6*(b*c - a*d)/d) + (10*I + 10)*36^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*
(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(6*I*b/d)) + (
-(2430*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c
 - a*d)/d) - (2430*I + 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)
*sin(-2*(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(2*I*b/d)) + ((2430*I + 2430)
*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-2*(b*c - a*d)/d) + (24
30*I - 2430)*4^(1/4)*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-2*(b*c - a
*d)/d))*erf(sqrt(d*x + c)*sqrt(-2*I*b/d)) + (-(10*I + 10)*36^(1/4)*sqrt(2)*
sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*cos(-6*(b*c - a*d)/d) - (10*I - 10)*36^(1/4)
*sqrt(2)*sqrt(pi)*b*d^2*(b^2/d^2)^(1/4)*sin(-6*(b*c - a*d)/d))*erf(sqrt(d*x
 + c)*sqrt(-6*I*b/d)))*d/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^3\,{\left(c+d\,x\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cos(b*x+a)**3*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.202 $\int x^3 \cos^2(x) \cot^2(x)\,dx$

Optimal. Leaf size=112

$$-3ix\text{Li}_2\left(e^{2ix}\right)+\frac{3}{2}\text{Li}_3\left(e^{2ix}\right)-\frac{3x^4}{8}-ix^3-x^3\cot(x)-\frac{1}{2}x^3\sin(x)\cos(x)+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)+\frac{3\cos^2}{8}$$

```
[Out] 3/8*x^2-I*x^3-3/8*x^4+3/8*cos(x)^2-3/4*x^2*cos(x)^2-x^3*cot(x)+3*x^2*ln(1-e
xp(2*I*x))-3*I*x*polylog(2,exp(2*I*x))+3/2*polylog(3,exp(2*I*x))+3/4*x*cos(
x)*sin(x)-1/2*x^3*cos(x)*sin(x)
```

Rubi [A] time = 0.19, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4408, 3311, 30, 3310, 3720, 3717, 2190, 2531, 2282, 6589}

$$-3ix\text{PolyLog}\left(2,e^{2ix}\right)+\frac{3}{2}\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3x^4}{8}-ix^3+\frac{3x^2}{8}+3x^2\log\left(1-e^{2ix}\right)-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)-\frac{1}{2}x^3\sin(x$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (3*x^2)/8 - I*x^3 - (3*x^4)/8 + (3*Cos[x]^2)/8 - (3*x^2*Cos[x]^2)/4 - x^3*C
ot[x] + 3*x^2*Log[1 - E^((2*I)*x)] - (3*I)*x*PolyLog[2, E^((2*I)*x)] + (3*P
olyLog[3, E^((2*I)*x)])/2 + (3*x*Cos[x]*Sin[x])/4 - (x^3*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\int x^3\cos^2(x)\cot^2(x)\,dx &= -\int x^3\cos^2(x)\,dx+\int x^3\cot^2(x)\,dx\\
&= -\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)-\frac{1}{2}x^3\cos(x)\sin(x)-\frac{\int x^3\,dx}{2}+\frac{3}{2}\int x\cos^2(x)\,dx+3\int x^2\\
&= -ix^3-\frac{3x^4}{8}+\frac{3\cos^2(x)}{8}-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)+\frac{3}{4}x\cos(x)\sin(x)-\frac{1}{2}x^3\cos(x)\sin\\
&= \frac{3x^2}{8}-ix^3-\frac{3x^4}{8}+\frac{3\cos^2(x)}{8}-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)+3x^2\log\left(1-e^{2ix}\right)+\frac{3}{4}x\cos(\\
&= \frac{3x^2}{8}-ix^3-\frac{3x^4}{8}+\frac{3\cos^2(x)}{8}-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)+3x^2\log\left(1-e^{2ix}\right)-3ix\mathrm{Li}_2\Big(\\
&= \frac{3x^2}{8}-ix^3-\frac{3x^4}{8}+\frac{3\cos^2(x)}{8}-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)+3x^2\log\left(1-e^{2ix}\right)-3ix\mathrm{Li}_2\Big(\\
&= \frac{3x^2}{8}-ix^3-\frac{3x^4}{8}+\frac{3\cos^2(x)}{8}-\frac{3}{4}x^2\cos^2(x)-x^3\cot(x)+3x^2\log\left(1-e^{2ix}\right)-3ix\mathrm{Li}_2\Big(
\end{aligned}$$

Mathematica [A] time = 0.17, size = 104, normalized size = 0.93

$$\frac{1}{16}\left(48ix\mathrm{Li}_2\left(e^{-2ix}\right)+24\mathrm{Li}_3\left(e^{-2ix}\right)-6x^4+16ix^3-4x^3\sin(2x)-16x^3\cot(x)+48x^2\log\left(1-e^{-2ix}\right)-6x^2\cos(2x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-2*I)*Pi^3 + (16*I)*x^3 - 6*x^4 + 3*Cos[2*x] - 6*x^2*Cos[2*x] - 16*x^3*Co
t[x] + 48*x^2*Log[1 - E^((-2*I)*x)] + (48*I)*x*PolyLog[2, E^((-2*I)*x)] + 2
4*PolyLog[3, E^((-2*I)*x)] + 6*x*Sin[2*x] - 4*x^3*Sin[2*x])/16
```

fricas [C] time = 0.51, size = 244, normalized size = 2.18

$$\frac{4\left(2\,x^3-3\,x\right)\cos(x)^3+24\,x^2\log\left(\cos(x)+i\,\sin(x)+1\right)\sin(x)+24\,x^2\log\left(\cos(x)-i\,\sin(x)+1\right)\sin(x)+24\,x^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(4*(2*x^3 - 3*x)*cos(x)^3 + 24*x^2*log(cos(x) + I*sin(x) + 1)*sin(x) +
 24*x^2*log(cos(x) - I*sin(x) + 1)*sin(x) + 24*x^2*log(-cos(x) + I*sin(x) +
 1)*sin(x) + 24*x^2*log(-cos(x) - I*sin(x) + 1)*sin(x) - 48*I*x*dilog(cos(x
) + I*sin(x))*sin(x) + 48*I*x*dilog(cos(x) - I*sin(x))*sin(x) + 48*I*x*dilo
g(-cos(x) + I*sin(x))*sin(x) - 48*I*x*dilog(-cos(x) - I*sin(x))*sin(x) - 12
*(2*x^3 - x)*cos(x) - 3*(2*x^4 + 2*(2*x^2 - 1)*cos(x)^2 - 2*x^2 + 1)*sin(x)
 + 48*polylog(3, cos(x) + I*sin(x))*sin(x) + 48*polylog(3, cos(x) - I*sin(x
))*sin(x) + 48*polylog(3, -cos(x) + I*sin(x))*sin(x) + 48*polylog(3, -cos(x
) - I*sin(x))*sin(x))/sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cos(x)^2\cot(x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^2, x)
```

maple [A] time = 0.12, size = 150, normalized size = 1.34

$$-\frac{3x^4}{8}+\frac{i\left(4x^3+6ix^2-6x-3i\right)e^{2ix}}{32}-\frac{i\left(4x^3-6ix^2-6x+3i\right)e^{-2ix}}{32}-\frac{2ix^3}{e^{2ix}-1}-2ix^3+3x^2\ln\left(1+e^{ix}\right)-6ix\,\mathrm{polylog}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x)
```

```
[Out] -3/8*x^4+1/32*I*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*I*(-6*I*x^2+4*x^3+3
*I-6*x)*exp(-2*I*x)-2*I*x^3/(exp(2*I*x)-1)-2*I*x^3+3*x^2*ln(1+exp(I*x))-6*I
*x*polylog(2,-exp(I*x))+6*polylog(3,-exp(I*x))+3*x^2*ln(1-exp(I*x))-6*I*x*p
olylog(2,exp(I*x))+6*polylog(3,exp(I*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cos(x)^2 \cot(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^2,x)
```

```
[Out] int(x^3*cos(x)^2*cot(x)^2, x)
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos^2 (x) \cot^2 (x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**3*cos(x)**2*cot(x)**2, x)
```

3.203 $\int x^2 \cos^2(x) \cot^2(x) \, dx$

Optimal. Leaf size=83

$$-i\text{Li}_2\left(e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] 1/4*x-I*x^2-1/2*x^3-1/2*x*cos(x)^2-x^2*cot(x)+2*x*ln(1-exp(2*I*x))-I*polylo
g(2,exp(2*I*x))+1/4*cos(x)*sin(x)-1/2*x^2*cos(x)*sin(x)
```

Rubi [A] time = 0.17, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {4408, 3311, 30, 2635, 8, 3720, 3717, 2190, 2279, 2391}

$$-i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{x^3}{2}-ix^2-x^2\cot(x)-\frac{1}{2}x^2\sin(x)\cos(x)+\frac{x}{4}+2x\log\left(1-e^{2ix}\right)-\frac{1}{2}x\cos^2(x)+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] x/4 - I*x^2 - x^3/2 - (x*Cos[x]^2)/2 - x^2*Cot[x] + 2*x*Log[1 - E^((2*I)*x)
] - I*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 - (x^2*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x^2 \cos^2(x)\cot^2(x)\,dx = -\int x^2\cos^2(x)\,dx + \int x^2\cot^2(x)\,dx$$
$$= -\frac{1}{2}x\cos^2(x) - x^2\cot(x) - \frac{1}{2}x^2\cos(x)\sin(x) - \frac{\int x^2\,dx}{2} + \frac{1}{2}\int\cos^2(x)\,dx + 2\int x\cot(x$$
$$= -ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos(x)\sin(x) - 4i\int\frac{e^{2ix}x}{1-e^{2}}$$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos($$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x) - \frac{1}{2}x^2\cos($$
$$= \frac{x}{4} - ix^2 - \frac{x^3}{2} - \frac{1}{2}x\cos^2(x) - x^2\cot(x) + 2x\log\left(1-e^{2ix}\right) - i\text{Li}_2\left(e^{2ix}\right) + \frac{1}{4}\cos(x)\sin(x$$

Mathematica [A] time = 0.10, size = 72, normalized size = 0.87

$$\frac{1}{8}\left(-8i\text{Li}_2\left(e^{2ix}\right) - 4x^3 - 8ix^2 - 2x^2\sin(2x) - 8x^2\cot(x) + 16x\log\left(1-e^{2ix}\right) + \sin(2x) - 2x\cos(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] ((-8*I)*x^2 - 4*x^3 - 2*x*Cos[2*x] - 8*x^2*Cot[x] + 16*x*Log[1 - E^((2*I)*x
)] - (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x] - 2*x^2*Sin[2*x])/8
```

fricas [B] time = 0.48, size = 162, normalized size = 1.95

$$\frac{\left(2x^2-1\right)\cos(x)^3 + 4x\log\left(\cos(x)+i\sin(x)+1\right)\sin(x) + 4x\log\left(\cos(x)-i\sin(x)+1\right)\sin(x) + 4x\log\left(-\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*x^2 - 1)*cos(x)^3 + 4*x*log(cos(x) + I*sin(x) + 1)*sin(x) + 4*x*log
(cos(x) - I*sin(x) + 1)*sin(x) + 4*x*log(-cos(x) + I*sin(x) + 1)*sin(x) + 4
*x*log(-cos(x) - I*sin(x) + 1)*sin(x) - (6*x^2 - 1)*cos(x) - (2*x^3 + 2*x*c
os(x)^2 - x)*sin(x) - 4*I*dilog(cos(x) + I*sin(x))*sin(x) + 4*I*dilog(cos(x
) - I*sin(x))*sin(x) + 4*I*dilog(-cos(x) + I*sin(x))*sin(x) - 4*I*dilog(-co
s(x) - I*sin(x))*sin(x))/sin(x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos(x)^2 \cot(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^2, x)
```

maple [A] time = 0.11, size = 112, normalized size = 1.35

$$-\frac{x^3}{2}+\frac{i\left(2x^2+2ix-1\right)\mathrm{e}^{2ix}}{16}-\frac{i\left(2x^2-2ix-1\right)\mathrm{e}^{-2ix}}{16}-\frac{2ix^2}{\mathrm{e}^{2ix}-1}+2x\ln\left(1+\mathrm{e}^{ix}\right)+2x\ln\left(1-\mathrm{e}^{ix}\right)-2ix^2-2i\,\mathrm{polylog}\left(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x)
```

```
[Out] -1/2*x^3+1/16*I*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*I*(-2*I*x+2*x^2-1)*exp(-2*I
*x)-2*I*x^2/(exp(2*I*x)-1)+2*x*ln(1+exp(I*x))+2*x*ln(1-exp(I*x))-2*I*x^2-2*
I*polylog(2,-exp(I*x))-2*I*polylog(2,exp(I*x))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos(x)^2 \cot(x)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^2,x)
```

```
[Out] int(x^2*cos(x)^2*cot(x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos^2(x) \cot^2(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**2, x)
```

3.204 $\int x\cos^2(x)\cot^2(x)\,dx$

Optimal. Leaf size=33

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

```
[Out] -3/4*x^2-1/4*cos(x)^2-x*cot(x)+ln(sin(x))-1/2*x*cos(x)*sin(x)
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4408, 3310, 30, 3720, 3475}

$$-\frac{3x^2}{4}-\frac{\cos^2(x)}{4}-x\cot(x)+\log(\sin(x))-\frac{1}{2}x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^2,x]
```

```
[Out] (-3*x^2)/4 - Cos[x]^2/4 - x*Cot[x] + Log[Sin[x]] - (x*Cos[x]*Sin[x])/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
```

```
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x\cos^2(x)\cot^2(x)\,dx &= -\int x\cos^2(x)\,dx + \int x\cot^2(x)\,dx\\ &= -\frac{1}{4}\cos^2(x) - x\cot(x) - \frac{1}{2}x\cos(x)\sin(x) - \frac{\int x\,dx}{2} - \int x\,dx + \int \cot(x)\,dx\\ &= -\frac{3x^2}{4} - \frac{\cos^2(x)}{4} - x\cot(x) + \log(\sin(x)) - \frac{1}{2}x\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{3x^2}{4} - \frac{1}{4}x\sin(2x) - \frac{1}{8}\cos(2x) - x\cot(x) + \log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^2,x]
[Out] (-3*x^2)/4 - Cos[2*x]/8 - x*Cot[x] + Log[Sin[x]] - (x*Sin[2*x])/4
```

fricas [A] time = 0.46, size = 45, normalized size = 1.36

$$\frac{4\,x\cos(x)^3 - 12\,x\cos(x) - \left(6\,x^2 + 2\,\cos(x)^2 - 1\right)\sin(x) + 8\,\log\left(\frac{1}{2}\,\sin(x)\right)\sin(x)}{8\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="fricas")
[Out] 1/8*(4*x*cos(x)^3 - 12*x*cos(x) - (6*x^2 + 2*cos(x)^2 - 1)*sin(x) + 8*log(1
/2*sin(x))*sin(x))/sin(x)
```

giac [B] time = 0.22, size = 206, normalized size = 6.24

$$-\frac{6\,x^2\tan\left(\frac{1}{2}\,x\right)^5-4\,x\tan\left(\frac{1}{2}\,x\right)^6-4\,\log\left(\frac{16\,\tan\left(\frac{1}{2}\,x\right)^2}{\tan\left(\frac{1}{2}\,x\right)^4+2\,\tan\left(\frac{1}{2}\,x\right)^2+1}\right)\tan\left(\frac{1}{2}\,x\right)^5+12\,x^2\tan\left(\frac{1}{2}\,x\right)^3-12\,x\tan\left(\frac{1}{2}\,x\right)^4+\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="giac")
```

```
[Out] -1/8*(6*x^2*tan(1/2*x)^5 - 4*x*tan(1/2*x)^6 - 4*log(16*tan(1/2*x)^2/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^5 + 12*x^2*tan(1/2*x)^3 - 12*x*tan
(1/2*x)^4 + tan(1/2*x)^5 - 8*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*
x)^2 + 1))*tan(1/2*x)^3 + 6*x^2*tan(1/2*x) + 12*x*tan(1/2*x)^2 - 6*tan(1/2*
x)^3 - 4*log(16*tan(1/2*x)^2/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x
) + 4*x + tan(1/2*x))/(tan(1/2*x)^5 + 2*tan(1/2*x)^3 + tan(1/2*x))
```

maple [B] time = 0.12, size = 76, normalized size = 2.30

$$\frac{-x-\frac{x^2\tan(x)}{2}}{2\tan(x)}-\frac{\ln\left(1+\tan^2(x)\right)}{2}+\ln\left(\tan(x)\right)+\frac{-\frac{\tan(x)}{2}-x-2x\left(\tan^2(x)\right)-x^2\tan(x)-x^2\left(\tan^3(x)\right)}{2\tan(x)\left(1+\tan^2(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x)
```

```
[Out] 1/2*(-x-1/2*x^2*tan(x))/tan(x)-1/2*ln(1+tan(x)^2)+ln(tan(x))+1/2*(-1/2*tan(
x)-x-2*x*tan(x)^2-x^2*tan(x)-x^2*tan(x)^3)/tan(x)/(1+tan(x)^2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

mupad [B] time = 1.23, size = 56, normalized size = 1.70

$$\ln\left(\mathrm{e}^{x\,2\mathrm{i}}-1\right)-\mathrm{e}^{-x\,2\mathrm{i}}\left(\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)+\mathrm{e}^{x\,2\mathrm{i}}\left(-\frac{1}{16}+\frac{x\,1\mathrm{i}}{8}\right)-\frac{3\,x^2}{4}-x\,2\mathrm{i}-\frac{x\,2\mathrm{i}}{\mathrm{e}^{x\,2\mathrm{i}}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^2,x)
```

```
[Out] log(exp(x*2i) - 1) - x*2i - exp(-x*2i)*((x*1i)/8 + 1/16) + exp(x*2i)*((x*1i
)/8 - 1/16) - (x*2i)/(exp(x*2i) - 1) - (3*x^2)/4
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^2(x)\cot^2(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**2,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**2, x)
```

3.205 $\int x^3 \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=180

$$3ix^2\text{Li}_2\left(e^{2ix}\right)-3x\text{Li}_3\left(e^{2ix}\right)-\frac{3}{2}i\text{Li}_2\left(e^{2ix}\right)-\frac{3}{2}i\text{Li}_4\left(e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-2x^3\log\left(1-e^{2ix}\right)+\frac{1}{2}x^3\sin^2(x)-\frac{1}{2}x^3\cot^2(x)-\frac{3i}{2}$$

```
[Out] 3/8*x+3*I*x^2*polylog(2,exp(2*I*x))-3/4*x^3+1/2*I*x^4-3/2*x^2*cot(x)-1/2*x^
3*cot(x)^2+3*x*ln(1-exp(2*I*x))-2*x^3*ln(1-exp(2*I*x))-3/2*I*polylog(2,exp(
2*I*x))-3/2*I*x^2-3*x*polylog(3,exp(2*I*x))-3/2*I*polylog(4,exp(2*I*x))-3/8
*cos(x)*sin(x)+3/4*x^2*cos(x)*sin(x)-3/4*x*sin(x)^2+1/2*x^3*sin(x)^2
```

Rubi [A] time = 0.40, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 15, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {4408, 3443, 3311, 30, 2635, 8, 3717, 2190, 2531, 6609, 2282, 6589, 3720, 2279, 2391}

$$3ix^2\text{PolyLog}\left(2,e^{2ix}\right)-3x\text{PolyLog}\left(3,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,e^{2ix}\right)-\frac{3}{2}i\text{PolyLog}\left(4,e^{2ix}\right)+\frac{ix^4}{2}-\frac{3x^3}{4}-\frac{3ix^2}{2}-2x^3\log\left(\right.$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (3*x)/8 - ((3*I)/2)*x^2 - (3*x^3)/4 + (I/2)*x^4 - (3*x^2*Cot[x])/2 - (x^3*C
ot[x]^2)/2 + 3*x*Log[1 - E^((2*I)*x)] - 2*x^3*Log[1 - E^((2*I)*x)] - ((3*I)
/2)*PolyLog[2, E^((2*I)*x)] + (3*I)*x^2*PolyLog[2, E^((2*I)*x)] - 3*x*PolyL
og[3, E^((2*I)*x)] - ((3*I)/2)*PolyLog[4, E^((2*I)*x)] - (3*Cos[x]*Sin[x])/
8 + (3*x^2*Cos[x]*Sin[x])/4 - (3*x*Sin[x]^2)/4 + (x^3*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int x^3\cos^2(x)\cot^3(x)\,dx = -\int x^3\cos^2(x)\cot(x)\,dx + \int x^3\cot^3(x)\,dx$$
$$= -\frac{1}{2}x^3\cot^2(x) + \frac{3}{2}\int x^2\cot^2(x)\,dx - 2\int x^3\cot(x)\,dx + \int x^3\cos(x)\sin(x)\,dx$$
$$= -\frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{1}{2}x^3\sin^2(x) - 2\left(-\frac{ix^4}{4} - 2i\int\frac{e^{2ix}x^3}{1-e^{2ix}}\,dx\right) - \frac{3\int x^2\,dx}{2} - \frac{3}{2}$$
$$= -\frac{3ix^2}{2} - \frac{x^3}{2} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + \frac{3}{4}x^2\cos(x)\sin(x) - \frac{3}{4}x\sin^2(x) + \frac{1}{2}x^3\sin^2(x)$$
$$= -\frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) + \frac{3}{4}x^2\cos$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{8}\cos(x)\sin(x) + \frac{3}{4}x$$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\text{Li}_2\left(e^{2ix}\right) - \frac{3}{8}\cos($$
$$= \frac{3x}{8} - \frac{3ix^2}{2} - \frac{3x^3}{4} - \frac{3}{2}x^2\cot(x) - \frac{1}{2}x^3\cot^2(x) + 3x\log\left(1-e^{2ix}\right) - \frac{3}{2}i\text{Li}_2\left(e^{2ix}\right) - 2\left(-\frac{ix}{4}\right.$$

Mathematica [A] time = 0.41, size = 159, normalized size = 0.88

$$\frac{1}{32}\left(-96ix^2\text{Li}_2\left(e^{-2ix}\right) - 96x\text{Li}_3\left(e^{-2ix}\right) - 48i\text{Li}_2\left(e^{2ix}\right) + 48i\text{Li}_4\left(e^{-2ix}\right) - 16ix^4 - 64x^3\log\left(1-e^{-2ix}\right) - 8x^3\cos(2x)\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I*Pi^4 - (48*I)*x^2 - (16*I)*x^4 + 12*x*Cos[2*x] - 8*x^3*Cos[2*x] - 48*x^2
*Cot[x] - 16*x^3*Csc[x]^2 - 64*x^3*Log[1 - E^((-2*I)*x)] + 96*x*Log[1 - E^(
(2*I)*x)] - (96*I)*x^2*PolyLog[2, E^((-2*I)*x)] - (48*I)*PolyLog[2, E^((2*I
)*x)] - 96*x*PolyLog[3, E^((-2*I)*x)] + (48*I)*PolyLog[4, E^((-2*I)*x)] - 6
*Sin[2*x] + 12*x^2*Sin[2*x])/32
```

fricas [C] time = 0.51, size = 508, normalized size = 2.82

$$-\frac{2\left(2x^3-3x\right)\cos(x)^4 - 2x^3 - 3\left(2x^3-3x\right)\cos(x)^2 - \left(\left(24ix^2-12i\right)\cos(x)^2 - 24ix^2 + 12i\right)\text{Li}_2\left(\cos(x) + i\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^3 - 3*x)*cos(x)^4 - 2*x^3 - 3*(2*x^3 - 3*x)*cos(x)^2 - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(cos(x) + I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(cos(x) - I*sin(x)) - ((-24*I*
x^2 + 12*I)*cos(x)^2 + 24*I*x^2 - 12*I)*dilog(-cos(x) + I*sin(x)) - ((24*I*
x^2 - 12*I)*cos(x)^2 - 24*I*x^2 + 12*I)*dilog(-cos(x) - I*sin(x)) - 4*(2*x^
3 - (2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (
2*x^3 - 3*x)*cos(x)^2 - 3*x)*log(cos(x) - I*sin(x) + 1) - 4*(2*x^3 - (2*x^3
 - 3*x)*cos(x)^2 - 3*x)*log(-cos(x) + I*sin(x) + 1) - 4*(2*x^3 - (2*x^3 - 3
*x)*cos(x)^2 - 3*x)*log(-cos(x) - I*sin(x) + 1) - (-48*I*cos(x)^2 + 48*I)*p
olylog(4, cos(x) + I*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, cos(x) - I
*sin(x)) - (48*I*cos(x)^2 - 48*I)*polylog(4, -cos(x) + I*sin(x)) - (-48*I*c
os(x)^2 + 48*I)*polylog(4, -cos(x) - I*sin(x)) + 48*(x*cos(x)^2 - x)*polylo
g(3, cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)*polylog(3, cos(x) - I*sin(x))
 + 48*(x*cos(x)^2 - x)*polylog(3, -cos(x) + I*sin(x)) + 48*(x*cos(x)^2 - x)
*polylog(3, -cos(x) - I*sin(x)) - 3*((2*x^2 - 1)*cos(x)^3 + (2*x^2 + 1)*cos
(x))*sin(x) - 3*x)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos(x)^2 \cot(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^3*cos(x)^2*cot(x)^3, x)
```

maple [A] time = 0.16, size = 240, normalized size = 1.33

$$6ix^2 \operatorname{polylog}\left(2, e^{ix}\right) - \frac{\left(4x^3 + 6ix^2 - 6x - 3i\right) e^{2ix}}{32} - \frac{\left(4x^3 - 6ix^2 - 6x + 3i\right) e^{-2ix}}{32} + \frac{x^2 \left(2x\, e^{2ix} - 3i e^{2ix} + 3i\right)}{\left(e^{2ix} - 1\right)^2} - 2x^3 \ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x)
```

```
[Out] -3*I*polylog(2,-exp(I*x))-1/32*(6*I*x^2+4*x^3-3*I-6*x)*exp(2*I*x)-1/32*(-6*
I*x^2+4*x^3+3*I-6*x)*exp(-2*I*x)+x^2*(2*x*exp(2*I*x)-3*I*exp(2*I*x)+3*I)/(e
xp(2*I*x)-1)^2-2*x^3*ln(1+exp(I*x))-2*x^3*ln(1-exp(I*x))-12*I*polylog(4,-ex
p(I*x))-12*x*polylog(3,exp(I*x))+3*x*ln(1+exp(I*x))+3*x*ln(1-exp(I*x))-12*x
*polylog(3,-exp(I*x))+6*I*x^2*polylog(2,exp(I*x))-3*I*x^2+6*I*x^2*polylog(2
,-exp(I*x))-3*I*polylog(2,exp(I*x))+1/2*I*x^4-12*I*polylog(4,exp(I*x))
```

maxima [B] time = 0.95, size = 3719, normalized size = 20.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -(4*x^3 + (4*x^3 + 6*I*x^2 - 6*x - 3*I)*cos(6*x)^2 - (-32*I*x^4 - 16*x^3 +
168*I*x^2 + 24*x + 12*I)*cos(4*x)^2 - (-32*I*x^4 + 56*x^3 + 96*I*x^2 + 12*x
)*cos(2*x)^2 - (4*x^3 + 6*I*x^2 - 6*x - 3*I)*sin(6*x)^2 - (32*I*x^4 + 16*x^
3 - 168*I*x^2 - 24*x - 12*I)*sin(4*x)^2 - (32*I*x^4 - 56*x^3 - 96*I*x^2 - 1
2*x)*sin(2*x)^2 - 6*I*x^2 - ((128*I*x^3 - 192*I*x)*cos(4*x)^2 + (128*I*x^3
- 192*I*x)*cos(2*x)^2 + (-128*I*x^3 + 192*I*x)*sin(4*x)^2 + (-128*I*x^3 + 1
92*I*x)*sin(2*x)^2 + (-64*I*x^3 + (-64*I*x^3 + 96*I*x)*cos(4*x) + (128*I*x^
3 - 192*I*x)*cos(2*x) + 32*(2*x^3 - 3*x)*sin(4*x) - 64*(2*x^3 - 3*x)*sin(2*
x) + 96*I*x)*cos(6*x) + (128*I*x^3 + (-320*I*x^3 + 480*I*x)*cos(2*x) + 160*
(2*x^3 - 3*x)*sin(2*x) - 192*I*x)*cos(4*x) + (-64*I*x^3 + 96*I*x)*cos(2*x)
+ (64*x^3 + 32*(2*x^3 - 3*x)*cos(4*x) - 64*(2*x^3 - 3*x)*cos(2*x) + (64*I*x
^3 - 96*I*x)*sin(4*x) + (-128*I*x^3 + 192*I*x)*sin(2*x) - 96*x)*sin(6*x) -
(128*x^3 + 128*(2*x^3 - 3*x)*cos(4*x) - 160*(2*x^3 - 3*x)*cos(2*x) - (320*I
*x^3 - 480*I*x)*sin(2*x) - 192*x)*sin(4*x) + 32*(2*x^3 - 4*(2*x^3 - 3*x)*co
s(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), cos(x) + 1) - ((-128*I*x^3 + 192*I*
x)*cos(4*x)^2 + (-128*I*x^3 + 192*I*x)*cos(2*x)^2 + (128*I*x^3 - 192*I*x)*s
in(4*x)^2 + (128*I*x^3 - 192*I*x)*sin(2*x)^2 + (64*I*x^3 + (64*I*x^3 - 96*I
*x)*cos(4*x) + (-128*I*x^3 + 192*I*x)*cos(2*x) - 32*(2*x^3 - 3*x)*sin(4*x)
+ 64*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*cos(6*x) + (-128*I*x^3 + (320*I*x^3 -
 480*I*x)*cos(2*x) - 160*(2*x^3 - 3*x)*sin(2*x) + 192*I*x)*cos(4*x) + (64*I
*x^3 - 96*I*x)*cos(2*x) - (64*x^3 + 32*(2*x^3 - 3*x)*cos(4*x) - 64*(2*x^3 -
 3*x)*cos(2*x) - (-64*I*x^3 + 96*I*x)*sin(4*x) - (128*I*x^3 - 192*I*x)*sin(
2*x) - 96*x)*sin(6*x) + (128*x^3 + 128*(2*x^3 - 3*x)*cos(4*x) - 160*(2*x^3
- 3*x)*cos(2*x) + (-320*I*x^3 + 480*I*x)*sin(2*x) - 192*x)*sin(4*x) - 32*(2
*x^3 - 4*(2*x^3 - 3*x)*cos(2*x) - 3*x)*sin(2*x))*arctan2(sin(x), -cos(x) +
1) - (16*I*x^4 + 8*x^3 - 12*I*x^2 + (16*I*x^4 + 16*x^3 - 72*I*x^2 - 24*x -
12*I)*cos(4*x) + (-32*I*x^4 + 52*x^3 + 90*I*x^2 + 18*x + 3*I)*cos(2*x) - (1
6*x^4 - 16*I*x^3 - 72*x^2 + 24*I*x - 12)*sin(4*x) + (32*x^4 + 52*I*x^3 - 90
*x^2 + 18*I*x - 3)*sin(2*x) - 12*x + 6*I)*cos(6*x) - (-32*I*x^4 - 20*x^3 +
30*I*x^2 + (80*I*x^4 - 104*x^3 - 276*I*x^2 - 36*x - 6*I)*cos(2*x) - (80*x^4
 + 104*I*x^3 - 276*x^2 + 36*I*x - 6)*sin(2*x) + 30*x - 15*I)*cos(4*x) - (16
*I*x^4 + 16*x^3 - 24*I*x^2 - 24*x + 12*I)*cos(2*x) - ((-384*I*x^2 + 192*I)*
cos(4*x)^2 + (-384*I*x^2 + 192*I)*cos(2*x)^2 + (384*I*x^2 - 192*I)*sin(4*x)
^2 + (384*I*x^2 - 192*I)*sin(2*x)^2 + (192*I*x^2 + (192*I*x^2 - 96*I)*cos(4
*x) + (-384*I*x^2 + 192*I)*cos(2*x) - 96*(2*x^2 - 1)*sin(4*x) + 192*(2*x^2
- 1)*sin(2*x) - 96*I)*cos(6*x) + (-384*I*x^2 + (960*I*x^2 - 480*I)*cos(2*x)
 - 480*(2*x^2 - 1)*sin(2*x) + 192*I)*cos(4*x) + (192*I*x^2 - 96*I)*cos(2*x)
 - (192*x^2 + 96*(2*x^2 - 1)*cos(4*x) - 192*(2*x^2 - 1)*cos(2*x) - (-192*I*
x^2 + 96*I)*sin(4*x) - (384*I*x^2 - 192*I)*sin(2*x) - 96)*sin(6*x) + (384*x
^2 + 384*(2*x^2 - 1)*cos(4*x) - 480*(2*x^2 - 1)*cos(2*x) + (-960*I*x^2 + 48
0*I)*sin(2*x) - 192)*sin(4*x) - 96*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin
```

```
(2*x))*dilog(-e^(I*x)) - ((-384*I*x^2 + 192*I)*cos(4*x)^2 + (-384*I*x^2 + 1
92*I)*cos(2*x)^2 + (384*I*x^2 - 192*I)*sin(4*x)^2 + (384*I*x^2 - 192*I)*sin
(2*x)^2 + (192*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-384*I*x^2 + 192*I)*c
os(2*x) - 96*(2*x^2 - 1)*sin(4*x) + 192*(2*x^2 - 1)*sin(2*x) - 96*I)*cos(6*
x) + (-384*I*x^2 + (960*I*x^2 - 480*I)*cos(2*x) - 480*(2*x^2 - 1)*sin(2*x)
+ 192*I)*cos(4*x) + (192*I*x^2 - 96*I)*cos(2*x) - (192*x^2 + 96*(2*x^2 - 1)
*cos(4*x) - 192*(2*x^2 - 1)*cos(2*x) - (-192*I*x^2 + 96*I)*sin(4*x) - (384*
I*x^2 - 192*I)*sin(2*x) - 96)*sin(6*x) + (384*x^2 + 384*(2*x^2 - 1)*cos(4*x
) - 480*(2*x^2 - 1)*cos(2*x) + (-960*I*x^2 + 480*I)*sin(2*x) - 192)*sin(4*x
) - 96*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*dilog(e^(I*x)) - (32*
(2*x^3 - 3*x)*cos(4*x)^2 + 32*(2*x^3 - 3*x)*cos(2*x)^2 - 32*(2*x^3 - 3*x)*s
in(4*x)^2 - 32*(2*x^3 - 3*x)*sin(2*x)^2 - (32*x^3 + 16*(2*x^3 - 3*x)*cos(4*
x) - 32*(2*x^3 - 3*x)*cos(2*x) - (-32*I*x^3 + 48*I*x)*sin(4*x) - (64*I*x^3
- 96*I*x)*sin(2*x) - 48*x)*cos(6*x) + (64*x^3 - 80*(2*x^3 - 3*x)*cos(2*x) +
 (-160*I*x^3 + 240*I*x)*sin(2*x) - 96*x)*cos(4*x) - 16*(2*x^3 - 3*x)*cos(2*
x) + (-32*I*x^3 + (-32*I*x^3 + 48*I*x)*cos(4*x) + (64*I*x^3 - 96*I*x)*cos(2
*x) + 16*(2*x^3 - 3*x)*sin(4*x) - 32*(2*x^3 - 3*x)*sin(2*x) + 48*I*x)*sin(6
*x) + (64*I*x^3 + (128*I*x^3 - 192*I*x)*cos(4*x) + (-160*I*x^3 + 240*I*x)*c
os(2*x) + 80*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*sin(4*x) + (-32*I*x^3 + (128*
I*x^3 - 192*I*x)*cos(2*x) + 48*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*c
os(x) + 1) - (32*(2*x^3 - 3*x)*cos(4*x)^2 + 32*(2*x^3 - 3*x)*cos(2*x)^2 - 3
2*(2*x^3 - 3*x)*sin(4*x)^2 - 32*(2*x^3 - 3*x)*sin(2*x)^2 - (32*x^3 + 16*(2*
x^3 - 3*x)*cos(4*x) - 32*(2*x^3 - 3*x)*cos(2*x) - (-32*I*x^3 + 48*I*x)*sin(
4*x) - (64*I*x^3 - 96*I*x)*sin(2*x) - 48*x)*cos(6*x) + (64*x^3 - 80*(2*x^3
- 3*x)*cos(2*x) + (-160*I*x^3 + 240*I*x)*sin(2*x) - 96*x)*cos(4*x) - 16*(2*
x^3 - 3*x)*cos(2*x) + (-32*I*x^3 + (-32*I*x^3 + 48*I*x)*cos(4*x) + (64*I*x^
3 - 96*I*x)*cos(2*x) + 16*(2*x^3 - 3*x)*sin(4*x) - 32*(2*x^3 - 3*x)*sin(2*x
) + 48*I*x)*sin(6*x) + (64*I*x^3 + (128*I*x^3 - 192*I*x)*cos(4*x) + (-160*I
*x^3 + 240*I*x)*cos(2*x) + 80*(2*x^3 - 3*x)*sin(2*x) - 96*I*x)*sin(4*x) + (
-32*I*x^3 + (128*I*x^3 - 192*I*x)*cos(2*x) + 48*I*x)*sin(2*x))*log(cos(x)^2
 + sin(x)^2 - 2*cos(x) + 1) - ((-384*I*cos(4*x) + 768*I*cos(2*x) + 384*sin(
4*x) - 768*sin(2*x) - 384*I)*cos(6*x) + (-1920*I*cos(2*x) + 1920*sin(2*x) +
 768*I)*cos(4*x) + 768*I*cos(4*x)^2 + 768*I*cos(2*x)^2 + (384*cos(4*x) - 76
8*cos(2*x) + 384*I*sin(4*x) - 768*I*sin(2*x) + 384)*sin(6*x) - (1536*cos(4*
x) - 1920*cos(2*x) - 1920*I*sin(2*x) + 768)*sin(4*x) - 768*I*sin(4*x)^2 - 3
84*(4*cos(2*x) - 1)*sin(2*x) - 768*I*sin(2*x)^2 - 384*I*cos(2*x))*polylog(4
, -e^(I*x)) - ((-384*I*cos(4*x) + 768*I*cos(2*x) + 384*sin(4*x) - 768*sin(2
*x) - 384*I)*cos(6*x) + (-1920*I*cos(2*x) + 1920*sin(2*x) + 768*I)*cos(4*x)
 + 768*I*cos(4*x)^2 + 768*I*cos(2*x)^2 + (384*cos(4*x) - 768*cos(2*x) + 384
*I*sin(4*x) - 768*I*sin(2*x) + 384)*sin(6*x) - (1536*cos(4*x) - 1920*cos(2*
x) - 1920*I*sin(2*x) + 768)*sin(4*x) - 768*I*sin(4*x)^2 - 384*(4*cos(2*x) -
 1)*sin(2*x) - 768*I*sin(2*x)^2 - 384*I*cos(2*x))*polylog(4, e^(I*x)) - (76
8*x*cos(4*x)^2 + 768*x*cos(2*x)^2 - 768*x*sin(4*x)^2 - 768*x*sin(2*x)^2 - (
384*x*cos(4*x) - 768*x*cos(2*x) + 384*I*x*sin(4*x) - 768*I*x*sin(2*x) + 384
*x)*cos(6*x) - 384*(5*x*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - 384*x*c
```

```
os(2*x) + (-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 384*x*sin(4*x) - 768*x*si
n(2*x) - 384*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1920*I*x*cos(2*x) + 1920*
x*sin(2*x) + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x) - 384*I*x)*sin(2*x))*po
lylog(3, -e^(I*x)) - (768*x*cos(4*x)^2 + 768*x*cos(2*x)^2 - 768*x*sin(4*x)^
2 - 768*x*sin(2*x)^2 - (384*x*cos(4*x) - 768*x*cos(2*x) + 384*I*x*sin(4*x)
- 768*I*x*sin(2*x) + 384*x)*cos(6*x) - 384*(5*x*cos(2*x) + 5*I*x*sin(2*x) -
 2*x)*cos(4*x) - 384*x*cos(2*x) + (-384*I*x*cos(4*x) + 768*I*x*cos(2*x) + 3
84*x*sin(4*x) - 768*x*sin(2*x) - 384*I*x)*sin(6*x) + (1536*I*x*cos(4*x) - 1
920*I*x*cos(2*x) + 1920*x*sin(2*x) + 768*I*x)*sin(4*x) + (1536*I*x*cos(2*x)
 - 384*I*x)*sin(2*x))*polylog(3, e^(I*x)) + (16*x^4 - 8*I*x^3 - 12*x^2 - (-
8*I*x^3 + 12*x^2 + 12*I*x - 6)*cos(6*x) + (16*x^4 - 16*I*x^3 - 72*x^2 + 24*
I*x - 12)*cos(4*x) - (32*x^4 + 52*I*x^3 - 90*x^2 + 18*I*x - 3)*cos(2*x) - (
-16*I*x^4 - 16*x^3 + 72*I*x^2 + 24*x + 12*I)*sin(4*x) - (32*I*x^4 - 52*x^3
- 90*I*x^2 - 18*x - 3*I)*sin(2*x) + 12*I*x + 6)*sin(6*x) - (32*x^4 - 20*I*x
^3 - 30*x^2 + (64*x^4 - 32*I*x^3 - 336*x^2 + 48*I*x - 24)*cos(4*x) - (80*x^
4 + 104*I*x^3 - 276*x^2 + 36*I*x - 6)*cos(2*x) + (-80*I*x^4 + 104*x^3 + 276
*I*x^2 + 36*x + 6*I)*sin(2*x) + 30*I*x + 15)*sin(4*x) + (16*x^4 - 16*I*x^3
- 24*x^2 - (64*x^4 + 112*I*x^3 - 192*x^2 + 24*I*x)*cos(2*x) + 24*I*x + 12)*
sin(2*x) - 6*x + 3*I)/((32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x) - 64*I*si
n(2*x) + 32)*cos(6*x) + (160*cos(2*x) + 160*I*sin(2*x) - 64)*cos(4*x) - 64*
cos(4*x)^2 - 64*cos(2*x)^2 - (-32*I*cos(4*x) + 64*I*cos(2*x) + 32*sin(4*x)
- 64*sin(2*x) - 32*I)*sin(6*x) - (128*I*cos(4*x) - 160*I*cos(2*x) + 160*sin
(2*x) + 64*I)*sin(4*x) + 64*sin(4*x)^2 - (128*I*cos(2*x) - 32*I)*sin(2*x) +
 64*sin(2*x)^2 + 32*cos(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \cos(x)^2 \cot(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x^3*cos(x)^2*cot(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cos^2(x) \cot^3(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**3*cos(x)**2*cot(x)**3, x)
```

3.206 $\int x^2 \cos^2(x) \cot^3(x) \, dx$

Optimal. Leaf size=106

$$2ix\text{Li}_2\left(e^{2ix}\right)-\text{Li}_3\left(e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot(x)+\log(\sin(x))$$

```
[Out] -3/4*x^2+2/3*I*x^3-x*cot(x)-1/2*x^2*cot(x)^2-2*x^2*ln(1-exp(2*I*x))+ln(sin(
x))+2*I*x*polylog(2,exp(2*I*x))-polylog(3,exp(2*I*x))+1/2*x*cos(x)*sin(x)-1
/4*sin(x)^2+1/2*x^2*sin(x)^2
```

Rubi [A] time = 0.28, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 11, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {4408, 3443, 3310, 30, 3717, 2190, 2531, 2282, 6589, 3720, 3475}

$$2ix\text{PolyLog}\left(2,e^{2ix}\right)-\text{PolyLog}\left(3,e^{2ix}\right)+\frac{2ix^3}{3}-\frac{3x^2}{4}-2x^2\log\left(1-e^{2ix}\right)+\frac{1}{2}x^2\sin^2(x)-\frac{1}{2}x^2\cot^2(x)-\frac{\sin^2(x)}{4}-x\cot$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x^2)/4 + ((2*I)/3)*x^3 - x*Cot[x] - (x^2*Cot[x]^2)/2 - 2*x^2*Log[1 - E^
((2*I)*x)] + Log[Sin[x]] + (2*I)*x*PolyLog[2, E^((2*I)*x)] - PolyLog[3, E^(
(2*I)*x)] + (x*Cos[x]*Sin[x])/2 - Sin[x]^2/4 + (x^2*Sin[x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int x^2\cos^2(x)\cot^3(x)\,dx = -\int x^2\cos^2(x)\cot(x)\,dx + \int x^2\cot^3(x)\,dx$$

$$= -\frac{1}{2}x^2\cot^2(x) - 2\int x^2\cot(x)\,dx + \int x\cot^2(x)\,dx + \int x^2\cos(x)\sin(x)\,dx$$

$$= -x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \frac{1}{2}x^2\sin^2(x) - 2\left(-\frac{ix^3}{3} - 2i\int\frac{e^{2ix}x^2}{1-e^{2ix}}\,dx\right) - \int x\,dx + \int \text{co}$$

$$= -\frac{x^2}{2} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2($$

$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2$$

$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) + \frac{1}{2}x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} + \frac{1}{2}x^2\sin^2$$

$$= -\frac{3x^2}{4} - x\cot(x) - \frac{1}{2}x^2\cot^2(x) + \log(\sin(x)) - 2\left(-\frac{ix^3}{3} + x^2\log\left(1-e^{2ix}\right) - ix\text{Li}_2\left(e^{2i\right.\right.$$

Mathematica [A] time = 0.33, size = 108, normalized size = 1.02

$$-2ix\text{Li}_2\left(e^{-2ix}\right) - \text{Li}_3\left(e^{-2ix}\right) - \frac{2ix^3}{3} - 2x^2\log\left(1-e^{-2ix}\right) - \frac{1}{4}x^2\cos(2x) - \frac{1}{2}x^2\csc^2(x) + \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x) - x\cot(x$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (I/12)*Pi^3 - ((2*I)/3)*x^3 + Cos[2*x]/8 - (x^2*Cos[2*x])/4 - x*Cot[x] - (x
^2*Csc[x]^2)/2 - 2*x^2*Log[1 - E^((-2*I)*x)] + Log[Sin[x]] - (2*I)*x*PolyLo
g[2, E^((-2*I)*x)] - PolyLog[3, E^((-2*I)*x)] + (x*Sin[2*x])/4
```

fricas [C] time = 0.54, size = 370, normalized size = 3.49

$$-\frac{2\left(2x^2-1\right)\cos(x)^4-3\left(2x^2-1\right)\cos(x)^2-2x^2-\left(16i\,x\cos(x)^2-16i\,x\right)\mathrm{Li}_2\left(\cos(x)+i\,\sin(x)\right)-\left(-16i\,x\cos(x)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*(2*x^2 - 1)*cos(x)^4 - 3*(2*x^2 - 1)*cos(x)^2 - 2*x^2 - (16*I*x*cos
(x)^2 - 16*I*x)*dilog(cos(x) + I*sin(x)) - (-16*I*x*cos(x)^2 + 16*I*x)*dilo
g(cos(x) - I*sin(x)) - (-16*I*x*cos(x)^2 + 16*I*x)*dilog(-cos(x) + I*sin(x)
) - (16*I*x*cos(x)^2 - 16*I*x)*dilog(-cos(x) - I*sin(x)) + 4*((2*x^2 - 1)*c
os(x)^2 - 2*x^2 + 1)*log(cos(x) + I*sin(x) + 1) + 4*((2*x^2 - 1)*cos(x)^2 -
 2*x^2 + 1)*log(cos(x) - I*sin(x) + 1) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) +
 1/2*I*sin(x) + 1/2) - 4*(cos(x)^2 - 1)*log(-1/2*cos(x) - 1/2*I*sin(x) + 1/
2) + 8*(x^2*cos(x)^2 - x^2)*log(-cos(x) + I*sin(x) + 1) + 8*(x^2*cos(x)^2 -
 x^2)*log(-cos(x) - I*sin(x) + 1) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) + I
*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, cos(x) - I*sin(x)) + 16*(cos(x)^2 -
 1)*polylog(3, -cos(x) + I*sin(x)) + 16*(cos(x)^2 - 1)*polylog(3, -cos(x) -
 I*sin(x)) - 4*(x*cos(x)^3 + x*cos(x))*sin(x) - 1)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x^2*cos(x)^2*cot(x)^3, x)
```

maple [A] time = 0.16, size = 170, normalized size = 1.60

$$\frac{2ix^3}{3}-\frac{\left(2x^2+2ix-1\right)e^{2ix}}{16}-\frac{\left(2x^2-2ix-1\right)e^{-2ix}}{16}+\frac{2x\left(x\,e^{2ix}-ie^{2ix}+i\right)}{\left(e^{2ix}-1\right)^2}+\ln\left(1+e^{ix}\right)+\ln\left(e^{ix}-1\right)-2\ln\left(e^{ix}\right)-2x^2\,\mathrm{l}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x)
```

```
[Out] 2/3*I*x^3-1/16*(2*I*x+2*x^2-1)*exp(2*I*x)-1/16*(-2*I*x+2*x^2-1)*exp(-2*I*x)
+2*x*(x*exp(2*I*x)-I*exp(2*I*x)+I)/(exp(2*I*x)-1)^2+ln(1+exp(I*x))+ln(exp(I
*x)-1)-2*ln(exp(I*x))-2*x^2*ln(1+exp(I*x))+4*I*x*polylog(2,-exp(I*x))-4*pol
```

```
ylog(3,-exp(I*x))-2*x^2*ln(1-exp(I*x))+4*I*x*polylog(2,exp(I*x))-4*polylog(
3,exp(I*x))
```

maxima [B] time = 0.75, size = 2855, normalized size = 26.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -(3*(2*x^2 + 2*I*x - 1)*cos(6*x)^2 - (-64*I*x^3 - 24*x^2 + 168*I*x + 12)*co
s(4*x)^2 - (-64*I*x^3 + 84*x^2 + 96*I*x + 6)*cos(2*x)^2 - 3*(2*x^2 + 2*I*x
- 1)*sin(6*x)^2 - (64*I*x^3 + 24*x^2 - 168*I*x - 12)*sin(4*x)^2 - (64*I*x^3
 - 84*x^2 - 96*I*x - 6)*sin(2*x)^2 + 6*x^2 - ((192*I*x^2 - 96*I)*cos(4*x)^2
 + (192*I*x^2 - 96*I)*cos(2*x)^2 + (-192*I*x^2 + 96*I)*sin(4*x)^2 + (-192*I
*x^2 + 96*I)*sin(2*x)^2 + (-96*I*x^2 + (-96*I*x^2 + 48*I)*cos(4*x) + (192*I
*x^2 - 96*I)*cos(2*x) + 48*(2*x^2 - 1)*sin(4*x) - 96*(2*x^2 - 1)*sin(2*x) +
 48*I)*cos(6*x) + (192*I*x^2 + (-480*I*x^2 + 240*I)*cos(2*x) + 240*(2*x^2 -
 1)*sin(2*x) - 96*I)*cos(4*x) + (-96*I*x^2 + 48*I)*cos(2*x) + (96*x^2 + 48*
(2*x^2 - 1)*cos(4*x) - 96*(2*x^2 - 1)*cos(2*x) + (96*I*x^2 - 48*I)*sin(4*x)
 + (-192*I*x^2 + 96*I)*sin(2*x) - 48)*sin(6*x) - (192*x^2 + 192*(2*x^2 - 1)
*cos(4*x) - 240*(2*x^2 - 1)*cos(2*x) - (480*I*x^2 - 240*I)*sin(2*x) - 96)*s
in(4*x) + 48*(2*x^2 - 4*(2*x^2 - 1)*cos(2*x) - 1)*sin(2*x))*arctan2(sin(x),
 cos(x) + 1) - ((48*I*cos(4*x) - 96*I*cos(2*x) - 48*sin(4*x) + 96*sin(2*x)
+ 48*I)*cos(6*x) + (240*I*cos(2*x) - 240*sin(2*x) - 96*I)*cos(4*x) - 96*I*c
os(4*x)^2 - 96*I*cos(2*x)^2 - (48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) -
96*I*sin(2*x) + 48)*sin(6*x) + (192*cos(4*x) - 240*cos(2*x) - 240*I*sin(2*x
) + 96)*sin(4*x) + 96*I*sin(4*x)^2 + 48*(4*cos(2*x) - 1)*sin(2*x) + 96*I*si
n(2*x)^2 + 48*I*cos(2*x))*arctan2(sin(x), cos(x) - 1) - (-192*I*x^2*cos(4*x
)^2 - 192*I*x^2*cos(2*x)^2 + 192*I*x^2*sin(4*x)^2 + 192*I*x^2*sin(2*x)^2 +
96*I*x^2*cos(2*x) + (96*I*x^2*cos(4*x) - 192*I*x^2*cos(2*x) - 96*x^2*sin(4*
x) + 192*x^2*sin(2*x) + 96*I*x^2)*cos(6*x) + (480*I*x^2*cos(2*x) - 480*x^2*
sin(2*x) - 192*I*x^2)*cos(4*x) - (96*x^2*cos(4*x) - 192*x^2*cos(2*x) + 96*I
*x^2*sin(4*x) - 192*I*x^2*sin(2*x) + 96*x^2)*sin(6*x) + 96*(4*x^2*cos(4*x)
- 5*x^2*cos(2*x) - 5*I*x^2*sin(2*x) + 2*x^2)*sin(4*x) + 96*(4*x^2*cos(2*x)
- x^2)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (32*I*x^3 + 12*x^2 + (32*I*
x^3 + 24*x^2 - 72*I*x - 12)*cos(4*x) + (-64*I*x^3 + 78*x^2 + 90*I*x + 9)*co
s(2*x) - (32*x^3 - 24*I*x^2 - 72*x + 12*I)*sin(4*x) + (64*x^3 + 78*I*x^2 -
90*x + 9*I)*sin(2*x) - 12*I*x - 6)*cos(6*x) - (-64*I*x^3 - 30*x^2 + (160*I*
x^3 - 156*x^2 - 276*I*x - 18)*cos(2*x) - (160*x^3 + 156*I*x^2 - 276*x + 18*
I)*sin(2*x) + 30*I*x + 15)*cos(4*x) - (32*I*x^3 + 24*x^2 - 24*I*x - 12)*cos
(2*x) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384*I*x*sin(4*x)^2 + 38
4*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x) - 192*x*sin(4*x) +
384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x) - 960*x*sin(2*x) - 3
84*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) - 384*x*cos(2*x) + 19
```

```
2*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + 192*(4*x*cos(4*x) - 5
*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + 192*(4*x*cos(2*x) - x)*sin(2
*x))*dilog(-e^(I*x)) - (-384*I*x*cos(4*x)^2 - 384*I*x*cos(2*x)^2 + 384*I*x*
sin(4*x)^2 + 384*I*x*sin(2*x)^2 + (192*I*x*cos(4*x) - 384*I*x*cos(2*x) - 19
2*x*sin(4*x) + 384*x*sin(2*x) + 192*I*x)*cos(6*x) + (960*I*x*cos(2*x) - 960
*x*sin(2*x) - 384*I*x)*cos(4*x) + 192*I*x*cos(2*x) - (192*x*cos(4*x) - 384*
x*cos(2*x) + 192*I*x*sin(4*x) - 384*I*x*sin(2*x) + 192*x)*sin(6*x) + 192*(4
*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) + 192*(4*x*cos(
2*x) - x)*sin(2*x))*dilog(e^(I*x)) - (48*(2*x^2 - 1)*cos(4*x)^2 + 48*(2*x^2
 - 1)*cos(2*x)^2 - 48*(2*x^2 - 1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(2*x)^2 -
(48*x^2 + 24*(2*x^2 - 1)*cos(4*x) - 48*(2*x^2 - 1)*cos(2*x) - (-48*I*x^2 +
24*I)*sin(4*x) - (96*I*x^2 - 48*I)*sin(2*x) - 24)*cos(6*x) + (96*x^2 - 120*
(2*x^2 - 1)*cos(2*x) + (-240*I*x^2 + 120*I)*sin(2*x) - 48)*cos(4*x) - 24*(2
*x^2 - 1)*cos(2*x) + (-48*I*x^2 + (-48*I*x^2 + 24*I)*cos(4*x) + (96*I*x^2 -
 48*I)*cos(2*x) + 24*(2*x^2 - 1)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x) + 24*I)
*sin(6*x) + (96*I*x^2 + (192*I*x^2 - 96*I)*cos(4*x) + (-240*I*x^2 + 120*I)*
cos(2*x) + 120*(2*x^2 - 1)*sin(2*x) - 48*I)*sin(4*x) + (-48*I*x^2 + (192*I*
x^2 - 96*I)*cos(2*x) + 24*I)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x) +
 1) - (48*(2*x^2 - 1)*cos(4*x)^2 + 48*(2*x^2 - 1)*cos(2*x)^2 - 48*(2*x^2 -
1)*sin(4*x)^2 - 48*(2*x^2 - 1)*sin(2*x)^2 - (48*x^2 + 24*(2*x^2 - 1)*cos(4*
x) - 48*(2*x^2 - 1)*cos(2*x) - (-48*I*x^2 + 24*I)*sin(4*x) - (96*I*x^2 - 48
*I)*sin(2*x) - 24)*cos(6*x) + (96*x^2 - 120*(2*x^2 - 1)*cos(2*x) + (-240*I*
x^2 + 120*I)*sin(2*x) - 48)*cos(4*x) - 24*(2*x^2 - 1)*cos(2*x) + (-48*I*x^2
 + (-48*I*x^2 + 24*I)*cos(4*x) + (96*I*x^2 - 48*I)*cos(2*x) + 24*(2*x^2 - 1
)*sin(4*x) - 48*(2*x^2 - 1)*sin(2*x) + 24*I)*sin(6*x) + (96*I*x^2 + (192*I*
x^2 - 96*I)*cos(4*x) + (-240*I*x^2 + 120*I)*cos(2*x) + 120*(2*x^2 - 1)*sin(
2*x) - 48*I)*sin(4*x) + (-48*I*x^2 + (192*I*x^2 - 96*I)*cos(2*x) + 24*I)*si
n(2*x))*log(cos(x)^2 + sin(x)^2 - 2*cos(x) + 1) + ((192*cos(4*x) - 384*cos(
2*x) + 192*I*sin(4*x) - 384*I*sin(2*x) + 192)*cos(6*x) + (960*cos(2*x) + 96
0*I*sin(2*x) - 384)*cos(4*x) - 384*cos(4*x)^2 - 384*cos(2*x)^2 - (-192*I*co
s(4*x) + 384*I*cos(2*x) + 192*sin(4*x) - 384*sin(2*x) - 192*I)*sin(6*x) - (
768*I*cos(4*x) - 960*I*cos(2*x) + 960*sin(2*x) + 384*I)*sin(4*x) + 384*sin(
4*x)^2 - (768*I*cos(2*x) - 192*I)*sin(2*x) + 384*sin(2*x)^2 + 192*cos(2*x))
*polylog(3, -e^(I*x)) + ((192*cos(4*x) - 384*cos(2*x) + 192*I*sin(4*x) - 38
4*I*sin(2*x) + 192)*cos(6*x) + (960*cos(2*x) + 960*I*sin(2*x) - 384)*cos(4*
x) - 384*cos(4*x)^2 - 384*cos(2*x)^2 - (-192*I*cos(4*x) + 384*I*cos(2*x) +
192*sin(4*x) - 384*sin(2*x) - 192*I)*sin(6*x) - (768*I*cos(4*x) - 960*I*cos
(2*x) + 960*sin(2*x) + 384*I)*sin(4*x) + 384*sin(4*x)^2 - (768*I*cos(2*x) -
 192*I)*sin(2*x) + 384*sin(2*x)^2 + 192*cos(2*x))*polylog(3, e^(I*x)) + (32
*x^3 - 12*I*x^2 - (-12*I*x^2 + 12*x + 6*I)*cos(6*x) + (32*x^3 - 24*I*x^2 -
72*x + 12*I)*cos(4*x) - (64*x^3 + 78*I*x^2 - 90*x + 9*I)*cos(2*x) - (-32*I*
x^3 - 24*x^2 + 72*I*x + 12)*sin(4*x) - (64*I*x^3 - 78*x^2 - 90*I*x - 9)*sin
(2*x) - 12*x + 6*I)*sin(6*x) - (64*x^3 - 30*I*x^2 + (128*x^3 - 48*I*x^2 - 3
36*x + 24*I)*cos(4*x) - (160*x^3 + 156*I*x^2 - 276*x + 18*I)*cos(2*x) + (-1
60*I*x^3 + 156*x^2 + 276*I*x + 18)*sin(2*x) - 30*x + 15*I)*sin(4*x) + (32*x
```

```
^3 - 24*I*x^2 - (128*x^3 + 168*I*x^2 - 192*x + 12*I)*cos(2*x) - 24*x + 12*I
)*sin(2*x) - 6*I*x - 3)/((48*cos(4*x) - 96*cos(2*x) + 48*I*sin(4*x) - 96*I*
sin(2*x) + 48)*cos(6*x) + (240*cos(2*x) + 240*I*sin(2*x) - 96)*cos(4*x) - 9
6*cos(4*x)^2 - 96*cos(2*x)^2 - (-48*I*cos(4*x) + 96*I*cos(2*x) + 48*sin(4*x
) - 96*sin(2*x) - 48*I)*sin(6*x) - (192*I*cos(4*x) - 240*I*cos(2*x) + 240*s
in(2*x) + 96*I)*sin(4*x) + 96*sin(4*x)^2 - (192*I*cos(2*x) - 48*I)*sin(2*x)
 + 96*sin(2*x)^2 + 48*cos(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \cos(x)^2 \cot(x)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x^2*cos(x)^2*cot(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cos^2(x) \cot^3(x) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x**2*cos(x)**2*cot(x)**3, x)
```

3.207 $\int x\cos^2(x)\cot^3(x)\,dx$

Optimal. Leaf size=73

$$i\mathrm{Li}_2\left(e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

```
[Out] -3/4*x+I*x^2-1/2*cot(x)-1/2*x*cot(x)^2-2*x*ln(1-exp(2*I*x))+I*polylog(2,exp
(2*I*x))+1/4*cos(x)*sin(x)+1/2*x*sin(x)^2
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 10, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4408, 3443, 2635, 8, 3717, 2190, 2279, 2391, 3720, 3473}

$$i\mathrm{PolyLog}\left(2,e^{2ix}\right)+ix^2-\frac{3x}{4}-2x\log\left(1-e^{2ix}\right)+\frac{1}{2}x\sin^2(x)-\frac{1}{2}x\cot^2(x)-\frac{\cot(x)}{2}+\frac{1}{4}\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] (-3*x)/4 + I*x^2 - Cot[x]/2 - (x*Cot[x]^2)/2 - 2*x*Log[1 - E^((2*I)*x)] + I
*PolyLog[2, E^((2*I)*x)] + (Cos[x]*Sin[x])/4 + (x*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int x\cos^2(x)\cot^3(x)\,dx = -\int x\cos^2(x)\cot(x)\,dx + \int x\cot^3(x)\,dx$$
$$= -\frac{1}{2}x\cot^2(x) + \frac{1}{2}\int \cot^2(x)\,dx - 2\int x\cot(x)\,dx + \int x\cos(x)\sin(x)\,dx$$
$$= -\frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} - 2i\int \frac{e^{2ix}x}{1-e^{2ix}}\,dx\right) - \frac{\int 1\,dx}{2} - \frac{1}{2}\int \sin^2(x$$
$$= -\frac{x}{2} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - \frac{\int 1\,dx}{4} - 2\left(-\frac{ix^2}{2} + x\log\left(1\right.\right.$$
$$= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) + \frac{1}{4}\cos(x)\sin(x) + \frac{1}{2}x\sin^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right)\right. -$$
$$= -\frac{3x}{4} - \frac{\cot(x)}{2} - \frac{1}{2}x\cot^2(x) - 2\left(-\frac{ix^2}{2} + x\log\left(1-e^{2ix}\right) - \frac{1}{2}i\text{Li}_2\left(e^{2ix}\right)\right) + \frac{1}{4}\cos(x)\sin(x$$

Mathematica [A] time = 0.11, size = 62, normalized size = 0.85

$$\frac{1}{8}\left(8i\text{Li}_2\left(e^{2ix}\right) + 8ix^2 - 16x\log\left(1-e^{2ix}\right) + \sin(2x) - 2x\cos(2x) - 4\cot(x) - 4x\csc^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[x]^2*Cot[x]^3,x]
```

```
[Out] ((8*I)*x^2 - 2*x*Cos[2*x] - 4*Cot[x] - 4*x*Csc[x]^2 - 16*x*Log[1 - E^((2*I)
*x)] + (8*I)*PolyLog[2, E^((2*I)*x)] + Sin[2*x])/8
```

fricas [B] time = 0.48, size = 203, normalized size = 2.78

$$-\frac{2\,x\cos(x)^4 - 3\,x\cos(x)^2 - \left(4i\,\cos(x)^2 - 4i\right)\text{Li}_2\left(\cos(x) + i\,\sin(x)\right) - \left(-4i\,\cos(x)^2 + 4i\right)\text{Li}_2\left(\cos(x) - i\,\sin(x)\right) -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*x*cos(x)^4 - 3*x*cos(x)^2 - (4*I*cos(x)^2 - 4*I)*dilog(cos(x) + I*s
in(x)) - (-4*I*cos(x)^2 + 4*I)*dilog(cos(x) - I*sin(x)) - (-4*I*cos(x)^2 +
4*I)*dilog(-cos(x) + I*sin(x)) - (4*I*cos(x)^2 - 4*I)*dilog(-cos(x) - I*sin
(x)) + 4*(x*cos(x)^2 - x)*log(cos(x) + I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*l
og(cos(x) - I*sin(x) + 1) + 4*(x*cos(x)^2 - x)*log(-cos(x) + I*sin(x) + 1)
+ 4*(x*cos(x)^2 - x)*log(-cos(x) - I*sin(x) + 1) - (cos(x)^3 + cos(x))*sin(
x) - x)/(cos(x)^2 - 1)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(x)^2*cot(x)^3, x)
```

maple [A] time = 0.14, size = 109, normalized size = 1.49

$$ix^2-\frac{(i+2x)\,e^{2ix}}{16}-\frac{(2x-i)\,e^{-2ix}}{16}+\frac{2x\,e^{2ix}-ie^{2ix}+i}{\left(e^{2ix}-1\right)^2}-2x\ln\left(1+e^{ix}\right)-2x\ln\left(1-e^{ix}\right)+2i\operatorname{polylog}\left(2,-e^{ix}\right)+2i\operatorname{poly}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x)
```

```
[Out] I*x^2-1/16*(I+2*x)*exp(2*I*x)-1/16*(2*x-I)*exp(-2*I*x)+(2*x*exp(2*I*x)-I*ex
p(2*I*x)+I)/(exp(2*I*x)-1)^2-2*x*ln(1+exp(I*x))-2*x*ln(1-exp(I*x))+2*I*poly
log(2,-exp(I*x))+2*I*polylog(2,exp(I*x))
```

maxima [B] time = 0.59, size = 1739, normalized size = 23.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)^2*cot(x)^3,x, algorithm="maxima")
```

```
[Out] -((2*x + I)*cos(6*x)^2 - (-32*I*x^2 - 8*x - 4*I)*cos(4*x)^2 - (-32*I*x^2 +
28*x - 16*I)*cos(2*x)^2 - (2*x + I)*sin(6*x)^2 - (32*I*x^2 + 8*x + 4*I)*sin
(4*x)^2 - (32*I*x^2 - 28*x + 16*I)*sin(2*x)^2 - (64*I*x*cos(4*x)^2 + 64*I*x
*cos(2*x)^2 - 64*I*x*sin(4*x)^2 - 64*I*x*sin(2*x)^2 + (-32*I*x*cos(4*x) + 6
4*I*x*cos(2*x) + 32*x*sin(4*x) - 64*x*sin(2*x) - 32*I*x)*cos(6*x) + (-160*I
*x*cos(2*x) + 160*x*sin(2*x) + 64*I*x)*cos(4*x) - 32*I*x*cos(2*x) + (32*x*c
os(4*x) - 64*x*cos(2*x) + 32*I*x*sin(4*x) - 64*I*x*sin(2*x) + 32*x)*sin(6*x
) - 32*(4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*x) - 32*(
4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), cos(x) + 1) - (-64*I*x*cos(4*x)
^2 - 64*I*x*cos(2*x)^2 + 64*I*x*sin(4*x)^2 + 64*I*x*sin(2*x)^2 + (32*I*x*co
s(4*x) - 64*I*x*cos(2*x) - 32*x*sin(4*x) + 64*x*sin(2*x) + 32*I*x)*cos(6*x)
 + (160*I*x*cos(2*x) - 160*x*sin(2*x) - 64*I*x)*cos(4*x) + 32*I*x*cos(2*x)
- (32*x*cos(4*x) - 64*x*cos(2*x) + 32*I*x*sin(4*x) - 64*I*x*sin(2*x) + 32*x
)*sin(6*x) + 32*(4*x*cos(4*x) - 5*x*cos(2*x) - 5*I*x*sin(2*x) + 2*x)*sin(4*
```

```
x) + 32*(4*x*cos(2*x) - x)*sin(2*x))*arctan2(sin(x), -cos(x) + 1) - (16*I*x
^2 + (16*I*x^2 + 8*x + 4*I)*cos(4*x) + (-32*I*x^2 + 26*x - 17*I)*cos(2*x) -
 4*(4*x^2 - 2*I*x + 1)*sin(4*x) + (32*x^2 + 26*I*x + 17)*sin(2*x) + 4*x + 1
4*I)*cos(6*x) - (-32*I*x^2 + (80*I*x^2 - 52*x + 34*I)*cos(2*x) - 2*(40*x^2
+ 26*I*x + 17)*sin(2*x) - 10*x - 27*I)*cos(4*x) - (16*I*x^2 + 8*x + 12*I)*c
os(2*x) - ((32*I*cos(4*x) - 64*I*cos(2*x) - 32*sin(4*x) + 64*sin(2*x) + 32*
I)*cos(6*x) + (160*I*cos(2*x) - 160*sin(2*x) - 64*I)*cos(4*x) - 64*I*cos(4*
x)^2 - 64*I*cos(2*x)^2 - (32*cos(4*x) - 64*cos(2*x) + 32*I*sin(4*x) - 64*I*
sin(2*x) + 32)*sin(6*x) + (128*cos(4*x) - 160*cos(2*x) - 160*I*sin(2*x) + 6
4)*sin(4*x) + 64*I*sin(4*x)^2 + 32*(4*cos(2*x) - 1)*sin(2*x) + 64*I*sin(2*x
)^2 + 32*I*cos(2*x))*dilog(-e^(I*x)) - ((32*I*cos(4*x) - 64*I*cos(2*x) - 32
*sin(4*x) + 64*sin(2*x) + 32*I)*cos(6*x) + (160*I*cos(2*x) - 160*sin(2*x) -
 64*I)*cos(4*x) - 64*I*cos(4*x)^2 - 64*I*cos(2*x)^2 - (32*cos(4*x) - 64*cos
(2*x) + 32*I*sin(4*x) - 64*I*sin(2*x) + 32)*sin(6*x) + (128*cos(4*x) - 160*
cos(2*x) - 160*I*sin(2*x) + 64)*sin(4*x) + 64*I*sin(4*x)^2 + 32*(4*cos(2*x)
 - 1)*sin(2*x) + 64*I*sin(2*x)^2 + 32*I*cos(2*x))*dilog(e^(I*x)) - (32*x*co
s(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*x)^2 - (16*x*cos(
4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) + 16*x)*cos(6*x) -
 16*(5*x*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - 16*x*cos(2*x) + (-16*I
*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2*x) - 16*I*x)*sin
(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x) + 32*I*x)*sin(4*
x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + sin(x)^2 + 2*cos(x
) + 1) - (32*x*cos(4*x)^2 + 32*x*cos(2*x)^2 - 32*x*sin(4*x)^2 - 32*x*sin(2*
x)^2 - (16*x*cos(4*x) - 32*x*cos(2*x) + 16*I*x*sin(4*x) - 32*I*x*sin(2*x) +
 16*x)*cos(6*x) - 16*(5*x*cos(2*x) + 5*I*x*sin(2*x) - 2*x)*cos(4*x) - 16*x*
cos(2*x) + (-16*I*x*cos(4*x) + 32*I*x*cos(2*x) + 16*x*sin(4*x) - 32*x*sin(2
*x) - 16*I*x)*sin(6*x) + (64*I*x*cos(4*x) - 80*I*x*cos(2*x) + 80*x*sin(2*x)
 + 32*I*x)*sin(4*x) + (64*I*x*cos(2*x) - 16*I*x)*sin(2*x))*log(cos(x)^2 + s
in(x)^2 - 2*cos(x) + 1) + (16*x^2 + 2*(2*I*x - 1)*cos(6*x) + 4*(4*x^2 - 2*I
*x + 1)*cos(4*x) - (32*x^2 + 26*I*x + 17)*cos(2*x) - (-16*I*x^2 - 8*x - 4*I
)*sin(4*x) - (32*I*x^2 - 26*x + 17*I)*sin(2*x) - 4*I*x + 14)*sin(6*x) - (32
*x^2 + 8*(8*x^2 - 2*I*x + 1)*cos(4*x) - 2*(40*x^2 + 26*I*x + 17)*cos(2*x) +
 (-80*I*x^2 + 52*x - 34*I)*sin(2*x) - 10*I*x + 27)*sin(4*x) + 4*(4*x^2 - 2*
(8*x^2 + 7*I*x + 4)*cos(2*x) - 2*I*x + 3)*sin(2*x) + 2*x - I)/((16*cos(4*x)
 - 32*cos(2*x) + 16*I*sin(4*x) - 32*I*sin(2*x) + 16)*cos(6*x) + (80*cos(2*x
) + 80*I*sin(2*x) - 32)*cos(4*x) - 32*cos(4*x)^2 - 32*cos(2*x)^2 - (-16*I*c
os(4*x) + 32*I*cos(2*x) + 16*sin(4*x) - 32*sin(2*x) - 16*I)*sin(6*x) - (64*
I*cos(4*x) - 80*I*cos(2*x) + 80*sin(2*x) + 32*I)*sin(4*x) + 32*sin(4*x)^2 -
 (64*I*cos(2*x) - 16*I)*sin(2*x) + 32*sin(2*x)^2 + 16*cos(2*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos(x)^2\cot(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(x)^2*cot(x)^3,x)
```

```
[Out] int(x*cos(x)^2*cot(x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos^2(x)\cot^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(x)**2*cot(x)**3,x)
```

```
[Out] Integral(x*cos(x)**2*cot(x)**3, x)
```

3.208 $\int (c+dx)^m \tan(a+bx)\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\tan(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \tan(a+bx)\,dx = \int (c+dx)^m \tan(a+bx)\,dx$$

Mathematica [A] time = 2.55, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x], x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sec(bx+a)\sin(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec (bx+a) \sin (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec (bx+a) \sin (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec (bx+a) \sin (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin (a+b\,x)\,(c+d\,x)^m}{\cos (a+b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin (a+bx) \sec (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*sec(a + b*x), x)
```

3.209 $\int (c+dx)^4 \tan(a+bx)\,dx$

Optimal. Leaf size=158

$$\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+d}{}$$

```
[Out] 1/5*I*(d*x+c)^5/d-(d*x+c)^4*ln(1+exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*
I*(b*x+a)))/b^5
```

Rubi [A] time = 0.21, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \tan(a+bx)\,dx = \frac{i(c+dx)^5}{5d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^4}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(4d)\int (c+dx)^3 \log\left(1+e^{2i(a+bx)}\right)\,dx}{b}$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{\left(6id^2\right)\int (c$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$
$$= \frac{i(c+dx)^5}{5d} - \frac{(c+dx)^4 \log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx$$

Mathematica [A] time = 0.07, size = 157, normalized size = 0.99

$$\frac{2id(c+dx)^3 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{3d^2\left(2b^2(c+dx)^2 \text{Li}_3\left(-e^{2i(a+bx)}\right) + d\left(2ib(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right) - d\text{Li}_5\left(-e^{2i(a+bx)}\right)\right)\right)}{2b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Tan[a + b*x],x]
```

```
[Out] ((I/5)*(c + d*x)^5)/d - ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b + ((2*
I)*d*(c + d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(2*b^2*(c +
 d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c + d*x)*PolyLog[4,
-E^((2*I)*(a + b*x))] - d*PolyLog[5, -E^((2*I)*(a + b*x))])))/(2*b^5)
```

fricas [C] time = 0.57, size = 1402, normalized size = 8.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*polylog(5, I
*cos(b*x + a) - sin(b*x + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) + sin(b*x
 + a)) + 24*d^4*polylog(5, -I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*
```

```
x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(
b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3
*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3
*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 1
2*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) -
(b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log
(cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*
c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) + I) -
 (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b
^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x + a)
+ sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 +
4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 +
 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*
a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x
^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d -
6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^4*c^4 - 4*a*b^3*c^3*d
 + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(b
*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
in(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^
2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^
3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x +
a)))/b^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(b*x + a), x)
```

maple [B] time = 0.11, size = 616, normalized size = 3.90

$$-\frac{3c^2d^2\,\mathrm{polylog}\left(3,-e^{2i(bx+a)}\right)}{b^3}-\frac{3d^4\,\mathrm{polylog}\left(3,-e^{2i(bx+a)}\right)x^2}{b^3}-\frac{d^4\ln\left(1+e^{2i(bx+a)}\right)x^4}{b}-ic^4x+\frac{2c^4\ln\left(e^{i(bx+a)}\right)}{b}+\frac{id}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))-3
/b^3*c^2*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x
+a)))*x^2+1/5*I*d^4*x^5+I*c*d^3*x^4+8*I/b*a*c^3*d*x-12*I/b^2*a^2*c^2*d^2*x+
8*I/b^3*c*d^3*a^3*x+6*I/b^4*c*d^3*a^4-8*I/b^3*a^3*c^2*d^2-2*I/b^4*d^4*a^4*x
+4*I/b^2*a^2*c^3*d-4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^3*polyl
og(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-
I*c^4*x-1/b*c^4*ln(1+exp(2*I*(b*x+a)))+2/b*c^4*ln(exp(I*(b*x+a)))-1/b*d^4*l
n(1+exp(2*I*(b*x+a)))*x^4-8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))-8/b^4*c*d^3*a^3*
ln(exp(I*(b*x+a)))+12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))+2*I*c^2*d^2*x^3+2*
I*c^3*d*x^2-4/b*c^3*d*ln(1+exp(2*I*(b*x+a)))*x-6/b^3*c*d^3*polylog(3,-exp(2
*I*(b*x+a)))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-8/5*I/b^5*d^4*a^5-3*I
/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x
+a)))*x^3-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+2*I/b^2*c^3*d*polylog(
2,-exp(2*I*(b*x+a)))
```

maxima [B] time = 0.60, size = 792, normalized size = 5.01

$$-\frac{15\,c^4\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{60\,ac^3d\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{90\,a^2c^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}-\frac{60\,a^3cd^3\log\left(-\sin(bx+a)^2+1\right)}{b^3}+\frac{15}{\ }}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/30*(15*c^4*log(-sin(b*x + a)^2 + 1) - 60*a*c^3*d*log(-sin(b*x + a)^2 + 1
)/b + 90*a^2*c^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 60*a^3*c*d^3*log(-sin(b
*x + a)^2 + 1)/b^3 + 15*a^4*d^4*log(-sin(b*x + a)^2 + 1)/b^4 + 2*(-3*I*(b*x
 + a)^5*d^4 + (-15*I*b*c*d^3 + 15*I*a*d^4)*(b*x + a)^4 - 45*d^4*polylog(5,
-e^(2*I*b*x + 2*I*a)) + (-30*I*b^2*c^2*d^2 + 60*I*a*b*c*d^3 - 30*I*a^2*d^4)
*(b*x + a)^3 + (-30*I*b^3*c^3*d + 90*I*a*b^2*c^2*d^2 - 90*I*a^2*b*c*d^3 + 3
0*I*a^3*d^4)*(b*x + a)^2 + (30*I*(b*x + a)^4*d^4 + (80*I*b*c*d^3 - 80*I*a*d
^4)*(b*x + a)^3 + (90*I*b^2*c^2*d^2 - 180*I*a*b*c*d^3 + 90*I*a^2*d^4)*(b*x
+ a)^2 + (60*I*b^3*c^3*d - 180*I*a*b^2*c^2*d^2 + 180*I*a^2*b*c*d^3 - 60*I*a
^3*d^4)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (-30*I
*b^3*c^3*d + 90*I*a*b^2*c^2*d^2 - 90*I*a^2*b*c*d^3 - 60*I*(b*x + a)^3*d^4 +
 30*I*a^3*d^4 + (-120*I*b*c*d^3 + 120*I*a*d^4)*(b*x + a)^2 + (-90*I*b^2*c^2
*d^2 + 180*I*a*b*c*d^3 - 90*I*a^2*d^4)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a
)) + 5*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3
*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (60*I*b*c*d^3 + 90*I*(b*x + a)*d^4 - 60*I*a
*d^4)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 15*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 +
```

```
6*(b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -
e^(2*I*b*x + 2*I*a)))/b^4)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)\,{(c+d\,x)}^4}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
[Out] int((sin(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \sin(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(b*x+a),x)
[Out] Integral((c + d*x)**4*sin(a + b*x)*sec(a + b*x), x)
```

3.210 $\int (c+dx)^3 \tan(a+bx)\,dx$

Optimal. Leaf size=132

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+\cdots}{4\cdots}$$

```
[Out] 1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*poly
log(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b
^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.18, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{(c+dx)^3\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] ((I/4)*(c + d*x)^4)/d - ((c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3
*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*
x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E
^((2*I)*(a + b*x))])/b^4
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \tan(a+bx)\,dx = \frac{i(c+dx)^4}{4d} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{(3d)\int (c+dx)^2\log\left(1+e^{2i(a+bx)}\right)dx}{b}$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{\left(3id^2\right)\int (c$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx$$
$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3d^2(c+dx$$

Mathematica [A] time = 0.08, size = 126, normalized size = 0.95

$$\frac{1}{4}i\left(\frac{3d\left(2b^2(c+dx)^2\text{Li}_2\left(-e^{2i(a+bx)}\right)+d\left(2ib(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)-d\text{Li}_4\left(-e^{2i(a+bx)}\right)\right)\right)}{b^4} + \frac{4i(c+dx)^3\log\left(1+e^{2i\right.}}{b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x],x]
```

```
[Out] (I/4)*((c + d*x)^4/d + ((4*I)*(c + d*x)^3*Log[1 + E^((2*I)*(a + b*x))])/b +
 (3*d*(2*b^2*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))] + d*((2*I)*b*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] - d*PolyLog[4, -E^((2*I)*(a + b*x))]
)))/b^4)
```

fricas [C] time = 0.54, size = 970, normalized size = 7.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*
d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x +
 a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(I*cos(b*x
```

```
+ a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*
dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x
 - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*s
in(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) -
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) -
sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log
(-cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*
d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*poly
log(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a), x)
```

maple [B] time = 0.08, size = 423, normalized size = 3.20

$$\frac{3ic^2d\,a^2}{b^2}+\frac{3ic^2d\,x^2}{2}+\frac{2ia^3d^3x}{b^3}-ic^3x-\frac{4ic\,d^2a^3}{b^3}-\frac{3id^3\,\mathrm{polylog}\left(4,-\mathrm{e}^{2i(bx+a)}\right)}{4b^4}-\frac{c^3\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}-\frac{2d^3a^3\ln\left(\mathrm{e}^{i(bx+a)}\right.}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3*I/b^2*a^2*c^2*d-4*I/b^3*a^3*c*d^2+2*I/b
^3*d^3*a^3*x-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))
)-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I
*(b*x+a)))*x-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+2/b*c^3*ln(exp(I*(b
*x+a)))+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+3/2*I*c^2*d*x^2+6/b^3*
c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x+a)))-1/b*d^3*ln(1+
exp(2*I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-6*I/b^
```

```
2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))+
3/2*I/b^4*d^3*a^4-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I
*(b*x+a)))*x^2
```

maxima [B] time = 0.57, size = 490, normalized size = 3.71

$$-\frac{6\,c^3\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{18\,ac^2d\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{18\,a^2cd^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}-\frac{6\,a^3d^3\log\left(-\sin(bx+a)^2+1\right)}{b^3}+\frac{-3i\,(bx\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/12*(6*c^3*log(-sin(b*x + a)^2 + 1) - 18*a*c^2*d*log(-sin(b*x + a)^2 + 1)
/b + 18*a^2*c*d^2*log(-sin(b*x + a)^2 + 1)/b^2 - 6*a^3*d^3*log(-sin(b*x + a
)^2 + 1)/b^3 + (-3*I*(b*x + a)^4*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x +
a)^3 + 12*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) + (-18*I*b^2*c^2*d + 36*I*
a*b*c*d^2 - 18*I*a^2*d^3)*(b*x + a)^2 + (16*I*(b*x + a)^3*d^3 + (36*I*b*c*d
^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*
d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (-18*I*b^
2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 - 18*I*a^2*d^3 + (-36*I*b*c
*d^2 + 36*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + 2*(4*(b*x + a)^
3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
 2*a) + 1) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b
*x + 2*I*a)))/b^3)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)\,{\left(c+d\,x\right)}^3}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^3\sin\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*sin(a + b*x)*sec(a + b*x), x)
```

3.211 $\int (c+dx)^2 \tan(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{d^2 \text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

```
[Out] 1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,
-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.357, Rules used = {3719, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b + (I*d
*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d^2*PolyLog[3, -E^((2*I
)*(a + b*x))])/(2*b^3)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\tan(a+bx)\,dx &= \frac{i(c+dx)^3}{3d}-2i\int\frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx\\
&= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(2d)\int(c+dx)\log\left(1+e^{2i(a+bx)}\right)\,dx}{b}\\
&= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{\left(id^2\right)\int\text{Li}_2\left(-\right.}{b}\\
&= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\,\text{Subst}\left(\int\right.}{}\\
&= \frac{i(c+dx)^3}{3d}-\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{Li}_3\left(-e^{2i(a\right.}}{2b^3}\end{aligned}$$

Mathematica [A] time = 0.04, size = 100, normalized size = 1.04

$$\frac{2ib^2(c+dx)^2\left(b(c+dx)+3id\log\left(1+e^{2i(a+bx)}\right)\right)+6ibd^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)-3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*b^2*(c + d*x)^2*(b*(c + d*x) + (3*I)*d*Log[1 + E^((2*I)*(a + b*x))])
 + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - 3*d^3*PolyLog[3
, -E^((2*I)*(a + b*x))])/(6*b^3*d)
```

fricas [C] time = 0.50, size = 594, normalized size = 6.19

$$-\frac{2\,d^2\text{polylog}\left(3,i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+2\,d^2\text{polylog}\left(3,i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)+2\,d^2\text{polylog}\left(3,\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - (-2*I*b*d^2*x - 2
*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*
dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*
sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*si
n(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*c
os(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2
*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a
) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a)
 + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec\left(bx+a\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a), x)
```

maple [B] time = 0.06, size = 257, normalized size = 2.68

$$\frac{id^2x^3}{3}-\frac{4id^2a^3}{3b^3}+\frac{id^2\,\text{polylog}\left(2,-e^{2i(bx+a)}\right)x}{b^2}-\frac{c^2\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{2c^2\ln\left(e^{i(bx+a)}\right)}{b}+\frac{2d^2a^2\ln\left(e^{i(bx+a)}\right)}{b^3}+icd\,x^2-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/3*I*d^2*x^3+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-2*I/b^2*a^2*d^2*x-1/
b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+a)))+2/b^3*d^2*a^2*ln(ex
p(I*(b*x+a)))+2*I/b^2*a^2*c*d-I*c^2*x-4/3*I/b^3*a^3*d^2-1/b*d^2*ln(1+exp(2*
I*(b*x+a)))*x^2+I*c*d*x^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*
d*a*ln(exp(I*(b*x+a)))+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+4*I/b*a*c*d*x
-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x
```

maxima [B] time = 0.53, size = 280, normalized size = 2.92

$$-\frac{3\,c^2\log\left(-\sin\left(bx+a\right)^2+1\right)-\frac{6\,acd\log\left(-\sin(bx+a)^2+1\right)}{b}+\frac{3\,a^2d^2\log\left(-\sin(bx+a)^2+1\right)}{b^2}+\frac{-2i\,(bx+a)^3d^2+\left(-6i\,bcd+6i\,ad^2\right)(bx+a)^2+3\,\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^2*log(-sin(b*x + a)^2 + 1) - 6*a*c*d*log(-sin(b*x + a)^2 + 1)/b +
 3*a^2*d^2*log(-sin(b*x + a)^2 + 1)/b^2 + (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c
*d + 6*I*a*d^2)*(b*x + a)^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (6*I
*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) + (-6*I*b*c*d - 6*I*(b*x + a)*d^2 + 6*I*a*d^2)
*dilog(-e^(2*I*b*x + 2*I*a)) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
)/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^2\sin\left(a+bx\right)\sec\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*sec(a + b*x), x)
```

3.212 $\int (c+dx)\tan(a+bx)\,dx$

Optimal. Leaf size=66

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2
*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.09, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b + ((I/2)
*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\tan(a+bx)\,dx &= \frac{i(c+dx)^2}{2d}-2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx\\ &= \frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{d\int \log\left(1+e^{2i(a+bx)}\right)\,dx}{b}\\ &= \frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(id)\,\mathrm{Subst}\left(\int \frac{\log(1+x)}{x}\,dx,x,e^{2i(a+bx)}\right)}{2b^2}\\ &= \frac{i(c+dx)^2}{2d}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 70, normalized size = 1.06

$$\frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{c\log(\cos(a+bx))}{b}-\frac{dx\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{1}{2}idx^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x],x]
```

```
[Out] (I/2)*d*x^2 - (d*x*Log[1 + E^((2*I)*(a + b*x))])/b - (c*Log[Cos[a + b*x]])/
b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2
```

fricas [B] time = 0.49, size = 310, normalized size = 4.70

$$\frac{-i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)+i\,d\mathrm{Li}_2\left(-i\,\cos\left(bx+a\right)+\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(-I*c
os(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
 + I) - (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*
```

```
log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)
- (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*log(-
cos(b*x + a) + I*sin(b*x + a) + I) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(
b*x + a) + I))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\sec(bx + a)\sin(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a), x)
```

maple [B] time = 0.07, size = 123, normalized size = 1.86

$$\frac{id\,x^2}{2} - icx - \frac{c\ln\left(1 + e^{2i(bx+a)}\right)}{b} + \frac{2c\ln\left(e^{i(bx+a)}\right)}{b} + \frac{2idax}{b} + \frac{id\,a^2}{b^2} - \frac{d\ln\left(1 + e^{2i(bx+a)}\right)x}{b} + \frac{id\,\mathrm{polylog}\left(2, -e^{2i(bx+a)}\right)}{2b^2} - \frac{2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] 1/2*I*d*x^2-I*c*x-1/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a)))+2*I
/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*polylog(2,-exp(
2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.53, size = 114, normalized size = 1.73

$$-\frac{-i\,b^2dx^2 - 2i\,b^2cx + (2i\,bdx + 2i\,bc)\arctan\left(\sin(2\,bx + 2\,a), \cos(2\,bx + 2\,a) + 1\right) - i\,d\mathrm{Li}_2\left(-e^{(2i\,bx+2i\,a)}\right) + (bdx}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(-I*b^2*d*x^2 - 2*I*b^2*c*x + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x
+ 2*a), cos(2*b*x + 2*a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + (b*d*x +
b*c)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1))
/b^2
```

mupad [B] time = 1.57, size = 148, normalized size = 2.24

$$\frac{c\,\ln\left(\tan(a + b\,x)^2 + 1\right)}{2\,b} - \frac{d\,\left(\pi\,\ln(\cos(b\,x)) + \mathrm{polylog}\left(2, -e^{-a\,2i}\,e^{-b\,x\,2i}\right)1i - \pi\,\ln\left(e^{-a\,2i}\,e^{-b\,x\,2i} + 1\right) + 2\,a\,\ln\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] (c*log(tan(a + b*x)^2 + 1))/(2*b) - (d*(polylog(2, -exp(-a*2i)*exp(-b*x*2i)
)*1i - pi*log(exp(b*x*2i) + 1) - pi*log(exp(-a*2i)*exp(-b*x*2i) + 1) + 2*a*
log(exp(-a*2i)*exp(-b*x*2i) + 1) + pi*log(cos(b*x)) + b^2*x^2*1i - log(cos(
a + b*x))*(2*a - pi) + 2*b*x*log(exp(-a*2i)*exp(-b*x*2i) + 1) + a*b*x*2i))/
(2*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \sin(a + bx) \sec(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)*sec(a + b*x), x)
```

3.213 $\int \frac{\tan(a+bx)}{c+dx} dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x),x]
[Out] Defer[Int][Tan[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{c+dx}\,dx = \int \frac{\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 3.53, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x),x]
[Out] Integrate[Tan[a + b*x]/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

3.214 $\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx=\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 5.30, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Tan[a + b*x]/(c + d*x)^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sec(b*x + a)*sin(b*x + a)/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{\sin(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(sin(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

3.215 $\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=148

$$\text{Int}\left(\sec(a+bx)(c+dx)^m,x\right)+\frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{ib(c+dx)}{d}\right)}{2b}$$

```
[Out] 1/2*I*exp(I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)
/d)^m)-1/2*I*(d*x+c)^m*GAMMA(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d
*x+c)/d)^m)+Unintegrable((d*x+c)^m*sec(b*x+a),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((I/2)*E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/
(b*(((-I)*b*(c + d*x))/d)^m) - ((I/2)*(c + d*x)^m*Gamma[1 + m, (I*b*(c + d*
x))/d])/(b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x
)^m*Sec[a + b*x], x]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m \cos(a+bx)\,dx+\int (c+dx)^m \sec(a+bx)\,dx\\
&= -\left(\frac{1}{2}\int e^{-i(a+bx)}(c+dx)^m\,dx\right)-\frac{1}{2}\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+dx)^m \sec(a+bx)\,dx\\
&= \frac{ie^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}-\frac{ie^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{ib(c+dx)}{d}\right)}{2b}+\int (c+dx)^m \sec(a+bx)\,dx
\end{aligned}$$

Mathematica [A] time = 6.62, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x], x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)\left(dx+c\right)^{m}\sec\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(dx+c\right)^{m}\sec\left(bx+a\right)\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int\left(dx+c\right)^{m}\sec\left(bx+a\right)\left(\sin^{2}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(dx+c\right)^{m}\sec\left(bx+a\right)\sin\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sin\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)^{m}}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.216 $\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=275

$$-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\cos(a+bx)}{b^4}-\frac{6d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b+6*d^3*cos(b*x+a)/b^4-3*d*(d*x+c)^2*
cos(b*x+a)/b^2+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+
c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+
a)))/b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3-6*I*d^3*polylog(4,-I
*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*
sin(b*x+a)/b^3-(d*x+c)^3*sin(b*x+a)/b
```

Rubi [A] time = 0.21, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4407, 3296, 2638, 4181, 2531, 6609, 2282, 6589}

$$-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b + (6*d^3*Cos[a + b*x])/b^4 -
 (3*d*(c + d*x)^2*Cos[a + b*x])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*
E^(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^3 + (6*d^2*(c +
 d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*
(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 + (6*d^2*(
c + d*x)*Sin[a + b*x])/b^3 - ((c + d*x)^3*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\,dx + \int (c+dx)^3\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^3\sin(a+bx)}{b} - \frac{(3d)\int (c+dx)^2 \mathrm{lo}}{}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id(c+dx)^2\mathrm{Li}}{b^2}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)}{b^2} + \frac{3id(c+dx)^2\mathrm{Li}}{b^2}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx}{b^2}$$
$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{6d^3\cos(a+bx)}{b^4} - \frac{3d(c+dx)^2\cos(a+bx}{b^2}$$

Mathematica [B] time = 1.47, size = 557, normalized size = 2.03

$$-\frac{b^3c^3\sin(a+bx) + 2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right) + 3b^3c^2dx\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -((((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^2*c^2*d*Cos[a + b*x] - 6*d^3
*Cos[a + b*x] + 6*b^2*c*d^2*x*Cos[a + b*x] + 3*b^2*d^3*x^2*Cos[a + b*x] - 3
*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a
 + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 +
I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x
^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] + 6*b
*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a
 + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3,
I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)*d^3
*PolyLog[4, I*E^(I*(a + b*x))] + b^3*c^3*Sin[a + b*x] - 6*b*c*d^2*Sin[a + b
*x] + 3*b^3*c^2*d*x*Sin[a + b*x] - 6*b*d^3*x*Sin[a + b*x] + 3*b^3*c*d^2*x^2
*Sin[a + b*x] + b^3*d^3*x^3*Sin[a + b*x])/b^4)
```

fricas [C] time = 0.56, size = 1071, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4,
 I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) + sin(
b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a) + (-3*I*b^2*d^3*x^2
- 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (
-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) -
sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-
I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b
^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^3*c^3 - 3*a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^3*c^
3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x +
 a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a
^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b*d^3*x + b*c*d^2)*pol
ylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) - 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*
(b^3*c^2*d - 2*b*d^3)*x)*sin(b*x + a))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec (bx+a) \sin (bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^2, x)
```

maple [B] time = 0.36, size = 901, normalized size = 3.28

$$-\frac{6id^2c \operatorname{polylog}\left(2, i e^{i(bx+a)}\right) x}{b^2}+\frac{6id^2c \operatorname{polylog}\left(2, -i e^{i(bx+a)}\right) x}{b^2}-\frac{3id^3 \operatorname{polylog}\left(2, i e^{i(bx+a)}\right) x^2}{b^2}+\frac{3id^3 \operatorname{polylog}\left(2, -\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*
b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))
+6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)
))/b^4-1/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,I*exp(I*(b*
x+a)))*x+1/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*
x^3-6/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+6/b^3*d^2*c*polylog(3,I*exp(I*
(b*x+a)))-6/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp
(I*(b*x+a)))-2*I/b*c^3*arctan(exp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*exp(I*(b*x+a
)))*x^2-3/b*d^2*c*ln(1+I*exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)
))*x+3/b^2*c^2*d*ln(1-I*exp(I*(b*x+a)))*a+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x
+a)))-3/b*c^2*d*ln(1+I*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))
*a-3/b^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*
(b*x+a)))+2*I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*
exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*pol
ylog(2,-I*exp(I*(b*x+a)))*x^2-1/2*I*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*
d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2
*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-6*I/b^2*d^2*c*polylog(2,I*exp(I*(b*x+a)))*x
-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*a*arctan(exp(I*(b*x
+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x
```

maxima [B] time = 0.67, size = 924, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 3*a*c^2*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b + 3*a^2*c*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x
 + a))/b^2 - a^3*d^3*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin
(b*x + a))/b^3 + (12*I*d^3*polylog(4, I*e^(I*b*x + I*a)) - 12*I*d^3*polylog
(4, -I*e^(I*b*x + I*a)) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3
)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*
arctan2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c
*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*
d^3)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 -
6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*dilog(I*e^(I*b*x + I*
a)) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 +
 (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*dilog(-I*e^(I*b*x + I*a)) + ((b*x +
 a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 +
```

```
a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) +
1) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin
(b*x + a) + 1) + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, I*e^(I*b*x
 + I*a)) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -I*e^(I*b*x + I*
a)) - 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x +
 a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(b*x + a)
)/b^3)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(a+b\,x)^2\,(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x)^3)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\sin^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*sin(a + b*x)**2*sec(a + b*x), x)
```

3.217 $\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=186

$$-\frac{2d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\sin(a+bx)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{2}{}$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-2*d*(d*x+c)*cos(b*x+a)/b^2+2*I*d*(d
*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x
+a)))/b^2-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b
*x+a)))/b^3+2*d^2*sin(b*x+a)/b^3-(d*x+c)^2*sin(b*x+a)/b
```

Rubi [A] time = 0.15, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {4407, 3296, 2637, 4181, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{2d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{PolyLog}\left(3\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (2*d*(c + d*x)*Cos[a + b*x
])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((2*I)*
d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (2*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (2*d^2*Sin
[a + b*x])/b^3 - ((c + d*x)^2*Sin[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\,dx + \int (c+dx)^2\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)^2\sin(a+bx)}{b} - \frac{(2d)\int (c+dx)\log}{}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2}{b^2}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2}{b^2}$$
$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d(c+dx)\cos(a+bx)}{b^2} + \frac{2id(c+dx)\mathrm{Li}_2}{b^2}$$

Mathematica [A] time = 0.83, size = 315, normalized size = 1.69

$$-\frac{b^2c^2\sin(a+bx) + 2ib^2c^2\tan^{-1}\left(e^{i(a+bx)}\right) - 2b^2cdx\log\left(1-ie^{i(a+bx)}\right) + 2b^2cdx\log\left(1+ie^{i(a+bx)}\right) + 2b^2cdx\sin(a+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -((((2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b*c*d*Cos[a + b*x] + 2*b*d^2*x
*Cos[a + b*x] - 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1
- I*E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + b^2*d^2*x^2
*Log[1 + I*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a +
 b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 2*d^2*PolyLog
[3, (-I)*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, I*E^(I*(a + b*x))] + b^2*c^2*S
in[a + b*x] - 2*d^2*Sin[a + b*x] + 2*b^2*c*d*x*Sin[a + b*x] + b^2*d^2*x^2*S
in[a + b*x])/b^3)
```

fricas [C] time = 0.52, size = 656, normalized size = 3.53

$$-\frac{2\,d^2\mathrm{polylog}\left(3, i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right) - 2\,d^2\mathrm{polylog}\left(3, i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right) + 2\,d^2\mathrm{polylog}\left(3, -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, I*
cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) + sin(b*x +
 a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 4*(b*d^2*x + b*c*
```

```
d)*cos(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x
 + a)) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) -
(2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (2*I*b*d^
2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c
*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d
^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x +
a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2 - 2*d^2)*sin(b*x + a))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec(bx+a)\sin(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^2, x)
```

maple [B] time = 0.26, size = 512, normalized size = 2.75

$$\frac{i\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)e^{i(bx+a)}}{2b^3}+\frac{4icda\arctan\left(e^{i(bx+a)}\right)}{b^2}-\frac{2cd\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{2d^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] 1/2*I*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp
(I*(b*x+a))+4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))-2/b*c*d*ln(1+I*exp(I*(b*x+
a)))*x-2*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-1/2*I*(d^2*x^2*b^2+2*b^2*c*d*
x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3*exp(-I*(b*x+a))-2*I/b^2*c*d*poly
log(2,I*exp(I*(b*x+a)))-2*I/b*c^2*arctan(exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp
(I*(b*x+a)))*x^2+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-2*I/b^2*d^2*polylog(
2,I*exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+2/b*c*d*ln(1-I*exp
(I*(b*x+a)))*x+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-2/b^2*c*d*ln(1+I*
exp(I*(b*x+a)))*a+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))-2*I/b^3*d^2*a^2*arct
an(exp(I*(b*x+a)))+2*I/b^2*c*d*polylog(2,-I*exp(I*(b*x+a)))+1/b*d^2*ln(1-I*
exp(I*(b*x+a)))*x^2-1/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))
```

maxima [B] time = 0.60, size = 510, normalized size = 2.74

$$c^2\Big(\log\left(\sin\left(bx+a\right)+1\right)-\log\left(\sin\left(bx+a\right)-1\right)-2\,\sin\left(bx+a\right)\Big)-\frac{2\,acd\Big(\log(\sin(bx+a)+1)-\log(\sin(bx+a)-1)-2\,\sin(bx+a)\Big)}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a)) -
 2*a*c*d*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))/b
 + a^2*d^2*(log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1) - 2*sin(b*x + a))
/b^2 + (4*d^2*polylog(3, I*e^(I*b*x + I*a)) - 4*d^2*polylog(3, -I*e^(I*b*x
+ I*a)) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*arcta
n2(cos(b*x + a), sin(b*x + a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - 4*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(b*x + a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*
I*a*d^2)*dilog(I*e^(I*b*x + I*a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*
d^2)*dilog(-I*e^(I*b*x + I*a)) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - ((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*sin(b*x + a) + 1) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) -
2*d^2)*sin(b*x + a))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2\,{\left(c+d\,x\right)}^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^2*(c + d*x)^2)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^{2}\sin^{2}\left(a+bx\right)\sec\left(a+bx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)**2*sec(a + b*x), x)
```

3.218 $\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{id\operatorname{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\operatorname{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] -2*I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-d*cos(b*x+a)/b^2+I*d*polylog(2,-I*exp
(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-(d*x+c)*sin(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4407, 3296, 2638, 4181, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\cos(a+bx)}{b^2}-\frac{(c+dx)\sin(a+bx)}{b}-\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - (d*Cos[a + b*x])/b^2 + (I*d*
PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, I*E^(I*(a + b*x))])
/b^2 - ((c + d*x)*Sin[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\sin(a+bx)\tan(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\,dx + \int (c+dx)\sec(a+bx)\,dx$$
$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{(c+dx)\sin(a+bx)}{b} - \frac{d\int \log\left(1-ie^{i(a+bx)}\right)}{b}$$
$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} - \frac{(c+dx)\sin(a+bx)}{b} + \frac{(id)}{}$$
$$= -\frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)}{b^2} + \frac{id\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2} - \frac{id\mathrm{Li}_2\left(\right.}{b}$$

Mathematica [B] time = 0.40, size = 213, normalized size = 2.07

$$\frac{d\left(i\left(\mathrm{Li}_2\left(-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\mathrm{Li}_2\left(e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)+\left(-a-bx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)-\left(\frac{\pi}{2}-\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (c*ArcTanh[Sin[a + b*x]])/b + (d*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a + Pi/
2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[(-a +
Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2, E^(I
```

```
*(-a + Pi/2 - b*x))]])))/b^2 - (d*Cos[b*x]*(Cos[a] + b*x*Sin[a]))/b^2 - (d*(
b*x*Cos[a] - Sin[a])*Sin[b*x]])/b^2 - (c*Sin[a + b*x])/b
```

fricas [B] time = 1.04, size = 331, normalized size = 3.21

$$-\frac{2\,d\cos(bx+a)+i\,d\mathrm{Li}_2\,(i\,\cos(bx+a)+\sin(bx+a))+i\,d\mathrm{Li}_2\,(i\,\cos(bx+a)-\sin(bx+a))-i\,d\mathrm{Li}_2\,(-i\,\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cos(b*x + a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dil
og(I*cos(b*x + a) - sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a
)) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c - a*d)*log(cos(b*x +
a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) +
I) - (b*d*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*l
og(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a*d)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
- (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d*x + b*c)*sin(b*x + a))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\sec(bx+a)\sin(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^2, x)
```

maple [B] time = 0.06, size = 209, normalized size = 2.03

$$-\frac{d\sin(bx+a)\,x}{b}-\frac{d\cos(bx+a)}{b^2}-\frac{c\sin(bx+a)}{b}-\frac{d\ln\left(1+ie^{i(bx+a)}\right)x}{b}+\frac{d\ln\left(1-ie^{i(bx+a)}\right)x}{b}-\frac{id\,\mathrm{dilog}\left(1-ie^{i(bx+}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x)
```

```
[Out] -1/b*d*sin(b*x+a)*x-d*cos(b*x+a)/b^2-1/b*c*sin(b*x+a)-1/b*d*ln(1+I*exp(I*(b
*x+a)))*x+1/b*d*ln(1-I*exp(I*(b*x+a)))*x-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))-
1/b^2*d*ln(1+I*exp(I*(b*x+a)))*a+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a+I/b^2*d*d
ilog(1+I*exp(I*(b*x+a)))-1/b^2*d*a*ln(sec(b*x+a)+tan(b*x+a))+1/b*c*ln(sec(b
*x+a)+tan(b*x+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
[Out] int((sin(a + b*x)^2*(c + d*x))/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**2,x)
[Out] Integral((c + d*x)*sin(a + b*x)**2*sec(a + b*x), x)
```

3.219 $\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=69

$$\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -Ci(b*c/d+b*x)*cos(a-b*c/d)/d+Si(b*c/d+b*x)*sin(a-b*c/d)/d+Unintegrable(sec
(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -((Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d) + (Sin[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][Sec[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)+\sin\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\text{s}}{}\\ &= -\frac{\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)}{c+dx}\,dx\end{aligned}$$

Mathematica [A] time = 6.04, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sec\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\left(\sin^{2}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^{2}}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
[Out] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c),x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

3.220 $\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$\mathrm{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)+\frac{b\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\cos(a+bx)}{d(c+dx)}$$

```
[Out] cos(b*x+a)/d/(d*x+c)+b*cos(a-b*c/d)*Si(b*c/d+b*x)/d^2+b*Ci(b*c/d+b*x)*sin(a
-b*c/d)/d^2+Unintegrable(sec(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Cos[a + b*x]/(d*(c + d*x)) + (b*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d]
)/d^2 + (b*Cos[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int][Se
c[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\int \frac{\sin(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\\
&= \frac{\cos(a+bx)}{d(c+dx)}+\frac{b\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int \frac{\sec(}{(c-}
\end{aligned}$$

Mathematica [A] time = 7.08, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sec\left(bx+a\right)}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.42, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\left(\sin^{2}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a+b\,x)^2}{\cos (a+b\,x)\,\,{(c+d\,x)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(sin(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}{\left (a + b x \right)} \sec{\left (a + b x \right)}}{\left (c + d x\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

3.221 $\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=152

$$\text{Int}\left(\tan(a+bx)(c+dx)^m,x\right)+\frac{2^{-m-3}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-m-3}e^{-2i\left(a-\frac{bc}{d}\right)}(c+dx\ldots}{}$$

```
[Out] 2^(-3-m)*exp(2*I*(a-b*c/d))*(d*x+c)^m*GAMMA(1+m,-2*I*b*(d*x+c)/d)/b/((-I*b*
(d*x+c)/d)^m)+2^(-3-m)*(d*x+c)^m*GAMMA(1+m,2*I*b*(d*x+c)/d)/b/exp(2*I*(a-b*
c/d))/((I*b*(d*x+c)/d)^m)+Unintegrable((d*x+c)^m*tan(b*x+a),x)
```

Rubi [A] time = 0.17, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (2^(-3 - m)*E^((2*I)*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-2*I)*b*(c +
 d*x))/d])/(b*(((-I)*b*(c + d*x))/d)^m) + (2^(-3 - m)*(c + d*x)^m*Gamma[1 +
 m, ((2*I)*b*(c + d*x))/d])/(b*E^((2*I)*(a - (b*c)/d))*((I*b*(c + d*x))/d)^
m) + Defer[Int][(c + d*x)^m*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m\sin^2(a+bx)\tan(a+bx)\,dx &= -\int (c+dx)^m\cos(a+bx)\sin(a+bx)\,dx+\int (c+dx)^m\tan(a+bx)\,d\ldots\\ &= -\int \frac{1}{2}(c+dx)^m\sin(2a+2bx)\,dx+\int (c+dx)^m\tan(a+bx)\,dx\\ &= -\left(\frac{1}{2}\int (c+dx)^m\sin(2a+2bx)\,dx\right)+\int (c+dx)^m\tan(a+bx)\,dx\\ &= -\left(\frac{1}{4}i\int e^{-i(2a+2bx)}(c+dx)^m\,dx\right)+\frac{1}{4}i\int e^{i(2a+2bx)}(c+dx)^m\,dx+\int (c\ldots\\ &= \frac{2^{-3-m}e^{2i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2ib(c+dx)}{d}\right)}{b}+\frac{2^{-3-m}e^{-\ldots}}{}\end{aligned}$$

Mathematica [A] time = 7.81, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]^2*Tan[a + b*x], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(bx+a\right)^{2}-1\right)(dx+c)^{m}\sec\left(bx+a\right)\sin\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*(d*x + c)^m*sec(b*x + a)*sin(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx+c)^{m}\sec\left(bx+a\right)\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

maple [A] time = 0.25, size = 0, normalized size = 0.00

$$\int(dx+c)^{m}\sec\left(bx+a\right)\left(\sin^{3}\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
[Out] int((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int(dx+c)^{m}\sec\left(bx+a\right)\sin\left(bx+a\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*sin(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3\,(c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((sin(a + b*x)^3*(c + d*x)^m)/cos(a + b*x), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*sin(b*x+a)**3,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.222 $\int (c+dx)^3 \sin^2(a+bx) \tan(a+bx)\,dx$

Optimal. Leaf size=251

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^3\sin(a+bx)\cos(a+bx)}{8b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\sin^2(a+bx)}{4b^3}+\frac{3id(c\ldots}{}$$

```
[Out] -3/8*d^3*x/b^3+1/4*(d*x+c)^3/b+1/4*I*(d*x+c)^4/d-(d*x+c)^3*ln(1+exp(2*I*(b*
x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)
*polylog(3,-exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^
4+3/8*d^3*cos(b*x+a)*sin(b*x+a)/b^4-3/4*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b
^2+3/4*d^2*(d*x+c)*sin(b*x+a)^2/b^3-1/2*(d*x+c)^3*sin(b*x+a)^2/b
```

Rubi [A] time = 0.30, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {4407, 4404, 3311, 32, 2635, 8, 3719, 2190, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (-3*d^3*x)/(8*b^3) + (c + d*x)^3/(4*b) + ((I/4)*(c + d*x)^4)/d - ((c + d*x)
^3*Log[1 + E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E
^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))
])/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (3*d^3*
Cos[a + b*x]*Sin[a + b*x])/(8*b^4) - (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a +
b*x])/(4*b^2) + (3*d^2*(c + d*x)*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)^3*Sin
[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^3\tan(a+bx)\,dx$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^3}{1+e^{2i(a+bx)}}\,dx + \frac{(3d}{}$$

$$= \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\cos(a+bx)\sin}{4b^2}$$

$$= \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2\mathrm{Li}}{2b}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(}{}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(}{}$$

$$= -\frac{3d^3x}{8b^3} + \frac{(c+dx)^3}{4b} + \frac{i(c+dx)^4}{4d} - \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{3id(}{}$$

<u>**Mathematica**</u> **[B]** time = 7.12, size = 1720, normalized size = 6.85

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-1/4*I)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
 - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a
))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*P
olyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x
))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] - (c^3*
Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Co
s[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Co
t[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
- ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*
x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*
I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a
]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(64*b^4) - ((I/64)*S
in[2*a + 2*b*x])/b^4)*(8*b^3*c^3*Cos[a] - (12*I)*b^2*c^2*d*Cos[a] - 12*b*c*
d^2*Cos[a] + (6*I)*d^3*Cos[a] + 24*b^3*c^2*d*x*Cos[a] - (24*I)*b^2*c*d^2*x*
Cos[a] - 12*b*d^3*x*Cos[a] + 24*b^3*c*d^2*x^2*Cos[a] - (12*I)*b^2*d^3*x^2*C
```

```
os[a] + 8*b^3*d^3*x^3*Cos[a] + (32*I)*b^4*c^3*x*Cos[a + 2*b*x] + (48*I)*b^4
*c^2*d*x^2*Cos[a + 2*b*x] + (32*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] + (8*I)*b^4
*d^3*x^4*Cos[a + 2*b*x] - (32*I)*b^4*c^3*x*Cos[3*a + 2*b*x] - (48*I)*b^4*c^
2*d*x^2*Cos[3*a + 2*b*x] - (32*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] - (8*I)*b^
4*d^3*x^4*Cos[3*a + 2*b*x] + 4*b^3*c^3*Cos[3*a + 4*b*x] + (6*I)*b^2*c^2*d*C
os[3*a + 4*b*x] - 6*b*c*d^2*Cos[3*a + 4*b*x] - (3*I)*d^3*Cos[3*a + 4*b*x] +
 12*b^3*c^2*d*x*Cos[3*a + 4*b*x] + (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] - 6*
b*d^3*x*Cos[3*a + 4*b*x] + 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] + (6*I)*b^2*d^
3*x^2*Cos[3*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] + 4*b^3*c^3*Cos[5*a
 + 4*b*x] + (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] - 6*b*c*d^2*Cos[5*a + 4*b*x] -
 (3*I)*d^3*Cos[5*a + 4*b*x] + 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] + (12*I)*b^2*
c*d^2*x*Cos[5*a + 4*b*x] - 6*b*d^3*x*Cos[5*a + 4*b*x] + 12*b^3*c*d^2*x^2*Co
s[5*a + 4*b*x] + (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] + 4*b^3*d^3*x^3*Cos[5*a
 + 4*b*x] - 32*b^4*c^3*x*Sin[a + 2*b*x] - 48*b^4*c^2*d*x^2*Sin[a + 2*b*x] -
 32*b^4*c*d^2*x^3*Sin[a + 2*b*x] - 8*b^4*d^3*x^4*Sin[a + 2*b*x] + 32*b^4*c^
3*x*Sin[3*a + 2*b*x] + 48*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] + 32*b^4*c*d^2*x^3
*Sin[3*a + 2*b*x] + 8*b^4*d^3*x^4*Sin[3*a + 2*b*x] + (4*I)*b^3*c^3*Sin[3*a
+ 4*b*x] - 6*b^2*c^2*d*Sin[3*a + 4*b*x] - (6*I)*b*c*d^2*Sin[3*a + 4*b*x] +
3*d^3*Sin[3*a + 4*b*x] + (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] - 12*b^2*c*d^2
*x*Sin[3*a + 4*b*x] - (6*I)*b*d^3*x*Sin[3*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2
*Sin[3*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[
3*a + 4*b*x] + (4*I)*b^3*c^3*Sin[5*a + 4*b*x] - 6*b^2*c^2*d*Sin[5*a + 4*b*x
] - (6*I)*b*c*d^2*Sin[5*a + 4*b*x] + 3*d^3*Sin[5*a + 4*b*x] + (12*I)*b^3*c^
2*d*x*Sin[5*a + 4*b*x] - 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] - (6*I)*b*d^3*x*Si
n[5*a + 4*b*x] + (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] - 6*b^2*d^3*x^2*Sin[
5*a + 4*b*x] + (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

fricas [C] time = 0.61, size = 1134, normalized size = 4.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/8*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 24*I*d^3*polylog(4, I*cos(b*x + a)
+ sin(b*x + a)) + 24*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 24*I
*d^3*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^3*polylog(4, -I*co
s(b*x + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3
 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*
c^2*d - b*d^3)*x - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*
dilog(I*cos(b*x + a) + sin(b*x + a)) - (12*I*b^2*d^3*x^2 + 24*I*b^2*c*d^2*x
 + 12*I*b^2*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (12*I*b^2*d^3*x^2
 + 24*I*b^2*c*d^2*x + 12*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - (-12*I*b^2*d^3*x^2 - 24*I*b^2*c*d^2*x - 12*I*b^2*c^2*d)*dilog(-I*cos(b*x
```

```
 + a) - sin(b*x + a)) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3
*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b
*x + a) - sin(b*x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*
d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*
x + a) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 4*
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*s
in(b*x + a) + I) + 4*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*lo
g(-cos(b*x + a) - I*sin(b*x + a) + I) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I
*cos(b*x + a) + sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x
 + a) - sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a)) + 24*(b*d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*
x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\sin(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(b*x + a)^3, x)
```

maple [B] time = 0.22, size = 641, normalized size = 2.55

$$-\frac{3d^3 \operatorname{polylog}\left(3, -e^{2i(bx+a)}\right) x}{2b^3} - \frac{d^3 \ln\left(1+e^{2i(bx+a)}\right) x^3}{b} - \frac{6ic\,d^2a^2x}{b^2} + \frac{6ic^2dax}{b} - \frac{2d^3a^3 \ln\left(e^{i(bx+a)}\right)}{b^4} + \frac{3ia^4d^3}{2b^4} + ic\,d^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/4*I*d^3*x^4+I*c*d^2*x^3-I*c^3*x+3*I/b^2*a^2*c^2*d-4*I/b^3*a^3*c*d^2+2*I/b
^3*d^3*a^3*x-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))
)+1/32*(4*d^3*x^3*b^3+6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*
b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*
I*(b*x+a))+1/32*(4*d^3*x^3*b^3-6*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*
d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b
^4*exp(-2*I*(b*x+a))-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3
*polylog(3,-exp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+
2/b*c^3*ln(exp(I*(b*x+a)))+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+3/2
```

```
*I*c^2*d*x^2+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a)))-6/b^2*c^2*d*a*ln(exp(I*(b*x
+a)))-1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(
b*x+a)))*x^2-6*I/b^2*a^2*c*d^2*x+6*I/b*a*c^2*d*x+3/2*I/b^2*c^2*d*polylog(2,
-exp(2*I*(b*x+a)))+3/2*I/b^4*d^3*a^4-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b
*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

maxima [B] time = 0.56, size = 685, normalized size = 2.73

$$-\frac{24\left(\sin\left(bx+a\right)^{2}+\log\left(\sin\left(bx+a\right)^{2}-1\right)\right)c^{3}-\frac{72\left(\sin(bx+a)^{2}+\log\left(\sin(bx+a)^{2}-1\right)\right)ac^{2}d}{b}+\frac{72\left(\sin(bx+a)^{2}+\log\left(\sin(bx+a)^{2}-1\right)\right)a^{2}c}{b^{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/48*(24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^3 - 72*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c^2*d/b + 72*(sin(b*x + a)^2 + log(sin(b*x
+ a)^2 - 1))*a^2*c*d^2/b^2 - 24*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*
a^3*d^3/b^3 + (-12*I*(b*x + a)^4*d^3 + (-48*I*b*c*d^2 + 48*I*a*d^3)*(b*x +
a)^3 + 48*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) + (-72*I*b^2*c^2*d + 144*I
*a*b*c*d^2 - 72*I*a^2*d^3)*(b*x + a)^2 + (64*I*(b*x + a)^3*d^3 + (144*I*b*c
*d^2 - 144*I*a*d^3)*(b*x + a)^2 + (144*I*b^2*c^2*d - 288*I*a*b*c*d^2 + 144*
I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(
2*(b*x + a)^3*d^3 - 3*b*c*d^2 + 3*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a
) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 96*I*(b*x + a)^2*d^3 - 72*I*a^2*d^
3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) +
 8*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1) + 24*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*pol
ylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^
2*d^3 + (2*a^2 - 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))/
b^3)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(a+b\,x\right)^{3}\,\left(c+d\,x\right)^{3}}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x)^3)/cos(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.223 $\int (c+dx)^2 \sin^2(a+bx) \tan(a+bx)\, dx$

Optimal. Leaf size=184

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\sin^2(a+bx)}{4b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}-\frac{(c+dx)^2\log}{b}$$

```
[Out] 1/2*c*d*x/b+1/4*d^2*x^2/b+1/3*I*(d*x+c)^3/d-(d*x+c)^2*ln(1+exp(2*I*(b*x+a))
)/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I
*(b*x+a)))/b^3-1/2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2+1/4*d^2*sin(b*x+a)^2
/b^3-1/2*(d*x+c)^2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.23, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4407, 4404, 3310, 3719, 2190, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\sin(a+bx)\cos(a+bx)}{2b^2}+\frac{d^2\sin^2(a+bx)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*d*x)/(2*b) + (d^2*x^2)/(4*b) + ((I/3)*(c + d*x)^3)/d - ((c + d*x)^2*Log[
1 + E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x)
)])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/(2*b^2) + (d^2*Sin[a + b*x]^2)/(4*b^3) - ((c + d*x)
^2*Sin[a + b*x]^2)/(2*b)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)^2\tan(a+bx)\,dx$$

$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx + \frac{d\int(c$$

$$= \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cos(a+bx)\sin(a}{2b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2}{b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2}{b^2}$$

$$= \frac{cdx}{2b} + \frac{d^2x^2}{4b} + \frac{i(c+dx)^3}{3d} - \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\mathrm{Li}_2}{b^2}$$

Mathematica [B] time = 6.47, size = 518, normalized size = 2.82

$$-\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\right.}\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-1/12*I)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I
)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*
Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot
[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)
] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi
*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLo
g[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*S
qrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^
2*Cos[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*
a] - 2*b*d^2*x*Sin[2*a]))/(8*b^3) - ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a]
 + 2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2
*Sin[2*a])*Sin[2*b*x])/(8*b^3) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

fricas [C] time = 0.54, size = 688, normalized size = 3.74

$$-\frac{b^2d^2x^2+2\,b^2cdx-\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(bx+a\right)^2+4\,d^2\text{polylog}\left(3,i\,\cos\left(bx+a\right)+\sin\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*x^2 + 2*b^2*c*d*x - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2
- d^2)*cos(b*x + a)^2 + 4*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4
*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*
x + a) + sin(b*x + a)) + 4*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) +
 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (-4*I*b*d^2*x - 4*I*b*c*d)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(I*c
os(b*x + a) - sin(b*x + a)) - (4*I*b*d^2*x + 4*I*b*c*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - (-4*I*b*d^2*x - 4*I*b*c*d)*dilog(-I*cos(b*x + a) - sin
(b*x + a)) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
 + a) + I) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x
+ a) + 1) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x
 + a) + I) + 2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*
x + a) + I))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(dx+c\right)^2\sec\left(bx+a\right)\sin\left(bx+a\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(b*x + a)^3, x)
```

maple [B] time = 0.33, size = 379, normalized size = 2.06

$$\frac{id^2x^3}{3}-\frac{2id^2a^2x}{b^2}+\frac{2icd\,a^2}{b^2}+\frac{\left(2d^2x^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2\right)e^{2i(bx+a)}}{16b^3}+\frac{\left(2d^2x^2b^2+4b^2cdx-2ib\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))+1/3*I*d^2*x^3-2*I/b^2*a^2*d^2*x+1/16
*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(2*
I*(b*x+a))+1/16*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2-
d^2)/b^3*exp(-2*I*(b*x+a))-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*
(b*x+a)))+2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,-exp(2*I*(b*
x+a)))*x+2*I/b^2*a^2*c*d+4*I/b*a*c*d*x-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-I
*c^2*x-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-4/b^2*c*d*a*ln(exp(I*(b*x+a
)))+I*c*d*x^2-4/3*I/b^3*a^3*d^2-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x
```

maxima [B] time = 0.53, size = 379, normalized size = 2.06

$$-\frac{12\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)c^2-\frac{24\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)acd}{b}+\frac{12\left(\sin(bx+a)^2+\log\left(\sin(bx+a)^2-1\right)\right)a^2d^2}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/24*(12*(sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1))*c^2 - 24*(sin(b*x + a)
^2 + log(sin(b*x + a)^2 - 1))*a*c*d/b + 12*(sin(b*x + a)^2 + log(sin(b*x +
a)^2 - 1))*a^2*d^2/b^2 + (-8*I*(b*x + a)^3*d^2 + (-24*I*b*c*d + 24*I*a*d^2)
*(b*x + a)^2 + 12*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) + (24*I*(b*x + a)^2*
d^2 + (48*I*b*c*d - 48*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*a), cos(2*
b*x + 2*a) + 1) - 3*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - d^2)
*cos(2*b*x + 2*a) + (-24*I*b*c*d - 24*I*(b*x + a)*d^2 + 24*I*a*d^2)*dilog(-
e^(2*I*b*x + 2*I*a)) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(b
*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)^3\,(c+d\,x)^2}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x)^2)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \sin^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(b*x+a)**3,x)
[Out] Integral((c + d*x)**2*sin(a + b*x)**3*sec(a + b*x), x)
```

3.224 $\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/4*d*x/b+1/2*I*(d*x+c)^2/d-(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b+1/2*I*d*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-1/4*d*cos(b*x+a)*sin(b*x+a)/b^2-1/2*(d*x+c)*sin(
b*x+a)^2/b
```

Rubi [A] time = 0.13, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4407, 4404, 2635, 8, 3719, 2190, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\sin(a+bx)\cos(a+bx)}{4b^2}-\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{(c+dx)\sin^2(a+bx)}{2b}+\frac{dx}{4b}+\frac{i(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x)/(4*b) + ((I/2)*(c + d*x)^2)/d - ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Cos[a + b*x]*Si
n[a + b*x])/(4*b^2) - ((c + d*x)*Sin[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int (c+dx)\sin^2(a+bx)\tan(a+bx)\,dx = -\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx + \int (c+dx)\tan(a+bx)\,dx$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\sin^2(a+bx)}{2b} - 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx + \frac{d\int \sin^2$$
$$= \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{(c$$
$$= \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\cos(a+bx)\sin(a+bx}{4b^2}$$
$$= \frac{dx}{4b} + \frac{i(c+dx)^2}{2d} - \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\,\text{co}$$

Mathematica [A] time = 0.30, size = 134, normalized size = 1.17

$$\frac{d\left(\frac{1}{2}i\text{Li}_2\left(-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2} - \frac{d\sin(2(a+bx))}{8b^2} + \frac{ad\log(\cos(a+bx))}{b^2} - \frac{c\left(\log(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (d*x*Cos[2*(a + b*x)])/(4*b) + (a*d*Log[Cos[a + b*x]])/b^2 - (c*(-1/2*Cos[a
 + b*x]^2 + Log[Cos[a + b*x]]))/b + (d*((I/2)*(a + b*x)^2 - (a + b*x)*Log[1
 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2 - (d
*Sin[2*(a + b*x)])/(8*b^2)
```

fricas [B] time = 0.49, size = 346, normalized size = 3.01

$$-\frac{bdx - 2\,(bdx+bc)\cos(bx+a)^2 + d\cos(bx+a)\sin(bx+a) + 2i\,d\text{Li}_2\left(i\cos(bx+a)+\sin(bx+a)\right) - 2i\,d\text{Li}_2\left(i\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 + d*cos(b*x + a)*sin(b*x + a)
+ 2*I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - 2*I*d*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + 2*I*d*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + 2*(b*c - a*d)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 2*(b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*(b*d
*x + a*d)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(I*co
s(b*x + a) - sin(b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) + sin(
```

```
b*x + a) + 1) + 2*(b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + 2
*(b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b*c - a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + I))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\sec (bx+a)\sin (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(b*x + a)^3, x)
```

maple [A] time = 0.34, size = 179, normalized size = 1.56

$$\frac{id\,x^2}{2}-icx+\frac{(2bdx+2cb+id)\,e^{2i(bx+a)}}{16b^2}+\frac{(2bdx+2cb-id)\,e^{-2i(bx+a)}}{16b^2}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}+\frac{2idax}{b}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x)
```

```
[Out] 1/2*I*d*x^2-I*c*x+1/16*(2*b*d*x+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))+1/16*(2*b*d
*x-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))-1/b*c*ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(e
xp(I*(b*x+a)))+2*I/b*d*a*x+I/b^2*d*a^2-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I
*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-2/b^2*d*a*ln(exp(I*(b*x+a)))
```

maxima [A] time = 0.52, size = 145, normalized size = 1.26

$$-\frac{-4i\,b^2dx^2-8i\,b^2cx+(8i\,bdx+8i\,bc)\arctan\left(\sin\left(2\,bx+2\,a\right),\cos\left(2\,bx+2\,a\right)+1\right)-2\,(bdx+bc)\cos\left(2\,bx+2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(-4*I*b^2*d*x^2 - 8*I*b^2*c*x + (8*I*b*d*x + 8*I*b*c)*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - 2*(b*d*x + b*c)*cos(2*b*x + 2*a) - 4*I*d*
dilog(-e^(2*I*b*x + 2*I*a)) + 4*(b*d*x + b*c)*log(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + d*sin(2*b*x + 2*a))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x),x)
```

```
[Out] int((sin(a + b*x)^3*(c + d*x))/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin^3(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sin(a + b*x)**3*sec(a + b*x), x)
```

3.225 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=82

$$\text{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)-\frac{\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] -1/2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d-1/2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b
*c/d)/d+Unintegrable(tan(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] -(CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/(2*d) - (Cos[2*a - (
2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Defer[Int][Tan[a + b*x]/(
c + d*x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= -\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx+\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx\right)+\int \frac{\tan(a+bx)}{c+dx}\,dx\\
&= -\left(\frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx\right)-\frac{1}{2}\sin\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2\right.}{c+dx}\\
&= -\frac{\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d}-\frac{\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{2d}+\int \frac{\tan(a+b}{c+dx}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^{2}-1\right)\sec\left(bx+a\right)\sin\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^{3}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c), x)
```

maple [A] time = 0.45, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\left(\sin^{3}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(i\,E_{1}\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_{1}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+8\,d\int\frac{\sin(2\,bx+2\,a)}{(dx+c)\left(\cos(2\,bx+2\,a)^{2}+\sin(2\,bx+2\,a)^{2}+2\,\cos(2\,bx+2\,a)+1\right)}\,dx+\left(\cdots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] 1/4*((I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(
2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*d*integrate(sin(2*b*x +
2*a)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2
*(d*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, (2*I*b*d*x + 2*I*
b*c)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d
))/d
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+bx)^3}{\cos(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)),x)
[Out] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.226 $\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$\mathrm{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}$$

```
[Out] -b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2+b*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c
/d)/d^2+1/2*sin(2*b*x+2*a)/d/(d*x+c)+Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2) + Sin[2*a +
2*b*x]/(2*d*(c + d*x)) + (b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*
b*x])/d^2 + Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= -\left(\frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx\right)+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d}+\int \frac{\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(2a+2bx)}{2d(c+dx)}-\frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}\\
&= -\frac{b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin(2a+2bx)}{2d(c+dx)}+\frac{b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}$$

Mathematica [A] time = 2.40, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(bx+a\right)^2-1\right)\sec\left(bx+a\right)\sin\left(bx+a\right)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(b*x + a)^2 - 1)*sec(b*x + a)*sin(b*x + a)/(d^2*x^2 + 2*c*d*x
 + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\sin\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(b*x + a)^3/(d*x + c)^2, x)
```

maple [A] time = 0.69, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)\left(\sin^3\left(bx+a\right)\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(iE_2\left(\frac{2ibdx+2ibc}{d}\right)-iE_2\left(-\frac{2ibdx+2ibc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+8\left(d^2x+cd\right)\int\frac{\sin(2bx+2a)}{(dx+c)^2\left(\cos(2bx+2a)^2+\sin(2bx+2a)^2+2\cos(2bx+2a)\right.}}{4\left(d^2x+cd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(
2*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 8*(d^2*x + c*d)*integrate(
sin(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x +
 2*a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 +
 2*c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, (2*I*b*d*x + 2*I
*b*c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/
d))/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(a+b\,x\right)^3}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(sin(a + b*x)^3/(cos(a + b*x)*(c + d*x)^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.227 $\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=23

$$\mathrm{Int}\left(\csc(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Mathematica [A] time = 5.89, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x], x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \csc(bx+a)\sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

maple [A] time = 0.06, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)\ \sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)),x)
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a),x)
[Out] Integral((c + d*x)**m*csc(a + b*x)*sec(a + b*x), x)
```

3.228 $\int (c+dx)^4 \csc(a+bx) \sec(a+bx) \, dx$

Optimal. Leaf size=247

$$\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{Li}_4\left(e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{Li}_3\left(\right.}{b^3}$$

```
[Out] -2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b+2*I*d*(d*x+c)^3*polylog(2,-exp(2*I
*(b*x+a)))/b^2-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c
)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog(3,exp(2*I*(b*x
+a)))/b^3-3*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3*I*d^3*(d*x+c)*
polylog(4,exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-3/
2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5
```

Rubi [A] time = 0.23, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^4*ArcTanh[E^((2*I)*(a + b*x))])/b + ((2*I)*d*(c + d*x)^3*Poly
Log[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyLog[2, E^((2*I
)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b
^3 + (3*d^2*(c + d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(
c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/
(2*b^5) - (3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^4 \csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(4d)\int (c+dx)^3 \log\left(1-e^{i(2a+2bx)}\right)dx}{b} +$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d$$
$$= -\frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{2id(c+dx)^3 \mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{2id(c+d$$

Mathematica [B] time = 1.29, size = 578, normalized size = 2.34

$$\frac{-4b^4c^4 \tanh^{-1}\left(e^{2i(a+bx)}\right) + 8b^4c^3dx \log\left(1-e^{2i(a+bx)}\right) - 8b^4c^3dx \log\left(1+e^{2i(a+bx)}\right) + 12b^4c^2d^2x^2 \log\left(1-e^{2i(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^4*c^4*ArcTanh[E^((2*I)*(a + b*x))] + 8*b^4*c^3*d*x*Log[1 - E^((2*I)*(
a + b*x))] + 12*b^4*c^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 8*b^4*c*d^3*
x^3*Log[1 - E^((2*I)*(a + b*x))] + 2*b^4*d^4*x^4*Log[1 - E^((2*I)*(a + b*x)
)] - 8*b^4*c^3*d*x*Log[1 + E^((2*I)*(a + b*x))] - 12*b^4*c^2*d^2*x^2*Log[1
+ E^((2*I)*(a + b*x))] - 8*b^4*c*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] - 2*b
^4*d^4*x^4*Log[1 + E^((2*I)*(a + b*x))] + (4*I)*b^3*d*(c + d*x)^3*PolyLog[2
, -E^((2*I)*(a + b*x))] - (4*I)*b^3*d*(c + d*x)^3*PolyLog[2, E^((2*I)*(a +
b*x))] - 6*b^2*c^2*d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - 12*b^2*c*d^3*x*Po
lyLog[3, -E^((2*I)*(a + b*x))] - 6*b^2*d^4*x^2*PolyLog[3, -E^((2*I)*(a + b*
x))] + 6*b^2*c^2*d^2*PolyLog[3, E^((2*I)*(a + b*x))] + 12*b^2*c*d^3*x*PolyL
og[3, E^((2*I)*(a + b*x))] + 6*b^2*d^4*x^2*PolyLog[3, E^((2*I)*(a + b*x))]
- (6*I)*b*c*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (6*I)*b*d^4*x*PolyLog[4,
 -E^((2*I)*(a + b*x))] + (6*I)*b*c*d^3*PolyLog[4, E^((2*I)*(a + b*x))] + (6
*I)*b*d^4*x*PolyLog[4, E^((2*I)*(a + b*x))] + 3*d^4*PolyLog[5, -E^((2*I)*(a
 + b*x))] - 3*d^4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5)
```

fricas [C] time = 0.66, size = 2600, normalized size = 10.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(24*d^4*polylog(5, cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*polylog(5,
cos(b*x + a) - I*sin(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) + sin(b*x
 + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*polylog(
5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a) - si
n(b*x + a)) + 24*d^4*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*po
lylog(5, -cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d
^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(cos(b*x + a) + I*sin(b*x
 + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b
^3*c^3*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b
^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) + s
in(b*x + a)) - (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x +
 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (4*I*b^3*d^4*x^3 + 1
2*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x +
a) + sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*
d^2*x - 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (4*I*b^3*d^4
*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-cos(
b*x + a) + I*sin(b*x + a)) - (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*
b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4
*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*l
og(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4
*c^4)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6
*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x +
a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*
x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d
^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3
- a^4*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*
d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2
*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (
b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3
*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^4*
c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-1/2
*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3
```

```
+ 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4
*a^3*b*c*d^3 - a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^
2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 -
 a^4*d^4)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*sin(
b*x + a) + I) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, cos(b*x + a) - I*s
in(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) + si
n(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a) - si
n(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) + s
in(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a) - s
in(b*x + a)) - (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) + I*
sin(b*x + a)) - (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -cos(b*x + a) - I*
sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, c
os(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d
^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d
^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4
*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x +
 a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylo
g(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^
2*c^2*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2
*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/b^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \csc(bx+a)\sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a), x)
```

maple [B] time = 0.17, size = 1242, normalized size = 5.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)
+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(
```

```
b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x
+a)))*x^2+12/b^3*d^4*polylog(3,-exp(I*(b*x+a)))*x^2-3/b^3*c^2*d^2*polylog(3
,-exp(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2-24*d^4*polyl
og(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-4/b*c*d^3*ln
(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+6*I
/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*c^4*ln(1+exp(2*I*(b*x+a)))+
1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)-1/b*d^4*ln(1+exp(
2*I*(b*x+a)))*x^4+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*
x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+a))+1
)*x^2+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(
I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp
(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^3*pol
ylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/b^4*d
^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+2
4*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(I*(b*
x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*
x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I*(b*x
+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-1
2*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(2,-ex
p(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*c*d^3
*polylog(2,exp(I*(b*x+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*c*d^3
*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-4/b*c^3*d*ln
(1+exp(2*I*(b*x+a)))*x-6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-6/b*c^2*d
^2*ln(1+exp(2*I*(b*x+a)))*x^2-3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*
I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)))*x^3-3*I/b^4*c*d^3*polylog(4,-exp(2*I
*(b*x+a)))+2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))
```

maxima [B] time = 0.71, size = 1779, normalized size = 7.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^4*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 12*a*c^3*d*(l
og(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 18*a^2*c^2*d^2*(log(sin(b
*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 12*a^3*c*d^3*(log(sin(b*x + a)^
2 - 1) - log(sin(b*x + a)^2))/b^3 + 3*a^4*d^4*(log(sin(b*x + a)^2 - 1) - lo
g(sin(b*x + a)^2))/b^4 - (18*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) - 144*d^4
*polylog(5, -e^(I*b*x + I*a)) - 144*d^4*polylog(5, e^(I*b*x + I*a)) - (12*I
*(b*x + a)^4*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*
d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a
*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) - (-6*I*(b*x + a)^4*d^4 + (-24*I*b*c*d^3 +
24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^
```

```
4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 +
 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*I*(b
*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2
 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^
2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*arctan2(sin(b*x + a
), -cos(b*x + a) + 1) - (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 - 36*I*a^2*b*
c*d^3 - 24*I*(b*x + a)^3*d^4 + 12*I*a^3*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*
(b*x + a)^2 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I
*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a
*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*
x + a))*dilog(-e^(I*b*x + I*a)) - (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72
*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I
*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(
b*x + a))*dilog(e^(I*b*x + I*a)) - 2*(3*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^
4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 6*(b
^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) +
3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*
c*d^3 - a^3*d^4)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x
 + a) + 1) - (24*I*b*c*d^3 + 36*I*(b*x + a)*d^4 - 24*I*a*d^4)*polylog(4, -e
^(2*I*b*x + 2*I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*
polylog(4, -e^(I*b*x + I*a)) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*
I*a*d^4)*polylog(4, e^(I*b*x + I*a)) - 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(
b*x + a)^2*d^4 + 3*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(
2*I*b*x + 2*I*a)) + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d
^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*polylog(3, -e^(I*b*x + I*a)) + 72*(b^2*
c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*
x + a))*polylog(3, e^(I*b*x + I*a)))/b^4)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^4}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^4/(cos(a + b*x)*sin(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \csc (a+bx) \sec (a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*csc(a + b*x)*sec(a + b*x), x)
```

3.229 $\int (c+dx)^3 \csc(a+bx) \sec(a+bx) \, dx$

Optimal. Leaf size=197

$$-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{Li}_2}{2b^2}$$

```
[Out] -2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))/b+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2
*I*(b*x+a)))/b^2-3/2*I*d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*
(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*
(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(
4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4419, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-3id$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^3*ArcTanh[E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d*(c + d*x)^2*
PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2,
 E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x
))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(2*b^3) -
(((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^3 \csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(3d)\int (c+dx)^2 \log\left(1-e^{i(2a+2bx)}\right)dx}{b} +$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d$$
$$= -\frac{2(c+dx)^3 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{3id(c+dx)^2 \text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{3id(c+d$$

Mathematica [A] time = 0.95, size = 350, normalized size = 1.78

$$\frac{-8b^3c^3 \tanh^{-1}\left(e^{2i(a+bx)}\right) + 12b^3c^2dx \log\left(1-e^{2i(a+bx)}\right) - 12b^3c^2dx \log\left(1+e^{2i(a+bx)}\right) + 12b^3cd^2x^2 \log\left(1-e^{2i(a+bx)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 - E^((2*I)*
(a + b*x))] + 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3
*Log[1 - E^((2*I)*(a + b*x))] - 12*b^3*c^2*d*x*Log[1 + E^((2*I)*(a + b*x))]
 - 12*b^3*c*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - 4*b^3*d^3*x^3*Log[1 + E^
((2*I)*(a + b*x))] + (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x)
)] - (6*I)*b^2*d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))] - 6*b*d^2*(c +
 d*x)*PolyLog[3, -E^((2*I)*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^((2*I)*(a +
 b*x))] + 6*b*d^3*x*PolyLog[3, E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyLog[4,
-E^((2*I)*(a + b*x))] + (3*I)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/(4*b^4)
```

fricas [C] time = 0.60, size = 1778, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*polylog(4,
 cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) + sin(b
*x + a)) - 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*poly
log(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*polylog(4, -I*cos(b*x + a)
 - sin(b*x + a)) - 6*I*d^3*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + 6*I
*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b
^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2
*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x
+ a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*
x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d
)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x
 + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
 (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*
c^2*d*x + b^3*c^3)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*
b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x +
a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d
^3)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a)
+ 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d
^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*
a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) +
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x +
 a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x
 + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*
x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a
) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)
) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*
d^3*x + b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
b*c*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)
*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*polylog
(3, -cos(b*x + a) - I*sin(b*x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a) \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a), x)
```

maple [B] time = 0.12, size = 816, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-1/b*c^3*ln(1+exp(2*I*(b*x+
a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a
)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6*I/b^4*d^3*polylog(4,-exp(I*(b
*x+a)))-3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-e
xp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+1/b*c^3*ln(ex
p(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)+3*I/b^2*c*d^2*polylog(2,-exp(2
*I*(b*x+a)))*x+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3*I/b^2*c^2*d*polylog(2
,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-3*I/b^2*d^3*polyl
og(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3/b*c^2
*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1
-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b*c*d^2*ln(1-exp(
I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(
b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-exp(I*(b*x+a)))*
a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+3/2
*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+3/2*I/b^2*c^2*d*polylog(2,-exp(
2*I*(b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*
x+a)))*x^2
```

maxima [B] time = 0.61, size = 1063, normalized size = 5.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/6*(3*c^3*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 9*a*c^2*d*(lo
g(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + 9*a^2*c*d^2*(log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 3*a^3*d^3*(log(sin(b*x + a)^2 - 1)
- log(sin(b*x + a)^2))/b^3 + (6*I*d^3*polylog(4, -e^(2*I*b*x + 2*I*a)) - 36
*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a))
+ (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^
2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*arctan2(sin(2*b*x + 2*a
), cos(2*b*x + 2*a) + 1) + (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I*a*
d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*x +
 a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + (18*I
*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*
I*a^2*d^3)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (-9*I*b^2*
c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^
2 + 18*I*a*d^3)*(b*x + a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*I*b^2*c^2*d -
36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I
*a*d^3)*(b*x + a))*dilog(-e^(I*b*x + I*a)) + (18*I*b^2*c^2*d - 36*I*a*b*c*d
^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x
 + a))*dilog(e^(I*b*x + I*a)) + (4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*log(cos(2*b*x
+ 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 3*((b*x + a)^3*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x +
 a) + 1) + 3*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*polylog(3, -e^(2*I*b*x
 + 2*I*a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, -e^(I*b*x + I*
a)) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*polylog(3, e^(I*b*x + I*a)))/b^3
)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^3}{\cos(a + b\,x)\,\sin(a + b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc(a + bx) \sec(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a),x)
[Out] Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x), x)
```

3.230 $\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=127

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\tanh^{-1}\left(e^2\right.}{b}$$

```
[Out] -2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*
x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-e
xp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.12, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4419, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b + (I*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
])/b^2 - (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E
^((2*I)*(a + b*x))])/(2*b^3)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)^2\csc(2a+2bx)\,dx$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{(2d)\int (c+dx)\log\left(1-e^{i(2a+2bx)}\right)dx}{b} +$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\text{L}}{b}$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\text{L}}{b}$$

$$= -\frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2} - \frac{id(c+dx)\text{L}}{b}$$

Mathematica [A] time = 0.59, size = 213, normalized size = 1.68

$$\frac{-4b^2c^2\tanh^{-1}\left(e^{2i(a+bx)}\right) + 4b^2cdx\log\left(1-e^{2i(a+bx)}\right) - 4b^2cdx\log\left(1+e^{2i(a+bx)}\right) + 2b^2d^2x^2\log\left(1-e^{2i(a+bx)}\right) - 2b^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 - E^((2*I)*(a
+ b*x))] + 2*b^2*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 +
 E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] + (2*I)*
b*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLo
g[2, E^((2*I)*(a + b*x))] - d^2*PolyLog[3, -E^((2*I)*(a + b*x))] + d^2*Poly
Log[3, E^((2*I)*(a + b*x))])/(2*b^3)
```

fricas [C] time = 0.56, size = 1090, normalized size = 8.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a
)) - 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*polylog(3, -I*
cos(b*x + a) + sin(b*x + a)) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
 a)) + 2*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(cos(b*x
+ a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(cos(b*x + a) - I*s
in(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*x +
a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I
*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x
 - 2*I*b*c*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*
c*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dil
og(-cos(b*x + a) - I*sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
log(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*
log(cos(b*x + a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b
*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b
^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b
*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos
(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*co
s(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*lo
g(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*
x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
 + 1) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a), x)
```

maple [B] time = 0.09, size = 469, normalized size = 3.69

$$-\frac{d^2 \operatorname{polylog}\left(3,-\mathrm{e}^{2i(bx+a)}\right)}{2b^3}+\frac{d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b^3}+\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-\mathrm{e}^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(\mathrm{e}^{i(bx+a)}+1\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1
)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(
b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-
2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a
)))*x-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2
*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(e
xp(I*(b*x+a))-1)-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))+I/b^2*d^2*polylog(2,
-exp(2*I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-2*I/b^2*c*d*pol
ylog(2,-exp(I*(b*x+a)))
```

maxima [B] time = 0.60, size = 590, normalized size = 4.65

$$-\frac{c^2\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)-\frac{2\,acd\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin(bx+a)^2\right)\right)}{b}+\frac{a^2d^2\left(\log\left(\sin(bx+a)^2-1\right)-\log\left(\sin\right.\right.}{b^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)) - 2*a*c*d*(log(si
n(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b + a^2*d^2*(log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^2 + (d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) - 4*d
^2*polylog(3, -e^(I*b*x + I*a)) - 4*d^2*polylog(3, e^(I*b*x + I*a)) + (2*I*
(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(2*b*x + 2*
```

```
a), cos(2*b*x + 2*a) + 1) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2
)*(b*x + a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (2*I*(b*x + a)^2*d^2
 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*arctan2(sin(b*x + a), -cos(b*x + a) +
 1) + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*dilog(-e^(2*I*b*x + 2*I*
a)) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*dilog(-e^(I*b*x + I*a)) +
 (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*dilog(e^(I*b*x + I*a)) + ((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2
)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - ((
b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*log(cos(b*x + a)^2 + sin(b*x
+ a)^2 - 2*cos(b*x + a) + 1))/b^2)/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cos(a+b\,x)\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)),x)
[Out] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a),x)
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x), x)
```

3.231 $\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=71

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

```
[Out] -2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))
/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.06, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4419, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] (-2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b + ((I/2)*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx = 2\int (c+dx)\csc(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\int \log\left(1-e^{i(2a+2bx)}\right)dx}{b} + \frac{d\int \log\left(1+\right.}{}$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{(id)\,\mathrm{Subst}\left(\int \frac{\log(1-x)}{x}dx, x, e^{i(2a+2bx)}\right)}{2b^2} -$$
$$= -\frac{2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} + \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{id\mathrm{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}$$

Mathematica [A] time = 0.11, size = 141, normalized size = 1.99

$$\frac{d\left(i\left(\mathrm{Li}_2\left(-e^{i(2a+2bx)}\right)-\mathrm{Li}_2\left(e^{i(2a+2bx)}\right)\right)+(2a+2bx)\left(\log\left(1-e^{i(2a+2bx)}\right)-\log\left(1+e^{i(2a+2bx)}\right)\right)-2a\log\left(\tan\left(\frac{1}{2}(2\right.\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x],x]
```

```
[Out] -((c*Log[Cos[a + b*x]])/b) + (c*Log[Sin[a + b*x]])/b + (d*((2*a + 2*b*x)*(L
og[1 - E^(I*(2*a + 2*b*x))] - Log[1 + E^(I*(2*a + 2*b*x))]) - 2*a*Log[Tan[(
2*a + 2*b*x)/2]] + I*(PolyLog[2, -E^(I*(2*a + 2*b*x))] - PolyLog[2, E^(I*(2
*a + 2*b*x))])))/(2*b^2)
```

fricas [B] time = 0.52, size = 554, normalized size = 7.80

$$\frac{-i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+i\,d\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*dilog(cos(b*x + a) - I
*sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(I*cos
```

```
(b*x + a) - sin(b*x + a)) + I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*dilog(-cos(b*x + a) + I*sin(b*
x + a)) - I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (b*d*x + b*c)*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a
*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d*x + a*d)*log(I*cos(b*x +
a) + sin(b*x + a) + 1) - (b*d*x + a*d)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-1/2*cos(b*x + a)
 + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) + (b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) -
 (b*c - a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\csc(bx + a)\sec(bx + a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a), x)
```

maple [B] time = 0.08, size = 208, normalized size = 2.93

$$\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{d\ln\left(1+e^{2i(bx+a)}\right)x}{b}+\frac{id\,\mathrm{polylog}\left(2,-e^{2i(bx+a)}\right)}{2b^2}+\frac{d\ln\left(e^{i}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] 1/b*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(1+exp(2*I*(b*x+a)))+1/b*c*ln(exp(I*(b*x
+a))+1)-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))
/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,-exp(I*(b*x+a)))/b^2+1/b*d*
ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a)))*a-I*d*polylog(2,exp(I*(
b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.70, size = 267, normalized size = 3.76

$$-\frac{2i\,bdx\arctan\left(\sin(bx+a),-\cos(bx+a)+1\right)-2i\,bc\arctan\left(\sin(bx+a),\cos(bx+a)-1\right)+\left(2i\,bdx+2i\,bc\right)a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(2*I*b*d*x*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 2*I*b*c*arctan2(
sin(b*x + a), cos(b*x + a) - 1) + (2*I*b*d*x + 2*I*b*c)*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) + (-2*I*b*d*x - 2*I*b*c)*arctan2(sin(b*x + a),
 cos(b*x + a) + 1) - I*d*dilog(-e^(2*I*b*x + 2*I*a)) + 2*I*d*dilog(-e^(I*b*
x + I*a)) + 2*I*d*dilog(e^(I*b*x + I*a)) + (b*d*x + b*c)*log(cos(2*b*x + 2*
a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b*d*x + b*c)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (b*d*x + b*c)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1))/b^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c+d\,x}{\cos\left(a+b\,x\right)\,\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)),x)
```

```
[Out] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x), x)
```

3.232 $\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=22

$$2\text{Int}\left(\frac{\csc(2a+2bx)}{c+dx},x\right)$$

```
[Out] 2*Unintegrable(csc(2*b*x+2*a)/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx = 2\int \frac{\csc(2a+2bx)}{c+dx}\,dx$$

Mathematica [A] time = 4.53, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

3.233 $\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=22

$$2\text{Int}\left(\frac{\csc(2a+2bx)}{(c+dx)^2}, x\right)$$

```
[Out] 2*Unintegrable(csc(2*b*x+2*a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] 2*Defer[Int][Csc[2*a + 2*b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2} dx = 2\int \frac{\csc(2a+2bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.72, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(csc(b*x + a)*sec(b*x + a)/(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)\,{(c+d\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

3.234 $\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\csc^2(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Mathematica [A] time = 17.79, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x], x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right) \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Timed out
```

3.235 $\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=350

$$-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}-\frac{6id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2}{}$$

```
[Out] -2*I*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)
))/b^2-(d*x+c)^3*csc(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^
3+3*I*d*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)^2*polylog(
2,I*exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-6*d^3
*polylog(3,-exp(I*(b*x+a)))/b^4-6*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/
b^3+6*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*
x+a)))/b^4-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,I*exp
(I*(b*x+a)))/b^4
```

Rubi [A] time = 0.64, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{6d^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E
^(I*(a + b*x))])/b^2 - ((c + d*x)^3*Csc[a + b*x])/b + ((6*I)*d^2*(c + d*x)*
PolyLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I)*E^
(I*(a + b*x))])/b^2 - ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b
^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 - (6*d^3*PolyLog
[3, -E^(I*(a + b*x))])/b^4 - (6*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x
))])/b^3 + (6*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + (6*d^3*Pol
yLog[3, E^(I*(a + b*x))])/b^4 - ((6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))]
)/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int (c+dx)$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - (3d)\int \frac{(c+dx}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c+dx)}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int \left((c+dx\right.}{}$$

$$= \frac{(c+dx)^3\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^3\csc(a+bx)}{b} - \frac{(3d)\int (c+dx)}{}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{\int b(c+dx)^3 \mathrm{se}}{b}$$

$$= -\frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3\csc(a+bx)}{b} + \frac{6id^2(c+dx)\mathrm{Li}}{b^3}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3}{}$$

$$= -\frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^3}{}$$

Mathematica [B] time = 6.41, size = 739, normalized size = 2.11

$$\frac{3d\left(\frac{2id\left(b(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx)}\right)+id\mathrm{Li}_3\left(-e^{i(a+bx)}\right)\right)}{b^2} + \frac{2d\left(d\mathrm{Li}_3\left(e^{i(a+bx)}\right)-ib(c+dx)\mathrm{Li}_2\left(e^{i(a+bx)}\right)\right)}{b^2} + (c+dx)^2\log\left(1-e^{i(a+bx)}\right) - (c+dx)^2\log\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^3*Csc[a])/b) + (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c
 + d*x)^2*Log[1 + E^(I*(a + b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I
*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d
*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2
 + ((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(I*(
a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1
 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3*c*
d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))]
 + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*d*(
c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(
a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog
[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)*d^
3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))
])/b^4 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^3*Sin[(b*x)/2]) - 3*c^2*d*x*Sin[
(b*x)/2] - 3*c*d^2*x^2*Sin[(b*x)/2] - d^3*x^3*Sin[(b*x)/2]))/(2*b) + (Csc[a
/2]*Csc[a/2 + (b*x)/2]*(c^3*Sin[(b*x)/2] + 3*c^2*d*x*Sin[(b*x)/2] + 3*c*d^2
*x^2*Sin[(b*x)/2] + d^3*x^3*Sin[(b*x)/2]))/(2*b)
```

fricas [C] time = 0.65, size = 1753, normalized size = 5.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 - 6*I*d^3
*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*I*d^3*polylog(4
, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b
*x + a) + sin(b*x + a))*sin(b*x + a) + 6*I*d^3*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) + I*sin(b*x + a
))*sin(b*x + a) - 6*d^3*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + 6*d^3*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3
*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x -
6*I*b*c*d^2)*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3
*x + 6*I*b*c*d^2)*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - (-3*I
*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*dilog(I*cos(b*x + a) + sin(
b*x + a))*sin(b*x + a) - (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*
d)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - (3*I*b^2*d^3*x^2 + 6
*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) - (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(
b*x + a) - sin(b*x + a))*sin(b*x + a) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(
-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) - (6*I*b*d^3*x + 6*I*b*c*d^2)*
dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2
```

```
*c*d^2*x + b^2*c^2*d)*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) -
 (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) + I*s
in(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)
*log(cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^3*c^3 - 3*a*b^2*c
^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(
b*x + a) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x +
a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
a^3*d^3)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
)*sin(b*x + a) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*log(-1/2*cos(b*x + a
) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 + 2*a*b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x +
 a) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a)
 + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*
b*c*d^2 - a^2*d^3)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*si
n(b*x + a) + I)*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b
*x + a) - sin(b*x + a))*sin(b*x + a) + 6*(b*d^3*x + b*c*d^2)*polylog(3, -I*
cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 6*(b*d^3*x + b*c*d^2)*polylog(3
, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a))/(b^4*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \csc (bx + a)^2 \sec (bx + a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a), x)
```

maple [B] time = 0.50, size = 1158, normalized size = 3.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] 6*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*x+a)))/b^
4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)-3/b^
```

```
2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-3/b^2*d^3*l
n(exp(I*(b*x+a))+1)*x^2+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*x^2+3/b^4*d^3*ln(1-e
xp(I*(b*x+a)))*a^2-6*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4-1/b^4*a^3*d^3*l
n(1+I*exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,I*exp(I*(b*x+a)))*x+1/b*d^3*ln(1-
I*exp(I*(b*x+a)))*x^3-1/b*d^3*ln(1+I*exp(I*(b*x+a)))*x^3-6/b^3*d^3*polylog(
3,-I*exp(I*(b*x+a)))*x+6/b^3*d^2*c*polylog(3,I*exp(I*(b*x+a)))-6/b^3*d^2*c*
polylog(3,-I*exp(I*(b*x+a)))+1/b^4*a^3*d^3*ln(1-I*exp(I*(b*x+a)))-2*I/b*c^3
*arctan(exp(I*(b*x+a)))+3/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-3/b*d^2*c*ln(1
+I*exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-
I*exp(I*(b*x+a)))*a+3/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))-3/b*c^2*d*ln(1+I
*exp(I*(b*x+a)))*x-3/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-3/b^3*a^2*c*d^2*ln(
1-I*exp(I*(b*x+a)))+3*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))+2*I/b^4*d^3*
a^3*arctan(exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))-3*I/b^
2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+3*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a
)))*x^2-2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)*exp(I*(b*x+a))/b/(exp(2*I*(
b*x+a))-1)-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)+6/b^3*d^3*ln(1-exp(I*(b*x+a))
)*a*x+6*I/b^3*d^2*c*dilog(exp(I*(b*x+a))+1)-6*I/b^4*d^3*a*dilog(exp(I*(b*x+
a))+1)+6*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,exp(I
*(b*x+a)))*a+6*I/b^3*dilog(exp(I*(b*x+a)))*c*d^2-6*I/b^4*dilog(exp(I*(b*x+a
)))*d^3*a-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-6/b^2*d^2*c*ln(exp(I*(b*x
+a))+1)*x+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^2*d^2*c*polylog(2,
I*exp(I*(b*x+a)))*x-6*I/b^3*c*d^2*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*c^2*d*
a*arctan(exp(I*(b*x+a)))+6*I/b^2*d^2*c*polylog(2,-I*exp(I*(b*x+a)))*x
```

maxima [B] time = 1.41, size = 3240, normalized size = 9.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 3*a*c^2*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b + 3*a^2*c*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a
) - 1))/b^2 - a^3*d^3*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x
 + a) - 1))/b^3 - 2*((2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 +
 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*
(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*si
n(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^
3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)
*(b*x + a) - 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a
)^3*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a
```

```
*b*c*d^2 - 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a),
-sin(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^
2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x
+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-
6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I
*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), c
os(b*x + a) + 1) - (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*a^2*d^3 - 6*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2
+ 6*I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) +
(6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^2*d^3 + (
-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x +
a), -cos(b*x + a) + 1) - 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)
^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2
*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3
)*(b*x + a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b
*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*
d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^2*c^2*d - 12*a*b*c
*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(
b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) - (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a
)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(2*b*x +
2*a))*dilog(-I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3
 - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (12*I*b*c*d^2 +
12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a))
+ (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*cos(2*b*x + 2*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3
)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^
2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a
) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6
*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(b^2*c^2*d - 2*a*b*
c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*
b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^
2 - 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^
2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a
) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a
*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
cos(b*x + a) + 1) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a
)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a) + (-I*(b*x +
a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a
*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a
```

```
) + 1) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3
*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a) + (I*(b*x + a)^3*d^3
+ (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 +
3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 + 3*(b*c*d^2 -
a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 12
*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*polylog(4, I*e^(I*b*
x + I*a)) - 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) - d^3)*polylo
g(4, -I*e^(I*b*x + I*a)) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3
+ (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(
b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x +
I*a)) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (12*I*b*c*d^2 +
12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)
*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a)) + (12*I*d^3*
cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, -e^(I*b*x
 + I*a)) + (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3
)*polylog(3, e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 + (-12*I*b*c*d^2 + 12
*I*a*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3)*(
b*x + a))*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a)
 + 2*I*b^3))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \csc^2(a + bx) \sec(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*csc(a + b*x)**2*sec(a + b*x), x)
```

3.236 $\int (c+dx)^2 \csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=226

$$\frac{2id^2\mathrm{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\mathrm{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{2d^2\mathrm{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2d^2\mathrm{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+$$

```
[Out] -2*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b-4*d*(d*x+c)*arctanh(exp(I*(b*x+a)))
/b^2-(d*x+c)^2*csc(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+2*I*d*(d
*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x
+a)))/b^2-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3-2*d^2*polylog(3,-I*exp(I*(b
*x+a)))/b^3+2*d^2*polylog(3,I*exp(I*(b*x+a)))/b^3
```

Rubi [A] time = 0.38, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2621, 321, 207, 4420, 6741, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b - (4*d*(c + d*x)*ArcTanh[E^(
I*(a + b*x))])/b^2 - ((c + d*x)^2*Csc[a + b*x])/b + ((2*I)*d^2*PolyLog[2, -
E^(I*(a + b*x))])/b^3 + ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))]
)/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*
PolyLog[2, E^(I*(a + b*x))])/b^3 - (2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])
/b^3 + (2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int (c+d$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - (2d)\int \frac{(c+}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int (c+dx}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int \left((c+\right.}{}$$

$$= \frac{(c+dx)^2\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)^2\csc(a+bx)}{b} - \frac{(2d)\int (c+dx}{}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{\int b(c+dx)^2 s}{b}$$

$$= -\frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2\csc(a+bx)}{b} + \frac{\left(2id^2\right)\text{Subst}\left(\right.}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2}{}$$

$$= -\frac{2i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{4d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{(c+dx)^2}{}$$

Mathematica [B] time = 6.27, size = 593, normalized size = 2.62

$$\frac{2d^2\left(-\frac{2\tan^{-1}(\tan(a))\tanh^{-1}\left(\frac{\sin(a)\tan\left(\frac{bx}{2}\right)-\cos(a)}{\sqrt{\sin^2(a)+\cos^2(a)}}\right)}{\sqrt{\sin^2(a)+\cos^2(a)}} + \frac{\sec(a)\left(i\left(\text{Li}_2\left(-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\text{Li}_2\left(e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)+\left(\tan^{-1}(\tan(a))+bx\right)\left(\log\left(1-e^{i\right.}\right.\right.}{\sqrt{\tan^2(a)+1}}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] -(((c + d*x)^2*Csc[a])/b) + ((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2
*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))]
```

```
- 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a +
b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(
c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*
x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((4*I)*c*d*ArcTan[(I*Cos[
a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sqrt[Cos[a]^2
+ Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]) - 2*c*d*x*
Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*
(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]))/(2*b) + (
2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[
a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x + ArcTan[Tan[a]])*(Lo
g[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*x + ArcTan[Tan[a]]))]
) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - PolyLog[2, E^(I*(b*x + A
rcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

fricas [C] time = 0.56, size = 1067, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*dilog(cos(b*x + a)
+ I*sin(b*x + a))*sin(b*x + a) - 2*I*d^2*dilog(cos(b*x + a) - I*sin(b*x + a
))*sin(b*x + a) + 2*I*d^2*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - 2*I*d^2*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 2*d^2*poly
log(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, I*cos
(b*x + a) - sin(b*x + a))*sin(b*x + a) + 2*d^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a))*sin(b*x + a) - 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x +
a))*sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) + sin(b*
x + a))*sin(b*x + a) - (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(I*cos(b*x + a) - si
n(b*x + a))*sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a)
+ sin(b*x + a))*sin(b*x + a) - (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x +
 a) - sin(b*x + a))*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*x + a) + I
*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b
*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 2*(b*d^2*x + b*c*d)*log(cos(b*
x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2
*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*
x + a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*
d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(
b*x + a) + 1/2)*sin(b*x + a) - 2*(b*c*d - a*d^2)*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(-cos(b*x + a
```

```
) + I*sin(b*x + a) + 1)*sin(b*x + a) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(
-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) - 2*(b*d^2*x + a*d^2)*log(
-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a))/(b^3*sin(b*x
+ a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a), x)
```

maple [B] time = 0.28, size = 556, normalized size = 2.46

$$\frac{4icda\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{2dc\ln\left(e^{i(bx+a)}-1\right)}{b^2}-\frac{2dc\ln\left(e^{i(bx+a)}+1\right)}{b^2}+\frac{2i\operatorname{dilog}\left(e^{i(bx+a)}\right)d^2}{b^3}-\frac{2ie^{i(bx+a)}\left(d^2x^2+2cdx\right.}{b\left(e^{2i(bx+a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] 4*I/b^2*c*d*a*arctan(exp(I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))-1)-2/b^2*d
*c*ln(exp(I*(b*x+a))+1)-2*I*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(exp(2*I
*(b*x+a))-1)+2*I/b^3*dilog(exp(I*(b*x+a)))*d^2-2*I/b^2*d^2*polylog(2,I*exp(
I*(b*x+a)))*x+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)-2*I/b*c^2*arctan(exp(I*(b
*x+a)))-2*I/b^3*d^2*a^2*arctan(exp(I*(b*x+a)))+2*I/b^2*c*d*polylog(2,-I*exp
(I*(b*x+a)))+2*I/b^2*d^2*polylog(2,-I*exp(I*(b*x+a)))*x-1/b^3*a^2*d^2*ln(1-
I*exp(I*(b*x+a)))-1/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-2/b^2*c*d*ln(1+I*exp(I
*(b*x+a)))*a+1/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))+2*d^2*polylog(3,I*exp(I*(
b*x+a)))/b^3-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)-2*d^2*polylog(3,-I*exp(I*(b*x
+a)))/b^3-2*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))-2/b^2*d^2*ln(exp(I*(b*x+a
))+1)*x+2/b*c*d*ln(1-I*exp(I*(b*x+a)))*x+2/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a
-2/b*c*d*ln(1+I*exp(I*(b*x+a)))*x+1/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2
```

maxima [B] time = 0.77, size = 1632, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))
- 2*a*c*d*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1))/
b + a^2*d^2*(2/sin(b*x + a) - log(sin(b*x + a) + 1) + log(sin(b*x + a) - 1)
)/b^2 - 2*((2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 2*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*I*(b*x + a)^2*d
^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x
+ a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a)
- 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (-2*
I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*a
rctan2(cos(b*x + a), -sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*
d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-4*I*b*c*d - 4*
I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*
x + a) + 1) - (4*b*c*d - 4*a*d^2 - 4*(b*c*d - a*d^2)*cos(2*b*x + 2*a) - (4*
I*b*c*d - 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) -
 1) - (4*(b*x + a)*d^2*cos(2*b*x + 2*a) + 4*I*(b*x + a)*d^2*sin(2*b*x + 2*a
) - 4*(b*x + a)*d^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*((b*x + a
)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(b*x + a) + (4*b*c*d + 4*(b*x + a
)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-4*
I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x
 + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 - 4*(b*c*d + (b*x + a)*d^2
- a*d^2)*cos(2*b*x + 2*a) - (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin
(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + 4*(d^2*cos(2*b*x + 2*a) + I*d^2*
sin(2*b*x + 2*a) - d^2)*dilog(-e^(I*b*x + I*a)) - 4*(d^2*cos(2*b*x + 2*a) +
 I*d^2*sin(2*b*x + 2*a) - d^2)*dilog(e^(I*b*x + I*a)) + (-2*I*b*c*d - 2*I*(
b*x + a)*d^2 + 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2)*cos(
2*b*x + 2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I*b*c*d + 2*I*(b*x +
 a)*d^2 - 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*cos(2*b*
x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b
*c*d - 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2
)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
 a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a)
 + 1) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (I*(b*x
+ a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x +
a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (-4*I*d^2*cos(2*b*x + 2*a) + 4*d
^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, I*e^(I*b*x + I*a)) + (4*I*d^2*cos
(2*b*x + 2*a) - 4*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, -I*e^(I*b*x +
I*a)) + (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(b*x
 + a))/(-2*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) + 2*I*b^2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc^2(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x), x)
```

3.237 $\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=131

$$\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b}-\frac{2}{}$$

```
[Out] -2*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2-d*x*arctanh(sin
(b*x+a))/b+(d*x+c)*arctanh(sin(b*x+a))/b-(d*x+c)*csc(b*x+a)/b+I*d*polylog(2
,-I*exp(I*(b*x+a)))/b^2-I*d*polylog(2,I*exp(I*(b*x+a)))/b^2
```

Rubi [A] time = 0.13, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2621, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770}

$$\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{(c+dx)\csc(a+bx)}{b}+\frac{(c+dx)\tanh^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] ((-2*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (d
*x*ArcTanh[Sin[a + b*x]])/b + ((c + d*x)*ArcTanh[Sin[a + b*x]])/b - ((c + d
*x)*Csc[a + b*x])/b + (I*d*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - (I*d*Pol
yLog[2, I*E^(I*(a + b*x))])/b^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx = \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - d\int\left(\frac{\tanh^{-1}(\sin(\right.$$
$$= \frac{(c+dx)\tanh^{-1}(\sin(a+bx))}{b} - \frac{(c+dx)\csc(a+bx)}{b} - \frac{d\int\tanh^{-1}(\sin(a}{b}$$
$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh^{-1}(\sin}{b}$$
$$= -\frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b} + \frac{(c+dx)\tanh^{-1}(\sin}{b}$$
$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b}$$
$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b}$$
$$= -\frac{2idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{dx\tanh^{-1}(\sin(a+bx))}{b}$$

Mathematica [C] time = 2.91, size = 517, normalized size = 3.95

$$\frac{d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} + \frac{d\left(a\cos\left(\frac{1}{2}(a+bx)\right)-(a+bx)\cos\left(\frac{1}{2}(a+bx)\right)\right)\csc\left(\frac{1}{2}(a+bx)\right)}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x],x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (d*x*(a*Log
[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(a
+ b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 + I) -
(1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 + I
/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2]
)]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2
])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*Ta
n[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1
- I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(b*(a - I*Log[1 - I*Tan[(a + b*x)/2]
] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*Sec[(a + b*x)/2]*(a*Sin[(a + b*x)/
2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

fricas [B] time = 0.50, size = 434, normalized size = 3.31

$$-\frac{2\,bdx + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)\sin\left(bx+a\right) + i\,d\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)\sin\left(bx+a\right) - i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + I*d*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + I*d
*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x +
 a) + sin(b*x + a))*sin(b*x + a) - I*d*dilog(-I*cos(b*x + a) - sin(b*x + a)
)*sin(b*x + a) - (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + d
*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*d*x + a*d)*log(I*cos(b*x + a
) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(I*cos(b*x + a) - sin
(b*x + a) + 1)*sin(b*x + a) - (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x +
 a) + 1)*sin(b*x + a) + (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) +
1)*sin(b*x + a) - d*log(-1/2*cos(b*x + a) + 1/2)*sin(b*x + a) - (b*c - a*d)
*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*c - a*d)*log(-co
s(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*c)/(b^2*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)\csc\left(bx+a\right)^{2}\sec\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a), x)
```

maple [A] time = 0.13, size = 235, normalized size = 1.79

$$-\frac{2ie^{i(bx+a)}\left(dx+c\right)}{b\left(e^{2i(bx+a)}-1\right)}+\frac{2ida\arctan\left(e^{i(bx+a)}\right)}{b^2}+\frac{d\ln\left(e^{i(bx+a)}-1\right)}{b^2}-\frac{d\ln\left(e^{i(bx+a)}+1\right)}{b^2}-\frac{2ic\arctan\left(e^{i(bx+a)}\right)}{b}-\frac{id\,\mathrm{dilog}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x)
```

```
[Out] -2*I*exp(I*(b*x+a))*(d*x+c)/b/(exp(2*I*(b*x+a))-1)+2*I/b^2*d*a*arctan(exp(I
*(b*x+a)))+d/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)-2*I/b*c*ar
ctan(exp(I*(b*x+a)))-I/b^2*d*dilog(1-I*exp(I*(b*x+a)))+I/b^2*d*dilog(1+I*ex
```

```
p(I*(b*x+a)))-1/b*d*ln(1+I*exp(I*(b*x+a)))*x-1/b^2*d*ln(1+I*exp(I*(b*x+a)))
*a+1/b*d*ln(1-I*exp(I*(b*x+a)))*x+1/b^2*d*ln(1-I*exp(I*(b*x+a)))*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a),x, algorithm="maxima")
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^2),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^2(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a),x)
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x), x)
```

3.238 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.55, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)/(d*x + c), x)
```

maple [A] time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)), x)
```

<u>**sympy**</u> **[A]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c),x)
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

3.239 $\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 11.08, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 3.12, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\sec\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(a+bx\right)\sec\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

3.240 $\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\csc^3(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)

Rubi [A] time = 0.24, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]

[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Mathematica [A] time = 18.29, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec(a+bx)\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x],x]

[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x], x]

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc\left(bx+a\right)^3 \sec\left(bx+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")

```
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right) \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)\,\sin(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3),x)
[Out] int((c + d*x)^m/(cos(a + b*x)*sin(a + b*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

3.241 $\int (c+dx)^3 \csc^3(a+bx) \sec(a+bx)\,dx$

Optimal. Leaf size=325

$$-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+bx}\right.}{2b^3}$$

```
[Out] -3/2*I*d*(d*x+c)^2/b^2-1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a))
)/b-3/2*d*(d*x+c)^2*cot(b*x+a)/b^2-1/2*(d*x+c)^3*cot(b*x+a)^2/b+3*d^2*(d*x+
c)*ln(1-exp(2*I*(b*x+a)))/b^3+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a))
)/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2
,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/2
*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polylog(4,-exp(2*I*(
b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.82, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3717, 2190, 2279, 2391, 32, 2551, 4183, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{3ia}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] (((-3*I)/2)*d*(c + d*x)^2)/b^2 - (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh
[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Cot[a + b*x])/(2*b^2) - ((c + d
*x)^3*Cot[a + b*x]^2)/(2*b) + (3*d^2*(c + d*x)*Log[1 - E^((2*I)*(a + b*x))]
)/b^3 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((
(3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - (((3*I)/2)*d*(c + d*x)^
2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2
*I)*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))]
)/(2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/
4)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int (c+dx)^2$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - (3d)\int \frac{(c+dx)}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int (c+dx)^2}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} - \frac{(3d)\int \left(-(c+d\right.}{}$$

$$= -\frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(3d)\int (c+dx)^2}{2}$$

$$= -\frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^3\csc(}{b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\cot(a+bx)}{2b^2} - \frac{(c+dx)^3\cot^2(a}{2b}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{2}$$

$$= -\frac{3id(c+dx)^2}{2b^2} - \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2}{2}$$

Mathematica [B] time = 6.97, size = 1477, normalized size = 4.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*((c + d*x)^3*Csc[a + b*x]^2)/b - (c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^
((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] +
(3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^(
```

```
(2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a +
 b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a +
b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(2*b^3) - (d^3*E^
(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[
1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*
(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))
] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a +
b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a
 + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)
*(a + b*x))]))/E^((2*I)*a)))/(4*b^4) + (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
+ d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E
^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLo
g[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)
*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a
) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1
+ E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*Pol
yLog[3, -E^((-2*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E
^((2*I)*a)))*Sec[a] - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c^3*Csc[a]*(-(b*x*Cos[a])
 + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)
) + (3*c*d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]
*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*Arc
Tan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*
I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]])
)] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I
*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])
/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (3*Csc[a]*Csc[a + b*x]*(c^2
*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*d*Cs
c[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]
) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*
(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x
+ ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a]
)/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^
3*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2
*I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[
b*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Ta
n[a])/Sqrt[1 + Tan[a]^2]))/(2*b^4*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.77, size = 3459, normalized size = 10.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) + (3*I*b^2*d^3*x^
2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2
*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) + I*
sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d
^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x
+ a)^2)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c
*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*
d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3
*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*
b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + (3*I*b^2*d^3*x^2 + 6*I*b^2*
c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x +
a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + (-3*I*b^2*d^3*x^
2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a)
- sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I
*d^3 + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*
x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^
2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x -
 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x +
 a)) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x
+ a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) + I*sin(b*x + a) + 1) +
(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 - (b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 3*b*c*d^2 + 3*(b^3*c^2*d + b*d^3)*x)*cos
(b*x + a)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(cos(b*x + a) - I*sin(b*x + a) +
1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^
2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin
(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2)*log(I*cos(b
*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b^3*d^3*x^3 + 3
*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*c
os(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3 - (b
^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^
2 + a^3*d^3)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3
*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3
*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-
```

```
1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - (b^3*c^3 - 3*a*b^2*c^2*d + 3
*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 +
1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I
*sin(b*x + a) + 1/2) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a
^2*b*c*d^2 + (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2
*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a
)^2 + 3*(b^3*c^2*d + b*d^3)*x)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + (b
^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d
 + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x +
 a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 +
 (a^3 + 3*a)*d^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2 + 3*(b^3
*c^2*d + b*d^3)*x)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + (b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3 - (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - a^3*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (
6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, cos(b*x + a) + I*sin(b*x + a))
 + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, cos(b*x + a) - I*sin(b*x
+ a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, I*cos(b*x + a) + sin(
b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, I*cos(b*x + a) -
 sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4, -I*cos(b*x
+ a) + sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylog(4, -I*cos
(b*x + a) - sin(b*x + a)) + (-6*I*d^3*cos(b*x + a)^2 + 6*I*d^3)*polylog(4,
-cos(b*x + a) + I*sin(b*x + a)) + (6*I*d^3*cos(b*x + a)^2 - 6*I*d^3)*polylo
g(4, -cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*
c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3
*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a)
- I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)
^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d
^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) +
 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*c
os(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*co
s(b*x + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c
*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) + I*sin
(b*x + a)) - 6*(b*d^3*x + b*c*d^2 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*pol
ylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a)^2 - b^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a), x)
```

maple [B] time = 0.19, size = 1223, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-1/b*c^3*ln(1+exp(2*I*(b*x+
a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a
)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)
))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+6*I/b^4*d^3*polylog(4,-exp(I*(b
*x+a)))-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/2/b^3*c*d^2*polylog(3,-exp(
2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(
4,-exp(2*I*(b*x+a)))/b^4+1/b*c^3*ln(exp(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x
+a))+1)+3*I/b^2*c*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(
b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d
^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a
))+3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))+6*I*c*d^2*x+3*I*c^2*d)/b^2/(exp(2*I
*(b*x+a))-1)^2+3/b^3*d^2*c*ln(exp(I*(b*x+a))-1)+3/b^3*d^2*c*ln(exp(I*(b*x+a
))+1)-6/b^3*d^2*c*ln(exp(I*(b*x+a)))+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3
*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3/b^4*d^3*a*ln
(exp(I*(b*x+a))-1)+6/b^4*d^3*a*ln(exp(I*(b*x+a)))-3*I/b^2*d^3*x^2-3*I/b^4*d
^3*a^2-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x
+a))-1)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-ex
p(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog
(2,-exp(I*(b*x+a)))*x^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp
(I*(b*x+a)))*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(
I*(b*x+a)))+3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+
1)*x^2-3/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+
1/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3
*ln(1+exp(2*I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-
6*I/b^3*d^3*a*x+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(1
+exp(2*I*(b*x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

maxima [B] time = 2.14, size = 5140, normalized size = 15.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 3*a*c^2*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a
```

```
)^2))/b + 3*a^2*c*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin
(b*x + a)^2))/b^2 - a^3*d^3*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - l
og(sin(b*x + a)^2))/b^3 - 2*(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 - (8*
(b*x + a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*
c*d^2 + a^2*d^3)*(b*x + a) + 2*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a
) - 4*(4*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d -
 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*(b*x + a)^3*d^3
 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^
2 + 18*I*a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + (-
36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2
- 36*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(
2*b*x + 2*a) + 1) + (6*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x
+ a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x +
 4*a) - 12*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) - (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^
2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^
2 - 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 36*I*b
*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c
^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (18*b*c*d^2 - 18*a*d^3 + 18*(b*c
*d^2 - a*d^3)*cos(4*b*x + 4*a) - 36*(b*c*d^2 - a*d^3)*cos(2*b*x + 2*a) - (-
18*I*b*c*d^2 + 18*I*a*d^3)*sin(4*b*x + 4*a) - (36*I*b*c*d^2 - 36*I*a*d^3)*s
in(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3*d
^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2
+ 1)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
+ 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) -
 12*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a
*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^
3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*
d^2 + (18*I*a^2 + 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-12*I*(b*x + a)
^3*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I
*a*b*c*d^2 + (-36*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(s
in(b*x + a), -cos(b*x + a) + 1) - 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)
*(b*x + a))*cos(4*b*x + 4*a) - (12*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*
b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (3
6*I*b^2*c^2*d - 36*(2*I*a + 1)*b*c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*
cos(2*b*x + 2*a) + (9*b^2*c^2*d - 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*a^2
*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b
*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a)
 - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*a^2*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 -
```

```
12*I*(b*x + a)^2*d^3 - 9*I*a^2*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a
))*sin(4*b*x + 4*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 24*I*(b*x + a)^2*d
^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)
^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*cos(4*b*x + 4*a) - 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a
^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c
^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 + (36*
I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72
*I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d
^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (18
*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 18*(a^2 + 1)*d^3 + 36*(b*c
*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (
a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 36*(b^2*c^
2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 + 1)*d^3 + 2*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x +
 a)^2*d^3 + (18*I*a^2 + 18*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*
sin(4*b*x + 4*a) + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*(b*x + a)^2*d^3
 + (-36*I*a^2 - 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b
*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 +
 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(
b*x + a) + (-4*I*(b*x + a)^3*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 +
 (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a
) + (8*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*
b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4
*(b*x + a)^3*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*(4*(b*x + a)^3*d^3 + 9*(b*c
*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1) - (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*
d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 +
 9*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*
I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I
*a^2 + 9*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I
*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b
^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*cos(2*b*x +
2*a) - 3*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
+ a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x +
 4*a) + 6*((b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x
 + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b
*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x +
a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x + a) + (
3*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(
```

```
b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 9*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 +
(-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^
2 + (-18*I*a^2 - 18*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^
3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*((b*x + a)^3*d
^3 + 3*b*c*d^2 - 3*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (6*d^3*cos(4*b*x + 4*a) - 12*d^
3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) - 12*I*d^3*sin(2*b*x + 2*a) +
 6*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + (36*d^3*cos(4*b*x + 4*a) - 72*d^
3*cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) - 72*I*d^3*sin(2*b*x + 2*a)
+ 36*d^3)*polylog(4, -e^(I*b*x + I*a)) + (36*d^3*cos(4*b*x + 4*a) - 72*d^3*
cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) - 72*I*d^3*sin(2*b*x + 2*a) +
36*d^3)*polylog(4, e^(I*b*x + I*a)) - (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 +
9*I*a*d^3 + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a
) + (18*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(
3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) - 6*(3*b*c*d^2 + 4*
(b*x + a)*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a))
 - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(
b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-72*I*b*c*d^2 - 72*I*(b*x +
a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3
)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))
*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a
*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) +
(-72*I*b*c*d^2 - 72*I*(b*x + a)*d^3 + 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*
c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 72*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) - (18*I*(b*x + a)
^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*(b*x
 + a)^3*d^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2
- (36*a - 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 +
36*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a)
 + 12*I*b^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) - 12*b^3*sin(2*b*x +
2*a) - 6*I*b^3))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Timed out
```

3.242 $\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=201

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\log(\sin(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-$$

```
[Out] -c*d*x/b-1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d*(d*x+c)*co
t(b*x+a)/b^2-1/2*(d*x+c)^2*cot(b*x+a)^2/b+d^2*ln(sin(b*x+a))/b^3+I*d*(d*x+c
)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(2,exp(2*I*(b*x+a)))/
b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*polylog(3,exp(2*I*(b*x
+a)))/b^3
```

Rubi [A] time = 0.44, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2620, 14, 4420, 6741, 12, 6742, 3720, 3475, 2551, 4183, 2531, 2282, 6589}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -((c*d*x)/b) - (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))
])/b - (d*(c + d*x)*Cot[a + b*x])/b^2 - ((c + d*x)^2*Cot[a + b*x]^2)/(2*b)
+ (d^2*Log[Sin[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*
x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Poly
Log[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x))
])/(2*b^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - (2d)\int (c+dx)$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - (2d)\int \frac{(c+dx)}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - \frac{d\int (c+dx)\left(-\right.}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} - \frac{d\int \left(-(c+dx)\right.}{}$$

$$= -\frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{d\int (c+dx)\cot}{b}$$

$$= -\frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2\csc(2a}{b}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{d(c+dx)\cot(a+bx)}{b^2} - \frac{(c+dx)^2\cot^2(a+bx)}{2b} + \frac{d^2\log}{}$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

$$= -\frac{cdx}{b} - \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d(c+dx)\cot(a+bx)}{b^2} -$$

Mathematica [B] time = 6.76, size = 872, normalized size = 4.34

$$-\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^2}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\,\mathrm{s}}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*((c + d*x)^2*Csc[a + b*x]^2)/b - (d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((
2*I)*a) + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3
```

```
*I)*b^2*(1 - E^((-2*I)*a))*x^2*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2
*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b
*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*
x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(6*b^3) + (x*(3*c^2
 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3
*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a)
)*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3,
-E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) +
(c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a])
)/(b*(Cos[a]^2 + Sin[a]^2)) + (d^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin
[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*(
(b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) -
Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x
 - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - Ar
cTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 +
Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Csc[a]*Csc
[a + b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^
(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a
]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2
]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.64, size = 1987, normalized size = 9.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b*d^2*x + b*c*d)*cos(b*x + a)
*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d
 + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin(b*x
 + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)
^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d + (2*I
*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a))
+ (-2*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dil
og(-I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2
*x - 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-2
*I*b*d^2*x - 2*I*b*c*d + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2)*dilog(-c
os(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2
*x^2 + 2*b^2*c*d*x + b^2*c^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*
```

```
cos(b*x + a)^2 + d^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2
*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(co
s(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + d^2)*cos(b*x + a)^2 + d^2)*log(cos(b*
x + a) - I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 -
2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I)
 + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x +
 a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - s
in(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-
I*cos(b*x + a) - sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 -
 (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2 - (b^2
*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-c
os(b*x + a) + I*sin(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a
) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2 - (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*si
n(b*x + a) + 1) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*c^2 - 2*a*b*c*d + a
^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 2*(d^2*co
s(b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(
b*x + a)^2 - d^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 2*(d^2*cos(b*
x + a)^2 - d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x
+ a)^2 - d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*(d^2*cos(b*x +
a)^2 - d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*(d^2*cos(b*x + a
)^2 - d^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*(d^2*cos(b*x + a)
^2 - d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*(d^2*cos(b*x + a)^
2 - d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^2 -
b^3)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a), x)
```

maple [B] time = 0.16, size = 632, normalized size = 3.14

$$\frac{d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}+\frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}-\frac{c^2\ln\left(1+e^{2i(bx+a)}\right)}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1
)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
log(3,exp(I*(b*x+a)))/b^3-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(
b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+
I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*c*d*polylog(2,-exp(2*I*(b*x+
a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*
c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(ex
p(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-2*I/b^2*d^2*polylo
g(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))-2*I/b^2*c*d*po
lylog(2,exp(I*(b*x+a)))+2*(b*d^2*x^2*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*
x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a)
)+I*d^2*x+I*d*c)/b^2/(exp(2*I*(b*x+a))-1)^2+1/b^3*d^2*ln(exp(I*(b*x+a))+1)-
2/b^3*d^2*ln(exp(I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.82, size = 2522, normalized size = 12.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2)
) - 2*a*c*d*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^
2))/b + a^2*d^2*(1/sin(b*x + a)^2 + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2))/b^2 + 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(b*x + a)*d^2*sin(4
*b*x + 4*a) - 4*b*c*d + 4*a*d^2 + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b
*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2
*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) +
(-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*(b*x + a)^2*d^2 + 4
*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(4*b*x + 4*a) - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)
*(b*x + a) + d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
```

```
4*I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(4*b*x + 4*a) - (4*I*(b*x + a)^2*d^2 + (
8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b
*x + a), cos(b*x + a) + 1) - (2*d^2*cos(4*b*x + 4*a) - 4*d^2*cos(2*b*x + 2*
a) + 2*I*d^2*sin(4*b*x + 4*a) - 4*I*d^2*sin(2*b*x + 2*a) + 2*d^2)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*
x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a)
 - 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (2*
I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + (
-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a)
)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + (4*I*(b*x + a)^2*d^2 + 4*b*c*d
 - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) -
(2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*co
s(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-2*I
*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*sin(4*b*x + 4*a) - (4*I*b*c*d + 4*I
*(b*x + a)*d^2 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) +
 (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I
*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*
I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (4
*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*
c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (-8*I*b*c*d - 8*I*(
b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-I*(b
*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 +
(-2*I*b*c*d + 2*I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2
 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 +
 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*
x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*
I*a*d^2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(
b*x + a) + I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d +
4*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*
c*d - a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^
2)*(b*x + a) + I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x +
a) + I*d^2)*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*
d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d -
a*d^2)*(b*x + a) + d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)
^2 - 2*cos(b*x + a) + 1) + (-I*d^2*cos(4*b*x + 4*a) + 2*I*d^2*cos(2*b*x + 2
*a) + d^2*sin(4*b*x + 4*a) - 2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^
(2*I*b*x + 2*I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) -
 4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -e^(
I*b*x + I*a)) + (4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
```

```
2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x
+ I*a)) - (4*(b*x + a)^2*d^2 - 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*
I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*c
os(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2
))/b
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc^3(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)**3*sec(a + b*x), x)
```

3.243 $\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx$

Optimal. Leaf size=141

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+bx))}{b}-\frac{2dx\tanh^{-1}(\ldots}{b}$$

```
[Out] -1/2*d*x/b-2*d*x*arctanh(exp(2*I*(b*x+a)))/b-1/2*d*cot(b*x+a)/b^2-1/2*(d*x+
c)*cot(b*x+a)^2/b-d*x*ln(tan(b*x+a))/b+(d*x+c)*ln(tan(b*x+a))/b+1/2*I*d*pol
ylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2620, 14, 4420, 3473, 8, 2548, 12, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\cot(a+bx)}{2b^2}-\frac{(c+dx)\cot^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+b\ldots}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -(d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Cot[a + b*x])/(2
*b^2) - ((c + d*x)*Cot[a + b*x]^2)/(2*b) - (d*x*Log[Tan[a + b*x]])/b + ((c
+ d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^
2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec(a+bx)\,dx = -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{(c+dx)\log(\tan(a+bx))}{b} - d\int\left(-\frac{\cot^2(a+bx)}{2b}\right.$$

$$= -\frac{(c+dx)\cot^2(a+bx)}{2b} + \frac{(c+dx)\log(\tan(a+bx))}{b} + \frac{d\int\cot^2(a+bx)\,d}{2b}$$

$$= -\frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)}{}$$

$$= -\frac{dx}{2b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c}{}$$

$$= -\frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} -$$

$$= -\frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} -$$

$$= -\frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d\cot(a+bx)}{2b^2} - \frac{(c+dx)\cot^2(a+bx)}{2b} -$$

Mathematica [A] time = 0.90, size = 210, normalized size = 1.49

$$\frac{d\left(\frac{1}{2}i\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\mathrm{L}\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x],x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (d*x*Csc[a + b*x]^2)/(2*b) + (a*d*Log[Cos[a + b
*x]])/b^2 - (c*(Csc[a + b*x]^2 + 2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]])
)/(2*b) - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a + b*x)^2 - (a + b*x)*L
og[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I)*(a + b*x))]))/b^2
+ (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((a + b*x)^2 + PolyLog
[2, E^((2*I)*(a + b*x))])))/b^2
```

fricas [B] time = 0.54, size = 942, normalized size = 6.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x + d*cos(b*x + a)*sin(b*x + a) + b*c + (-I*d*cos(b*x + a)^2 + I*d
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(c
os(b*x + a) - I*sin(b*x + a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(I*cos(b*x
 + a) + sin(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(I*cos(b*x + a) - s
in(b*x + a)) + (I*d*cos(b*x + a)^2 - I*d)*dilog(-I*cos(b*x + a) + sin(b*x +
 a)) + (-I*d*cos(b*x + a)^2 + I*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(I*d*cos(b*x + a)^2 - I*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-I*d*co
s(b*x + a)^2 + I*d)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b*d*x - (b*d*x
 + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*
c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) + I*sin(b*x + a) + I)
 - (b*d*x - (b*d*x + b*c)*cos(b*x + a)^2 + b*c)*log(cos(b*x + a) - I*sin(b*
x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*log(I*cos
(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*d)*cos(b*x +
 a)^2 + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x - (b*d*x + a*
d)*cos(b*x + a)^2 + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + ((b*c -
a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a)
 + 1/2) + ((b*c - a*d)*cos(b*x + a)^2 - b*c + a*d)*log(-1/2*cos(b*x + a) -
1/2*I*sin(b*x + a) + 1/2) - (b*d*x - (b*d*x + a*d)*cos(b*x + a)^2 + a*d)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + 1) - ((b*c - a*d)*cos(b*x + a)^2 - b*c +
 a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - (b*d*x - (b*d*x + a*d)*cos(
b*x + a)^2 + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - ((b*c - a*d)*co
s(b*x + a)^2 - b*c + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*cos
(b*x + a)^2 - b^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a), x)
```

maple [B] time = 0.12, size = 270, normalized size = 1.91

$$\frac{2bdx\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}+id}{b^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}+\frac{c\ln\left(e^{i(bx+a)}+1\right)}{b}-\frac{d\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))+I*d)/
b^2/(exp(2*I*(b*x+a))-1)^2+1/b*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(1+exp(2*I*(b
*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,
-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a
)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.62, size = 1035, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a),x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2
*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) - 4*(b*d*x + b*c)*cos(2*b
*x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (4*I*b*d*x + 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) - 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) - 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) - 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) - 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) - 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) - 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
 - 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
 + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*cos(2*b*x +
2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b
*d*x + 4*b*c - 2*I*d)*sin(2*b*x + 2*a) - 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) +
4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a)
- 2*I*b^2)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc^3 (a + bx) \sec (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a),x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x), x)
```

3.244 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx=\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 14.07, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)/(d*x + c), x)
```

maple [A] time = 3.38, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\sec(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)\,\sin(a+b\,x)^3\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x), x)
```

3.245 $\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\operatorname{Int}\left(\frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 16.23, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.64, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 5.59, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\sec\left(bx+a\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)*sin(a + b*x)^3*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(a+bx\right)\sec\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)/(c + d*x)**2, x)
```

3.246 $\int (c+dx)^m \sec(a+bx) \tan(a+bx) \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\tan(a+bx)\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

Rubi [A] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sec(a+bx) \tan(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx) \tan(a+bx) \, dx = \int (c+dx)^m \sec(a+bx) \tan(a+bx) \, dx$$

Mathematica [A] time = 2.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec(a+bx) \tan(a+bx) \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x], x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sec(bx+a)\tan(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)\tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)\ (c+d\,x)^m}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x), x)
```

3.247 $\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=227

$$\frac{24id^4\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^5}-\frac{24id^4\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^5}+\frac{24d^3(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx}{}$$

```
[Out] 8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-12*I*d^2*(d*x+c)^2*polylog(2,-I*
exp(I*(b*x+a)))/b^3+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3+24*d
^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3*(d*x+c)*polylog(3,I*exp(
I*(b*x+a)))/b^4+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(
4,I*exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

Rubi [A] time = 0.19, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4409, 4181, 2531, 6609, 2282, 6589}

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{12id^2(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - ((12*I)*d^2*(c + d*x)^2
*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)*d^2*(c + d*x)^2*PolyLog[2,
 I*E^(I*(a + b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x)
)])/b^4 - (24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((24*I)*d^
4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I)*d^4*PolyLog[4, I*E^(I*(a
+ b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec(a+bx)}{b} - \frac{(4d)\int (c+dx)^3\sec(a+bx)\,dx}{b}$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4\sec(a+bx)}{b} + \frac{\left(12d^2\right)\int (c+dx$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c$$
$$= \frac{8id(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c$$

Mathematica [A] time = 1.17, size = 428, normalized size = 1.89

$$\frac{(c+dx)^4\sec(a+bx)}{b} - \frac{4d\left(-2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right) + 3b^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - I*E^(
I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*d^3*x^3*Lo
g[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 3*b^3
*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x
))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] - (3*I)*b^2*
d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*c*d^2*Poly
Log[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*
x))]))/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

fricas [C] time = 0.57, size = 1186, normalized size = 5.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

[Out] (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4 - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 12*I*d^4*cos(b*x + a)*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 12*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 12*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b^2*d^4*x^2 + 12*I*b^2*c*d^3*x + 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b^2*d^4*x^2 - 12*I*b^2*c*d^3*x - 6*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 2*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(b*d^4*x + b*c*d^3)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/(b^5*cos(b*x + a))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)\tan(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")

[Out] integrate((d*x + c)^4*sec(b*x + a)*tan(b*x + a), x)

maple [B] time = 0.20, size = 767, normalized size = 3.38

$$\frac{24id^3c\,a^2\arctan\left(e^{i(bx+a)}\right)}{b^4}-\frac{24id^2c^2a\arctan\left(e^{i(bx+a)}\right)}{b^3}-\frac{24id^3c\,\mathrm{polylog}\left(2,-ie^{i(bx+a)}\right)x}{b^3}+\frac{24id^3c\,\mathrm{polylog}\left(2,ie^{i(bx+}\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(4,I*exp(I*(b*x+a
)))/b^5+12/b^2*d^3*c*ln(1+I*exp(I*(b*x+a)))*x^2-12/b^2*d^3*c*ln(1-I*exp(I*(
b*x+a)))*x^2+12/b^4*d^3*a^2*c*ln(1-I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*
exp(I*(b*x+a)))*x-12/b^3*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*a-12/b^4*d^3*a^2*c*
ln(1+I*exp(I*(b*x+a)))+12/b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12/b^3*d^2*c
^2*ln(1+I*exp(I*(b*x+a)))*a-12*I/b^3*d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+1
2*I/b^3*d^4*polylog(2,I*exp(I*(b*x+a)))*x^2+12*I/b^3*d^2*c^2*polylog(2,I*ex
p(I*(b*x+a)))-8*I/b^5*d^4*a^3*arctan(exp(I*(b*x+a)))-12*I/b^3*d^2*c^2*polyl
og(2,-I*exp(I*(b*x+a)))+8*I/b^2*d*c^3*arctan(exp(I*(b*x+a)))-24*I/b^3*d^3*c
*polylog(2,-I*exp(I*(b*x+a)))*x+24*I/b^3*d^3*c*polylog(2,I*exp(I*(b*x+a)))*
x+24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a)))-24*I/b^3*d^2*c^2*a*arctan(exp(I
*(b*x+a)))+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*x^2+4*c^3*d*x+c^
4)/b/(1+exp(2*I*(b*x+a)))+24/b^4*d^4*polylog(3,-I*exp(I*(b*x+a)))*x-24/b^4*
d^4*polylog(3,I*exp(I*(b*x+a)))*x+4/b^5*d^4*a^3*ln(1+I*exp(I*(b*x+a)))+24/b
^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))-24/b^4*d^3*c*polylog(3,I*exp(I*(b*x+a
)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*x^3-4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x
^3-4/b^5*d^4*a^3*ln(1-I*exp(I*(b*x+a)))
```

maxima [B] time = 0.79, size = 2944, normalized size = 12.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] (2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*c^3
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2*a
)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b*x
 + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2
*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*a*c
^2*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b^2) + 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x
 + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos
```

```
(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1
))*a^2*c*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) - 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin
(2*b*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x +
a) + 1))*a^3*d^4/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*b^4) + c^4/cos(b*x + a) - 4*a*c^3*d/(b*cos(b*x + a)) + 6*a^2*c^2*
d^2/(b^2*cos(b*x + a)) - 4*a^3*c*d^3/(b^3*cos(b*x + a)) + a^4*d^4/(b^4*cos(
b*x + a)) + ((4*(b*x + a)^3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*
d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a
))*cos(2*b*x + 2*a) - (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*
(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a)
)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*(b*x + a)^
3*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a) + 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 +
 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-4*
I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c
^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan
2(cos(b*x + a), -sin(b*x + a) + 1) - (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 -
8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)
*(b*x + a)^2)*cos(b*x + a) + (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^
2*d^4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a
*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2
*b*x + 2*a) - (-12*I*b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*(b*x + a)^2*d^4 -
12*I*a^2*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*di
log(I*e^(I*b*x + I*a)) - (12*b^2*c^2*d^2 - 24*a*b*c*d^3 + 12*(b*x + a)^2*d^
4 + 12*a^2*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a) + 12*(b^2*c^2*d^2 - 2*a*b*c
*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x
 + 2*a) + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*(b*x + a)^2*d^4 + 12*I*
a^2*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I
*e^(I*b*x + I*a)) - (-2*I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x
 + a)^2 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a) + (-2
*I*(b*x + a)^3*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a)^2 + (-6*I*b^2*c^2
*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 2*((b*x
+ a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) - (2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*
(b*x + a)^2 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a) +
(2*I*(b*x + a)^3*d^4 + (6*I*b*c*d^3 - 6*I*a*d^4)*(b*x + a)^2 + (6*I*b^2*c^2
*d^2 - 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 2*((b*x
+ a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3
 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
```

```
2 - 2*sin(b*x + a) + 1) - 24*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a)
 + d^4)*polylog(4, I*e^(I*b*x + I*a)) + 24*(d^4*cos(2*b*x + 2*a) + I*d^4*si
n(2*b*x + 2*a) + d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (-24*I*b*c*d^3 - 24*
I*(b*x + a)*d^4 + 24*I*a*d^4 + (-24*I*b*c*d^3 - 24*I*(b*x + a)*d^4 + 24*I*a
*d^4)*cos(2*b*x + 2*a) + 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2
*a))*polylog(3, I*e^(I*b*x + I*a)) - (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 2
4*I*a*d^4 + (24*I*b*c*d^3 + 24*I*(b*x + a)*d^4 - 24*I*a*d^4)*cos(2*b*x + 2*
a) - 24*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(3, -I*e
^(I*b*x + I*a)) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*
(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2)*sin(b*x + a))/(-I*b^4*co
s(2*b*x + 2*a) + b^4*sin(2*b*x + 2*a) - I*b^4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^4}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \tan(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x), x)
```

3.248 $\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=159

$$\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{6id(c+dx)^2\tan^{-1}}{b^2}$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylog(2,-I*exp
(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*poly
log(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^
3*sec(b*x+a)/b
```

Rubi [A] time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4409, 4181, 2531, 2282, 6589}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{PolyLo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*Po
lyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(
I*(a + b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*
PolyLog[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\sec(a+bx)\,dx}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{\left(6d^2\right)\int (c+dx)}{}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+}{}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+}{}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+}{}$$

Mathematica [A] time = 0.81, size = 256, normalized size = 1.61

$$\frac{(c+dx)^3\sec(a+bx)}{b} - \frac{3d\left(-2ib^2c^2\tan^{-1}\left(e^{i(a+bx)}\right) + 2b^2cdx\log\left(1-ie^{i(a+bx)}\right) - 2b^2cdx\log\left(1+ie^{i(a+bx)}\right) + b^2d^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

fricas [C] time = 0.53, size = 779, normalized size = 4.90

$$\frac{2\,b^3d^3x^3 + 6\,b^3cd^2x^2 + 6\,b^3c^2dx + 2\,b^3c^3 + 6\,d^3\cos\left(bx+a\right)\operatorname{polylog}\left(3, i\,\cos\left(bx+a\right) + \sin\left(bx+a\right)\right) - 6\,d^3\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b
*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(I*
cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*di
log(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x
+ a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d
- 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I
) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*lo
g(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a
*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c
*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*
x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b
*x + a) - I*sin(b*x + a) + I))/(b^4*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec\left(bx+a\right)\tan\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a), x)
```

maple [B] time = 0.13, size = 463, normalized size = 2.91

$$\frac{2\,\mathrm{e}^{i(bx+a)}\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}-\frac{6d^2c\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)a}{b^3}+\frac{3d^3\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)x^2}{b^2}+\frac{6id\,c^2\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 2*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))
-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+
6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6
/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/
b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b
^3+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/
b^4+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))-12*I/b^3*d^2*c*a*arctan(exp(I*(b
*x+a)))-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*c*d^2*polylog(2,I*ex
p(I*(b*x+a)))/b^3-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+6/b^2*d^2*c*ln(1+I*
exp(I*(b*x+a)))*x+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3
```

maxima [B] time = 0.62, size = 1774, normalized size = 11.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*c^2*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*
b) - 6*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x +
 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))
*a*c*d^2/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
)*b^2) + 3*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b
*x + 2*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + s
in(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
a)^2 + 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*c
os(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
```

```
 1))*a^2*d^3/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
 + 1)*b^3) + 2*c^3/cos(b*x + a) - 6*a*c^2*d/(b*cos(b*x + a)) + 6*a^2*c*d^2/
(b^2*cos(b*x + a)) - 2*a^3*d^3/(b^3*cos(b*x + a)) + 2*((6*(b*x + a)^2*d^3 +
 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I
*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1
) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^
3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3
 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*
x + a), -sin(b*x + a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*
d^3)*(b*x + a)^2)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3
+ 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 -
12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a))
 - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3
- a*d^3)*cos(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3
)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) - (-3*I*(b*x + a)^2*d^3 + (-6
*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 +
 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 -
 a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
2*sin(b*x + a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x
 + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b
*x + 2*a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-12*I*d
^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*polylog(3, I*e^(I
*b*x + I*a)) - (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3*sin(2*b*x + 2*a) + 12*I*
d^3)*polylog(3, -I*e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2)*sin(b*x + a))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x
 + 2*a) - 2*I*b^3))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,(c+d\,x)^3}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3\tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x), x)
```

3.249 $\int (c+dx)^2 \sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=97

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)
))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4409, 4181, 2279, 2391}

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I
)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + (
(c + d*x)^2*Sec[a + b*x])/b
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\sec(a+bx)\,dx}{b}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{\left(2d^2\right)\int \log\left(1-\right.}{b^2}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{\left(2id^2\right)\text{Subst}\left(\int\right.}{}$$
$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}$$

Mathematica [A] time = 1.66, size = 174, normalized size = 1.79

$$\frac{b^2(c+dx)^2\sec(a+bx) - 4bcd\tanh^{-1}\left(\cos(a)\tan\left(\frac{bx}{2}\right)+\sin(a)\right) + \frac{2d^2\csc(a)\left(i\text{Li}_2\left(-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-i\text{Li}_2\left(e^{i\left(bx-\tan^{-1}(\cot(a\right.}\right.\right.}{}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a + b*x])/b^3
```

fricas [B] time = 0.50, size = 446, normalized size = 4.60

$$\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos\left(bx+a\right)\text{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right) + i\,d^2\cos\left(bx+a\right)\text{Li}_2\left(i\,\cos\left(bx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b*c*d - a*d^2)*cos(b*x + a
)*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log
(cos(b*x + a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*
cos(b*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos
(b*x + a) - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b
*x + a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x
 + a) - sin(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*
sin(b*x + a) + I))/(b^3*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec(bx+a)\tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a), x)
```

maple [B] time = 0.02, size = 239, normalized size = 2.46

$$\frac{d^2x^2}{b\cos(bx+a)} + \frac{2d^2\ln\left(1+ie^{i(bx+a)}\right)x}{b^2} + \frac{2d^2\ln\left(1+ie^{i(bx+a)}\right)a}{b^3} - \frac{2d^2\ln\left(1-ie^{i(bx+a)}\right)x}{b^2} - \frac{2d^2\ln\left(1-ie^{i(bx+a)}\right)a}{b^3} + \frac{2id}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 1/b*d^2/cos(b*x+a)*x^2+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I*
exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/b^3*d^2*ln(1-I*exp(I
*(b*x+a)))*a+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*d^2*dilog(1+I*ex
p(I*(b*x+a)))+2/b^3*a*d^2*ln(sec(b*x+a)+tan(b*x+a))+2/b*c*d/cos(b*x+a)*x-2/
b^2*c*d*ln(sec(b*x+a)+tan(b*x+a))+1/b/cos(b*x+a)*c^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,\left(c+d\,x\right)^2}{\cos(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x),x)
[Out] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a),x)
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x), x)
```

3.250 $\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=29

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}$$

```
[Out] -d*arctanh(sin(b*x+a))/b^2+(d*x+c)*sec(b*x+a)/b
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4409, 3770}

$$\frac{(c+dx)\sec(a+bx)}{b}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Sec[a + b*x])/b
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\sec(a+bx)\tan(a+bx)\,dx &= \frac{(c+dx)\sec(a+bx)}{b}-\frac{d\int\sec(a+bx)\,dx}{b}\\ &= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}\end{aligned}$$

Mathematica [B] time = 0.04, size = 93, normalized size = 3.21

$$\frac{d\log\left(\cos\left(\frac{a}{2}+\frac{bx}{2}\right)-\sin\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}-\frac{d\log\left(\sin\left(\frac{a}{2}+\frac{bx}{2}\right)+\cos\left(\frac{a}{2}+\frac{bx}{2}\right)\right)}{b^2}+\frac{c\sec(a+bx)}{b}+\frac{dx\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x],x]
```

```
[Out] (d*Log[Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2]])/b^2 - (d*Log[Cos[a/2 + (b*
x)/2] + Sin[a/2 + (b*x)/2]])/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b*x])/
b
```

fricas [B] time = 0.47, size = 60, normalized size = 2.07

$$\frac{2\,bdx - d\cos\left(bx + a\right)\log\left(\sin\left(bx + a\right) + 1\right) + d\cos\left(bx + a\right)\log\left(-\sin\left(bx + a\right) + 1\right) + 2\,bc}{2\,b^{2}\cos\left(bx + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - d*cos(b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-s
in(b*x + a) + 1) + 2*b*c)/(b^2*cos(b*x + a))
```

giac [B] time = 1.31, size = 1537, normalized size = 53.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2*tan(1/2*a)^
2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*
tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan
(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^2*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)
^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a
)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2 + 2*b*d*x*ta
n(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 +
2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^
4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1
/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) +
 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)
^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*
b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*
tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2
*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2 - 4*d*log(2*(tan(1/2*b*x)^4*tan
```

```
(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(ta
n(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*
tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b
*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*t
an(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)
 + 2*b*c*tan(1/2*a)^2 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2
*a)^2 + 1))*tan(1/2*a)^2 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2
*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan
(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 +
 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(
1/2*a)^2 + 1))*tan(1/2*a)^2 + 2*b*d*x + 2*b*c + d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x
)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
 + 1)/(tan(1/2*a)^2 + 1)))/(b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 - b^2*tan(1/2*b
*x)^2 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) - b^2*tan(1/2*a)^2 + b^2)
```

maple [A] time = 0.02, size = 49, normalized size = 1.69

$$\frac{dx}{b\cos\left(bx+a\right)}-\frac{d\ln\left(\sec\left(bx+a\right)+\tan\left(bx+a\right)\right)}{b^{2}}+\frac{c}{b\cos\left(bx+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] 1/b*d/cos(b*x+a)*x-1/b^2*d*ln(sec(b*x+a)+tan(b*x+a))+1/b*c/cos(b*x+a)
```

maxima [B] time = 0.45, size = 259, normalized size = 8.93

$$\frac{\left(4\,(bx+a)\cos(2\,bx+2\,a)\cos(bx+a)+4\,(bx+a)\sin(2\,bx+2\,a)\sin(bx+a)+4\,(bx+a)\cos(bx+a)-\left(\cos(2\,bx+2\,a)^{2}+\sin(2\,bx+2\,a)^{2}+2\,\cos(2\,bx+2\,a)+1\right)\log\left(\cos\right.\right.}{\left(\cos(2\,bx+2\,a)^{2}+\sin(2\,bx+2\,a)^{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*((4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*sin(2*b*x + 2
*a)*sin(b*x + a) + 4*(b*x + a)*cos(b*x + a) - (cos(2*b*x + 2*a)^2 + sin(2*b
*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 +
 2*sin(b*x + a) + 1) + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1))*d
/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) + 2
*c/cos(b*x + a) - 2*a*d/(b*cos(b*x + a)))/b
```

mupad [B] time = 2.58, size = 78, normalized size = 2.69

$$\frac{d\ \ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}-\mathrm{i}\right)}{b^2}-\frac{d\ \ln\left(\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}+1\mathrm{i}\right)}{b^2}+\frac{2\,\mathrm{e}^{a\,1\mathrm{i}+b\,x\,1\mathrm{i}}\ (c+d\,x)}{b\ \left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2
 + (2*exp(a*1i + b*x*1i)*(c + d*x))/(b*(exp(a*2i + b*x*2i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan (a+bx)\sec (a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x), x)
```

3.251 $\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 11.05, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x), x)
```

3.252 $\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 19.12, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)*tan(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)/(c + d*x)**2, x)
```

3.253 $\int (c+dx)^m \tan^2(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\tan^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \tan^2(a+bx)\,dx = \int (c+dx)^m \tan^2(a+bx)\,dx$$

Mathematica [A] time = 2.88, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \tan(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*tan(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\tan^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^2,x)
[Out] int((d*x+c)^m*tan(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*tan(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \tan(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^m,x)
[Out] int(tan(a + b*x)^2*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*tan(a + b*x)**2, x)
```

3.254 $\int (c+dx)^3 \tan^2(a+bx)\,dx$

Optimal. Leaf size=128

$$\frac{3d^3\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}-\frac{i(c+dx)^3}{b}$$

```
[Out] -I*(d*x+c)^3/b-1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^2-3*I
*d^2*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^3+3/2*d^3*polylog(3,-exp(2*I*(b
*x+a)))/b^4+(d*x+c)^3*tan(b*x+a)/b
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3719, 2190, 2531, 2282, 6589, 32}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3d(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tan(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^3)/b - (c + d*x)^4/(4*d) + (3*d*(c + d*x)^2*Log[1 + E^((2*I
)*(a + b*x))])/b^2 - ((3*I)*d^2*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])
/b^3 + (3*d^3*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^4) + ((c + d*x)^3*Tan[
a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \tan^2(a+bx)\,dx = \frac{(c+dx)^3 \tan(a+bx)}{b} - \frac{(3d)\int (c+dx)^2 \tan(a+bx)\,dx}{b} - \int (c+dx)^3\,dx$$
$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \tan(a+bx)}{b} + \frac{(6id)\int \frac{e^{2i(a+bx)}(c+dx)^2}{1+e^{2i(a+bx)}}\,dx}{b}$$
$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3 \tan(a+bx)}{b} -$$
$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+\right.}}{b^3}$$
$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+\right.}}{b^3}$$
$$= -\frac{i(c+dx)^3}{b} - \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{3id^2(c+dx)\mathrm{Li}_2\left(-e^{2i(a+\right.}}{b^3}$$

Mathematica [B] time = 6.55, size = 424, normalized size = 3.31

$$\frac{3c^2 d \sec(a)(bx\sin(a) + \cos(a)\log(\cos(a)\cos(bx) - \sin(a)\sin(bx)))}{b^2\left(\sin^2(a)+\cos^2(a)\right)} + \frac{ie^{-ia}d^3\sec(a)\left(2b^2x^2\left(2bx - 3i\left(1+e^{2ia}\right)\log\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^2,x]
```

```
[Out] -1/4*(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)) + ((I/4)*d^3*(2*b^2*x^
2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1
+ E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a)
)*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^4*E^(I*a)) + (3*c^2*d*Sec[a
]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a
]^2 + Sin[a]^2)) + (3*c*d^2*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a
]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - A
rcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*
(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(
Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(c^3*Sin[b*x] + 3*c^2*d*x*Sin
[b*x] + 3*c*d^2*x^2*Sin[b*x] + d^3*x^3*Sin[b*x]))/b
```

fricas [C] time = 0.46, size = 373, normalized size = 2.91

$$-\frac{b^4d^3x^4 + 4\,b^4cd^2x^3 + 6\,b^4c^2dx^2 + 4\,b^4c^3x - 3\,d^3\text{polylog}\left(3, \frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right) - 3\,d^3\text{polylog}\left(3, \frac{\tan(bx+a)^2}{\tan(}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*b^4*c^2*d*x^2 + 4*b^4*c^3*x - 3*d^3
*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) -
 3*d^3*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 +
 1)) - (6*I*b*d^3*x + 6*I*b*c*d^2)*dilog(2*(I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1) - (-6*I*b*d^3*x - 6*I*b*c*d^2)*dilog(2*(-I*tan(b*x + a) - 1)
/(tan(b*x + a)^2 + 1) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*lo
g(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^3*x^2 + 2*b^2*c*
d^2*x + b^2*c^2*d)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) - 4*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \tan(bx+a)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^2, x)
```

maple [B] time = 0.13, size = 348, normalized size = 2.72

$$-\frac{d^3x^4}{4} - c\,d^2x^3 - \frac{3c^2d\,x^2}{2} - c^3x - \frac{6id^2c\,a^2}{b^3} + \frac{3d\,c^2\ln\left(1+e^{2i(bx+a)}\right)}{b^2} - \frac{6d\,c^2\ln\left(e^{i(bx+a)}\right)}{b^2} - \frac{6d^3a^2\ln\left(e^{i(bx+a)}\right)}{b^4} - \frac{6id^2c\,x^2}{b} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^2,x)
```

```
[Out] -1/4*d^3*x^4-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x-6*I*d^2/b^3*c*a^2+3*d/b^2*c^2*ln
(1+exp(2*I*(b*x+a)))-6*d/b^2*c^2*ln(exp(I*(b*x+a)))-6*d^3/b^4*a^2*ln(exp(I*
(b*x+a)))-6*I*d^2/b*c*x^2+6*I*d^3/b^3*a^2*x+2*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*
d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+3*d^3/b^2*ln(1+exp(2*I*(b*x+a)))*x^2+4*I*d^
3/b^4*a^3+3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+12*d^2/b^3*c*a*ln(exp(I*
(b*x+a)))-3*I*d^3/b^3*polylog(2,-exp(2*I*(b*x+a)))*x-2*I*d^3/b*x^3-3*I*d^2/
```

```
b^3*c*polylog(2,-exp(2*I*(b*x+a)))+6*d^2/b^2*c*ln(1+exp(2*I*(b*x+a)))*x-12*
I*d^2/b^2*c*a*x
```

maxima [B] time = 0.63, size = 1363, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c^3 - 6*(b*x + a - tan(b*x + a))*a*c^2*d/b
 + 6*(b*x + a - tan(b*x + a))*a^2*c*d^2/b^2 - 2*(b*x + a - tan(b*x + a))*a^
3*d^3/b^3 + 3*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a
)^2 + 2*(b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 +
sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^2
*d/((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b) -
 6*((b*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*
x + a)^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)
^2 + 2*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^2/((cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^2) + 3*((b*
x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)^
2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*
cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*a^2*d^3/((cos(2*b*x +
 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b^3) - 2*(I*(b*x + a
)^4*d^3 + (4*I*b*c*d^2 - 4*I*a*d^3)*(b*x + a)^3 + (12*(b*x + a)^2*d^3 + 24*
(b*c*d^2 - a*d^3)*(b*x + a) + 12*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*
x + a))*cos(2*b*x + 2*a) + (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a)
 + 1) + (I*(b*x + a)^4*d^3 + (4*I*b*c*d^2 - 4*(I*a + 2)*d^3)*(b*x + a)^3 -
24*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(2*b*x + 2*a) - (12*b*c*d^2 + 12*(b*x
+ a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(2*b*x + 2*a))*dilo
g(-e^(2*I*b*x + 2*I*a)) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d
^3)*(b*x + a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x +
 a))*cos(2*b*x + 2*a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*
x + 2*a) + 1) + (-6*I*d^3*cos(2*b*x + 2*a) + 6*d^3*sin(2*b*x + 2*a) - 6*I*d
^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - ((b*x + a)^4*d^3 + (4*b*c*d^2 - (4*a
 - 8*I)*d^3)*(b*x + a)^3 - (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a)^2)*sin(2*
b*x + 2*a))/(-4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(2*b*x + 2*a) - 4*I*b^3))
/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+bx)^2 (c+dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^3,x)
[Out] int(tan(a + b*x)^2*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan^2(a+bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*tan(a + b*x)**2, x)
```

3.255 $\int (c+dx)^2 \tan^2(a+bx)\,dx$

Optimal. Leaf size=96

$$-\frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

```
[Out] -I*(d*x+c)^2/b-1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^2-I*d^2
*polylog(2,-exp(2*I*(b*x+a)))/b^3+(d*x+c)^2*tan(b*x+a)/b
```

Rubi [A] time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{id^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^3}+\frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2}+\frac{(c+dx)^2\tan(a+bx)}{b}-\frac{i(c+dx)^2}{b}-\frac{(c+dx)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] ((-I)*(c + d*x)^2)/b - (c + d*x)^3/(3*d) + (2*d*(c + d*x)*Log[1 + E^((2*I)*
(a + b*x))])/b^2 - (I*d^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^3 + ((c + d*x
)^2*Tan[a + b*x])/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\tan^2(a+bx)\,dx = \frac{(c+dx)^2\tan(a+bx)}{b} - \frac{(2d)\int (c+dx)\tan(a+bx)\,dx}{b} - \int (c+dx)^2\,dx$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2\tan(a+bx)}{b} + \frac{(4id)\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx}{b}$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} -$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tan(a+bx)}{b} +$$

$$= -\frac{i(c+dx)^2}{b} - \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^2} - \frac{id^2\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^3} + \frac{(c}{}$$

Mathematica [B] time = 6.37, size = 276, normalized size = 2.88

$$\frac{2cd\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^2\left(\sin^2(a)+\cos^2(a)\right)} + \frac{d^2\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^2,x]
```

```
[Out] -1/3*(x*(3*c^2 + 3*c*d*x + d^2*x^2)) + (2*c*d*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^2*(Cos[a]^2 + Sin[a]^2)) + (d^2*
Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot
[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2
*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[
b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/S
qrt[1 + Cot[a]^2])*Sec[a])/(b^3*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Se
c[a]*Sec[a + b*x]*(c^2*Sin[b*x] + 2*c*d*x*Sin[b*x] + d^2*x^2*Sin[b*x]))/b
```

fricas [B] time = 0.47, size = 210, normalized size = 2.19

$$-\frac{2\,b^{3}d^{2}x^{3}+6\,b^{3}cdx^{2}+6\,b^{3}c^{2}x-3i\,d^{2}\mathrm{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}+1\right)+3i\,d^{2}\mathrm{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^{2}+1}+1\right)-6\left(bd^{2}x+bcd\right)\mathrm{lo}}{6\,b^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*I*d^2*dilog(2*(I*tan(
b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 3*I*d^2*dilog(2*(-I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) - 6*(b*d^2*x + b*c*d)*log(-2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 6*(b*d^2*x + b*c*d)*log(-2*(-I*tan(b*x + a) -
 1)/(tan(b*x + a)^2 + 1)) - 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*tan(b*x
 + a))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{2}\tan(bx+a)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^2, x)
```

maple [B] time = 0.09, size = 191, normalized size = 1.99

$$-\frac{d^{2}x^{3}}{3}-cd\,x^{2}-c^{2}x+\frac{2i\left(d^{2}x^{2}+2cdx+c^{2}\right)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}+\frac{2dc\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b^{2}}-\frac{4dc\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b^{2}}-\frac{2id^{2}x^{2}}{b}-\frac{4id^{2}ax}{b^{2}}-\frac{2id^{2}a^{2}}{b^{3}}+\frac{2d^{2}\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^2,x)
```

```
[Out] -1/3*d^2*x^3-c*d*x^2-c^2*x+2*I*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))
+2*d/b^2*c*ln(1+exp(2*I*(b*x+a)))-4/b^2*d*c*ln(exp(I*(b*x+a)))-2*I/b*d^2*x^
2-4*I/b^2*d^2*a*x-2*I/b^3*d^2*a^2+2*d^2/b^2*ln(1+exp(2*I*(b*x+a)))*x-I*d^2*
polylog(2,-exp(2*I*(b*x+a)))/b^3+4/b^3*d^2*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.60, size = 417, normalized size = 4.34

$$\frac{i b^3 d^2 x^3 + 3i b^3 c d x^2 + 3i b^3 c^2 x + 6 b^2 c^2 + \left(6 b d^2 x + 6 b c d + 6 \left(b d^2 x + b c d\right) \cos\left(2 b x + 2 a\right) + \left(6i b d^2 x + 6i b c d\right) s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (I*b^3*d^2*x^3 + 3*I*b^3*c*d*x^2 + 3*I*b^3*c^2*x + 6*b^2*c^2 + (6*b*d^2*x +
 6*b*c*d + 6*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a) + (6*I*b*d^2*x + 6*I*b*c*d)
*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + (I*b^3
*d^2*x^3 + (3*I*b^3*c*d - 6*b^2*d^2)*x^2 - 3*(-I*b^3*c^2 + 4*b^2*c*d)*x)*co
s(2*b*x + 2*a) - 3*(d^2*cos(2*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*di
log(-e^(2*I*b*x + 2*I*a)) + (-3*I*b*d^2*x - 3*I*b*c*d + (-3*I*b*d^2*x - 3*I
*b*c*d)*cos(2*b*x + 2*a) + 3*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))*log(cos(2*
b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (b^3*d^2*x^3
+ 3*(b^3*c*d + 2*I*b^2*d^2)*x^2 + (3*b^3*c^2 + 12*I*b^2*c*d)*x)*sin(2*b*x +
 2*a))/(-3*I*b^3*cos(2*b*x + 2*a) + 3*b^3*sin(2*b*x + 2*a) - 3*I*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(a + b\,x\right)^2\,\left(c + d\,x\right)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^2 \tan^2\left(a + bx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2, x)
```

3.256 $\int (c+dx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=40

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

```
[Out] -c*x-1/2*d*x^2+d*ln(cos(b*x+a))/b^2+(d*x+c)*tan(b*x+a)/b
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3720, 3475}

$$\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}-cx-\frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -(c*x) - (d*x^2)/2 + (d*Log[Cos[a + b*x]])/b^2 + ((c + d*x)*Tan[a + b*x])/b
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\tan^2(a+bx)\,dx &= \frac{(c+dx)\tan(a+bx)}{b}-\frac{d\int \tan(a+bx)\,dx}{b}-\int (c+dx)\,dx\\ &= -cx-\frac{dx^2}{2}+\frac{d\log(\cos(a+bx))}{b^2}+\frac{(c+dx)\tan(a+bx)}{b}\end{aligned}$$

Mathematica [A] time = 0.28, size = 76, normalized size = 1.90

$$\frac{d\log(\cos(a+bx))}{b^2}-\frac{c\tan^{-1}(\tan(a+bx))}{b}+\frac{c\tan(a+bx)}{b}+\frac{dx\sec(a)\sin(bx)\sec(a+bx)}{b}-\frac{dx\sec(a)(bx\cos(a)-2}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^2,x]
```

```
[Out] -((c*ArcTan[Tan[a + b*x]])/b) + (d*Log[Cos[a + b*x]])/b^2 - (d*x*Sec[a]*(b*
x*Cos[a] - 2*Sin[a]))/(2*b) + (d*x*Sec[a]*Sec[a + b*x]*Sin[b*x])/b + (c*Tan
[a + b*x])/b
```

fricas [A] time = 0.44, size = 53, normalized size = 1.32

$$-\frac{b^2dx^2+2\,b^2cx-d\log\left(\frac{1}{\tan(bx+a)^2+1}\right)-2\,(bdx+bc)\tan\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*x^2 + 2*b^2*c*x - d*log(1/(tan(b*x + a)^2 + 1)) - 2*(b*d*x + b*
c)*tan(b*x + a))/b^2
```

giac [B] time = 0.56, size = 223, normalized size = 5.58

$$-\frac{b^2dx^2\tan\left(bx\right)\tan(a)+2\,b^2cx\tan\left(bx\right)\tan(a)-b^2dx^2-2\,b^2cx+2\,bdx\tan\left(bx\right)+2\,bdx\tan(a)-d\log\left(\frac{4\left(\tan(bx\right.}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(b^2*d*x^2*tan(b*x)*tan(a) + 2*b^2*c*x*tan(b*x)*tan(a) - b^2*d*x^2 - 2
*b^2*c*x + 2*b*d*x*tan(b*x) + 2*b*d*x*tan(a) - d*log(4*(tan(b*x)^4*tan(a)^2
 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + tan(b*x)^2 - 2*tan(b*x)*tan(
a) + 1)/(tan(a)^2 + 1))*tan(b*x)*tan(a) + 2*b*c*tan(b*x) + 2*b*c*tan(a) + d
*log(4*(tan(b*x)^4*tan(a)^2 - 2*tan(b*x)^3*tan(a) + tan(b*x)^2*tan(a)^2 + t
an(b*x)^2 - 2*tan(b*x)*tan(a) + 1)/(tan(a)^2 + 1)))/(b^2*tan(b*x)*tan(a) -
b^2)
```

maple [A] time = 0.05, size = 47, normalized size = 1.18

$$-\frac{d\,x^2}{2}-cx+\frac{d\tan\left(bx+a\right)x}{b}+\frac{d\ln\left(\cos\left(bx+a\right)\right)}{b^2}+\frac{c\tan\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^2,x)
```

```
[Out] -1/2*d*x^2-c*x+1/b*d*tan(b*x+a)*x+d*ln(cos(b*x+a))/b^2+1/b*c*tan(b*x+a)
```

maxima [B] time = 0.46, size = 237, normalized size = 5.92

$$-\frac{2\,(bx+a-\tan(bx+a))c-\frac{2\,(bx+a-\tan(bx+a))ad}{b}+\frac{\left((bx+a)^2\cos(2\,bx+2\,a)^2+(bx+a)^2\sin(2\,bx+2\,a)^2+2\,(bx+a)^2\cos(2\,bx+2\,a)+(bx+a)^2-\cdots\right.}{(c\cdots}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(b*x + a - tan(b*x + a))*c - 2*(b*x + a - tan(b*x + a))*a*d/b + ((b
*x + a)^2*cos(2*b*x + 2*a)^2 + (b*x + a)^2*sin(2*b*x + 2*a)^2 + 2*(b*x + a)
^2*cos(2*b*x + 2*a) + (b*x + a)^2 - (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2
*cos(2*b*x + 2*a) + 1) - 4*(b*x + a)*sin(2*b*x + 2*a))*d/((cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*b))/b
```

mupad [B] time = 1.44, size = 52, normalized size = 1.30

$$-c\,x-\frac{d\,x^2}{2}-\frac{\frac{d\,\ln\left(\tan(a+b\,x)^2+1\right)}{2}-b\,\left(c\tan\left(a+b\,x\right)+d\,x\tan\left(a+b\,x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2*(c + d*x),x)
```

```
[Out] - c*x - (d*x^2)/2 - ((d*log(tan(a + b*x)^2 + 1))/2 - b*(c*tan(a + b*x) + d*
x*tan(a + b*x)))/b^2
```

sympy [A] time = 0.25, size = 65, normalized size = 1.62

$$\begin{cases} -cx-\frac{dx^2}{2}+\frac{c\tan\left(a+bx\right)}{b}+\frac{dx\tan\left(a+bx\right)}{b}-\frac{d\log\left(\tan^2\left(a+bx\right)+1\right)}{2b^2} & \text{for } b\neq 0 \\ \left(cx+\frac{dx^2}{2}\right)\tan^2\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**2,x)
```

```
[Out] Piecewise((-c*x - d*x**2/2 + c*tan(a + b*x)/b + d*x*tan(a + b*x)/b - d*log(
tan(a + b*x)**2 + 1)/(2*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*tan(a)**2, True
))
```

3.257 $\int \frac{\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tan^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx=\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 3.83, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Tan[a + b*x]^2/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan\left(bx+a\right)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^2/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\left(bdx + (bdx+bc)\cos(2\,bx+2\,a)^2 + (bdx+bc)\sin(2\,bx+2\,a)^2 + bc + 2\,(bdx+bc)\cos(2\,bx+2\,a)\right)\log(dx+c}{bd^2x + bcd + \left(bd^2x+bcd\right)\cos(2\,bx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] (2*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a)^2 + (b*d^3*x +
 b*c*d^2)*sin(2*b*x + 2*a)^2 + 2*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a))*inte
grate(sin(2*b*x + 2*a)/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2 + 2*b*c*
d*x + b*c^2)*cos(2*b*x + 2*a)^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sin(2*b*x
 + 2*a)^2 + 2*(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cos(2*b*x + 2*a)), x) - (b*d*
x + (b*d*x + b*c)*cos(2*b*x + 2*a)^2 + (b*d*x + b*c)*sin(2*b*x + 2*a)^2 + b
*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a))*log(d*x + c) + 2*d*sin(2*b*x + 2*a))
/(b*d^2*x + b*c*d + (b*d^2*x + b*c*d)*cos(2*b*x + 2*a)^2 + (b*d^2*x + b*c*d
)*sin(2*b*x + 2*a)^2 + 2*(b*d^2*x + b*c*d)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan\left(a+b\,x\right)^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^2/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x), x)
```

3.258 $\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tan^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^2/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx = \int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 5.07, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^2/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]^2/(c + d*x)^2, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(tan(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.24, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^2/(d*x+c)^2,x)
[Out] int(tan(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(c + d*x)^2,x)
[Out] int(tan(a + b*x)^2/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**2/(c + d*x)**2, x)
```

3.259 $\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=150

$$\text{Int}\left(\tan(a+bx)\sec(a+bx)(c+dx)^m,x\right)+\frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx)^m$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a),x)+1/2*exp(I*(a-b*c/d))*(d*
x+c)^m*GAMMA(1+m,-I*b*(d*x+c)/d)/b/((-I*b*(d*x+c)/d)^m)+1/2*(d*x+c)^m*GAMMA
(1+m,I*b*(d*x+c)/d)/b/exp(I*(a-b*c/d))/((I*b*(d*x+c)/d)^m)
```

Rubi [A] time = 0.16, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (E^(I*(a - (b*c)/d))*(c + d*x)^m*Gamma[1 + m, ((-I)*b*(c + d*x))/d])/(2*b*(
((-I)*b*(c + d*x))/d)^m) + ((c + d*x)^m*Gamma[1 + m, (I*b*(c + d*x))/d])/(2
*b*E^(I*(a - (b*c)/d))*((I*b*(c + d*x))/d)^m) + Defer[Int][(c + d*x)^m*Sec[
a + b*x]*Tan[a + b*x], x]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx &= -\int (c+dx)^m \sin(a+bx)\,dx+\int (c+dx)^m \sec(a+bx)\tan(a+bx)\,d\\ &= -\left(\frac{1}{2}i\int e^{-i(a+bx)}(c+dx)^m\,dx\right)+\frac{1}{2}i\int e^{i(a+bx)}(c+dx)^m\,dx+\int (c+d\\ &= \frac{e^{i\left(a-\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{ib(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{ib(c+dx)}{d}\right)}{2b}+\frac{e^{-i\left(a-\frac{bc}{d}\right)}(c+dx\end{aligned}$$

Mathematica [A] time = 23.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Sin[a + b*x]*Tan[a + b*x]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(dx+c\right)^{m}\sin\left(bx+a\right)\tan\left(bx+a\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\sin\left(bx+a\right)\left(\tan^{2}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
[Out] int((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sin(b*x + a)*tan(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,\tan\left(a+b\,x\right)^{2}\,\left(c+d\,x\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m,x)
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sin(a+bx) \tan^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sin(b*x+a)*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*sin(a + b*x)*tan(a + b*x)**2, x)
```

3.260 $\int (c+dx)^3 \sin(a+bx) \tan^2(a+bx) \, dx$

Optimal. Leaf size=228

$$\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\sin(a+bx)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\cdots$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*cos(b*x+a)/b^3+(d*
x+c)^3*cos(b*x+a)/b-6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^
2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a))
)/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+(d*x+c)^3*sec(b*x+a)/b+6*d^3*si
n(b*x+a)/b^4-3*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A] time = 0.21, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {4407, 3296, 2637, 4409, 4181, 2531, 2282, 6589}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{PolyLo}\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*Cos[a
+ b*x])/b^3 + ((c + d*x)^3*Cos[a + b*x])/b - ((6*I)*d^2*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(a +
b*x))])/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog
[3, I*E^(I*(a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b + (6*d^3*Sin[a +
 b*x])/b^4 - (3*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^3 \sin(a+bx)\,dx + \int (c+dx)^3 \sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+dx)^2\cos(a-}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\cos(a+bx)}{b} + \frac{(c+dx)^3\sec(a-}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(}{b}$$
$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{(c+dx)^3\cos(}{b}$$

Mathematica [B] time = 1.54, size = 532, normalized size = 2.33

$$\frac{\sec(a+bx)\left(b^3c^3\cos(2(a+bx)) + 3b^3c^2dx\cos(2(a+bx)) + 3b^3cd^2x^2\cos(2(a+bx)) + b^3d^3x^3\cos(2(a+bx)) - 3b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b^3*c^3 - 6*b*c*d^2 + 9*b^3*c^2*d*x - 6*b*d^3*x + 9*b^3*c*
d^2*x^2 + 3*b^3*d^3*x^3 + (12*I)*b^2*c^2*d*ArcTan[E^(I*(a + b*x))]*Cos[a +
b*x] + b^3*c^3*Cos[2*(a + b*x)] - 6*b*c*d^2*Cos[2*(a + b*x)] + 3*b^3*c^2*d*
x*Cos[2*(a + b*x)] - 6*b*d^3*x*Cos[2*(a + b*x)] + 3*b^3*c*d^2*x^2*Cos[2*(a
+ b*x)] + b^3*d^3*x^3*Cos[2*(a + b*x)] - 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1
- I*E^(I*(a + b*x))] - 6*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*E^(I*(a + b*x))
] + 12*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] + 6*b^2*d^3*x^2*
Cos[a + b*x]*Log[1 + I*E^(I*(a + b*x))] - (12*I)*b*d^2*(c + d*x)*Cos[a + b*
x]*PolyLog[2, (-I)*E^(I*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Cos[a + b*x]*P
olyLog[2, I*E^(I*(a + b*x))] + 12*d^3*Cos[a + b*x]*PolyLog[3, (-I)*E^(I*(a
+ b*x))] - 12*d^3*Cos[a + b*x]*PolyLog[3, I*E^(I*(a + b*x))] - 3*b^2*c^2*d*
Sin[2*(a + b*x)] + 6*d^3*Sin[2*(a + b*x)] - 6*b^2*c*d^2*x*Sin[2*(a + b*x)]
- 3*b^2*d^3*x^2*Sin[2*(a + b*x)]))/(2*b^4)
```

fricas [C] time = 0.57, size = 892, normalized size = 3.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -I
*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
a) - sin(b*x + a)) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2
 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 + (6*I*b*d^3*x + 6*I*b*c*d^2)*
cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*
d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*
I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3
*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
 + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x
 + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
- a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3
*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x +
a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2
*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2
 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a)
- sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^
3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*x
+ a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sin(bx+a) \tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sin(b*x + a)*tan(b*x + a)^2, x)
```

maple [B] time = 0.12, size = 677, normalized size = 2.97

$$\frac{\left(d^3x^3b^3 + 3b^3c\,d^2x^2 + 3ib^2d^3x^2 + 3b^3c^2dx + 6ib^2c\,d^2x + b^3c^3 + 3ib^2c^2d - 6b\,d^3x - 6c\,d^2b - 6id^3\right)e^{i(bx+a)}}{2b^4} + \frac{\left(d^3x^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*
d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+1
/2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d
^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))+2
*exp(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+
3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2-6
/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+6*
I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a
)))+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)
))+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I/b^2*d*c^2*arctan(exp(I*(b*
x+a)))-6*I*c*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-12*I/b^3*d^2*c*a*arctan(e
xp(I*(b*x+a)))-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2-6*d^3*polylog(3,I*exp(I
*(b*x+a)))/b^4-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+6/b^2*d^2*c*ln(1+
I*exp(I*(b*x+a)))*x+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4
```

maxima [B] time = 2.38, size = 11054, normalized size = 48.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c^3*(1/cos(b*x + a) + cos(b*x + a)) - 6*a*c^2*d*(1/cos(b*x + a) + co
s(b*x + a))/b + 6*a^2*c*d^2*(1/cos(b*x + a) + cos(b*x + a))/b^2 - 2*a^3*d^3
*(1/cos(b*x + a) + cos(b*x + a))/b^3 + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a)
 + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 +
((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*
(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos
(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x
+ a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(
b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) +
 (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
 + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x +
a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x +
 a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13
*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*
x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 +
2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + c
os(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*
(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*
x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x +
a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*co
s(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x
 + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2
```

```
*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
+ 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 +
sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*s
in(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x
+ a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x +
 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
 a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(
b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*
x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) +
 sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1)
+ (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 +
 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(
b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*
b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*
(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*
cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x
 + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^
2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(
b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*c^2*d
/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x +
3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*
x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin
(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(
b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a
) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b) - 6*((b*x + (b*x +
a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^3 + 6*(b*x + a
)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + 2*(4*(b*x + a)*c
os(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x + a) + (3*(b*x + a)*sin(
b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)^2 + ((b*x + a)*
cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8*(b*x + a)*sin(2*b*x +
2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a
))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x
 + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))*sin(3*b*x + 3*a)^2 + (
(b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2*a)^2 + ((b*x + a)*cos(
2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x + a)*sin(2*b*x + 2*a)^2
 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*cos(b*x + a)^2 + (b*x + a
```

```
)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (12*(b*x + a)*cos(b*x +
a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a) + a)*co
s(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(b*x + a)*cos(b*x + a)*s
in(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a) + (
b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos
(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x
+ 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2
*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(c
os(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*
b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*
log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) + ((cos(2*b*x + 2
*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (
cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 +
 sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x
 + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*
x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a)
+ cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(
2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*
b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))
*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2
*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*c
os(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + 2*(((b*x + a)*
sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)*cos(b*x + a) + s
in(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(
3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) - cos(b*x + a)^2 - s
in(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x + 2*a)^2 - cos(b*x + a)^2 +
 ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x + a)^2)*sin(2*b*x + 2*a)
- sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b*x + 3*a) + 6*((b*x + a)*
cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x + a)^3)*sin(2*b*x + 2*a) -
sin(b*x + a))*a*c*d^2/(((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*
b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(
2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2
*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x +
 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*
(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(co
s(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b
*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*b
^2) + 3*((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*b*
x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - cos
(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x +
a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x +
```

```
 a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x
 + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 + (8
*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2*a
) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a) -
sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a))
*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 2
*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*x
+ a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*co
s(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) + (
12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)*s
in(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3*(
b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x + a
))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin(2
*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^
2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin
(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(
b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a
))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)
+ cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*
sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x +
3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a)
 + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2
+ (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*
b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(co
s(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b
*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2
 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^
2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b
*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(b*
x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((b*
x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x +
a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x +
a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x + 2*
a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x +
a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b*x
 + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x + a)
^3)*sin(2*b*x + 2*a) - sin(b*x + a))*a^2*d^3/(((cos(2*b*x + 2*a)^2 + sin(2*
b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2
 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2
*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(
b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b
```

```
*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a)
)*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) +
 cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*s
in(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3
*a) + sin(b*x + a)^2)*b^3) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a + 6*I)*d
^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2 + ((b*x + a)^3*d^3 - 6*b*c*d
^2 + (6*a - 6*I)*d^3 + (3*b*c*d^2 - (3*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c
*d^2 - 6*(I*a + 1)*d^3)*(b*x + a))*cos(3*b*x + 3*a)^2 + 6*((b*x + a)^3*d^3
- 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*
cos(b*x + a)^2 - ((b*x + a)^3*d^3 - 6*b*c*d^2 + (6*a - 6*I)*d^3 + (3*b*c*d^
2 - (3*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - 6*(-I*a - 1)*d^3)*(b*x +
 a))*sin(3*b*x + 3*a)^2 + (12*I*(b*x + a)^3*d^3 - 24*I*b*c*d^2 - 24*I*(b*x
+ a)*d^3 + 24*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2)*cos(b*x +
a)*sin(b*x + a) - 6*((b*x + a)^3*d^3 - 2*b*c*d^2 - 2*(b*x + a)*d^3 + 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*sin(b*x + a)^2 - 6*(I*b*c*d^2 + (-I*a
+ 1)*d^3)*(b*x + a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(
b*x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*co
s(2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*
b*x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I
*a*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*
(b*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*
c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b*c*
d^2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(
b*x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(b*c
*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*
d^2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - 6*((b*x + a)^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(cos(b*x + a),
sin(b*x + a) + 1) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*
x + a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(
2*b*x + 2*a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*
x + 2*a))*cos(3*b*x + 3*a) + ((6*I*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a
*d^3)*(b*x + a))*cos(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b
*x + a))*sin(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^3 + (12*I*b*c*
d^2 - 12*I*a*d^3)*(b*x + a))*cos(b*x + a) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^
2 - a*d^3)*(b*x + a) + 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(2*b*x + 2*a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (6*((b*x + a)^2*d^3 + 2*(b*c*d
^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^
2 + 12*I*a*d^3)*(b*x + a))*sin(b*x + a))*sin(2*b*x + 2*a) - 6*((b*x + a)^2*
d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b*x + a))*arctan2(cos(b*x + a), -s
in(b*x + a) + 1) + ((7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)*d^3 + (2
1*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 + (6*I*b*c*d^2 - 6*(I*a + 3)*d^3)
*(b*x + a))*cos(b*x + a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a
- 1)*d^3 + (21*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6
```

```
*a - 18*I)*d^3)*(b*x + a))*sin(b*x + a))*cos(3*b*x + 3*a) + ((b*x + a)^3*d^
3 - 6*b*c*d^2 + (6*a + 6*I)*d^3 + (3*b*c*d^2 - (3*a + 3*I)*d^3)*(b*x + a)^2
 - 6*(I*b*c*d^2 + (-I*a + 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + ((12*I*b*c*
d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2
*b*x + 2*a))*cos(3*b*x + 3*a) + ((12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*
a*d^3)*cos(b*x + a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*co
s(2*b*x + 2*a) + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*cos(b*x +
 a) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d
^3 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a
*d^3)*sin(2*b*x + 2*a))*sin(3*b*x + 3*a) - (12*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*cos(b*x + a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(
b*x + a))*sin(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x +
 a))*dilog(I*e^(I*b*x + I*a)) + ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a)
 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
+ ((-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + 12*(b*c
*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*cos(2*b*x + 2*a) + (-12*I*b*c*d
^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(b*x + a) + (12*b*c*d^2 + 12*(b*x
+ a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a)
 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*sin(3
*b*x + 3*a) + (12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(b*x + a) + (12*I*b*
c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(b*x + a))*sin(2*b*x + 2*a) + 1
2*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(b*x + a))*dilog(-I*e^(I*b*x + I*a))
 - ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + 3*((b*x + a)^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^3
+ (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*x + 3*a)
+ (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a) - (-3*I
*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*sin(b*x + a))*cos(
2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x
+ a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I
*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) +
 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*sin(
3*b*x + 3*a) - ((-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a
))*cos(b*x + a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(b
*x + a))*sin(2*b*x + 2*a) - (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d
^3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*
x + a) + 1) + ((3*(b*x + a)^2*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a) + 3*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (3*I*(b*x +
 a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*cos(3*b*
x + 3*a) + (3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(b*x + a
) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*sin(b*x + a
))*cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(b*x + a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) +
(3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a
```

```
) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*s
in(3*b*x + 3*a) + ((3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x +
a))*cos(b*x + a) - 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(
b*x + a))*sin(2*b*x + 2*a) + (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^
3)*(b*x + a))*sin(b*x + a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x
 + a) + 1) - (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b*x + a) + 12*(d^3*cos(2*b
*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*cos(3*b*x + 3*a) + 12*(d^3*cos(b*
x + a) + I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) - (-12*I*d^3*cos(2*b*x + 2*a)
 + 12*d^3*sin(2*b*x + 2*a) - 12*I*d^3)*sin(3*b*x + 3*a) - (-12*I*d^3*cos(b*
x + a) + 12*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a
)) + (12*d^3*cos(b*x + a) + 12*I*d^3*sin(b*x + a) + 12*(d^3*cos(2*b*x + 2*a
) + I*d^3*sin(2*b*x + 2*a) + d^3)*cos(3*b*x + 3*a) + 12*(d^3*cos(b*x + a) +
 I*d^3*sin(b*x + a))*cos(2*b*x + 2*a) + (12*I*d^3*cos(2*b*x + 2*a) - 12*d^3
*sin(2*b*x + 2*a) + 12*I*d^3)*sin(3*b*x + 3*a) + (12*I*d^3*cos(b*x + a) - 1
2*d^3*sin(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x + I*a)) + ((2*
I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 - 12*(-I*a - 1)*d^3 + (6*I*b*c*d^2 - 6*(I*
a + 1)*d^3)*(b*x + a)^2 - (12*b*c*d^2 - (12*a - 12*I)*d^3)*(b*x + a))*cos(3
*b*x + 3*a) + (7*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 - 6*(-3*I*a - 1)*d^3 + (2
1*I*b*c*d^2 - 3*(7*I*a + 1)*d^3)*(b*x + a)^2 - (6*b*c*d^2 - (6*a - 18*I)*d^
3)*(b*x + a))*cos(b*x + a) - (7*(b*x + a)^3*d^3 - 18*b*c*d^2 + (18*a - 6*I)
*d^3 + (21*b*c*d^2 - (21*a - 3*I)*d^3)*(b*x + a)^2 - (-6*I*b*c*d^2 - 6*(-I*
a - 3)*d^3)*(b*x + a))*sin(b*x + a))*sin(3*b*x + 3*a) + (I*(b*x + a)^3*d^3
- 6*I*b*c*d^2 - 6*(-I*a + 1)*d^3 - 3*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)
^2 + (6*b*c*d^2 - (6*a + 6*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(2*b^3*cos(
b*x + a) + 2*I*b^3*sin(b*x + a) + (2*b^3*cos(2*b*x + 2*a) + 2*I*b^3*sin(2*b
*x + 2*a) + 2*b^3)*cos(3*b*x + 3*a) + 2*(b^3*cos(b*x + a) + I*b^3*sin(b*x +
 a))*cos(2*b*x + 2*a) - (-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a)
 - 2*I*b^3)*sin(3*b*x + 3*a) - (-2*I*b^3*cos(b*x + a) + 2*b^3*sin(b*x + a))
*sin(2*b*x + 2*a)))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(a+bx)\, \tan(a+bx)^2\,(c+dx)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \sin(a+bx)\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sin(b*x+a)*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*sin(a + b*x)*tan(a + b*x)**2, x)
```

3.261 $\int (c+dx)^2 \sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=145

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\cos(a+bx)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{(}{}$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*d^2*cos(b*x+a)/b^3+(d*x+c)^2*cos
(b*x+a)/b-2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(
I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b-2*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A] time = 0.13, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4407, 3296, 2638, 4409, 4181, 2279, 2391}

$$-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\sin(a+bx)}{b^2}+\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{2d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*d^2*Cos[a + b*x])/b^3
+ ((c + d*x)^2*Cos[a + b*x])/b - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))
])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 + ((c + d*x)^2*Sec[a
 + b*x])/b - (2*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2\sin(a+bx)\,dx + \int (c+dx)^2\sec(a+bx)\tan(a+bx)\,dx$$

$$= \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+dx)\cos(a}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\cos(a+bx)}{b} + \frac{(c+dx)^2\sec(a}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{(c+dx)^2\cos(a+bx)}{b}$$

Mathematica [B] time = 3.12, size = 362, normalized size = 2.50

$$\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)-\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(c+d\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + (2*d^2*Csc[a]*((b*x - ArcTan[Cot[a]])*(
Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]])
)]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + b^2*(c + d*x)^2*Sec[a] + Cos[b*x]*((
-2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - (2*b*d*(c + d*
x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] + (b^2*(c + d*x)^2*
Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2]))
 - (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
+ Sin[(a + b*x)/2])))/b^3
```

fricas [B] time = 0.53, size = 511, normalized size = 3.52

$$b^2d^2x^2+2\,b^2cdx+b^2c^2+i\,d^2\cos\left(bx+a\right){\rm Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d^2\cos\left(bx+a\right){\rm Li}_2\left(i\,\cos\left(bx+a\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x +
 a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a
)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(b
*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a)
 + I) - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\sin\left(bx+a\right)\tan\left(bx+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sin(b*x + a)*tan(b*x + a)^2, x)
```

maple [B] time = 0.13, size = 345, normalized size = 2.38

$$\frac{\left(d^2x^2b^2 + 2b^2cdx + 2ib\,d^2x + b^2c^2 + 2ibcd - 2d^2\right)\mathrm{e}^{i(bx+a)}}{2b^3} + \frac{\left(d^2x^2b^2 + 2b^2cdx - 2ib\,d^2x + b^2c^2 - 2ibcd - 2d^2\right)\mathrm{e}^{-i}}{2b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I
*(b*x+a))+1/2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)
/b^3*exp(-I*(b*x+a))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b
*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))
*x+2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x-2/
b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I/b^3*d^2*dilog(1+I*exp(I*(b*x+a)))+2*I/
b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(b*x+a)))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(a+b\,x\right)\,{\tan\left(a+b\,x\right)}^2\,{\left(c+d\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c + dx\right)^{2} \sin{\left(a + bx \right)} \tan^{2}{\left(a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sin(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*sin(a + b*x)*tan(a + b*x)**2, x)
```

3.262 $\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=56

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] -d*arctanh(sin(b*x+a))/b^2+(d*x+c)*cos(b*x+a)/b+(d*x+c)*sec(b*x+a)/b-d*sin(
b*x+a)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4407, 3296, 2637, 4409, 3770}

$$-\frac{d\sin(a+bx)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\cos(a+bx)}{b}+\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -((d*ArcTanh[Sin[a + b*x]])/b^2) + ((c + d*x)*Cos[a + b*x])/b + ((c + d*x)*
Sec[a + b*x])/b - (d*Sin[a + b*x])/b^2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sin(a+bx)\,dx + \int (c+dx)\sec(a+bx)\tan(a+bx)\,dx$$
$$= \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} - \frac{d\int \cos(a+bx)\,dx}{b} - \frac{d\int s}{}$$
$$= -\frac{d\tanh^{-1}(\sin(a+bx))}{b^2} + \frac{(c+dx)\cos(a+bx)}{b} + \frac{(c+dx)\sec(a+bx)}{b} -$$

Mathematica [A] time = 0.30, size = 107, normalized size = 1.91

$$\frac{\sec(a+bx)\left(b(c+dx)\cos(2(a+bx)) - d\sin(2(a+bx)) + 2d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right) - \sin\left(\frac{1}{2}(a+bx)\right)\right.\right.\right.}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sin[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (Sec[a + b*x]*(3*b*c + 3*b*d*x + b*(c + d*x)*Cos[2*(a + b*x)] + 2*d*Cos[a +
 b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]]) - d*Sin[2*(a + b*x)]))/(2*b^2)
```

fricas [A] time = 0.48, size = 93, normalized size = 1.66

$$\frac{2\,bdx + 2\,(bdx+bc)\cos(bx+a)^2 - d\cos(bx+a)\log(\sin(bx+a)+1) + d\cos(bx+a)\log(-\sin(bx+a)+1) -}{2\,b^2\cos(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x + 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
+ a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 2*d*cos(b*x + a)*sin(b*
x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

giac [B] time = 7.90, size = 2762, normalized size = 49.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b*c*tan(1/2*b*x)^4*tan(1/2*a)^
4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*
tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan
(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*
b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)
^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*
b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*ta
n(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a
)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^
3 - 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x
)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1
/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan
(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*t
an(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*ta
n(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a
)^3 + 4*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4
*b*d*x*tan(1/2*b*x)^4 + 16*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 48*b*d*x*tan(1
/2*b*x)^2*tan(1/2*a)^2 + 16*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 4*b*d*x*tan(1
/2*a)^4 + 4*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 +
 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)
/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
- 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x
)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
 + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 16*b*c*tan(1/2*b*x)^3*tan(1/2*a)
 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2
*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)
^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + ta
n(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2
*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x
)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2
```

*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*
a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 4*d*tan(1/2*b*x)^4*tan(1/2*a) + 48*b
*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*d*tan(1/2*b*x)^3*tan(1/2*a)^2 + 16*b*c*
tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(
1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*
tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^
2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(t
an(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - 24*d*tan(1/
2*b*x)^2*tan(1/2*a)^3 + 4*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/
2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(
1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b
*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan
(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 + d*log(2*(tan(1/2*b*x)^4*tan
(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + t
an(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/
2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*
tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 - 4*d*tan(1/2*b*x)*tan(1/2
*a)^4 - 16*b*d*x*tan(1/2*b*x)*tan(1/2*a) + 4*d*tan(1/2*b*x)^3 - 16*b*c*tan(
1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*
x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/
2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*
tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2
*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^
2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b
*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*t
an(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*
a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 24*d*tan(1/2*b*x)^2*t
an(1/2*a) + 24*d*tan(1/2*b*x)*tan(1/2*a)^2 + 4*d*tan(1/2*a)^3 + 4*b*d*x + 4
*b*c + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) +
 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 +
tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*l
og(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2
*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*t
an(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)
^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - 4*d*tan(1/2*b
*x) - 4*d*tan(1/2*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)
^3*tan(1/2*a)^3 - b^2*tan(1/2*b*x)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*
b^2*tan(1/2*b*x)*tan(1/2*a)^3 - b^2*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)*tan(1
/2*a) + b^2)

maple [C] time = 0.16, size = 123, normalized size = 2.20

$$\frac{(bdx + cb + id)\,\mathrm{e}^{i(bx+a)}}{2b^2} + \frac{(bdx + cb - id)\,\mathrm{e}^{-i(bx+a)}}{2b^2} + \frac{2\,\mathrm{e}^{i(bx+a)}\,(dx + c)}{b\left(1 + \mathrm{e}^{2i(bx+a)}\right)} - \frac{d\ln\left(\mathrm{e}^{i(bx+a)} + i\right)}{b^2} + \frac{d\ln\left(\mathrm{e}^{i(bx+a)} - i\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 1/2*(b*d*x+c*b+I*d)/b^2*exp(I*(b*x+a))+1/2*(b*d*x+c*b-I*d)/b^2*exp(-I*(b*x+
a))+2*exp(I*(b*x+a))*(d*x+c)/b/(1+exp(2*I*(b*x+a)))-d/b^2*ln(exp(I*(b*x+a))
+I)+d/b^2*ln(exp(I*(b*x+a))-I)
```

maxima [B] time = 0.48, size = 2123, normalized size = 37.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*c*(1/cos(b*x + a) + cos(b*x + a)) - 2*a*d*(1/cos(b*x + a) + cos(b*x
+ a))/b + ((b*x + (b*x + a)*cos(2*b*x + 2*a) + a + sin(2*b*x + 2*a))*cos(3*
b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)^3 + ((b*x + a)*sin(2*b*x + 2*a) - c
os(2*b*x + 2*a) - 1)*sin(3*b*x + 3*a)^3 + 6*(b*x + a)*cos(b*x + a)*sin(b*x
+ a)^2 + 2*(4*(b*x + a)*cos(2*b*x + 2*a)*cos(b*x + a) + 4*(b*x + a)*cos(b*x
 + a) + (3*(b*x + a)*sin(b*x + a) + cos(b*x + a))*sin(2*b*x + 2*a))*cos(3*b
*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*cos(2*b*x + 2*a)^2 +
(8*(b*x + a)*sin(2*b*x + 2*a)*sin(b*x + a) + (b*x + (b*x + a)*cos(2*b*x + 2
*a) + a + sin(2*b*x + 2*a))*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)
- sin(b*x + a))*cos(2*b*x + 2*a) + 6*(b*x + a)*cos(b*x + a) - 2*sin(b*x + a
))*sin(3*b*x + 3*a)^2 + ((b*x + a)*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 2*a)^2 + ((b*x + a)*cos(2*b*x + 2*a)^2 + 13*(b*x + a)*cos(b*x + a)^2 + (b*
x + a)*sin(2*b*x + 2*a)^2 + (b*x + a)*sin(b*x + a)^2 + b*x + (13*(b*x + a)*
cos(b*x + a)^2 + (b*x + a)*sin(b*x + a)^2 + 2*b*x + 2*a)*cos(2*b*x + 2*a) +
 (12*(b*x + a)*cos(b*x + a)*sin(b*x + a) + cos(b*x + a)^2 + sin(b*x + a)^2)
*sin(2*b*x + 2*a) + a)*cos(3*b*x + 3*a) + 2*(3*(b*x + a)*cos(b*x + a)^3 + 3
*(b*x + a)*cos(b*x + a)*sin(b*x + a)^2 + (b*x + a)*cos(b*x + a) - sin(b*x +
 a))*cos(2*b*x + 2*a) + (b*x + a)*cos(b*x + a) - ((cos(2*b*x + 2*a)^2 + sin
(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a
)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + co
s(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x +
 a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a
```

```
) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^
2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x
+ 3*a) + sin(b*x + a)^2)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x +
a) + 1) + ((cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) +
1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a)^
2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*sin(
3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 2*a)^2 + 2*(
cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 2*a)^2 + 2*cos(2
*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)
^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos(b*x + a)^2 + 2*(cos(2*b*x + 2*a
)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b*x + a) + 2*cos(2*b*x + 2*a)*sin
(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a) + sin(b*x + a)^2)*log(cos(b*x +
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (((b*x + a)*sin(2*b*x + 2*a)
- cos(2*b*x + 2*a) - 1)*cos(3*b*x + 3*a)^2 + 12*(b*x + a)*cos(b*x + a)*sin(
b*x + a) + 2*(((b*x + a)*sin(b*x + a) - cos(b*x + a))*cos(2*b*x + 2*a) + ((
b*x + a)*cos(b*x + a) + sin(b*x + a))*sin(2*b*x + 2*a) + (b*x + a)*sin(b*x
+ a) - cos(b*x + a))*cos(3*b*x + 3*a) + (12*(b*x + a)*cos(b*x + a)*sin(b*x
+ a) - cos(b*x + a)^2 - sin(b*x + a)^2 - 2)*cos(2*b*x + 2*a) - cos(2*b*x +
2*a)^2 - cos(b*x + a)^2 + ((b*x + a)*cos(b*x + a)^2 + 13*(b*x + a)*sin(b*x
+ a)^2)*sin(2*b*x + 2*a) - sin(2*b*x + 2*a)^2 - sin(b*x + a)^2 - 1)*sin(3*b
*x + 3*a) + 6*((b*x + a)*cos(b*x + a)^2*sin(b*x + a) + (b*x + a)*sin(b*x +
a)^3)*sin(2*b*x + 2*a) - sin(b*x + a))*d/(((cos(2*b*x + 2*a)^2 + sin(2*b*x
+ 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + s
in(b*x + a)^2)*cos(2*b*x + 2*a)^2 + (cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^
2 + 2*cos(2*b*x + 2*a) + 1)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 2*a)^2 + 2*(cos(2*b*x + 2*a)^2*cos(b*x + a) + cos(b*x +
 a)*sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)*cos(b*x + a) + cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 2*a) + cos
(b*x + a)^2 + 2*(cos(2*b*x + 2*a)^2*sin(b*x + a) + sin(2*b*x + 2*a)^2*sin(b
*x + a) + 2*cos(2*b*x + 2*a)*sin(b*x + a) + sin(b*x + a))*sin(3*b*x + 3*a)
+ sin(b*x + a)^2)*b))/b
```

mupad [B] time = 1.16, size = 151, normalized size = 2.70

$$e^{a\,1i+b\,x\,1i}\left(\frac{b\,c+d\,1i}{2\,b^2}+\frac{d\,x}{2\,b}\right)-e^{-a\,1i-b\,x\,1i}\left(\frac{-b\,c+d\,1i}{2\,b^2}-\frac{d\,x}{2\,b}\right)+\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}-i\right)}{b^2}-\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}+1i\right)}{b^2}+\frac{e^{a\,1i+b\,x\,1i}\cdots}{b\,\left(e^{a\,2i+\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(a + b*x)*tan(a + b*x)^2*(c + d*x),x)
```

```
[Out] exp(a*1i + b*x*1i)*((d*1i + b*c)/(2*b^2) + (d*x)/(2*b)) - exp(- a*1i - b*x*
1i)*((d*1i - b*c)/(2*b^2) - (d*x)/(2*b)) + (d*log(exp(a*1i + b*x*1i) - 1i))
/b^2 - (d*log(exp(a*1i + b*x*1i) + 1i))/b^2 + (exp(a*1i + b*x*1i)*(c + d*x)
*2i)/(b*(exp(a*2i + b*x*2i)*1i + 1i))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sin(b*x+a)*tan(b*x+a)**2,x)
[Out] Integral((c + d*x)*sin(a + b*x)*tan(a + b*x)**2, x)
```

3.263 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=76

$$\mathrm{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)-\frac{\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)-cos(a-b*c/d)*Si(b*c/d+b*x)
/d-Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -((CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d) - (Cos[a - (b*c)/d]*SinI
ntegral[(b*c)/d + b*x])/d + Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x
), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx &= -\int \frac{\sin(a+bx)}{c+dx}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{c+dx}\,dx\\ &= -\left(\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\right)-\sin\left(a-\frac{bc}{d}\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\int \frac{\mathrm{se}}{}\\ &= -\frac{\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d}+\int \frac{\sec(a+bx)\tan(a+}{c+dx}\end{aligned}$$

Mathematica [A] time = 3.78, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.19, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(bx+a\right)\left(\tan^{2}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (a + b\,x)\ \tan (a + b\,x)^2}{c + d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x),x)
```

```
[Out] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (a + bx) \tan^2 (a + bx)}{c + dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x), x)
```

3.264 $\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=94

$$\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)-\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}$$

```
[Out] CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)-b*Ci(b*c/d+b*x)*cos(a-b*
c/d)/d^2+b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+sin(b*x+a)/d/(d*x+c)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -((b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2) + Sin[a + b*x]/(d*(c
 + d*x)) + (b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + Defer[Int]
[(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx &= -\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}+\int \frac{\sec(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx\\
&= \frac{\sin(a+bx)}{d(c+dx)}-\frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}+\frac{\left(b\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d}\\
&= -\frac{b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{\sin(a+bx)}{d(c+dx)}+\frac{b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\int
\end{aligned}$$

Mathematica [A] time = 4.07, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sin[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}}{d^{2}x^{2}+2\,cdx+c^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sin(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)\tan\left(bx+a\right)^{2}}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sin(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(bx+a\right)\left(\tan^{2}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(a+b\,x)\,\tan(a+b\,x)^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2,x)
[Out] int((sin(a + b*x)*tan(a + b*x)^2)/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
[Out] Integral(sin(a + b*x)*tan(a + b*x)**2/(c + d*x)**2, x)
```

3.265 $\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\csc(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

Rubi [A] time = 0.25, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A] time = 24.57, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)\sec(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\left(\sec^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2\,\sin(a+b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.266 $\int (c+dx)^4 \csc(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=469

$$\frac{24id^4\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^5}-\frac{24id^4\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^5}+\frac{24d^4\text{Li}_5\left(-e^{i(a+bx)}\right)}{b^5}-\frac{24d^4\text{Li}_5\left(e^{i(a+bx)}\right)}{b^5}+\frac{24d^3(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-$$

```
[Out] 8*I*d*(d*x+c)^3*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^4*arctanh(exp(I*(b*x+a
)))/b-12*I*d^2*(d*x+c)^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-24*I*d^4*polylog(
4,I*exp(I*(b*x+a)))/b^5+12*I*d^2*(d*x+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^3-
24*I*d^3*(d*x+c)*polylog(4,-exp(I*(b*x+a)))/b^4-12*d^2*(d*x+c)^2*polylog(3,
-exp(I*(b*x+a)))/b^3+24*d^3*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^4-24*d^3
*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^4+12*d^2*(d*x+c)^2*polylog(3,exp(I*(
b*x+a)))/b^3+24*I*d^3*(d*x+c)*polylog(4,exp(I*(b*x+a)))/b^4+4*I*d*(d*x+c)^3
*polylog(2,-exp(I*(b*x+a)))/b^2-4*I*d*(d*x+c)^3*polylog(2,exp(I*(b*x+a)))/b
^2+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,-exp(I*(b*x+a
)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5+(d*x+c)^4*sec(b*x+a)/b
```

Rubi [A] time = 0.79, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$\frac{24d^3(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^4}-\frac{24id^3(c+dx)\text{PolyLog}\left(4,-e^{i(a+bx)}\right)}{b^4}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((8*I)*d*(c + d*x)^3*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^4*ArcTanh[
E^(I*(a + b*x))])/b + ((4*I)*d*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((12*I)*d^2*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((12*I)
*d^2*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((4*I)*d*(c + d*x)^3*
PolyLog[2, E^(I*(a + b*x))])/b^2 - (12*d^2*(c + d*x)^2*PolyLog[3, -E^(I*(a
+ b*x))])/b^3 + (24*d^3*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (
24*d^3*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^4 + (12*d^2*(c + d*x)^2*P
olyLog[3, E^(I*(a + b*x))])/b^3 - ((24*I)*d^3*(c + d*x)*PolyLog[4, -E^(I*(a
 + b*x))])/b^4 + ((24*I)*d^4*PolyLog[4, (-I)*E^(I*(a + b*x))])/b^5 - ((24*I
)*d^4*PolyLog[4, I*E^(I*(a + b*x))])/b^5 + ((24*I)*d^3*(c + d*x)*PolyLog[4,
 E^(I*(a + b*x))])/b^4 + (24*d^4*PolyLog[5, -E^(I*(a + b*x))])/b^5 - (24*d^
4*PolyLog[5, E^(I*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^4 \tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4 \sec(a+bx)}{b} - (4d)\int (c+a$$

$$= -\frac{(c+dx)^4 \tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4 \sec(a+bx)}{b} - (4d)\int \frac{(c+}{}$$

$$= -\frac{(c+dx)^4 \tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4 \sec(a+bx)}{b} - \frac{(4d)\int (c+d}{}$$

$$= -\frac{(c+dx)^4 \tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4 \sec(a+bx)}{b} - \frac{(4d)\int \left(-(c-\right.}{}$$

$$= -\frac{(c+dx)^4 \tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^4 \sec(a+bx)}{b} + \frac{(4d)\int (c+d}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^4 \sec(a+bx)}{b} + \frac{\int b(c+dx)^4 \csc}{b}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{12id^2(c+dx)^2 \mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{12id^2(c-}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{12id^2(c+a}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx}{}$$

$$= \frac{8id(c+dx)^3 \tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4id(c+dx}{}$$

Mathematica [A] time = 3.37, size = 694, normalized size = 1.48

$$\frac{b^4(c+dx)^4\log\left(1-e^{i(a+bx)}\right)-b^4(c+dx)^4\log\left(1+e^{i(a+bx)}\right)+b^4(c+dx)^4\sec(a+bx)-4d\left(-2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (b^4*(c + d*x)^4*Log[1 - E^(I*(a + b*x))] - b^4*(c + d*x)^4*Log[1 + E^(I*(a
 + b*x))] - 4*d*((-2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log
[1 - I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] + b^3*
d^3*x^3*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x)
)] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 + I*E^(
I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))] -
(3*I)*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3
, (-I)*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*
c*d^2*PolyLog[3, I*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x)
)] - (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, I*E^
(I*(a + b*x))]) + (4*I)*d*(b^3*(c + d*x)^3*PolyLog[2, -E^(I*(a + b*x))] + (
3*I)*b^2*d*(c + d*x)^2*PolyLog[3, -E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*Po
lyLog[4, -E^(I*(a + b*x))] + I*d*PolyLog[5, -E^(I*(a + b*x))])) - (4*I)*d*(
b^3*(c + d*x)^3*PolyLog[2, E^(I*(a + b*x))] + (3*I)*b^2*d*(c + d*x)^2*PolyL
og[3, E^(I*(a + b*x))] - 6*d^2*(b*(c + d*x)*PolyLog[4, E^(I*(a + b*x))] + I
*d*PolyLog[5, E^(I*(a + b*x))])) + b^4*(c + d*x)^4*Sec[a + b*x])/b^5
```

fricas [C] time = 0.81, size = 2507, normalized size = 5.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 + 8*b^4*c^3*d*x +
 2*b^4*c^4 - 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) + I*sin(b*x + a))
- 24*d^4*cos(b*x + a)*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*co
s(b*x + a)*polylog(5, -cos(b*x + a) + I*sin(b*x + a)) + 24*d^4*cos(b*x + a)
*polylog(5, -cos(b*x + a) - I*sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog
(4, I*cos(b*x + a) + sin(b*x + a)) - 24*I*d^4*cos(b*x + a)*polylog(4, I*cos
(b*x + a) - sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a
) + sin(b*x + a)) + 24*I*d^4*cos(b*x + a)*polylog(4, -I*cos(b*x + a) - sin(
b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4
*I*b^3*c^3*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I*b^3*
d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*cos(b*x
+ a)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (12*I*b^2*d^4*x^2 + 24*I*b^2*c*
```

```
d^3*x + 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))
 + (12*I*b^2*d^4*x^2 + 24*I*b^2*c*d^3*x + 12*I*b^2*c^2*d^2)*cos(b*x + a)*di
log(I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b^2*d^4*x^2 - 24*I*b^2*c*d^3*x
- 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-
12*I*b^2*d^4*x^2 - 24*I*b^2*c*d^3*x - 12*I*b^2*c^2*d^2)*cos(b*x + a)*dilog(
-I*cos(b*x + a) - sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 -
12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) + I*si
n(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x +
4*I*b^3*c^3*d)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^4*d^
4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(
b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x
+ a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*
d*x + b^4*c^4)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3
*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(cos(b*
x + a) - I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(I*cos(
b*x + a) - sin(b*x + a) + 1) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2
*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) + 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^
2*d^2*x + 3*a*b^2*c^2*d^2 - 3*a^2*b*c*d^3 + a^3*d^4)*cos(b*x + a)*log(-I*co
s(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d
^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) +
1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x +
 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*
log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 +
3*a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)*lo
g(-cos(b*x + a) - I*sin(b*x + a) + 1) + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*
a^2*b*c*d^3 - a^3*d^4)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I)
 + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b*x + a) + I*s
in(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)*polylog(4, cos(b
*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)*poly
log(4, -cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos
(b*x + a)*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x
 + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylo
g(3, cos(b*x + a) - I*sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(b*x + a)*p
olylog(3, I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3)*cos(b*x +
 a)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 24*(b*d^4*x + b*c*d^3)*cos(
```

```
b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 24*(b*d^4*x + b*c*d^3
)*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2
 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*s
in(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)*
polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^5*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \csc(bx+a)\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^2, x)
```

maple [B] time = 0.60, size = 1866, normalized size = 3.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)-12/b^3*c^2*d^2*polylog(3,-exp(I*(b*x+a))
)+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b*x+a)
))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2-12/b^3*d^4*polylog(3,-exp(I*(b*
x+a)))*x^2+24*I*d^4*polylog(4,-I*exp(I*(b*x+a)))/b^5+24*d^4*polylog(5,-exp(
I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^5-24*I*d^4*polylog(4,I*e
xp(I*(b*x+a)))/b^5+12*I/b^3*c^2*d^2*dilog(1-I*exp(I*(b*x+a)))-12*I/b^5*a^2*
d^4*polylog(2,I*exp(I*(b*x+a)))-12*I/b^5*a^2*d^4*dilog(1+I*exp(I*(b*x+a)))+
12*I/b^5*a^2*d^4*dilog(1-I*exp(I*(b*x+a)))+12*I/b^5*a^2*d^4*polylog(2,-I*ex
p(I*(b*x+a)))-12*I/b^3*c^2*d^2*dilog(1+I*exp(I*(b*x+a)))+12/b^2*d^3*c*ln(1+
I*exp(I*(b*x+a)))*x^2-12/b^2*d^3*c*ln(1-I*exp(I*(b*x+a)))*x^2+12/b^4*d^3*a^
2*c*ln(1-I*exp(I*(b*x+a)))-12/b^2*d^2*c^2*ln(1-I*exp(I*(b*x+a)))*x-12/b^3*d
^2*c^2*ln(1-I*exp(I*(b*x+a)))*a-12/b^4*d^3*a^2*c*ln(1+I*exp(I*(b*x+a)))+12/
b^2*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*x+12/b^3*d^2*c^2*ln(1+I*exp(I*(b*x+a)))*
a-12*I/b^3*d^4*polylog(2,-I*exp(I*(b*x+a)))*x^2+12*I/b^3*d^4*polylog(2,I*ex
p(I*(b*x+a)))*x^2-8*I/b^5*d^4*a^3*arctan(exp(I*(b*x+a)))+8*I/b^2*d*c^3*arct
an(exp(I*(b*x+a)))-24*I/b^3*d^3*c*polylog(2,-I*exp(I*(b*x+a)))*x+24*I/b^3*d
^3*c*polylog(2,I*exp(I*(b*x+a)))*x+24*I/b^4*d^3*c*a^2*arctan(exp(I*(b*x+a))
)-24*I/b^3*d^2*c^2*a*arctan(exp(I*(b*x+a)))-1/b*c^4*ln(exp(I*(b*x+a))+1)+1/
b*c^4*ln(exp(I*(b*x+a))-1)+2*exp(I*(b*x+a))*(d^4*x^4+4*c*d^3*x^3+6*c^2*d^2*
x^2+4*c^3*d*x+c^4)/b/(1+exp(2*I*(b*x+a)))-24*I/b^4*d^4*polylog(4,-exp(I*(b*
x+a)))*x-24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+4*I/b^2*c^3*d*polylog(2,
-exp(I*(b*x+a)))+4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*x^3+24/b^4*d^4*poly
```

```
log(3,-I*exp(I*(b*x+a)))*x-24/b^4*d^4*polylog(3,I*exp(I*(b*x+a)))*x+4/b^5*d
^4*a^3*ln(1+I*exp(I*(b*x+a)))+24/b^4*d^3*c*polylog(3,-I*exp(I*(b*x+a)))-24/
b^4*d^3*c*polylog(3,I*exp(I*(b*x+a)))+4/b^2*d^4*ln(1+I*exp(I*(b*x+a)))*x^3-
4/b^2*d^4*ln(1-I*exp(I*(b*x+a)))*x^3-4/b^5*d^4*a^3*ln(1-I*exp(I*(b*x+a)))-4
/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c^3*
d*ln(1-exp(I*(b*x+a)))*a-6/b*c^2*d^2*ln(exp(I*(b*x+a))+1)*x^2-24/b^3*c*d^3*
polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c^2
*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(3,exp(I*(b*x+a)))*x+24*I
/b^4*c*d^3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*
x^3+24*I/b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,exp(I*
(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(
b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*
x^4-1/b*d^4*ln(exp(I*(b*x+a))+1)*x^4-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+
a)))*x-12*I/b^2*c*d^3*polylog(2,exp(I*(b*x+a)))*x^2-4/b*c*d^3*ln(exp(I*(b*x
+a))+1)*x^3+4/b*c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+
a)))*a^3+12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+12*I/b^2*c^2*d^2*pol
ylog(2,-exp(I*(b*x+a)))*x-24*I/b^4*c*d^3*polylog(2,-I*exp(I*(b*x+a)))*a+24*
I/b^4*a*c*d^3*dilog(1+I*exp(I*(b*x+a)))-24*I/b^4*a*c*d^3*dilog(1-I*exp(I*(b
*x+a)))+24*I/b^4*c*d^3*polylog(2,I*exp(I*(b*x+a)))*a
```

maxima [B] time = 2.57, size = 5695, normalized size = 12.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 4*a*c^3*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 6*a^2*c^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x +
a) - 1))/b^2 - 4*a^3*c*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(co
s(b*x + a) - 1))/b^3 + a^4*d^4*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + lo
g(cos(b*x + a) - 1))/b^4 + 2*((8*b^3*c^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*
d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24
*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x +
2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x +
 a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I
*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*
arctan2(cos(b*x + a), sin(b*x + a) + 1) + (8*b^3*c^3*d - 24*a*b^2*c^2*d^2 +
 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x
 + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 8*(b^3*c^3*d
 - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3
 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*
```

```
cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^2*d^2 - 24*I*a^2*b*c*d^3
- 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a
)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a^2*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^4*d^4 +
 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a) + 2*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2
+ 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d
^4 + (8*I*b*c*d^3 - 8*I*a*d^4)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c
*d^3 + 12*I*a^2*d^4)*(b*x + a)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24
*I*a^2*b*c*d^3 - 8*I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x
+ a), cos(b*x + a) + 1) - (2*(b*x + a)^4*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a
)^3 + 12*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 8*(b^3*c^3*d -
 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 2*((b*x + a)^4*d^4
+ 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b
*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 8*I*a*d^4
)*(b*x + a)^3 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a
)^2 + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2 + 24*I*a^2*b*c*d^3 - 8*I*a^3*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4
*I*(b*x + a)^4*d^4 + (16*I*b*c*d^3 - 16*I*a*d^4)*(b*x + a)^3 + (24*I*b^2*c^
2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)*(b*x + a)^2 + (16*I*b^3*c^3*d - 48*I
*a*b^2*c^2*d^2 + 48*I*a^2*b*c*d^3 - 16*I*a^3*d^4)*(b*x + a))*cos(b*x + a) +
 (24*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*(b*x + a)^2*d^4 + 24*a^2*d^4 + 48*(b*c
*d^3 - a*d^4)*(b*x + a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 +
 a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-24*I*b^2*c^2
*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*b*c*d^
3 + 48*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (24
*b^2*c^2*d^2 - 48*a*b*c*d^3 + 24*(b*x + a)^2*d^4 + 24*a^2*d^4 + 48*(b*c*d^3
 - a*d^4)*(b*x + a) + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2
*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (24*I*b^2*c^2*d^2
- 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2*d^4 + (48*I*b*c*d^3 - 48
*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (8*b^3*c
^3*d - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 +
24*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*(b*x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d
^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d
^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-8*I*b^3*c^3*d + 24*I*a*b^2*c^
2*d^2 - 24*I*a^2*b*c*d^3 - 8*I*(b*x + a)^3*d^4 + 8*I*a^3*d^4 + (-24*I*b*c*d
^3 + 24*I*a*d^4)*(b*x + a)^2 + (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*a
^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (8*b^3*c^3*d
 - 24*a*b^2*c^2*d^2 + 24*a^2*b*c*d^3 + 8*(b*x + a)^3*d^4 - 8*a^3*d^4 + 24*(
b*c*d^3 - a*d^4)*(b*x + a)^2 + 24*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*
x + a) + 8*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 -
```

```
a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (8*I*b^3*c^3*d - 24*I*a*b^2*c^2*d^2
+ 24*I*a^2*b*c*d^3 + 8*I*(b*x + a)^3*d^4 - 8*I*a^3*d^4 + (24*I*b*c*d^3 - 2
4*I*a*d^4)*(b*x + a)^2 + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*a^2*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^4*d^4
+ (-4*I*b*c*d^3 + 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d
^3 - 6*I*a^2*d^4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I
*a^2*b*c*d^3 + 4*I*a^3*d^4)*(b*x + a) + (-I*(b*x + a)^4*d^4 + (-4*I*b*c*d^3
+ 4*I*a*d^4)*(b*x + a)^3 + (-6*I*b^2*c^2*d^2 + 12*I*a*b*c*d^3 - 6*I*a^2*d^
4)*(b*x + a)^2 + (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 +
4*I*a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^4*d^4 + 4*(b*c*d^3 -
a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 +
4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(2*
b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*
(b*x + a)^4*d^4 + (4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2
- 12*I*a*b*c*d^3 + 6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c
^2*d^2 + 12*I*a^2*b*c*d^3 - 4*I*a^3*d^4)*(b*x + a) + (I*(b*x + a)^4*d^4 + (
4*I*b*c*d^3 - 4*I*a*d^4)*(b*x + a)^3 + (6*I*b^2*c^2*d^2 - 12*I*a*b*c*d^3 +
6*I*a^2*d^4)*(b*x + a)^2 + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b
*c*d^3 - 4*I*a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^4*d^4 + 4*(b
*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x
+ a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a
))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) +
1) - (-4*I*b^3*c^3*d + 12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x +
a)^3*d^4 + 4*I*a^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*
b^2*c^2*d^2 + 24*I*a*b*c*d^3 - 12*I*a^2*d^4)*(b*x + a) + (-4*I*b^3*c^3*d +
12*I*a*b^2*c^2*d^2 - 12*I*a^2*b*c*d^3 - 4*I*(b*x + a)^3*d^4 + 4*I*a^3*d^4 +
(-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2 + (-12*I*b^2*c^2*d^2 + 24*I*a*b*c
*d^3 - 12*I*a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 4*(b^3*c^3*d - 3*a*b^2*c
^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b
*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x +
2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*I*b^3*
c^3*d - 12*I*a*b^2*c^2*d^2 + 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a
^3*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I
*a*b*c*d^3 + 12*I*a^2*d^4)*(b*x + a) + (4*I*b^3*c^3*d - 12*I*a*b^2*c^2*d^2
+ 12*I*a^2*b*c*d^3 + 4*I*(b*x + a)^3*d^4 - 4*I*a^3*d^4 + (12*I*b*c*d^3 - 12
*I*a*d^4)*(b*x + a)^2 + (12*I*b^2*c^2*d^2 - 24*I*a*b*c*d^3 + 12*I*a^2*d^4)*
(b*x + a))*cos(2*b*x + 2*a) - 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^
3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a
)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (48*I*d^4*cos(2*b*x + 2*a) - 4
8*d^4*sin(2*b*x + 2*a) + 48*I*d^4)*polylog(5, -e^(I*b*x + I*a)) - (-48*I*d^
4*cos(2*b*x + 2*a) + 48*d^4*sin(2*b*x + 2*a) - 48*I*d^4)*polylog(5, e^(I*b*
x + I*a)) - 48*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a) + d^4)*polylo
g(4, I*e^(I*b*x + I*a)) + 48*(d^4*cos(2*b*x + 2*a) + I*d^4*sin(2*b*x + 2*a)
```

```
 + d^4)*polylog(4, -I*e^(I*b*x + I*a)) - (48*b*c*d^3 + 48*(b*x + a)*d^4 - 4
8*a*d^4 + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) + (48*I*b*c
*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*
b*x + I*a)) + (48*b*c*d^3 + 48*(b*x + a)*d^4 - 48*a*d^4 + 48*(b*c*d^3 + (b*
x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4
+ 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) - (-48*I*b*c*d^
3 - 48*I*(b*x + a)*d^4 + 48*I*a*d^4 + (-48*I*b*c*d^3 - 48*I*(b*x + a)*d^4 +
 48*I*a*d^4)*cos(2*b*x + 2*a) + 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*
b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (48*I*b*c*d^3 + 48*I*(b*x + a)*
d^4 - 48*I*a*d^4 + (48*I*b*c*d^3 + 48*I*(b*x + a)*d^4 - 48*I*a*d^4)*cos(2*b
*x + 2*a) - 48*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*sin(2*b*x + 2*a))*polylog(
3, -I*e^(I*b*x + I*a)) - (-24*I*b^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x +
a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-24*I*b
^2*c^2*d^2 + 48*I*a*b*c*d^3 - 24*I*(b*x + a)^2*d^4 - 24*I*a^2*d^4 + (-48*I*
b*c*d^3 + 48*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 24*(b^2*c^2*d^2 - 2*a*b
*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b
*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^
3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x
+ a) + (24*I*b^2*c^2*d^2 - 48*I*a*b*c*d^3 + 24*I*(b*x + a)^2*d^4 + 24*I*a^2
*d^4 + (48*I*b*c*d^3 - 48*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 24*(b^2*c^
2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x
+ a))*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^4*d^4 +
4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(
b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x
 + a))*sin(b*x + a))/(-2*I*b^4*cos(2*b*x + 2*a) + 2*b^4*sin(2*b*x + 2*a) -
2*I*b^4))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.267 $\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\,dx$

Optimal. Leaf size=343

$$\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}-\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c}{}$$

```
[Out] 6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^3*arctanh(exp(I*(b*x+a
)))/b+3*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d^2*(d*x+c)*polylo
g(2,-I*exp(I*(b*x+a)))/b^3+6*I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-
3*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-6*d^2*(d*x+c)*polylog(3,-exp(
I*(b*x+a)))/b^3+6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*ex
p(I*(b*x+a)))/b^4+6*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-6*I*d^3*polyl
og(4,-exp(I*(b*x+a)))/b^4+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+(d*x+c)^3*s
ec(b*x+a)/b
```

Rubi [A] time = 0.57, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 14, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.636, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4181}

$$-\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^3*ArcTanh[
E^(I*(a + b*x))])/b + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*x))])/b^
2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*d^2
*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(c + d*x)^2*PolyLo
g[2, E^(I*(a + b*x))])/b^2 - (6*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])
/b^3 + (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E
^(I*(a + b*x))])/b^4 + (6*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 -
((6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((6*I)*d^3*PolyLog[4, E^(I*(
a + b*x))])/b^4 + ((c + d*x)^3*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - (3d)\int (c+$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - (3d)\int \frac{(c+}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int (c+}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{(3d)\int \left(-(c\right.}{}$$

$$= -\frac{(c+dx)^3\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{(3d)\int (c+}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^3\sec(a+bx)}{b} - \frac{\int b(-c-dx)^3 c}{b}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6id^2(c+dx)\mathrm{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3} + \frac{6id^2(c+}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6id^2(c+}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+d}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+d}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+d}{}$$

$$= \frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id(c+d}{}$$

Mathematica [A] time = 1.34, size = 473, normalized size = 1.38

$$\frac{b^3(c+dx)^3\sec(a+bx) - 2b^3(c+dx)^3\tanh^{-1}(\cos(a+bx)+i\sin(a+bx)) - 3d\left(-2ib^2c^2\tan^{-1}\left(e^{i(a+bx)}\right) + 2b^2cd\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] - 3*d*((-2*I)*b^
2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] + b^
2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x)
)] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2
, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))]
 - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3, I*E^(I*(a + b*
x))]) + (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]]
 + (2*I)*b*d*(c + d*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*P
olyLog[4, -Cos[a + b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyL
og[2, Cos[a + b*x] + I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a
 + b*x] + I*Sin[a + b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]
) + b^3*(c + d*x)^3*Sec[a + b*x])/b^4
```

fricas [C] time = 0.65, size = 1697, normalized size = 4.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*I*d^3*
cos(b*x + a)*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x +
a)*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)*polylog
(4, -cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)*polylog(4, -cos(
b*x + a) - I*sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) +
 sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a
)) + 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*
cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin
(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x +
a)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b
*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (6*I*b*d^3*x + 6*I*b*c*d^2)*
cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*d^3*x - 6*I*b*c
*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-6*I*b*d^3*x -
6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*c
os(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a)) - (b^3*d^3*x^3 + 3*b^3*c*
d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*
x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*
x + a) + I*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x +
a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
 a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
```

```
 + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) +
1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*l
og(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2
*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
+ (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2
*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a
) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d -
 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) +
 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a)
+ I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*s
in(b*x + a) + 1) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-
cos(b*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)*pol
ylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)
*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x
+ a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \csc (bx + a) \sec (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^2, x)
```

maple [B] time = 0.47, size = 1152, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^
4-6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^3*d^2*c*a*arctan(exp(I*(
b*x+a)))+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+6*I*d^3*x*polylog(2,I*exp(I*
(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-1/b^4*d^3*a^3*ln(e
xp(I*(b*x+a))-1)-6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog
(3,exp(I*(b*x+a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-6/b^3*d^3*polylog(
3,-exp(I*(b*x+a)))*x-6*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4-6*I/b^3*d^2*c*d
ilog(1+I*exp(I*(b*x+a)))+6*I/b^3*d^2*c*dilog(1-I*exp(I*(b*x+a)))+6*I/b^4*a*
d^3*dilog(1+I*exp(I*(b*x+a)))-6*I/b^4*a*d^3*dilog(1-I*exp(I*(b*x+a)))+6*I/b
```

```
^4*d^3*polylog(2,I*exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)
))*a+1/b*c^3*ln(exp(I*(b*x+a))-1)-1/b*c^3*ln(exp(I*(b*x+a))+1)+2*exp(I*(b*x
+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))+6*I/b^2*c*d
^2*polylog(2,-exp(I*(b*x+a)))*x+6/b^2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x+6*I/b^
4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*d*c^2*arctan(exp(I*(b*x+a)))+3*I/b
^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a
)))-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))
*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1
)-3*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*d^3*polylog(2,exp(I*(b*x+
a)))*x^2-3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+
3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+3/b
*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2*c^
2*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln(1-
exp(I*(b*x+a)))*a^3-1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3+3/b^2*d^3*ln(1+I*exp(I
*(b*x+a)))*x^2-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))-3/b^2*d^3*ln(1-I*exp(I*
(b*x+a)))*x^2+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))
```

maxima [B] time = 1.14, size = 3205, normalized size = 9.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 3*a*c^2*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b + 3*a^2*c*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a)
 - 1))/b^2 - a^3*d^3*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x
+ a) - 1))/b^3 + 2*((6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2
*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
 a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6
*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*
b*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), si
n(b*x + a) + 1) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d
^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a
)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I
*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*
c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin
(b*x + a) + 1) - (2*(b*x + a)^3*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 + 3*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))
*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x +
 a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*a^2*d^3)*(b*x + a))*sin(2*b*x
 + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^3*d^3 + 6*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x +
```

```
a) + 2*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d -
2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (2*I*(b*x + a)^3*d^3
+ (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 +
6*I*a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x +
a) + 1) - (4*I*(b*x + a)^3*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a)^2 +
(12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*(b*x + a))*cos(b*x + a) +
(12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)
*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^
3 - 12*a*d^3 + 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (12*
I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(
I*b*x + I*a)) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3
+ 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b
^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*
d^2 + 12*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) - (6
*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*a^2*d^3 + 12*(b*c*d^2 - a
*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b
*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b
*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 + (
-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3
*I*a^2*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b
*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*a^2*d^3)*(b*x + a))*cos(2
*b*x + 2*a) + ((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^3*d^3 + (3*I*b*c*d
^2 - 3*I*a*d^3)*(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)
*(b*x + a) + (I*(b*x + a)^3*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*x + a)^2 + (
3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) -
((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c
*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x +
a)^2 - 2*cos(b*x + a) + 1) - (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x +
a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a) + (-3*I*b^2*c
^2*d + 6*I*a*b*c*d^2 - 3*I*(b*x + a)^2*d^3 - 3*I*a^2*d^3 + (-6*I*b*c*d^2 +
6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x
+ a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (3*I*b^2*c^2*d - 6
*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3
)*(b*x + a) + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + 3*I*(b*x + a)^2*d^3 + 3*I*a^
2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*(b^2*c^2*
d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a)
)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) +
1) - 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*polylog(4, -e
^(I*b*x + I*a)) + 12*(d^3*cos(2*b*x + 2*a) + I*d^3*sin(2*b*x + 2*a) + d^3)*
```

```
polylog(4, e^(I*b*x + I*a)) - (-12*I*d^3*cos(2*b*x + 2*a) + 12*d^3*sin(2*b*
x + 2*a) - 12*I*d^3)*polylog(3, I*e^(I*b*x + I*a)) - (12*I*d^3*cos(2*b*x +
2*a) - 12*d^3*sin(2*b*x + 2*a) + 12*I*d^3)*polylog(3, -I*e^(I*b*x + I*a)) -
 (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3 + (-12*I*b*c*d^2 - 12*I*(
b*x + a)*d^3 + 12*I*a*d^3)*cos(2*b*x + 2*a) + 12*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (12*I*b*c*d^2 + 12
*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3 - 12*I*a
*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2
*a))*polylog(3, e^(I*b*x + I*a)) + 4*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(b*x + a
))/(-2*I*b^3*cos(2*b*x + 2*a) + 2*b^3*sin(2*b*x + 2*a) - 2*I*b^3))/b
```

mupad **[F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] \text{Hanged}
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \csc(a+bx) \sec^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)**3*csc(a + b*x)*sec(a + b*x)**2, x)
```

3.268 $\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=219

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{2d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{2d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c$$

```
[Out] 4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2-2*(d*x+c)^2*arctanh(exp(I*(b*x+a))
)/b+2*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2*polylog(2,-I*exp(I
*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-2*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(
3,exp(I*(b*x+a)))/b^3+(d*x+c)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.38, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 15, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.682, Rules used = {2622, 321, 207, 4420, 6741, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4181, 2279, 2391}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 - (2*(c + d*x)^2*ArcTanh[E^
(I*(a + b*x))])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 -
((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*
E^(I*(a + b*x))])/b^3 - ((2*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2
 - (2*d^2*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (2*d^2*PolyLog[3, E^(I*(a + b
*x))])/b^3 + ((c + d*x)^2*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
```

```
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - (2d)\int (c+d$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - (2d)\int \frac{(c+}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int (c+d}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{(2d)\int \left(-(c+\right.}{}$$

$$= -\frac{(c+dx)^2\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{(2d)\int (c+d}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{\int b(c+dx)^2\csc(}{b}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\sec(a+bx)}{b} - \frac{\left(2id^2\right)\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2id^2\mathrm{Li}_2\left(-i\right.}{b^3}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)}{}$$

$$= \frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2id(c+dx)}{}$$

Mathematica [A] time = 2.47, size = 317, normalized size = 1.45

$$\frac{b^2(c+dx)^2\log\left(1-e^{i(a+bx)}\right) - b^2(c+dx)^2\log\left(1+e^{i(a+bx)}\right) + b^2(c+dx)^2\sec(a+bx) + 2id\left(b(c+dx)\mathrm{Li}_2\left(-e^{i(a+bx}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - 4*d^2*ArcTan[Cot[a]]*ArcT
anh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))
] - b^2*(c + d*x)^2*Log[1 + E^(I*(a + b*x))] + (2*d^2*Csc[a]*((b*x - ArcTan
```

```
[Cot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcT
an[Cot[a]]))]) + I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2,
 E^(I*(b*x - ArcTan[Cot[a]]))]))/Sqrt[Csc[a]^2] + (2*I)*d*(b*(c + d*x)*Poly
Log[2, -E^(I*(a + b*x))] + I*d*PolyLog[3, -E^(I*(a + b*x))]) + 2*d*((-I)*b*
(c + d*x)*PolyLog[2, E^(I*(a + b*x))] + d*PolyLog[3, E^(I*(a + b*x))]) + b^
2*(c + d*x)^2*Sec[a + b*x])/b^3
```

fricas [C] time = 0.57, size = 1031, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*I*d^2*cos(b*x + a)*dilog(I
*cos(b*x + a) + sin(b*x + a)) + 2*I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) -
 sin(b*x + a)) - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))
 - 2*I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b
*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)*poly
log(3, cos(b*x + a) - I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(
b*x + a) + I*sin(b*x + a)) - 2*d^2*cos(b*x + a)*polylog(3, -cos(b*x + a) -
I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)*dilog(cos(b*x + a
) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)*dilog(cos(b*x
+ a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)*dilog(-cos
(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)*dilog(
-cos(b*x + a) - I*sin(b*x + a)) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos
(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2)*cos(b*
x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(b*c*d -
 a*d^2)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d^2*x +
a*d^2)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a
*d^2)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 2*(b*d^2*x + a*
d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2*(b*d^2*x + a*
d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) +
1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 2*(b*c*d - a*d^2
)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b
^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x
+ a) + 1) + 2*(b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
a) + I))/(b^3*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \csc(bx + a) \sec(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")

[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^2, x)

maple [B] time = 0.27, size = 568, normalized size = 2.59

$$\frac{d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}+\frac{d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}-\frac{d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}+\frac{2d^2\ln\left(1+ie^{i(bx+a)}\right)x}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x)

[Out] 1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d
^2*ln(1-exp(I*(b*x+a)))*a^2-1/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-2*d^2*polylog(
3,-exp(I*(b*x+a)))/b^3+2*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^2*ln(1-I
exp(I(b*x+a)))*a+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*d^2*dilog(
1+I*exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln(1+I*exp
(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x+1/b*c^2*ln(exp(I*(b*x+a))
-1)-1/b*c^2*ln(exp(I*(b*x+a))+1)+2/b*c*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*c*d*l
n(1-exp(I*(b*x+a)))*a-2/b*c*d*ln(exp(I*(b*x+a))+1)*x-2/b^2*c*d*a*ln(exp(I*(
b*x+a))-1)-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,ex
p(I*(b*x+a)))+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I
*(b*x+a)))+2*exp(I*(b*x+a))*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))+2*
I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+2*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)
))

maxima [B] time = 0.69, size = 1598, normalized size = 7.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")

[Out] 1/2*(c^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1)) -
 2*a*c*d*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))/b
 + a^2*d^2*(2/cos(b*x + a) - log(cos(b*x + a) + 1) + log(cos(b*x + a) - 1))
/b^2 + 2*((4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 -
 a*d^2)*cos(2*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin
(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (4*b*c*d + 4*(b*x
+ a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (
-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b
*x + a), -sin(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x +

```
 a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) +
(2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*d -
a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*
b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d
^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*cos(b*x + a) + 4*(d^2*cos(2*b*x + 2
*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(I*e^(I*b*x + I*a)) - 4*(d^2*cos(2
*b*x + 2*a) + I*d^2*sin(2*b*x + 2*a) + d^2)*dilog(-I*e^(I*b*x + I*a)) + (4*
b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2
*b*x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a)
)*dilog(-e^(I*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d
 + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2
 - 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-I*(b*x + a)^2*d^
2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d
+ 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
os(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) +
 (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) -
 ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-2*I*b*c*d - 2*I*(b*x
+ a)*d^2 + 2*I*a*d^2 + (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*cos(2*b
*x + 2*a) + 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
 + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (2*I*b*c*d + 2*I*(b*x + a)
*d^2 - 2*I*a*d^2 + (2*I*b*c*d + 2*I*(b*x + a)*d^2 - 2*I*a*d^2)*cos(2*b*x +
2*a) - 2*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-4*I*d^2*cos(2*b*x + 2*a) + 4*
d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (4*I*d^2*cos
(2*b*x + 2*a) - 4*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x + I*a
)) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(b*x + a))/(-2*I*
b^2*cos(2*b*x + 2*a) + 2*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \csc(a + bx) \sec^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**2, x)
```

3.269 $\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=113

$$\frac{id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{c\sec(a+bx)}{b}-\frac{c\tanh^{-1}(\cos(a+bx))}{b}+\frac{dx\sec(a+bx)}{b}$$

```
[Out] -2*d*x*arctanh(exp(I*(b*x+a)))/b-c*arctanh(cos(b*x+a))/b-d*arctanh(sin(b*x+
a))/b^2+I*d*polylog(2,-exp(I*(b*x+a)))/b^2-I*d*polylog(2,exp(I*(b*x+a)))/b^
2+c*sec(b*x+a)/b+d*x*sec(b*x+a)/b
```

Rubi [A] time = 0.13, antiderivative size = 122, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2622, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770}

$$\frac{id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{(c+dx)\sec(a+bx)}{b}-\frac{(c+dx)\tanh^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] (-2*d*x*ArcTanh[E^(I*(a + b*x))])/b + (d*x*ArcTanh[Cos[a + b*x]])/b - ((c +
 d*x)*ArcTanh[Cos[a + b*x]])/b - (d*ArcTanh[Sin[a + b*x]])/b^2 + (I*d*PolyL
og[2, -E^(I*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + ((c
+ d*x)*Sec[a + b*x])/b
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec^2(a+bx)\,dx = -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)\sec(a+bx)}{b} - d\int\left(-\frac{\tanh^{-1}\right.$$
$$= -\frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} + \frac{(c+dx)\sec(a+bx)}{b} + \frac{d\int\tanh^{-1}(\mathrm{co}}{b}$$
$$= \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} - \frac{d\tanh^{-1}(\sin}{b^2}$$
$$= \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\cos(a+bx))}{b} - \frac{d\tanh^{-1}(\sin}{b^2}$$
$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\mathrm{co}}{b}$$
$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\mathrm{co}}{b}$$
$$= -\frac{2dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{dx\tanh^{-1}(\cos(a+bx))}{b} - \frac{(c+dx)\tanh^{-1}(\mathrm{co}}{b}$$

Mathematica [A] time = 0.49, size = 212, normalized size = 1.88

$$\frac{d\left(i\left(\mathrm{Li}_2\left(-e^{i(a+bx)}\right)-\mathrm{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{b^2}-\frac{ad\log\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2,x]
```

```
[Out] -((c*Log[Cos[(a + b*x)/2]])/b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]
])/b^2 + (c*Log[Sin[(a + b*x)/2]])/b - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b
*x)/2]])/b^2 - (a*d*Log[Tan[(a + b*x)/2]])/b^2 + (d*((a + b*x)*(Log[1 - E^(
I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))]
 - PolyLog[2, E^(I*(a + b*x))])))/b^2 + (c*Sec[a + b*x])/b + (d*x*Sec[a + b
*x])/b
```

fricas [B] time = 0.47, size = 366, normalized size = 3.24

$$2\,bdx - i\,d\cos\left(bx+a\right)\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right) + i\,d\cos\left(bx+a\right)\mathrm{Li}_2\left(\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right) - i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - I*d*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*
cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)*dilog(
-cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x + a)*dilog(-cos(b*x + a) - I*
sin(b*x + a)) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a
) + 1) - (b*d*x + b*c)*cos(b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + 1)
+ (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2
) + (b*c - a*d)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1
/2) + (b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) +
(b*d*x + a*d)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - d*cos(
b*x + a)*log(sin(b*x + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) + 2*
b*c)/(b^2*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)\csc(bx + a)\sec(bx + a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^2, x)
```

maple [A] time = 0.11, size = 160, normalized size = 1.42

$$\frac{2\,\mathrm{e}^{i(bx+a)}(dx+c)}{b\left(1+\mathrm{e}^{2i(bx+a)}\right)}+\frac{c\ln\left(\mathrm{e}^{i(bx+a)}-1\right)}{b}-\frac{c\ln\left(\mathrm{e}^{i(bx+a)}+1\right)}{b}+\frac{2id\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+a)}\right)}{b^2}+\frac{id\,\mathrm{dilog}\left(\mathrm{e}^{i(bx+}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x)
```

```
[Out] 2*exp(I*(b*x+a))*(d*x+c)/b/(1+exp(2*I*(b*x+a)))+1/b*c*ln(exp(I*(b*x+a))-1)-
1/b*c*ln(exp(I*(b*x+a))+1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+I/b^2*d*dilog(e
xp(I*(b*x+a)))+I/b^2*d*dilog(exp(I*(b*x+a))+1)-1/b*d*ln(exp(I*(b*x+a))+1)*x
-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.62, size = 806, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2*a) + d)*arctan2(2*(cos(b*x + 2*
a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*
```

```
sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (c
os(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2
 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)) + (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(2*b*x + 2*a)
- (-2*I*b*d*x - 2*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x +
a) + 1) - (2*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(2*b*x + 2*a) + 2*b*c)*arcta
n2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*
sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (-4*
I*b*d*x - 4*I*b*c)*cos(b*x + a) - 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x + 2
*a) + d)*dilog(-e^(I*b*x + I*a)) + 2*(d*cos(2*b*x + 2*a) + I*d*sin(2*b*x +
2*a) + d)*dilog(e^(I*b*x + I*a)) - (I*b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos
(2*b*x + 2*a) - (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 + 2*cos(b*x + a) + 1) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos
(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*
x + a)^2 - 2*cos(b*x + a) + 1) - (-I*d*cos(2*b*x + 2*a) + d*sin(2*b*x + 2*a
) - I*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b
*x + 2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a
)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a)
+ sin(a)^2)) - 4*(b*d*x + b*c)*sin(b*x + a))/(-2*I*b^2*cos(2*b*x + 2*a) + 2
*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc (a + bx) \sec^2 (a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**2, x)
```

3.270 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

`[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)`

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

`[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]`

`[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]`

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 9.65, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

`[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x),x]`

`[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x), x]`

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\sec(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 2.33, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\left(\sec^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (a+bx)\sec ^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

3.271 $\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.18, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 9.76, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] sage0*x
```

maple [A] time = 2.56, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^2\left(bx+a\right)\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
[Out] int(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(a+bx\right)\sec^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

3.272 $\int (c+dx)^m \csc^2(a+bx) \sec^2(a+bx)\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\csc^2(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A] time = 2.63, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

maple [A] time = 0.08, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right)\left(\sec^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.273 $\int (c+dx)^3 \csc^2(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=118

$$\frac{3d^3\text{Li}_3\left(e^{4i(a+bx)}\right)}{8b^4}-\frac{3id^2(c+dx)\text{Li}_2\left(e^{4i(a+bx)}\right)}{2b^3}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2a+2bx)}{b}-\frac{2i(c+d$$

```
[Out] -2*I*(d*x+c)^3/b-2*(d*x+c)^3*cot(2*b*x+2*a)/b+3*d*(d*x+c)^2*ln(1-exp(4*I*(b
*x+a)))/b^2-3/2*I*d^2*(d*x+c)*polylog(2,exp(4*I*(b*x+a)))/b^3+3/8*d^3*polyl
og(3,exp(4*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.28, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4419, 4184, 3717, 2190, 2531, 2282, 6589}

$$-\frac{3id^2(c+dx)\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{3d^3\text{PolyLog}\left(3,e^{4i(a+bx)}\right)}{8b^4}+\frac{3d(c+dx)^2\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^3\cot(2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^3)/b - (2*(c + d*x)^3*Cot[2*a + 2*b*x])/b + (3*d*(c + d*x
)^2*Log[1 - E^((4*I)*(a + b*x))])/b^2 - (((3*I)/2)*d^2*(c + d*x)*PolyLog[2,
 E^((4*I)*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, E^((4*I)*(a + b*x))])/(8*b^4
)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sec^2(a+bx)\,dx = 4\int (c+dx)^3 \csc^2(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{(6d)\int (c+dx)^2\cot(2a+2bx)\,dx}{b}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} - \frac{(12id)\int \frac{e^{2i(2a+2bx)}(c+dx)^2}{1-e^{2i(2a+2bx)}}\,dx}{b}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+}\right.}{b^2}$$
$$= -\frac{2i(c+dx)^3}{b} - \frac{2(c+dx)^3\cot(2a+2bx)}{b} + \frac{3d(c+dx)^2\log\left(1-e^{4i(a+}\right.}{b^2}$$

Mathematica [B] time = 2.13, size = 285, normalized size = 2.42

$$-\frac{8ib^3(c+dx)^3}{-1+e^{4ia}} + 4b^3\csc(2a)\sin(2bx)(c+dx)^3\csc(2(a+bx)) + 6b^2d(c+dx)^2\log\left(1-e^{-i(a+bx)}\right) + 6b^2d(c+dx)^2\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-8*I)*b^3*(c + d*x)^3)/(-1 + E^((4*I)*a)) + 6*b^2*d*(c + d*x)^2*Log[1 -
E^((-I)*(a + b*x))] + 6*b^2*d*(c + d*x)^2*Log[1 + E^((-I)*(a + b*x))] + 6*b
^2*d*(c + d*x)^2*Log[1 + E^((-2*I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*Pol
yLog[2, -E^((-I)*(a + b*x))] + (12*I)*b*d^2*(c + d*x)*PolyLog[2, E^((-I)*(a
 + b*x))] + (6*I)*b*d^2*(c + d*x)*PolyLog[2, -E^((-2*I)*(a + b*x))] + 12*d^
3*PolyLog[3, -E^((-I)*(a + b*x))] + 12*d^3*PolyLog[3, E^((-I)*(a + b*x))] +
 3*d^3*PolyLog[3, -E^((-2*I)*(a + b*x))] + 4*b^3*(c + d*x)^3*Csc[2*a]*Csc[2
*(a + b*x)]*Sin[2*b*x])/(2*b^4)
```

fricas [C] time = 0.62, size = 1627, normalized size = 13.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*co
s(b*x + a)*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + 6*d^3*c
os(b*x + a)*polylog(3, cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) + 6*d^3*
cos(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*d^3
*cos(b*x + a)*polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) + 6*d^
3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 6*
d^3*cos(b*x + a)*polylog(3, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a)
+ 6*d^3*cos(b*x + a)*polylog(3, -cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a
) + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(b*
x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(cos(b
*x + a) - I*sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*
x + a)*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x -
6*I*b*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
 + (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*
x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*co
s(b*x + a) - sin(b*x + a))*sin(b*x + a) + (6*I*b*d^3*x + 6*I*b*c*d^2)*cos(b
*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (-6*I*b*d^3*x
- 6*I*b*c*d^2)*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x
+ a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*
d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*log(cos(b*x + a) - I*
sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(
b*x + a)*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x
^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a)
 + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*
c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*
x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x +
a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b
*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*
(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-1/2*cos(b*x + a) - 1/
2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a
*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*si
n(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-cos(b*
x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
+ 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) +
1)*sin(b*x + a) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(-c
os(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) - 4*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2)/(b^4*cos(b*x + a)*sin(b*x
 + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \csc (bx + a)^2 \sec (bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

maple [B] time = 0.13, size = 687, normalized size = 5.82

$$-\frac{24id^2cax}{b^2}+\frac{3d\,c^2\ln\left(1+e^{2i(bx+a)}\right)}{b^2}-\frac{12d\,c^2\ln\left(e^{i(bx+a)}\right)}{b^2}-\frac{12d^3a^2\ln\left(e^{i(bx+a)}\right)}{b^4}+\frac{3d^3\ln\left(1+e^{2i(bx+a)}\right)x^2}{b^2}+\frac{8id^3a^3}{b^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] 3/2*d^3*polylog(3,-exp(2*I*(b*x+a)))/b^4+6*d^3*polylog(3,-exp(I*(b*x+a)))/b
^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3*d/b^2*c^2*ln(1+exp(2*I*(b*x+a)))-1
2*d/b^2*c^2*ln(exp(I*(b*x+a)))-12*d^3/b^4*a^2*ln(exp(I*(b*x+a)))+3*d^3/b^2*
ln(1+exp(2*I*(b*x+a)))*x^2+3/b^2*c^2*d*ln(exp(I*(b*x+a))-1)+3/b^2*c^2*d*ln(
exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)+3/b^2*d^3*ln(exp(I*(b*
x+a))+1)*x^2+3/b^2*d^3*ln(1-exp(I*(b*x+a)))*x^2-3/b^4*d^3*ln(1-exp(I*(b*x+a
)))*a^2-6/b^3*c*d^2*a*ln(exp(I*(b*x+a))-1)+8*I*d^3/b^4*a^3-4*I*d^3/b*x^3-6*
I*d^2/b^3*c*polylog(2,-exp(I*(b*x+a)))+12*I*d^3/b^3*a^2*x-12*I*d^2/b*c*x^2-
12*I*d^2/b^3*c*a^2-6*I*d^3/b^3*polylog(2,-exp(I*(b*x+a)))*x-6*I/b^3*d^3*pol
ylog(2,exp(I*(b*x+a)))*x+6/b^2*d^2*c*ln(exp(I*(b*x+a))+1)*x-6*I/b^3*d^2*c*p
olylog(2,exp(I*(b*x+a)))-24*I*d^2/b^2*c*a*x-4*I*(d^3*x^3+3*c*d^2*x^2+3*c^2*
d*x+c^3)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a))-1)+6/b^2*d^2*c*ln(1-exp(I
*(b*x+a)))*x+6/b^3*d^2*c*ln(1-exp(I*(b*x+a)))*a+24*d^2/b^3*c*a*ln(exp(I*(b*
x+a)))+6*d^2/b^2*c*ln(1+exp(2*I*(b*x+a)))*x-3*I*d^3/b^3*polylog(2,-exp(2*I*
(b*x+a)))*x-3*I*d^2/b^3*c*polylog(2,-exp(2*I*(b*x+a)))
```

maxima [B] time = 0.74, size = 2355, normalized size = 19.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c^3*(1/tan(b*x + a) - tan(b*x + a)) - 6*a*c^2*d*(1/tan(b*x + a) - t
an(b*x + a))/b + 6*a^2*c*d^2*(1/tan(b*x + a) - tan(b*x + a))/b^2 - 2*a^3*d^
```

```
3*(1/tan(b*x + a) - tan(b*x + a))/b^3 - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x
+ 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a
)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 -
2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a
) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)
*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*si
n(4*b*x + 4*a))*c^2*d/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*b) + 6*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b
*x + 4*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2
+ sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a*c*
d^2/((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^2
) - 3*((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*l
og(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (cos
(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 + sin(4
*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x + a)^2
 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*a^2*d^3/((cos(4*b*x
+ 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b^3) + 2*((6*(b*x +
 a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^
2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (6*I*(b*x + a)^2*d^3 + (12*I*b*c*d
^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (6*(b*x + a)^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) -
6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (6*I
*(b*x + a)^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))
*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*(b*x + a)^2*d^3 + 12*(b*c*d^2
 - a*d^3)*(b*x + a) - 6*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*c
os(4*b*x + 4*a) + (-6*I*(b*x + a)^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x
 + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) + 8*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2)*cos(4*b*x + 4*a) - (6*b*c*d^
2 + 6*(b*x + a)*d^3 - 6*a*d^3 - 6*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b
*x + 4*a) + (-6*I*b*c*d^2 - 6*I*(b*x + a)*d^3 + 6*I*a*d^3)*sin(4*b*x + 4*a)
)*dilog(-e^(2*I*b*x + 2*I*a)) - (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 -
 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 1
2*I*(b*x + a)*d^3 + 12*I*a*d^3)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) -
 (12*b*c*d^2 + 12*(b*x + a)*d^3 - 12*a*d^3 - 12*(b*c*d^2 + (b*x + a)*d^3 -
a*d^3)*cos(4*b*x + 4*a) + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)
*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c
*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2
+ 2*cos(2*b*x + 2*a) + 1) - (3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3
)*(b*x + a) + (-3*I*(b*x + a)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))
```

```
*cos(4*b*x + 4*a) + 3*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin
(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) -
(3*I*(b*x + a)^2*d^3 + (6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a) + (-3*I*(b*x + a
)^2*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 3*((b*x
+ a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x +
 a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-3*I*d^3*cos(4*b*x + 4*a) +
 3*d^3*sin(4*b*x + 4*a) + 3*I*d^3)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-12*
I*d^3*cos(4*b*x + 4*a) + 12*d^3*sin(4*b*x + 4*a) + 12*I*d^3)*polylog(3, -e^
(I*b*x + I*a)) - (-12*I*d^3*cos(4*b*x + 4*a) + 12*d^3*sin(4*b*x + 4*a) + 12
*I*d^3)*polylog(3, e^(I*b*x + I*a)) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^
2 + 24*I*a*d^3)*(b*x + a)^2)*sin(4*b*x + 4*a))/(-2*I*b^3*cos(4*b*x + 4*a) +
 2*b^3*sin(4*b*x + 4*a) + 2*I*b^3))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cos\left(a+b\,x\right)^2\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2),x)
[Out] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**2,x)
[Out] Timed out
```

3.274 $\int (c+dx)^2 \csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{id^2\text{Li}_2\left(e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

```
[Out] -2*I*(d*x+c)^2/b-2*(d*x+c)^2*cot(2*b*x+2*a)/b+2*d*(d*x+c)*ln(1-exp(4*I*(b*x
+a)))/b^2-1/2*I*d^2*polylog(2,exp(4*I*(b*x+a)))/b^3
```

Rubi [A] time = 0.19, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4419, 4184, 3717, 2190, 2279, 2391}

$$-\frac{id^2\text{PolyLog}\left(2,e^{4i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\log\left(1-e^{4i(a+bx)}\right)}{b^2}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{2i(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] ((-2*I)*(c + d*x)^2)/b - (2*(c + d*x)^2*Cot[2*a + 2*b*x])/b + (2*d*(c + d*x
)*Log[1 - E^((4*I)*(a + b*x))])/b^2 - ((I/2)*d^2*PolyLog[2, E^((4*I)*(a + b
*x))])/b^3
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3717

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[(I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^
m*E^(2*I*k*Pi)*E^(2*I*(e + f*x)))/(1 + E^(2*I*k*Pi)*E^(2*I*(e + f*x))), x],
 x] /; FreeQ[{c, d, e, f}, x] && IntegerQ[4*k] && IGtQ[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\csc^2(a+bx)\sec^2(a+bx)\,dx &= 4\int (c+dx)^2\csc^2(2a+2bx)\,dx\\
&= -\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{(4d)\int (c+dx)\cot(2a+2bx)\,dx}{b}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}-\frac{(8id)\int \frac{e^{2i(2a+2bx)}(c+dx)}{1-e^{2i(2a+2bx)}}\,dx}{b}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{4i(a+b}\right.}{b^2}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{4i(a+b}\right.}{b^2}\\
&= -\frac{2i(c+dx)^2}{b}-\frac{2(c+dx)^2\cot(2a+2bx)}{b}+\frac{2d(c+dx)\log\left(1-e^{4i(a+b}\right.}{b^2}
\end{aligned}$$

Mathematica [B] time = 1.72, size = 277, normalized size = 3.15

$$\frac{2b^2\csc(2a)\sin(2bx)(c+dx)^2\csc(2(a+bx)) - \frac{ie^{4ia}\left(4e^{-4ia}b^2(c+dx)^2+2i\left(1-e^{-4ia}\right)bd(c+dx)\log\left(1-e^{-i(a+bx)}\right)+2i\left(1-e^{-4ia}\right)bd(c+dx)\log\left(\right.\right.}{}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (((-I)*E^((4*I)*a)*((4*b^2*(c + d*x)^2)/E^((4*I)*a) + (2*I)*b*d*(1 - E^((-4
*I)*a))*(c + d*x)*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a)
)*(c + d*x)*Log[1 + E^((-I)*(a + b*x))] + (2*I)*b*d*(1 - E^((-4*I)*a))*(c +
 d*x)*Log[1 + E^((-2*I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, -
E^((-I)*(a + b*x))] - 2*d^2*(1 - E^((-4*I)*a))*PolyLog[2, E^((-I)*(a + b*x)
)] - d^2*(1 - E^((-4*I)*a))*PolyLog[2, -E^((-2*I)*(a + b*x))]))/(-1 + E^((4
*I)*a)) + 2*b^2*(c + d*x)^2*Csc[2*a]*Csc[2*(a + b*x)]*Sin[2*b*x])/b^3
```

fricas [B] time = 0.56, size = 950, normalized size = 10.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (b^2*d^2*x^2 + 2*b^2*c*d*x - I*d^2*cos(b*x + a)*dilog(cos(b*x + a) + I*sin(
b*x + a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)
)*sin(b*x + a) - I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a))*si
n(b*x + a) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b
*x + a) + I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x
+ a) + I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a
) - I*d^2*cos(b*x + a)*dilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) +
 b^2*c^2 + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*x + a)
 + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x + a) + I*sin(
b*x + a) + I)*sin(b*x + a) + (b*d^2*x + b*c*d)*cos(b*x + a)*log(cos(b*x + a
) - I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b*x +
a)*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x + a*d^2)*
cos(b*x + a)*log(I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + (b*d^2*x
 + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a)
 + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2)*sin(b*x + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-1/2*cos(
b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + (b*d^2*x + a*d^2)*cos(b
*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (b*c*d - a*d
^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + (b*
d^2*x + a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x
 + a) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2)/(b
^3*cos(b*x + a)*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \csc(bx + a)^2 \sec(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^2, x)
```

maple [B] time = 0.12, size = 351, normalized size = 3.99

$$-\frac{4i\left(d^2x^2 + 2cdx + c^2\right)}{b\left(1 + e^{2i(bx+a)}\right)\left(e^{2i(bx+a)} - 1\right)} + \frac{2dc\ln\left(e^{i(bx+a)} - 1\right)}{b^2} + \frac{2dc\ln\left(1 + e^{2i(bx+a)}\right)}{b^2} + \frac{2dc\ln\left(e^{i(bx+a)} + 1\right)}{b^2} - \frac{8dc\ln\left(e^{i(bx+a}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] -2*I/b^3*d^2*polylog(2,-exp(I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))-1)+2*d/
b^2*c*ln(1+exp(2*I*(b*x+a)))+2/b^2*d*c*ln(exp(I*(b*x+a))+1)-8/b^2*d*c*ln(ex
p(I*(b*x+a)))-2*I*d^2*polylog(2,exp(I*(b*x+a)))/b^3+2/b^2*d^2*ln(exp(I*(b*x
+a))+1)*x-4*I*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*I*(b*x+a)))/(exp(2*I*(b*x+a)
)-1)-4*I*d^2/b^3*a^2+2*d^2/b^2*ln(1+exp(2*I*(b*x+a)))*x+2/b^2*d^2*ln(1-exp(
I*(b*x+a)))*x+2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a-I*d^2*polylog(2,-exp(2*I*(b*
x+a)))/b^3-4*I*d^2/b*x^2-8*I*d^2/b^2*a*x-2/b^3*d^2*a*ln(exp(I*(b*x+a))-1)+8
/b^3*d^2*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.69, size = 777, normalized size = 8.83

$$-\frac{4\,b^2c^2 + \left(2\,bd^2x + 2\,bcd - 2\left(bd^2x + bcd\right)\cos\left(4\,bx + 4\,a\right) - \left(2i\,bd^2x + 2i\,bcd\right)\sin\left(4\,bx + 4\,a\right)\right)\arctan\left(\sin\left(2\,bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -(4*b^2*c^2 + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(4*b*x + 4*a) -
 (2*I*b*d^2*x + 2*I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(
2*b*x + 2*a) + 1) + (2*b*d^2*x + 2*b*c*d - 2*(b*d^2*x + b*c*d)*cos(4*b*x +
4*a) - (2*I*b*d^2*x + 2*I*b*c*d)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), co
s(b*x + a) + 1) - (2*b*c*d*cos(4*b*x + 4*a) + 2*I*b*c*d*sin(4*b*x + 4*a) -
2*b*c*d)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d^2*x*cos(4*b*x + 4
*a) + 2*I*b*d^2*x*sin(4*b*x + 4*a) - 2*b*d^2*x)*arctan2(sin(b*x + a), -cos(
b*x + a) + 1) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x)*cos(4*b*x + 4*a) + (d^2*cos(4
```

```
*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(2*I*b*x + 2*I*a)) + 2
*(d^2*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(-e^(I*b*x + I*
a)) + 2*(d^2*cos(4*b*x + 4*a) + I*d^2*sin(4*b*x + 4*a) - d^2)*dilog(e^(I*b*
x + I*a)) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b*c*d)*cos(4*b*x + 4*a)
+ (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x +
2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*d^2*x - I*b
*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log(cos(b*x +
a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*b*d^2*x + I*b*c*d + (-I*b*
d^2*x - I*b*c*d)*cos(4*b*x + 4*a) + (b*d^2*x + b*c*d)*sin(4*b*x + 4*a))*log
(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I*b^2*d^2*x^2
- 8*I*b^2*c*d*x)*sin(4*b*x + 4*a))/(-I*b^3*cos(4*b*x + 4*a) + b^3*sin(4*b*x
 + 4*a) + I*b^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2),x)
[Out] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2\csc^2(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

3.275 $\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

```
[Out] -2*(d*x+c)*cot(2*b*x+2*a)/b+d*ln(sin(2*b*x+2*a))/b^2
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4419, 4184, 3475}

$$\frac{d\log(\sin(2a+2bx))}{b^2}-\frac{2(c+dx)\cot(2a+2bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*(c + d*x)*Cot[2*a + 2*b*x])/b + (d*Log[Sin[2*a + 2*b*x]])/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec^2(a+bx)\,dx = 4\int (c+dx)\csc^2(2a+2bx)\,dx$$
$$= -\frac{2(c+dx)\cot(2a+2bx)}{b} + \frac{(2d)\int \cot(2a+2bx)\,dx}{b}$$
$$= -\frac{2(c+dx)\cot(2a+2bx)}{b} + \frac{d\log(\sin(2a+2bx))}{b^2}$$

Mathematica [**A**] time = 0.20, size = 32, normalized size = 0.91

$$\frac{d\log(\sin(2(a+bx))) - 2b(c+dx)\cot(2(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^2,x]
```

```
[Out] (-2*b*(c + d*x)*Cot[2*(a + b*x)] + d*Log[Sin[2*(a + b*x)]])/b^2
```

fricas [**B**] time = 0.45, size = 75, normalized size = 2.14

$$\frac{d\cos(bx+a)\log\left(-\frac{1}{2}\cos(bx+a)\sin(bx+a)\right)\sin(bx+a) + bdx - 2(bdx+bc)\cos(bx+a)^2 + bc}{b^2\cos(bx+a)\sin(bx+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (d*cos(b*x + a)*log(-1/2*cos(b*x + a)*sin(b*x + a))*sin(b*x + a) + b*d*x -
2*(b*d*x + b*c)*cos(b*x + a)^2 + b*c)/(b^2*cos(b*x + a)*sin(b*x + a))
```

giac [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [**B**] time = 0.09, size = 182, normalized size = 5.20

$$\frac{\frac{c}{2b} + \frac{dx}{2b} - \frac{3c\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} + \frac{c\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b} - \frac{3dx\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b} + \frac{dx\left(\tan^4\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{2b}}{\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\left(\tan^2\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)} + \frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right)}{b^2} + \frac{d\ln\left(\tan\left(\frac{bx}{2}+\frac{a}{2}\right)\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x)
```

```
[Out] (1/2*c/b+1/2*d*x/b-3*c/b*tan(1/2*b*x+1/2*a)^2+1/2*c/b*tan(1/2*b*x+1/2*a)^4-
3/b*d*x*tan(1/2*b*x+1/2*a)^2+1/2/b*d*x*tan(1/2*b*x+1/2*a)^4)/tan(1/2*b*x+1/
2*a)/(tan(1/2*b*x+1/2*a)^2-1)+d/b^2*ln(tan(1/2*b*x+1/2*a))+d/b^2*ln(tan(1/2
*b*x+1/2*a)-1)+d/b^2*ln(tan(1/2*b*x+1/2*a)+1)-2*d/b^2*ln(1+tan(1/2*b*x+1/2*
a)^2)
```

maxima [B] time = 0.48, size = 308, normalized size = 8.80

$$-\frac{2\,c\left(\frac{1}{\tan(bx+a)}-\tan\left(bx+a\right)\right)-\frac{2\,ad\left(\frac{1}{\tan(bx+a)}-\tan(bx+a)\right)}{b}-\frac{\left(\left(\cos(4\,bx+4\,a)^2+\sin(4\,bx+4\,a)^2-2\,\cos(4\,bx+4\,a)+1\right)\log\left(\cos(2\,bx+2\,a)^2+s\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*c*(1/tan(b*x + a) - tan(b*x + a)) - 2*a*d*(1/tan(b*x + a) - tan(b*x
 + a))/b - ((cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) +
 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) +
 (cos(4*b*x + 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos
(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (cos(4*b*x + 4*a)^2 +
sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 - 2*cos(b*x + a) + 1) - 8*(b*x + a)*sin(4*b*x + 4*a))*d/((cos(4*b*x +
 4*a)^2 + sin(4*b*x + 4*a)^2 - 2*cos(4*b*x + 4*a) + 1)*b))/b
```

mupad [B] time = 1.66, size = 55, normalized size = 1.57

$$\frac{d\,\ln\left({\mathrm{e}}^{a\,4{}\mathrm{i}}\,{\mathrm{e}}^{b\,x\,4{}\mathrm{i}}-1\right)}{b^2}-\frac{\left(c+d\,x\right)\,4{}\mathrm{i}}{b\,\left({\mathrm{e}}^{a\,4{}\mathrm{i}+b\,x\,4{}\mathrm{i}}-1\right)}-\frac{d\,x\,4{}\mathrm{i}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^2),x)
```

```
[Out] (d*log(exp(a*4i)*exp(b*x*4i) - 1))/b^2 - ((c + d*x)*4i)/(b*(exp(a*4i + b*x*
4i) - 1)) - (d*x*4i)/b
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)\csc^2\left(a+bx\right)\sec^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**2, x)
```

3.276 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$4\text{Int}\left(\frac{\csc^2(2a+2bx)}{c+dx},x\right)$$

```
[Out] 4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx = 4\int \frac{\csc^2(2a+2bx)}{c+dx}\,dx$$

Mathematica [A] time = 7.06, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc (bx+a)^2 \sec (bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2 (bx+a)\right)\left(\sec^2 (bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos (a+b\,x)^2 \sin (a+b\,x)^2\, (c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x), x)
```

3.277 $\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=24

$$4\text{Int}\left(\frac{\csc^2(2a+2bx)}{(c+dx)^2}, x\right)$$

```
[Out] 4*Unintegrable(csc(2*b*x+2*a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] 4*Defer[Int][Csc[2*a + 2*b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx = 4\int \frac{\csc^2(2a+2bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 7.33, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^2\sec(bx+a)^2}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2 \sec(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sec(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\left(\sec^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+b\,x)^2\,\sin(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**2/(c + d*x)**2, x)
```

3.278 $\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\csc^3(a+bx)\sec^2(a+bx)(c+dx)^m, x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A] time = 25.51, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.279 $\int (c+dx)^3 \csc^3(a+bx) \sec^2(a+bx)\,dx$

Optimal. Leaf size=601

$$\frac{3id^3\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{9id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{9id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}$$

```
[Out] -3*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^
4+9*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-3*c*d^2*x*csc(b*x+a)/b^2-6*I*c*d^2*
polylog(2,-I*exp(I*(b*x+a)))/b^3-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3
-9/2*I*d*(d*x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2+6*I*d^3*x^2*arctan(exp(I*(
b*x+a)))/b^2+6*I*c*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3+6*I*d^3*x*polylog(2,
I*exp(I*(b*x+a)))/b^3-9*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a)))/b^3+9*d^2*(d
*x+c)*polylog(3,exp(I*(b*x+a)))/b^3+12*I*c*d^2*x*arctan(exp(I*(b*x+a)))/b^2
+9/2*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2+6*d^3*polylog(3,-I*exp(I*
(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(2,-exp(
I*(b*x+a)))/b^4+3/2*(d*x+c)^3*sec(b*x+a)/b-6*d^3*x*arctanh(exp(I*(b*x+a)))/
b^3-3*c*d^2*arctanh(cos(b*x+a))/b^3-3*c^2*d*arctanh(sin(b*x+a))/b^2-3/2*c^2
*d*csc(b*x+a)/b^2-3/2*d^3*x^2*csc(b*x+a)/b^2-1/2*(d*x+c)^3*csc(b*x+a)^2*sec
(b*x+a)/b-9*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4
```

Rubi [A] time = 2.31, antiderivative size = 601, normalized size of antiderivative = 1.00, number of steps used = 64, number of rules used = 24, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 6609, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770, 14}

$$-\frac{6icd^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6icd^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{9d^2(c+dx)\text{PolyL}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((12*I)*c*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 + ((6*I)*d^3*x^2*ArcTan[E^(I*(
a + b*x))])/b^2 - (6*d^3*x*ArcTanh[E^(I*(a + b*x))])/b^3 - (3*(c + d*x)^3*A
rcTanh[E^(I*(a + b*x))])/b - (3*c*d^2*ArcTanh[Cos[a + b*x]])/b^3 - (3*c^2*d
*ArcTanh[Sin[a + b*x]])/b^2 - (3*c^2*d*Csc[a + b*x])/(2*b^2) - (3*c*d^2*x*C
sc[a + b*x])/b^2 - (3*d^3*x^2*Csc[a + b*x])/(2*b^2) + ((3*I)*d^3*PolyLog[2,
 -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^(I*(a + b*
x))])/b^2 - ((6*I)*c*d^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^3
*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*c*d^2*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 + ((6*I)*d^3*x*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*
d^3*PolyLog[2, E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2,
E^(I*(a + b*x))])/b^2 - (9*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3
+ (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*d^3*PolyLog[3, I*E^(I*(
```

```
a + b*x))])/b^4 + (9*d^2*(c + d*x)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I
)*d^3*PolyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, E^(I*(a + b
*x))])/b^4 + (3*(c + d*x)^3*Sec[a + b*x])/(2*b) - ((c + d*x)^3*Csc[a + b*x]
^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
```

```
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b} - \frac{3\int b(-}{}$$

$$= \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\csc^2(a+bx)\sec(a+bx)}{2b} - \frac{3}{2}\int (-$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3(c+dx)^3\sec(a+bx)}{2b} - \frac{(c+dx)^3\operatorname{cs}}{}$$

$$= -\frac{3(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3c^2d\csc(a+bx)}{2b^2} + \frac{9id(c+dx)^2\operatorname{Li}_2\left(-\right.}{2b^2}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tanh^{-1}}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tanh^{-1}}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^3\tanh^{-1}}{b}$$

$$= \frac{18icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{9id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+\right.}{b^3}$$

$$= \frac{12icd^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6id^3x^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d^3x\tanh^{-1}\left(e^{i(a+\right.}{b^3}$$

Mathematica [A] time = 8.35, size = 907, normalized size = 1.51

$$-\frac{\sec(a+bx)\left(-bc^3+3b\cos(2a+2bx)c^3-3bdxc^2+9bdx\cos(2a+2bx)c^2+3d\sin(2a+2bx)c^2-3bd^2x^2c+9bd}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*((-2*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + 2*b^2*c*d*x*Log[1 - I*E^(I*
(a + b*x))] + b^2*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - 2*b^2*c*d*x*Log[1 +
I*E^(I*(a + b*x))] - b^2*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + (2*I)*b*d*(c
+ d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2, I*
E^(I*(a + b*x))] - 2*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 2*d^2*PolyLog[3
, I*E^(I*(a + b*x))]))/b^4 + (3*(b^3*c^3*Log[1 - E^(I*(a + b*x))] + 2*b*c*d
^2*Log[1 - E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 - E^(I*(a + b*x))] + 2*b*
d^3*x*Log[1 - E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 - E^(I*(a + b*x))] +
 b^3*d^3*x^3*Log[1 - E^(I*(a + b*x))] - b^3*c^3*Log[1 + E^(I*(a + b*x))] -
2*b*c*d^2*Log[1 + E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Log[1 + E^(I*(a + b*x))]
 - 2*b*d^3*x*Log[1 + E^(I*(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 + E^(I*(a + b
*x))] - b^3*d^3*x^3*Log[1 + E^(I*(a + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)
^2)*PolyLog[2, -E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[
2, E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, -E^(I*(a + b*x))] - 6*b*d^3*x*Po
lyLog[3, -E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, E^(I*(a + b*x))] + 6*b*d^
3*x*PolyLog[3, E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, -E^(I*(a + b*x))] +
(6*I)*d^3*PolyLog[4, E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]^2*Sec[a + b
*x]*(-(b*c^3) - 3*b*c^2*d*x - 3*b*c*d^2*x^2 - b*d^3*x^3 + 3*b*c^3*Cos[2*a +
 2*b*x] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 3
*b*d^3*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a
+ 2*b*x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

fricas [C] time = 0.81, size = 3173, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 4*b^3*c^3 - 6*(b^
3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 6*(
b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - ((-9*I
*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^3 +
 (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a
))*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^
2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^3 + (-9*I*b^2*d^3*x^2 - 18*I*b^
```

```
2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a))*dilog(cos(b*x + a) - I*s
in(b*x + a)) - ((12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^3 + (-12*I*b*d^3
*x - 12*I*b*c*d^2)*cos(b*x + a))*dilog(I*cos(b*x + a) + sin(b*x + a)) - ((1
2*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)
*cos(b*x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) - ((-12*I*b*d^3*x - 12*
I*b*c*d^2)*cos(b*x + a)^3 + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a))*dil
og(-I*cos(b*x + a) + sin(b*x + a)) - ((-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*
x + a)^3 + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a))*dilog(-I*cos(b*x + a
) - sin(b*x + a)) - ((-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d -
 6*I*d^3)*cos(b*x + a)^3 + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^
2*d + 6*I*d^3)*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((9*I*
b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^3 +
(-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a
))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^
2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^3 - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x
)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*c
os(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a
)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 2*b*c*d^2 + (3*b^3*c^2*d +
 2*b*d^3)*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 6*((b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 +
 a^2*d^3)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) + 6*((b^2*d^
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^
2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(I*cos(b*x + a)
 + sin(b*x + a) + 1) - 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*
d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)
*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 6*((b^2*d^3*x^2 + 2
*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a))*log(-I*cos(b*x + a) + sin(b
*x + a) + 1) - 6*((b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos
(b*x + a)^3 - (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x
 + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((b^3*c^3 - 3*a*b^2*c^2*
d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*
b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*c
os(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b^3*c^3 - 3*a*b^2*c^2*d + (3*
a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^3 - (b^3*c^3 - 3*a*b^2*c^2
*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a))*log(-1/2*cos(b*x
+ a) - 1/2*I*sin(b*x + a) + 1/2) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*
b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*co
s(b*x + a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d
^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*
x + a) + I*sin(b*x + a) + 1) + 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b
*x + a)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x
```

```
+ a) + I*sin(b*x + a) + I) - 3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^
2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x
+ a)^3 - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (
a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a))*log(-cos(b*x + a)
 - I*sin(b*x + a) + 1) - 6*((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a
)^3 - (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a))*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) - (18*I*d^3*cos(b*x + a)^3 - 18*I*d^3*cos(b*x + a))*po
lylog(4, cos(b*x + a) + I*sin(b*x + a)) - (-18*I*d^3*cos(b*x + a)^3 + 18*I*
d^3*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) - (18*I*d^3*cos
(b*x + a)^3 - 18*I*d^3*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x +
 a)) - (-18*I*d^3*cos(b*x + a)^3 + 18*I*d^3*cos(b*x + a))*polylog(4, -cos(b
*x + a) - I*sin(b*x + a)) - 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3
*x + b*c*d^2)*cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 18*
((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*pol
ylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(d^3*cos(b*x + a)^3 - d^3*cos(b
*x + a))*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 12*(d^3*cos(b*x + a)^3
 - d^3*cos(b*x + a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(d^3*co
s(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a))
 + 12*(d^3*cos(b*x + a)^3 - d^3*cos(b*x + a))*polylog(3, -I*cos(b*x + a) -
sin(b*x + a)) + 18*((b*d^3*x + b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2
)*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) + 18*((b*d^3*x +
 b*c*d^2)*cos(b*x + a)^3 - (b*d^3*x + b*c*d^2)*cos(b*x + a))*polylog(3, -co
s(b*x + a) - I*sin(b*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [B] time = 0.54, size = 1613, normalized size = 2.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^
4-12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))+3*I*d^3*polylog(2,-exp(I*(b*x+a))
)/b^4+6*I*d^3*x*polylog(2,I*exp(I*(b*x+a)))/b^3+9*I*d^3*polylog(4,exp(I*(b*
x+a)))/b^4-6*I*d^3*x*polylog(2,-I*exp(I*(b*x+a)))/b^3-3/2/b^4*d^3*a^3*ln(ex
```

```
p(I*(b*x+a))-1)-9/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+9/b^3*c*d^2*polylog(
3,exp(I*(b*x+a)))+9/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-9/b^3*d^3*polylog(3
,-exp(I*(b*x+a)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-9*I*d^3*polylog(4
,-exp(I*(b*x+a)))/b^4+1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*
d^3*x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c^2*d*x*b*exp(5
*I*(b*x+a))-2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x+a))-6*c*d^2*x
^2*b*exp(3*I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a))-6*c^2*d*x*b*exp(3*I*(b*x+a
))+3*d^3*x^3*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*x+a))-2*c^3*b*exp(3*I*
(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+9*c^2*d*x*
b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))+3*I*c^
2*d*exp(I*(b*x+a))+3*I*d^3*x^2*exp(I*(b*x+a)))-6*I/b^3*d^2*c*dilog(1+I*exp(
I*(b*x+a)))+6*I/b^3*d^2*c*dilog(1-I*exp(I*(b*x+a)))+6*I/b^4*a*d^3*dilog(1+I
*exp(I*(b*x+a)))-6*I/b^4*a*d^3*dilog(1-I*exp(I*(b*x+a)))+6*I/b^4*d^3*polylo
g(2,I*exp(I*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,-I*exp(I*(b*x+a)))*a+3/2/b*c^
3*ln(exp(I*(b*x+a))-1)-3/2/b*c^3*ln(exp(I*(b*x+a))+1)+3/b^3*d^2*c*ln(exp(I*
(b*x+a))-1)-3/b^3*d^2*c*ln(exp(I*(b*x+a))+1)-3/b^3*d^3*ln(exp(I*(b*x+a))+1)
*x+3/b^3*d^3*ln(1-exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3/b^4*
d^3*a*ln(exp(I*(b*x+a))-1)+9/2*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))-9/2*I
/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+9/2*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)
))*x^2-9/2*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+6/b^2*d^2*c*ln(1+I*exp(I
*(b*x+a)))*x+6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*I/b^2*d*c^2*arctan(ex
p(I*(b*x+a)))-6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+6/b^3*d^2*c*ln(1+I*exp(I
*(b*x+a)))*a-6/b^2*d^2*c*ln(1-I*exp(I*(b*x+a)))*x+9/2/b^3*c*d^2*a^2*ln(exp(
I*(b*x+a))-1)-9/2/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+9/2/b*c^2*d*ln(1-exp(I*(b*
x+a)))*x+9/2/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-9/2/b^3*c*d^2*a^2*ln(1-exp(I*
(b*x+a)))+9/2/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/2/b*c*d^2*ln(exp(I*(b*x+a)
)+1)*x^2-9/2/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+3/2/b*d^3*ln(1-exp(I*(b*x+a))
)*x^3+3/2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-3/2/b*d^3*ln(exp(I*(b*x+a))+1)*x
^3+3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2-3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a))
)-3/b^2*d^3*ln(1-I*exp(I*(b*x+a)))*x^2+3/b^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))
-9*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+9*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x
```

maxima [B] time = 6.03, size = 8043, normalized size = 13.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*cos(b*x + a)
^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(c
os(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3
 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 -
```

a^3*d^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^3 + 4*((12*b^2*c^2*d - 24*a*
b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a)
+ 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 12*(b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a
)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x
+ 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2
*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*I*b^2*
c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^
2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x
+ a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 12*a^2*d^3
+ 24*(b*c*d^2 - a*d^3)*(b*x + a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^
2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*(b^2
*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(4*b*x + 4*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a
^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^2*c^2*d
 + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b*c*d^2 +
24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2
+ 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x +
a))*sin(4*b*x + 4*a) - (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*
d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a
))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (6*(b*x + a)^3*d^3 + 12*b*c*d
^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +
 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
d^3 + 3(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 -
2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3
*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*
c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^
2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) +
 (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*
a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)
d^3)(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 +
12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d +
36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arcta
n2(sin(b*x + a), cos(b*x + a) + 1) + (12*b*c*d^2 - 12*a*d^3 + 12*(b*c*d^2 -
 a*d^3)*cos(6*b*x + 6*a) - 12*(b*c*d^2 - a*d^3)*cos(4*b*x + 4*a) - 12*(b*c*
d^2 - a*d^3)*cos(2*b*x + 2*a) - (-12*I*b*c*d^2 + 12*I*a*d^3)*sin(6*b*x + 6*
a) - (12*I*b*c*d^2 - 12*I*a*d^3)*sin(4*b*x + 4*a) - (12*I*b*c*d^2 - 12*I*a*
d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) - (6*(b*x +

```
a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c*d^2
+ (3*a^2 + 2)*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*
x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b
*x + 6*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c
^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x
 + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2
+ (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18
*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (
18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 +
 (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d
^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^
3*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*
a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(si
n(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*
a*b*c*d^2 + 12*a^2*d^3 - 12*(-3*I*b*c*d^2 + (3*I*a - 1)*d^3)*(b*x + a)^2 +
(36*I*b^2*c^2*d - 24*(3*I*a - 1)*b*c*d^2 + (36*I*a^2 - 24*a)*d^3)*(b*x + a)
)*cos(5*b*x + 5*a) - (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(
b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3)*(b*x + a))*c
os(3*b*x + 3*a) - (12*I*(b*x + a)^3*d^3 - 12*b^2*c^2*d + 24*a*b*c*d^2 - 12*
a^2*d^3 + (36*I*b*c*d^2 - 12*(3*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d
 - 24*(3*I*a + 1)*b*c*d^2 + (36*I*a^2 + 24*a)*d^3)*(b*x + a))*cos(b*x + a)
+ (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 -
 a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x +
 4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) - (-24*I*b*c*
d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(6*b*x + 6*a) - (24*I*b*c*d^2 + 2
4*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(4*b*x + 4*a) - (24*I*b*c*d^2 + 24*I*(b*
x + a)*d^3 - 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (24*b
*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)
*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(4*b*x + 4*a) -
 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (24*I*b*c*d^2 + 24
*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(6*b*x + 6*a) + (-24*I*b*c*d^2 - 24*I*(b*
x + a)*d^3 + 24*I*a*d^3)*sin(4*b*x + 4*a) + (-24*I*b*c*d^2 - 24*I*(b*x + a)
*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (18*b^2*c^
2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^2 -
 a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a
^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*(3*b^2*c^
2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^
3)*(b*x + a))*cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)
^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a)
- (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1
2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (18*I
*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3
+ (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (18*I*b^2*c^2*d
 - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (36*I*b*
c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) -
```

```
(18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*
(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*
d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6
*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*
d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3
*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b
*x + 2*a) + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I
*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a)
+ (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 1
2*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (-18*
I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)*d^
3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*
x + I*a)) - (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2
 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 -
6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9
*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-
9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (3*I*(b*x + a)^3*d^3 + 6*
I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^
2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(
b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x
 + a))*sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*c
*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(
b*x + a))*sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(
b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(
b*x + a) + 1) - (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d
^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 +
6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I
*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*
a^2 + 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b
*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2
*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)
*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*
x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*
(b*x + a))*sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 + 3*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)*d^
3)*(b*x + a))*sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^3 +
 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 + 2)
*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*
cos(b*x + a) + 1) - (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3
```

```
- 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (-6*I*b^2*c^2*d +
12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*
a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*
(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(
4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*
a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 6*(b^2*
c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x
+ a))*sin(6*b*x + 6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2
*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a
*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (6
*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b
*c*d^2 - 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x
 + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(6*b*
x + 6*a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2
*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*I*b^2
*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b*c*d^
2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 +
(b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a)
 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*
d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2
*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b
*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (36*d^3*cos(6*b*x + 6*a)
 - 36*d^3*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x +
 6*a) - 36*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*pol
ylog(4, -e^(I*b*x + I*a)) + (36*d^3*cos(6*b*x + 6*a) - 36*d^3*cos(4*b*x + 4
*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) - 36*I*d^3*sin(4*
b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*polylog(4, e^(I*b*x + I*a)
) - (-24*I*d^3*cos(6*b*x + 6*a) + 24*I*d^3*cos(4*b*x + 4*a) + 24*I*d^3*cos(
2*b*x + 2*a) + 24*d^3*sin(6*b*x + 6*a) - 24*d^3*sin(4*b*x + 4*a) - 24*d^3*s
in(2*b*x + 2*a) - 24*I*d^3)*polylog(3, I*e^(I*b*x + I*a)) - (24*I*d^3*cos(6
*b*x + 6*a) - 24*I*d^3*cos(4*b*x + 4*a) - 24*I*d^3*cos(2*b*x + 2*a) - 24*d^
3*sin(6*b*x + 6*a) + 24*d^3*sin(4*b*x + 4*a) + 24*d^3*sin(2*b*x + 2*a) + 24
*I*d^3)*polylog(3, -I*e^(I*b*x + I*a)) - (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^
3 + 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(6*b*
x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a
) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(2*b*x + 2*a) + 36*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a
)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(
2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*c*d^2 + 36*I*(b*x + a)
*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(6*
b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(4*b*x +
4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3)*cos(2*b*x + 2*a) -
 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*x + 6*a) + 36*(b*c*d^2 + (b*x
 + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*
```

```
sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (12*(b*x + a)^3*d^3 - 12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d^3
)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a)*
d^3)*(b*x + a))*sin(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(3*b*x +
 3*a) + (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^
3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a -
24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*
cos(6*b*x + 6*a) + 4*I*b^3*cos(4*b*x + 4*a) + 4*I*b^3*cos(2*b*x + 2*a) + 4*
b^3*sin(6*b*x + 6*a) - 4*b^3*sin(4*b*x + 4*a) - 4*b^3*sin(2*b*x + 2*a) - 4*
I*b^3))/b
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.280 $\int (c+dx)^2 \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=305

$$-\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{3d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(a+bx))}{b^3}+\frac{3id(c+$$

```
[Out] 4*I*d^2*x*arctan(exp(I*(b*x+a)))/b^2-3*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-
d^2*arctanh(cos(b*x+a))/b^3-2*c*d*arctanh(sin(b*x+a))/b^2-c*d*csc(b*x+a)/b^
2-d^2*x*csc(b*x+a)/b^2+3*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*d^2
*polylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*d^2*polylog(2,I*exp(I*(b*x+a)))/b^3-3
*I*d*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^2-3*d^2*polylog(3,-exp(I*(b*x+a)))
/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3+3/2*(d*x+c)^2*sec(b*x+a)/b-1/2*(d*
x+c)^2*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.87, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 36, number of rules used = 22, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.917, Rules used = {2622, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4183, 2531, 2282, 6589, 4133, 453, 206, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2id^2\text{PolyLog}}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*d^2*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*(c + d*x)^2*ArcTanh[E^(I*(a
+ b*x))])/b - (d^2*ArcTanh[Cos[a + b*x]])/b^3 - (2*c*d*ArcTanh[Sin[a + b*x]
])/b^2 - (c*d*Csc[a + b*x])/b^2 - (d^2*x*Csc[a + b*x])/b^2 + ((3*I)*d*(c +
d*x)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^3 + ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*d*(
c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (3*d^2*PolyLog[3, -E^(I*(a + b*
x))])/b^3 + (3*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 + (3*(c + d*x)^2*Sec[a
+ b*x])/(2*b) - ((c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4133

```
Int[((a_) + (b_.)*sec[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*sin[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f
, Subst[Int[((1 - ff^2*x^2)^((m - 1)/2)*(b + a*(ff*x)^n)^p)/(ff*x)^(n*p), x
], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[(m - 1)/2
] && IntegerQ[n] && IntegerQ[p]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2 \cdots}{\cdots}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2 \cdots}{\cdots}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2 \cdots}{\cdots}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2 \cdots}{\cdots}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2 \cdots}{\cdots}$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int b(c \cdots}{\cdots}$$

$$= \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int (c \cdots$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3(c+dx)^2\sec(a+bx)}{2b} - \frac{(c+dx)^2\csc \cdots}{\cdots}$$

$$= -\frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{cd\csc(a+bx)}{b^2} + \frac{3id(c+dx)\text{Li}_2\left(-e^{i(a \cdots}\right.}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2cd\tanh^{-1}(\sin(\cdots}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2cd\tanh^{-1}(\sin(\cdots}{b^2}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a \cdots}{b^3}$$

$$= \frac{6id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a \cdots}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a \cdots}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a \cdots}{b^3}$$

$$= \frac{4id^2x\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(a \cdots}{b^3}$$

Mathematica [B] time = 7.97, size = 889, normalized size = 2.91

$$-\frac{2\left(\frac{2\tan^{-1}(\cot(a))\tanh^{-1}\left(\frac{\sin(a)+\cos(a)\tan\left(\frac{bx}{2}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}\right)}{\sqrt{\cos^2(a)+\sin^2(a)}}-\frac{\csc(a)\left(\left(bx-\tan^{-1}(\cot(a))\right)\left(\log\left(1-e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)-\log\left(1+e^{i\left(bx-\tan^{-1}(\cot(a))\right)}\right)\right)+i\left(\mathrm{Li}_2\left(-e^{i(b}\right.\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a/2 + (b*x)/2]^2)/(8*b) + (3*b^2*c^2*Log[1
- E^(I*(a + b*x))] + 2*d^2*Log[1 - E^(I*(a + b*x))] + 6*b^2*c*d*x*Log[1 - E
^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*c^2*Log[1
+ E^(I*(a + b*x))] - 2*d^2*Log[1 + E^(I*(a + b*x))] - 6*b^2*c*d*x*Log[1 + E
^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 + E^(I*(a + b*x))] + (6*I)*b*d*(c + d
*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, E^(I*(a +
 b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))] + 6*d^2*PolyLog[3, E^(I*(a + b
*x))])/(2*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a/2 + (b*x)/2]^2)/(8*b) + (
(c + d*x)*Csc[a]*Sec[a]*(-(d*Cos[a]) + b*c*Sin[a] + b*d*x*Sin[a]))/b^2 - ((
4*I)*c*d*ArcTan[((-I)*Sin[a] - I*Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a
]^2]])/(b^2*Sqrt[Cos[a]^2 + Sin[a]^2]) - (2*d^2*(-((Csc[a]*((b*x - ArcTan[C
ot[a]])*(Log[1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan
[Cot[a]]))]) + I*(PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - PolyLog[2, E^
(I*(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2]) + (2*ArcTan[Cot[a]]*ArcT
anh[(Sin[a] + Cos[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^
2 + Sin[a]^2]))/b^3 + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c*d*Sin[(b*x)/2]) - d
^2*x*Sin[(b*x)/2]))/(2*b^2) + (Csc[a/2]*Csc[a/2 + (b*x)/2]*(c*d*Sin[(b*x)/2
] + d^2*x*Sin[(b*x)/2]))/(2*b^2) + (c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2]
 + d^2*x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin
[a/2 + (b*x)/2])) + (-(c^2*Sin[(b*x)/2]) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*S
in[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)
/2]))
```

fricas [C] time = 0.66, size = 1801, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(4*b^2*d^2*x^2 + 8*b^2*c*d*x + 4*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
```

```
 - ((-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*c
os(b*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*c
*d)*cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a))*dilog(cos(b*x
 + a) - I*sin(b*x + a)) - (4*I*d^2*cos(b*x + a)^3 - 4*I*d^2*cos(b*x + a))*d
ilog(I*cos(b*x + a) + sin(b*x + a)) - (4*I*d^2*cos(b*x + a)^3 - 4*I*d^2*cos
(b*x + a))*dilog(I*cos(b*x + a) - sin(b*x + a)) - (-4*I*d^2*cos(b*x + a)^3
+ 4*I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (-4*I*d^2*c
os(b*x + a)^3 + 4*I*d^2*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a))
 - ((-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^3 + (6*I*b*d^2*x + 6*I*b*c*d)*c
os(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b*d^2*x + 6*I*b*
c*d)*cos(b*x + a)^3 + (-6*I*b*d^2*x - 6*I*b*c*d)*cos(b*x + a))*dilog(-cos(b
*x + a) - I*sin(b*x + a)) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d
^2)*cos(b*x + a)^3 - (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(
b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b
*x + a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a)
 + I) + ((3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a)^3 -
 (3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*d^2)*cos(b*x + a))*log(cos(b*
x + a) - I*sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d -
 a*d^2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) + 4*((b*d^2*x
+ a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(I*cos(b*x + a
) + sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*
d^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 4*((b*d^2*x + a
*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^2)*cos(b*x + a))*log(-I*cos(b*x + a)
+ sin(b*x + a) + 1) - 4*((b*d^2*x + a*d^2)*cos(b*x + a)^3 - (b*d^2*x + a*d^
2)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((3*b^2*c^2 - 6*
a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2
 + 2)*d^2)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)
- ((3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^3 - (3*b^2*c^2 -
6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*si
n(b*x + a) + 1/2) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^3 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 4*((b*c*d - a*d^2)*cos(b*x +
a)^3 - (b*c*d - a*d^2)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I
) - 3*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^3 - (
b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a))*log(-cos(b*x
 + a) - I*sin(b*x + a) + 1) - 4*((b*c*d - a*d^2)*cos(b*x + a)^3 - (b*c*d -
a*d^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 6*(d^2*cos(b
*x + a)^3 - d^2*cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 6
*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*
x + a)) + 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3, -cos(b*x + a
) + I*sin(b*x + a)) + 6*(d^2*cos(b*x + a)^3 - d^2*cos(b*x + a))*polylog(3,
-cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [B] time = 0.31, size = 802, normalized size = 2.63

$$\frac{3d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{2b^3}+\frac{3d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{2b}-\frac{3d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{2b^3}-\frac{3d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{2b}+\frac{2d^2\ln\left(1+ie^{i(bx}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] 3/2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+3/2/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/2
/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2-3/2/b*d^2*ln(exp(I*(b*x+a))+1)*x^2-3*d^2*
polylog(3,-exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,exp(I*(b*x+a)))/b^3-2/b^3*d^
2*ln(1-I*exp(I*(b*x+a)))*a+2*I/b^3*d^2*dilog(1-I*exp(I*(b*x+a)))-2*I/b^3*d^
2*dilog(1+I*exp(I*(b*x+a)))+2/b^2*d^2*ln(1+I*exp(I*(b*x+a)))*x+2/b^3*d^2*ln
(1+I*exp(I*(b*x+a)))*a-2/b^2*d^2*ln(1-I*exp(I*(b*x+a)))*x+3/2/b*c^2*ln(exp(
I*(b*x+a))-1)-3/2/b*c^2*ln(exp(I*(b*x+a))+1)-3*I/b^2*c*d*polylog(2,exp(I*(b
*x+a)))+3*I/b^2*c*d*polylog(2,-exp(I*(b*x+a)))+3/b*c*d*ln(1-exp(I*(b*x+a)))
*x+3/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-3/b*c*d*ln(exp(I*(b*x+a))+1)*x-3/b^2*c*
d*a*ln(exp(I*(b*x+a))-1)-1/b^3*d^2*ln(exp(I*(b*x+a))+1)+1/b^3*d^2*ln(exp(I*
(b*x+a))-1)+4*I*d/b^2*c*arctan(exp(I*(b*x+a)))-4*I*d^2/b^3*a*arctan(exp(I*(
b*x+a)))+3*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x-3*I/b^2*polylog(2,exp(I*(
b*x+a)))*d^2*x+1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*d^2*x^2
*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*x+a))+3*c^2*b*exp(5*I*(b*x+a))-2*d
^2*x^2*b*exp(3*I*(b*x+a))-4*c*d*x*b*exp(3*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x
+a))-2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b*exp(I*(b*x+a))-2*I*c*d*exp(5*I*(b
*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*exp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x
+a))+2*I*d*c*exp(I*(b*x+a)))
```

maxima [B] time = 1.62, size = 3820, normalized size = 12.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 2*a*c*d*(2*(3*cos(b*x + a)^2
 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos
(b*x + a) - 1))/b + a^2*d^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos
(b*x + a)) - 3*log(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1))/b^2 + 4*((8
*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(
6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*
d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d
^2 + 8*I*a*d^2)*sin(6*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d
^2)*sin(4*b*x + 4*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*
x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*
d^2 - 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 8*(b*c*d + (b*x + a)*d^2 - a
*d^2)*cos(2*b*x + 2*a) - (-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(6
*b*x + 6*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(4*b*x + 4*a)
 - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(cos
(b*x + a), -sin(b*x + a) + 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*
x + a) + 4*d^2 + 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2
)*cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2
*d^2)*cos(4*b*x + 4*a) - 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a)
 + 2*d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^
2)*(b*x + a) + 4*I*d^2)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b
*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^
2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(2*b*x + 2*a))*a
rctan2(sin(b*x + a), cos(b*x + a) + 1) + (4*d^2*cos(6*b*x + 6*a) - 4*d^2*co
s(4*b*x + 4*a) - 4*d^2*cos(2*b*x + 2*a) + 4*I*d^2*sin(6*b*x + 6*a) - 4*I*d^
2*sin(4*b*x + 4*a) - 4*I*d^2*sin(2*b*x + 2*a) + 4*d^2)*arctan2(sin(b*x + a)
, cos(b*x + a) - 1) - (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 6
*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x
 + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a) - 6*((b*x + a)^
2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^
2 + (12*I*b*c*d - 12*I*a*d^2)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)
^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*
x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*arct
an2(sin(b*x + a), -cos(b*x + a) + 1) + 4*(-3*I*(b*x + a)^2*d^2 - 2*b*c*d +
2*a*d^2 + 2*(-3*I*b*c*d + (3*I*a - 1)*d^2)*(b*x + a))*cos(5*b*x + 5*a) - (-
8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a))*cos(3*b*x + 3*a
) - (12*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (24*I*b*c*d - 8*(3*I*a + 1)
*d^2)*(b*x + a))*cos(b*x + a) + (8*d^2*cos(6*b*x + 6*a) - 8*d^2*cos(4*b*x +
 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*sin(6*b*x + 6*a) - 8*I*d^2*sin(4*b
*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) + 8*d^2)*dilog(I*e^(I*b*x + I*a)) - (8
*d^2*cos(6*b*x + 6*a) - 8*d^2*cos(4*b*x + 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8
*I*d^2*sin(6*b*x + 6*a) - 8*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*
a) + 8*d^2)*dilog(-I*e^(I*b*x + I*a)) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a
*d^2 + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b
```

```
*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*
cos(2*b*x + 2*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(6*b*
x + 6*a) - (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(4*b*x + 4*a)
- (12*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e
^(I*b*x + I*a)) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 + 12*(b*c*d + (b*
x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*c
os(4*b*x + 4*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (12
*I*b*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(6*b*x + 6*a) + (-12*I*b*c*d
 - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) + (-12*I*b*c*d - 12*I*
(b*x + a)*d^2 + 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) - (-3*
I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2 + (-3*I*(b
*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x + 6
*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2)*c
os(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a)
+ 2*I*d^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x +
 a) + 2*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x
 + a) + 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b
*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*
cos(b*x + a) + 1) - (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a
) + 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*
I*d^2)*cos(6*b*x + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*
(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d
 + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*d^2 +
6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2
+ 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2*d^
2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2
 + 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(6*b*x + 6*a
) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(4*b*x + 4*a) + (4*I*b*c
*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(2*b*x + 2*a) + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2)*sin(6*b*x + 6*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(4*b*
x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*I*b*c*d + 4*I*(b*x + a)*
d^2 - 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(6*b*x + 6
*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(4*b*x + 4*a) + (-4*I
*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(2*b*x + 2*a) - 4*(b*c*d + (b*x
+ a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(
4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2*b*x + 2*a))*log(cos(
b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-12*I*d^2*cos(6*b*x +
6*a) + 12*I*d^2*cos(4*b*x + 4*a) + 12*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(6
*b*x + 6*a) - 12*d^2*sin(4*b*x + 4*a) - 12*d^2*sin(2*b*x + 2*a) - 12*I*d^2)
*polylog(3, -e^(I*b*x + I*a)) - (12*I*d^2*cos(6*b*x + 6*a) - 12*I*d^2*cos(4
*b*x + 4*a) - 12*I*d^2*cos(2*b*x + 2*a) - 12*d^2*sin(6*b*x + 6*a) + 12*d^2*
sin(4*b*x + 4*a) + 12*d^2*sin(2*b*x + 2*a) + 12*I*d^2)*polylog(3, e^(I*b*x
+ I*a)) + (12*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a +
```

```
 8*I)*d^2)*(b*x + a))*sin(5*b*x + 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*
d^2)*(b*x + a))*sin(3*b*x + 3*a) + (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*
d^2 + (24*b*c*d - (24*a - 8*I)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^2*cos(
6*b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*
sin(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^
2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.281 $\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=154

$$\frac{3idLi_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3idLi_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\csc^2}{}$$

```
[Out] -3*d*x*arctanh(exp(I*(b*x+a)))/b-3/2*c*arctanh(cos(b*x+a))/b-d*arctanh(sin(
b*x+a))/b^2-1/2*d*csc(b*x+a)/b^2+3/2*I*d*polylog(2,-exp(I*(b*x+a)))/b^2-3/2
*I*d*polylog(2,exp(I*(b*x+a)))/b^2+3/2*(d*x+c)*sec(b*x+a)/b-1/2*(d*x+c)*csc
(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.19, antiderivative size = 174, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{d\csc(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3(c+dx)\sec(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*d*x*ArcTanh[E^(I*(a + b*x))])/b + (3*d*x*ArcTanh[Cos[a + b*x]])/(2*b) -
 (3*(c + d*x)*ArcTanh[Cos[a + b*x]])/(2*b) - (d*ArcTanh[Sin[a + b*x]])/b^2
- (d*Csc[a + b*x])/(2*b^2) + (((3*I)/2)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2
 - (((3*I)/2)*d*PolyLog[2, E^(I*(a + b*x))])/b^2 + (3*(c + d*x)*Sec[a + b*x
])/(2*b) - ((c + d*x)*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
```

```
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\,\mathrm{cs}}{}$$

$$= -\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3(c+dx)\sec(a+bx)}{2b}-\frac{(c+dx)\,\mathrm{cs}}{}$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\tanh}{}$$

$$= \frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3d\tanh}{}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}}{2b}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}}{2b}$$

$$= -\frac{3dx\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{3dx\tanh^{-1}(\cos(a+bx))}{2b}-\frac{3(c+dx)\tanh^{-1}}{2b}$$

Mathematica [B] time = 4.92, size = 520, normalized size = 3.38

$$\frac{3d\left(i\left(\mathrm{Li}_2\left(-e^{i(a+bx)}\right)-\mathrm{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{2b^2}-\frac{d\tan\left(\frac{1}{2}(a+bx)\right)}{4b^2}-\frac{d\cot}{}$$

Antiderivative was successfully verified.

[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^2,x]

[Out] (d*x)/b - (d*Cot[(a + b*x)/2])/(4*b^2) - (c*Csc[(a + b*x)/2]^2)/(8*b) - (d*x*Csc[(a + b*x)/2]^2)/(8*b) - (3*c*Log[Cos[(a + b*x)/2]])/(2*b) + (d*Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]])/b^2 + (3*c*Log[Sin[(a + b*x)/2]])/(2*b) - (d*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]])/b^2 - (3*a*d*Log[Tan[(a + b*x)/2]])/(2*b^2) + (3*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/(2*b^2) + (c*Sec[(a + b*x)/2]^2)/(8*b) + (d*x*Sec[(a + b*x)/2]^2)/(8*b) + (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (c*Sin[(a + b*x)/2])/(b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*(-(a*Sin[(a + b*x)/2]) + (a + b*x)*Sin[(a + b*x)/2]))/(b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*Tan[(a + b*x)/2])/(4*b^2)

fricas [B] time = 0.52, size = 621, normalized size = 4.03

$$-\frac{4\,bdx - 6\,(bdx + bc)\cos\left(bx + a\right)^{2} - 2\,d\cos\left(bx + a\right)\sin\left(bx + a\right) + 4\,bc - \left(-3i\,d\cos\left(bx + a\right)^{3} + 3i\,d\cos\left(bx + a\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")

[Out] -1/4*(4*b*d*x - 6*(b*d*x + b*c)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 4*b*c - (-3*I*d*cos(b*x + a)^3 + 3*I*d*cos(b*x + a))*dilog(cos(b*x + a) + I*sin(b*x + a)) - (3*I*d*cos(b*x + a)^3 - 3*I*d*cos(b*x + a))*dilog(cos(b*x + a) - I*sin(b*x + a)) - (-3*I*d*cos(b*x + a)^3 + 3*I*d*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) - (3*I*d*cos(b*x + a)^3 - 3*I*d*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) + 3*((b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*d*x + b*c)*cos(b*x + a)^3 - (b*d*x + b*c)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 3*((b*c - a*d)*cos(b*x + a)^3 - (b*c - a*d)*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - 3*((b*d*x + a*d)*cos(b*x + a)^3 - (b*d*x + a*d)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(sin(b*x + a) + 1) - 2*(d*cos(b*x + a)^3 - d*cos(b*x + a))*log(-sin(b*x + a) + 1))/(b^2*cos(b*x + a)^3 - b^2*cos(b*x + a))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc(bx+a)^3\sec(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [A] time = 0.16, size = 267, normalized size = 1.73

$$\frac{3bdx\,e^{5i(bx+a)}+3cb\,e^{5i(bx+a)}-2bdx\,e^{3i(bx+a)}-2cb\,e^{3i(bx+a)}-id\,e^{5i(bx+a)}+3bdx\,e^{i(bx+a)}+3cb\,e^{i(bx+a)}+id\,e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(1+e^{2i(bx+a)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*d*x*exp(5*I*(b*x+a))
+3*c*b*exp(5*I*(b*x+a))-2*b*d*x*exp(3*I*(b*x+a))-2*c*b*exp(3*I*(b*x+a))-I*d
*exp(5*I*(b*x+a))+3*b*d*x*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*(b*
x+a)))+3/2/b*c*ln(exp(I*(b*x+a))-1)-3/2/b*c*ln(exp(I*(b*x+a))+1)-3/2/b^2*d*
a*ln(exp(I*(b*x+a))-1)+2*I/b^2*d*arctan(exp(I*(b*x+a)))+3/2*I/b^2*d*dilog(e
xp(I*(b*x+a)))+3/2*I/b^2*d*dilog(exp(I*(b*x+a))+1)-3/2/b*d*ln(exp(I*(b*x+a)
)+1)*x
```

maxima [B] time = 0.90, size = 1503, normalized size = 9.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -((4*d*cos(6*b*x + 6*a) - 4*d*cos(4*b*x + 4*a) - 4*d*cos(2*b*x + 2*a) + 4*I
*d*sin(6*b*x + 6*a) - 4*I*d*sin(4*b*x + 4*a) - 4*I*d*sin(2*b*x + 2*a) + 4*d
)*arctan2(2*(cos(b*x + 2*a)*cos(a) + sin(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)
^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*
a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)^2 - cos(a)^2 + sin(b*x + 2*a)^2 - si
n(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (6*b*d*x + 6*b*c + 6*(b*d*x
 + b*c)*cos(6*b*x + 6*a) - 6*(b*d*x + b*c)*cos(4*b*x + 4*a) - 6*(b*d*x + b*
c)*cos(2*b*x + 2*a) - (-6*I*b*d*x - 6*I*b*c)*sin(6*b*x + 6*a) - (6*I*b*d*x
+ 6*I*b*c)*sin(4*b*x + 4*a) - (6*I*b*d*x + 6*I*b*c)*sin(2*b*x + 2*a))*arcta
```

```
n2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*c*cos(6*b*x + 6*a) - 6*b*c*cos(4*
b*x + 4*a) - 6*b*c*cos(2*b*x + 2*a) + 6*I*b*c*sin(6*b*x + 6*a) - 6*I*b*c*si
n(4*b*x + 4*a) - 6*I*b*c*sin(2*b*x + 2*a) + 6*b*c)*arctan2(sin(b*x + a), co
s(b*x + a) - 1) + (6*b*d*x*cos(6*b*x + 6*a) - 6*b*d*x*cos(4*b*x + 4*a) - 6*
b*d*x*cos(2*b*x + 2*a) + 6*I*b*d*x*sin(6*b*x + 6*a) - 6*I*b*d*x*sin(4*b*x +
 4*a) - 6*I*b*d*x*sin(2*b*x + 2*a) + 6*b*d*x)*arctan2(sin(b*x + a), -cos(b*
x + a) + 1) - (-12*I*b*d*x - 12*I*b*c - 4*d)*cos(5*b*x + 5*a) - (8*I*b*d*x
+ 8*I*b*c)*cos(3*b*x + 3*a) - (-12*I*b*d*x - 12*I*b*c + 4*d)*cos(b*x + a) -
 (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6*d*cos(2*b*x + 2*a) + 6*I*
d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I*d*sin(2*b*x + 2*a) + 6*d)
*dilog(-e^(I*b*x + I*a)) + (6*d*cos(6*b*x + 6*a) - 6*d*cos(4*b*x + 4*a) - 6
*d*cos(2*b*x + 2*a) + 6*I*d*sin(6*b*x + 6*a) - 6*I*d*sin(4*b*x + 4*a) - 6*I
*d*sin(2*b*x + 2*a) + 6*d)*dilog(e^(I*b*x + I*a)) - (3*I*b*d*x + 3*I*b*c +
(3*I*b*d*x + 3*I*b*c)*cos(6*b*x + 6*a) + (-3*I*b*d*x - 3*I*b*c)*cos(4*b*x +
 4*a) + (-3*I*b*d*x - 3*I*b*c)*cos(2*b*x + 2*a) - 3*(b*d*x + b*c)*sin(6*b*x
 + 6*a) + 3*(b*d*x + b*c)*sin(4*b*x + 4*a) + 3*(b*d*x + b*c)*sin(2*b*x + 2*
a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (-3*I*b*d*x
 - 3*I*b*c + (-3*I*b*d*x - 3*I*b*c)*cos(6*b*x + 6*a) + (3*I*b*d*x + 3*I*b*c
)*cos(4*b*x + 4*a) + (3*I*b*d*x + 3*I*b*c)*cos(2*b*x + 2*a) + 3*(b*d*x + b*
c)*sin(6*b*x + 6*a) - 3*(b*d*x + b*c)*sin(4*b*x + 4*a) - 3*(b*d*x + b*c)*si
n(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) -
 (-2*I*d*cos(6*b*x + 6*a) + 2*I*d*cos(4*b*x + 4*a) + 2*I*d*cos(2*b*x + 2*a)
 + 2*d*sin(6*b*x + 6*a) - 2*d*sin(4*b*x + 4*a) - 2*d*sin(2*b*x + 2*a) - 2*I
*d)*log((cos(b*x + 2*a)^2 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x +
2*a)^2 + 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - (12*b*d*x + 12*b*c - 4*I*d)*sin(5*b*x + 5*a) + 8*(b*d*x + b*c)*si
n(3*b*x + 3*a) - (12*b*d*x + 12*b*c + 4*I*d)*sin(b*x + a))/(-4*I*b^2*cos(6*
b*x + 6*a) + 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*si
n(6*b*x + 6*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \csc^3(a + bx) \sec^2(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Integral((c + d*x)*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

3.282 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]

[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 22.22, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x),x]

[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x), x]

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d*x + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 2.67, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\left(\sec^{2}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^2\,\sin\left(a+b\,x\right)^3\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(a+bx\right)\sec^{2}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x), x)
```

3.283 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A] time = 0.22, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 30.37, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\csc\left(bx+a\right)^3\sec\left(bx+a\right)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 5.20, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\left(\sec^{2}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(a+b\,x\right)}^2\,{\sin\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^2*sin(a + b*x)^3*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(a + bx \right)} \sec^{2}{\left(a + bx \right)}}{\left(c + dx\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/(c + d*x)**2, x)
```

3.284 $\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(x^m \csc^3(a+bx)\sec^2(a+bx),x\right)$$

```
[Out] CannotIntegrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

Rubi [A] time = 0.92, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
[Out] Defer[Int][x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

Rubi steps

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx = \int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Mathematica [A] time = 39.65, size = 0, normalized size = 0.00

$$\int x^m \csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
[Out] Integrate[x^m*Csc[a + b*x]^3*Sec[a + b*x]^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(x^m \csc(bx+a)^3 \sec(bx+a)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
[Out] integral(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [A] time = 0.11, size = 0, normalized size = 0.00

$$\int x^m \left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] int(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
[Out] integrate(x^m*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{x^m}{\cos(a+b\,x)^2 \sin(a+b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x^m/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Timed out
```

3.285 $\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=387

$$\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^4}-\frac{3i\text{Li}_2\left(e^{i(a+bx)}\right)}{b^4}+\frac{6\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{6\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{9i\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{9i\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}-\frac{6ix\text{Li}_2\left(-i\right.}{b^3}$$

```
[Out] 9*I*polylog(4,exp(I*(b*x+a)))/b^4-6*x*arctanh(exp(I*(b*x+a)))/b^3-3*x^3*arc
tanh(exp(I*(b*x+a)))/b-3/2*x^2*csc(b*x+a)/b^2-9*I*polylog(4,-exp(I*(b*x+a))
)/b^4-6*I*x*polylog(2,-I*exp(I*(b*x+a)))/b^3+9/2*I*x^2*polylog(2,-exp(I*(b*
x+a)))/b^2+6*I*x^2*arctan(exp(I*(b*x+a)))/b^2+3*I*polylog(2,-exp(I*(b*x+a))
)/b^4+6*I*x*polylog(2,I*exp(I*(b*x+a)))/b^3-9*x*polylog(3,-exp(I*(b*x+a)))/
b^3+6*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*polylog(3,I*exp(I*(b*x+a)))/b^4+9*
x*polylog(3,exp(I*(b*x+a)))/b^3-9/2*I*x^2*polylog(2,exp(I*(b*x+a)))/b^2-3*I
*polylog(2,exp(I*(b*x+a)))/b^4+3/2*x^3*sec(b*x+a)/b-1/2*x^3*csc(b*x+a)^2*se
c(b*x+a)/b
```

Rubi [A] time = 0.96, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 40, number of rules used = 18, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 6609, 2282, 6589, 6742, 4181, 2621, 2279, 2391}

$$\frac{9ix^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{9ix^2\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{6ix\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{6ix\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{9x\text{P}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((6*I)*x^2*ArcTan[E^(I*(a + b*x))])/b^2 - (6*x*ArcTanh[E^(I*(a + b*x))])/b^
3 - (3*x^3*ArcTanh[E^(I*(a + b*x))])/b - (3*x^2*Csc[a + b*x])/(2*b^2) + ((3
*I)*PolyLog[2, -E^(I*(a + b*x))])/b^4 + (((9*I)/2)*x^2*PolyLog[2, -E^(I*(a
+ b*x))])/b^2 - ((6*I)*x*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((6*I)*x*P
olyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*PolyLog[2, E^(I*(a + b*x))])/b^4
 - (((9*I)/2)*x^2*PolyLog[2, E^(I*(a + b*x))])/b^2 - (9*x*PolyLog[3, -E^(I*
(a + b*x))])/b^3 + (6*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 - (6*PolyLog[3,
 I*E^(I*(a + b*x))])/b^4 + (9*x*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((9*I)*P
olyLog[4, -E^(I*(a + b*x))])/b^4 + ((9*I)*PolyLog[4, E^(I*(a + b*x))])/b^4
+ (3*x^3*Sec[a + b*x])/(2*b) - (x^3*Csc[a + b*x]^2*Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
```

```
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
```

```
x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^3 \csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$= -\frac{3x^3\tanh^{-1}(\cos(a+bx))}{2b} + \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3\int bx^3\csc(a+bx)\,dx}{2b} + \frac{3}{}$$

$$= \frac{3x^3\sec(a+bx)}{2b} - \frac{x^3\csc^2(a+bx)\sec(a+bx)}{2b} + \frac{3}{2}\int x^3\csc(a+bx)\,dx + \frac{3\int}{}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2}{}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2}{}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3x^2\tanh^{-1}(\sin(a+bx))}{2b^2} - \frac{3x^2}{}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

$$= \frac{9ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

$$= \frac{6ix^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3x^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3x^2\csc(}{2b}$$

Mathematica [A] time = 7.21, size = 672, normalized size = 1.74

$$\frac{3x^2\csc\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\csc\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2}-\frac{3x^2\sec\left(\frac{a}{2}\right)\sin\left(\frac{bx}{2}\right)\sec\left(\frac{a}{2}+\frac{bx}{2}\right)}{4b^2}+\frac{x^2\csc(a)\sec(a)(2bx\sin(a)-3\cos(a))}{2b^2}+\frac{6(}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^3*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -1/8*(x^3*Csc[a/2 + (b*x)/2]^2)/b + (6*(I*b^2*x^2*ArcTan[Cos[a + b*x] + I*S
in[a + b*x]] + I*b*x*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - I*b*x*Poly
Log[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] - PolyLog[3, I*Cos[a + b*x] - Sin[
a + b*x]] + PolyLog[3, (-I)*Cos[a + b*x] + Sin[a + b*x]]))/b^4 + (3*(2*b*x*
Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] + b^3*x^3*Log[1 - Cos[a + b*x] - I*S
in[a + b*x]] - 2*b*x*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] - b^3*x^3*Log[1
 + Cos[a + b*x] + I*Sin[a + b*x]] + I*(2 + 3*b^2*x^2)*PolyLog[2, -Cos[a + b
*x] - I*Sin[a + b*x]] - I*(2 + 3*b^2*x^2)*PolyLog[2, Cos[a + b*x] + I*Sin[a
 + b*x]] - 6*b*x*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] + 6*b*x*PolyLog
[3, Cos[a + b*x] + I*Sin[a + b*x]] - (6*I)*PolyLog[4, -Cos[a + b*x] - I*Sin
[a + b*x]] + (6*I)*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]]))/(2*b^4) + (x
^3*Sec[a/2 + (b*x)/2]^2)/(8*b) + (x^2*Csc[a]*Sec[a]*(-3*Cos[a] + 2*b*x*Sin[
a]))/(2*b^2) + (3*x^2*Csc[a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) - (
3*x^2*Sec[a/2]*Sec[a/2 + (b*x)/2]*Sin[(b*x)/2])/(4*b^2) + (x^3*Sin[(b*x)/2]
)/(b*(Cos[a/2] - Sin[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^
3*Sin[(b*x)/2])/(b*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b
*x)/2]))
```

fricas [C] time = 0.62, size = 1735, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^3*x^3*cos(b*x + a)^2 - 4*b^3*x^3 + 6*b^2*x^2*cos(b*x + a)*sin(b*x
+ a) + ((-9*I*b^2*x^2 - 6*I)*cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*cos(b*x +
 a))*dilog(cos(b*x + a) + I*sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*cos(b*x +
a)^3 + (-9*I*b^2*x^2 - 6*I)*cos(b*x + a))*dilog(cos(b*x + a) - I*sin(b*x +
a)) + (12*I*b*x*cos(b*x + a)^3 - 12*I*b*x*cos(b*x + a))*dilog(I*cos(b*x + a
) + sin(b*x + a)) + (12*I*b*x*cos(b*x + a)^3 - 12*I*b*x*cos(b*x + a))*dilog
(I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*x*cos(b*x + a)^3 + 12*I*b*x*cos(
b*x + a))*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-12*I*b*x*cos(b*x + a)^3
 + 12*I*b*x*cos(b*x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + ((-9*I*b^
2*x^2 - 6*I)*cos(b*x + a)^3 + (9*I*b^2*x^2 + 6*I)*cos(b*x + a))*dilog(-cos(
```

```
b*x + a) + I*sin(b*x + a)) + ((9*I*b^2*x^2 + 6*I)*cos(b*x + a)^3 + (-9*I*b^
2*x^2 - 6*I)*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*((b^3*
x^3 + 2*b*x)*cos(b*x + a)^3 - (b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x +
 a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(c
os(b*x + a) + I*sin(b*x + a) + I) - 3*((b^3*x^3 + 2*b*x)*cos(b*x + a)^3 - (
b^3*x^3 + 2*b*x)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 6*(
a^2*cos(b*x + a)^3 - a^2*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) +
I) - 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(
I*cos(b*x + a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b
^2*x^2 - a^2)*cos(b*x + a))*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*((b^
2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-I*cos(b*x
+ a) + sin(b*x + a) + 1) + 6*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a
^2)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 3*((a^3 + 2*a)*
cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*si
n(b*x + a) + 1/2) - 3*((a^3 + 2*a)*cos(b*x + a)^3 - (a^3 + 2*a)*cos(b*x + a
))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^3*x^3 + a^3 +
2*b*x + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*l
og(-cos(b*x + a) + I*sin(b*x + a) + 1) - 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*
x + a))*log(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*((b^3*x^3 + a^3 + 2*b*x
 + 2*a)*cos(b*x + a)^3 - (b^3*x^3 + a^3 + 2*b*x + 2*a)*cos(b*x + a))*log(-c
os(b*x + a) - I*sin(b*x + a) + 1) + 6*(a^2*cos(b*x + a)^3 - a^2*cos(b*x + a
))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + (18*I*cos(b*x + a)^3 - 18*I*co
s(b*x + a))*polylog(4, cos(b*x + a) + I*sin(b*x + a)) + (-18*I*cos(b*x + a)
^3 + 18*I*cos(b*x + a))*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (18*I*c
os(b*x + a)^3 - 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) + I*sin(b*x + a
)) + (-18*I*cos(b*x + a)^3 + 18*I*cos(b*x + a))*polylog(4, -cos(b*x + a) -
I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, cos
(b*x + a) + I*sin(b*x + a)) + 18*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*po
lylog(3, cos(b*x + a) - I*sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(b*x + a)
)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 - cos(b*x
+ a))*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(cos(b*x + a)^3 - cos(
b*x + a))*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*(cos(b*x + a)^3 -
 cos(b*x + a))*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 18*(b*x*cos(b*x
 + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 18
*(b*x*cos(b*x + a)^3 - b*x*cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b
*x + a)))/(b^4*cos(b*x + a)^3 - b^4*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^3*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [F] time = 1.62, size = 0, normalized size = 0.00

$$\int x^3 \left(\csc^3 (bx + a)\right) \left(\sec^2 (bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
[Out] int(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

maxima [B] time = 1.24, size = 3989, normalized size = 10.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*(a^3*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log
(cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) - 4*((12*(b*x + a)^2 - 24*(b*
x + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(6*b*x + 6*a)
 - 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) - 12*((b*x + a)^
2 - 2*(b*x + a)*a + a^2)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^2 + 24*I*(b*x
+ a)*a - 12*I*a^2)*sin(6*b*x + 6*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*a
+ 12*I*a^2)*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2 - 24*I*(b*x + a)*a + 12*I*
a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (12*(b*x +
 a)^2 - 24*(b*x + a)*a + 12*a^2 + 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*co
s(6*b*x + 6*a) - 12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(4*b*x + 4*a) -
12*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*cos(2*b*x + 2*a) - (-12*I*(b*x + a)^
2 + 24*I*(b*x + a)*a - 12*I*a^2)*sin(6*b*x + 6*a) - (12*I*(b*x + a)^2 - 24*
I*(b*x + a)*a + 12*I*a^2)*sin(4*b*x + 4*a) - (12*I*(b*x + a)^2 - 24*I*(b*x
+ a)*a + 12*I*a^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) +
1) - (6*(b*x + a)^3 - 18*(b*x + a)^2*a + 6*(3*a^2 + 2)*(b*x + a) + 6*((b*x
+ a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(6*b*x + 6*a) -
6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos(4*b*x +
 4*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*cos
(2*b*x + 2*a) + (6*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + (18*I*a^2 + 12*I)*(
b*x + a) - 12*I*a)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3 + 18*I*(b*x + a)^2*
a + (-18*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(4*b*x + 4*a) + (-6*I*(b*x +
a)^3 + 18*I*(b*x + a)^2*a + (-18*I*a^2 - 12*I)*(b*x + a) + 12*I*a)*sin(2*b*
x + 2*a) - 12*a)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (12*a*cos(6*b*x
+ 6*a) - 12*a*cos(4*b*x + 4*a) - 12*a*cos(2*b*x + 2*a) + 12*I*a*sin(6*b*x +
 6*a) - 12*I*a*sin(4*b*x + 4*a) - 12*I*a*sin(2*b*x + 2*a) + 12*a)*arctan2(s
in(b*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^3 - 18*(b*x + a)^2*a + 6*(3*a
```

```
^2 + 2)*(b*x + a) + 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x + a
))*cos(6*b*x + 6*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x +
 a))*cos(4*b*x + 4*a) - 6*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(b*x
 + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3 - 18*I*(b*x + a)^2*a + (18*I*a^2
 + 12*I)*(b*x + a))*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^3 + 18*I*(b*x + a)^2
*a + (-18*I*a^2 - 12*I)*(b*x + a))*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^3 + 1
8*I*(b*x + a)^2*a + (-18*I*a^2 - 12*I)*(b*x + a))*sin(2*b*x + 2*a))*arctan2
(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^3 - 12*(b*x + a)^2*(3*I
*a - 1) + (36*I*a^2 - 24*a)*(b*x + a) + 12*a^2)*cos(5*b*x + 5*a) - (-8*I*(b
*x + a)^3 + 24*I*(b*x + a)^2*a - 24*I*(b*x + a)*a^2)*cos(3*b*x + 3*a) - (12
*I*(b*x + a)^3 - 12*(b*x + a)^2*(3*I*a + 1) + (36*I*a^2 + 24*a)*(b*x + a) -
 12*a^2)*cos(b*x + a) + (24*b*x*cos(6*b*x + 6*a) - 24*b*x*cos(4*b*x + 4*a)
- 24*b*x*cos(2*b*x + 2*a) + 24*I*b*x*sin(6*b*x + 6*a) - 24*I*b*x*sin(4*b*x
+ 4*a) - 24*I*b*x*sin(2*b*x + 2*a) + 24*b*x)*dilog(I*e^(I*b*x + I*a)) - (24
*b*x*cos(6*b*x + 6*a) - 24*b*x*cos(4*b*x + 4*a) - 24*b*x*cos(2*b*x + 2*a) +
 24*I*b*x*sin(6*b*x + 6*a) - 24*I*b*x*sin(4*b*x + 4*a) - 24*I*b*x*sin(2*b*x
 + 2*a) + 24*b*x)*dilog(-I*e^(I*b*x + I*a)) + (18*(b*x + a)^2 - 36*(b*x + a
)*a + 18*a^2 + 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*
a) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - 6*(3*
(b*x + a)^2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(2*b*x + 2*a) - (-18*I*(b*x + a
)^2 + 36*I*(b*x + a)*a - 18*I*a^2 - 12*I)*sin(6*b*x + 6*a) - (18*I*(b*x + a
)^2 - 36*I*(b*x + a)*a + 18*I*a^2 + 12*I)*sin(4*b*x + 4*a) - (18*I*(b*x + a
)^2 - 36*I*(b*x + a)*a + 18*I*a^2 + 12*I)*sin(2*b*x + 2*a) + 12)*dilog(-e^(
I*b*x + I*a)) - (18*(b*x + a)^2 - 36*(b*x + a)*a + 18*a^2 + 6*(3*(b*x + a)^
2 - 6*(b*x + a)*a + 3*a^2 + 2)*cos(6*b*x + 6*a) - 6*(3*(b*x + a)^2 - 6*(b*x
 + a)*a + 3*a^2 + 2)*cos(4*b*x + 4*a) - 6*(3*(b*x + a)^2 - 6*(b*x + a)*a +
3*a^2 + 2)*cos(2*b*x + 2*a) + (18*I*(b*x + a)^2 - 36*I*(b*x + a)*a + 18*I*a
^2 + 12*I)*sin(6*b*x + 6*a) + (-18*I*(b*x + a)^2 + 36*I*(b*x + a)*a - 18*I*
a^2 - 12*I)*sin(4*b*x + 4*a) + (-18*I*(b*x + a)^2 + 36*I*(b*x + a)*a - 18*I
*a^2 - 12*I)*sin(2*b*x + 2*a) + 12)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*x + a
)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + (-3*I*(b*x + a)^3 +
9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + 6*I*a)*cos(6*b*x + 6*a) +
(3*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*c
os(4*b*x + 4*a) + (3*I*(b*x + a)^3 - 9*I*(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b
*x + a) - 6*I*a)*cos(2*b*x + 2*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a
^2 + 2)*(b*x + a) - 2*a)*sin(6*b*x + 6*a) - 3*((b*x + a)^3 - 3*(b*x + a)^2*
a + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(4*b*x + 4*a) - 3*((b*x + a)^3 - 3*(b*x
 + a)^2*a + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) + 6*I*a)*log(cos(
b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b*x + a)^3 - 9*I*
(b*x + a)^2*a + (9*I*a^2 + 6*I)*(b*x + a) + (3*I*(b*x + a)^3 - 9*I*(b*x + a
)^2*a + (9*I*a^2 + 6*I)*(b*x + a) - 6*I*a)*cos(6*b*x + 6*a) + (-3*I*(b*x +
a)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) + 6*I*a)*cos(4*b*x +
4*a) + (-3*I*(b*x + a)^3 + 9*I*(b*x + a)^2*a + (-9*I*a^2 - 6*I)*(b*x + a) +
 6*I*a)*cos(2*b*x + 2*a) - 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^2 + 2)*(
b*x + a) - 2*a)*sin(6*b*x + 6*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a + (3*a^
```

```
2 + 2)*(b*x + a) - 2*a)*sin(4*b*x + 4*a) + 3*((b*x + a)^3 - 3*(b*x + a)^2*a
 + (3*a^2 + 2)*(b*x + a) - 2*a)*sin(2*b*x + 2*a) - 6*I*a)*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-6*I*(b*x + a)^2 + 12*I*(b*x +
a)*a - 6*I*a^2 + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6*I*a^2)*cos(6*b*x
+ 6*a) + (6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2)*cos(4*b*x + 4*a) +
(6*I*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2)*cos(2*b*x + 2*a) + 6*((b*x +
 a)^2 - 2*(b*x + a)*a + a^2)*sin(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a
)*a + a^2)*sin(4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (6*I
*(b*x + a)^2 - 12*I*(b*x + a)*a + 6*I*a^2 + (6*I*(b*x + a)^2 - 12*I*(b*x +
a)*a + 6*I*a^2)*cos(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6
*I*a^2)*cos(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a - 6*I*a^2)*
cos(2*b*x + 2*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(6*b*x + 6*a) +
 6*((b*x + a)^2 - 2*(b*x + a)*a + a^2)*sin(4*b*x + 4*a) + 6*((b*x + a)^2 -
2*(b*x + a)*a + a^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2
- 2*sin(b*x + a) + 1) - (36*cos(6*b*x + 6*a) - 36*cos(4*b*x + 4*a) - 36*cos
(2*b*x + 2*a) + 36*I*sin(6*b*x + 6*a) - 36*I*sin(4*b*x + 4*a) - 36*I*sin(2*
b*x + 2*a) + 36)*polylog(4, -e^(I*b*x + I*a)) + (36*cos(6*b*x + 6*a) - 36*c
os(4*b*x + 4*a) - 36*cos(2*b*x + 2*a) + 36*I*sin(6*b*x + 6*a) - 36*I*sin(4*
b*x + 4*a) - 36*I*sin(2*b*x + 2*a) + 36)*polylog(4, e^(I*b*x + I*a)) - (-24
*I*cos(6*b*x + 6*a) + 24*I*cos(4*b*x + 4*a) + 24*I*cos(2*b*x + 2*a) + 24*si
n(6*b*x + 6*a) - 24*sin(4*b*x + 4*a) - 24*sin(2*b*x + 2*a) - 24*I)*polylog(
3, I*e^(I*b*x + I*a)) - (24*I*cos(6*b*x + 6*a) - 24*I*cos(4*b*x + 4*a) - 24
*I*cos(2*b*x + 2*a) - 24*sin(6*b*x + 6*a) + 24*sin(4*b*x + 4*a) + 24*sin(2*
b*x + 2*a) + 24*I)*polylog(3, -I*e^(I*b*x + I*a)) - (-36*I*b*x*cos(6*b*x +
6*a) + 36*I*b*x*cos(4*b*x + 4*a) + 36*I*b*x*cos(2*b*x + 2*a) + 36*b*x*sin(6
*b*x + 6*a) - 36*b*x*sin(4*b*x + 4*a) - 36*b*x*sin(2*b*x + 2*a) - 36*I*b*x)
*polylog(3, -e^(I*b*x + I*a)) - (36*I*b*x*cos(6*b*x + 6*a) - 36*I*b*x*cos(4
*b*x + 4*a) - 36*I*b*x*cos(2*b*x + 2*a) - 36*b*x*sin(6*b*x + 6*a) + 36*b*x*
sin(4*b*x + 4*a) + 36*b*x*sin(2*b*x + 2*a) + 36*I*b*x)*polylog(3, e^(I*b*x
+ I*a)) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a + 12*I) + 12*(3*a^2 + 2*I*a)*
(b*x + a) - 12*I*a^2)*sin(5*b*x + 5*a) - 8*((b*x + a)^3 - 3*(b*x + a)^2*a +
 3*(b*x + a)*a^2)*sin(3*b*x + 3*a) + (12*(b*x + a)^3 - (b*x + a)^2*(36*a -
12*I) + 12*(3*a^2 - 2*I*a)*(b*x + a) + 12*I*a^2)*sin(b*x + a))/(-4*I*cos(6*
b*x + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*a) + 4*sin(6*b*x + 6*
a) - 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\cos(a+bx)^2 \sin(a+bx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(x^3/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Timed out
```

3.286 $\int x^2 \csc^3(a+bx)\sec^2(a+bx)\,dx$

Optimal. Leaf size=235

$$-\frac{2i\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}-\frac{3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}+\frac{3ix\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-$$

```
[Out] 4*I*x*arctan(exp(I*(b*x+a)))/b^2-3*x^2*arctanh(exp(I*(b*x+a)))/b-arctanh(co
s(b*x+a))/b^3-x*csc(b*x+a)/b^2+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-2*I*pol
ylog(2,-I*exp(I*(b*x+a)))/b^3+2*I*polylog(2,I*exp(I*(b*x+a)))/b^3-3*I*x*pol
ylog(2,exp(I*(b*x+a)))/b^2-3*polylog(3,-exp(I*(b*x+a)))/b^3+3*polylog(3,exp
(I*(b*x+a)))/b^3+3/2*x^2*sec(b*x+a)/b-1/2*x^2*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.54, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 19, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.950, Rules used = {2622, 288, 321, 207, 4420, 14, 6273, 12, 4183, 2531, 2282, 6589, 6742, 4181, 2279, 2391, 2621, 6271, 3770}

$$\frac{3ix\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3ix\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{2i\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}+\frac{2i\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-3\text{PolyI}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] ((4*I)*x*ArcTan[E^(I*(a + b*x))])/b^2 - (3*x^2*ArcTanh[E^(I*(a + b*x))])/b
- ArcTanh[Cos[a + b*x]]/b^3 - (x*Csc[a + b*x])/b^2 + ((3*I)*x*PolyLog[2, -E
^(I*(a + b*x))])/b^2 - ((2*I)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^3 + ((2*I
)*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((3*I)*x*PolyLog[2, E^(I*(a + b*x))]
)/b^2 - (3*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (3*PolyLog[3, E^(I*(a + b*x)
)])/b^3 + (3*x^2*Sec[a + b*x])/(2*b) - (x^2*Csc[a + b*x]^2*Sec[a + b*x])/(2
*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int x^2\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$= -\frac{3x^2\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}$$

$$= \frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{\int\left(-3x\sec(a+bx)+x\csc\right.}{b}$$

$$= \frac{3x^2\sec(a+bx)}{2b}-\frac{x^2\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{3}{2}\int x^2\csc(a+bx)\,dx+\int$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{x\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{x\csc}{}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{x\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{x\csc}{}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b}$$

$$= \frac{6ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b}$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b}$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b}$$

$$= \frac{4ix\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{3x^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\cos(a+bx))}{b^3}-\frac{x\csc(a}{b}$$

Mathematica [B] time = 6.62, size = 613, normalized size = 2.61

$$-\frac{2\left(i\left(\text{Li}_2\left(-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\text{Li}_2\left(e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)+\left(-a-bx+\frac{\pi}{2}\right)\left(\log\left(1-e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)-\log\left(1+e^{i\left(-a-bx+\frac{\pi}{2}\right)}\right)\right)-\left(\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] -1/8*(x^2*Csc[a/2 + (b*x)/2]^2)/b - (2*((-a + Pi/2 - b*x)*(Log[1 - E^(I*(-a
 + Pi/2 - b*x))] - Log[1 + E^(I*(-a + Pi/2 - b*x))]) - (-a + Pi/2)*Log[Tan[
(-a + Pi/2 - b*x)/2]] + I*(PolyLog[2, -E^(I*(-a + Pi/2 - b*x))] - PolyLog[2
, E^(I*(-a + Pi/2 - b*x))])))/b^3 + (2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x
]] + 3*b^2*x^2*Log[1 - Cos[a + b*x] - I*Sin[a + b*x]] - 2*Log[1 + Cos[a + b
*x] + I*Sin[a + b*x]] - 3*b^2*x^2*Log[1 + Cos[a + b*x] + I*Sin[a + b*x]] +
(6*I)*b*x*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] - (6*I)*b*x*PolyLog[2,
 Cos[a + b*x] + I*Sin[a + b*x]] - 6*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*
x]] + 6*PolyLog[3, Cos[a + b*x] + I*Sin[a + b*x]])/(2*b^3) + (x^2*Sec[a/2 +
 (b*x)/2]^2)/(8*b) + (x*Csc[a]*Sec[a]*(-Cos[a] + b*x*Sin[a]))/b^2 + (x*Csc[
a/2]*Csc[a/2 + (b*x)/2]*Sin[(b*x)/2])/(2*b^2) - (x*Sec[a/2]*Sec[a/2 + (b*x)
/2]*Sin[(b*x)/2])/(2*b^2) + (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] - Sin[a/2])*(Co
s[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) - (x^2*Sin[(b*x)/2])/(b*(Cos[a/2] +
 Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2]))
```

fricas [C] time = 0.56, size = 1229, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b^2*x^2*cos(b*x + a)^2 - 4*b^2*x^2 + 4*b*x*cos(b*x + a)*sin(b*x + a)
 + (-6*I*b*x*cos(b*x + a)^3 + 6*I*b*x*cos(b*x + a))*dilog(cos(b*x + a) + I*
sin(b*x + a)) + (6*I*b*x*cos(b*x + a)^3 - 6*I*b*x*cos(b*x + a))*dilog(cos(b
*x + a) - I*sin(b*x + a)) + (4*I*cos(b*x + a)^3 - 4*I*cos(b*x + a))*dilog(I
*cos(b*x + a) + sin(b*x + a)) + (4*I*cos(b*x + a)^3 - 4*I*cos(b*x + a))*dil
og(I*cos(b*x + a) - sin(b*x + a)) + (-4*I*cos(b*x + a)^3 + 4*I*cos(b*x + a)
)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-4*I*cos(b*x + a)^3 + 4*I*cos(b*
x + a))*dilog(-I*cos(b*x + a) - sin(b*x + a)) + (-6*I*b*x*cos(b*x + a)^3 +
6*I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*x*cos(
b*x + a)^3 - 6*I*b*x*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) -
((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*x^2 + 2)*cos(b*x + a))*log(cos(b*x
 + a) + I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(cos
(b*x + a) + I*sin(b*x + a) + I) - ((3*b^2*x^2 + 2)*cos(b*x + a)^3 - (3*b^2*
x^2 + 2)*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*
x + a)^3 - a*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + I) - 4*((b*x
 + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x + a))*log(I*co
s(b*x + a) - sin(b*x + a) + 1) - 4*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*co
s(b*x + a))*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 4*((b*x + a)*cos(b*x
+ a)^3 - (b*x + a)*cos(b*x + a))*log(-I*cos(b*x + a) - sin(b*x + a) + 1) +
((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*cos(b*x + a))*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2) + ((3*a^2 + 2)*cos(b*x + a)^3 - (3*a^2 + 2)*
cos(b*x + a))*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*((b^2*x
```

```
^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 - a^2)*cos(b*x + a))*log(-cos(b*x + a)
+ I*sin(b*x + a) + 1) + 4*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-cos(b*x
+ a) + I*sin(b*x + a) + I) + 3*((b^2*x^2 - a^2)*cos(b*x + a)^3 - (b^2*x^2 -
 a^2)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 4*(a*cos(b*x
+ a)^3 - a*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + I) + 6*(cos(b
*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(co
s(b*x + a)^3 - cos(b*x + a))*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*
(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
- 6*(cos(b*x + a)^3 - cos(b*x + a))*polylog(3, -cos(b*x + a) - I*sin(b*x +
a)))/(b^3*cos(b*x + a)^3 - b^3*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \csc(bx+a)^3 \sec(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^2*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [B] time = 0.22, size = 429, normalized size = 1.83

$$\frac{x\left(3bx\,e^{5i(bx+a)}-2bx\,e^{3i(bx+a)}-2ie^{5i(bx+a)}+3bx\,e^{i(bx+a)}+2ie^{i(bx+a)}\right)}{b^2\left(e^{2i(bx+a)}-1\right)^2\left(1+e^{2i(bx+a)}\right)}-\frac{3a^2\ln\left(1-e^{i(bx+a)}\right)}{2b^3}-\frac{3ix\,\mathrm{polylog}\left(2,e^{i(bx+}\right.}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] x/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-2*I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+2*I*exp(I*(
b*x+a)))-3/2/b^3*a^2*ln(1-exp(I*(b*x+a)))-2*I/b^3*dilog(1+I*exp(I*(b*x+a)))
+3*polylog(3,exp(I*(b*x+a)))/b^3-3*polylog(3,-exp(I*(b*x+a)))/b^3+2/b^2*ln(
1+I*exp(I*(b*x+a)))*x+3*I*x*polylog(2,-exp(I*(b*x+a)))/b^2-3/2/b*ln(exp(I*(
b*x+a))+1)*x^2-3*I*x*polylog(2,exp(I*(b*x+a)))/b^2+3/2/b*ln(1-exp(I*(b*x+a)
))*x^2-2/b^2*ln(1-I*exp(I*(b*x+a)))*x+2/b^3*ln(1+I*exp(I*(b*x+a)))*a-2/b^3*
ln(1-I*exp(I*(b*x+a)))*a+1/b^3*ln(exp(I*(b*x+a))-1)-1/b^3*ln(exp(I*(b*x+a))
+1)+3/2/b^3*a^2*ln(exp(I*(b*x+a))-1)-4*I/b^3*a*arctan(exp(I*(b*x+a)))+2*I/b
^3*dilog(1-I*exp(I*(b*x+a)))
```

maxima [B] time = 0.74, size = 2219, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")

[Out] 1/4*(a^2*(2*(3*cos(b*x + a)^2 - 2)/(cos(b*x + a)^3 - cos(b*x + a)) - 3*log(
cos(b*x + a) + 1) + 3*log(cos(b*x + a) - 1)) + 4*((8*b*x*cos(6*b*x + 6*a) -
 8*b*x*cos(4*b*x + 4*a) - 8*b*x*cos(2*b*x + 2*a) + 8*I*b*x*sin(6*b*x + 6*a)
 - 8*I*b*x*sin(4*b*x + 4*a) - 8*I*b*x*sin(2*b*x + 2*a) + 8*b*x)*arctan2(cos
(b*x + a), sin(b*x + a) + 1) + (8*b*x*cos(6*b*x + 6*a) - 8*b*x*cos(4*b*x +
4*a) - 8*b*x*cos(2*b*x + 2*a) + 8*I*b*x*sin(6*b*x + 6*a) - 8*I*b*x*sin(4*b*
x + 4*a) - 8*I*b*x*sin(2*b*x + 2*a) + 8*b*x)*arctan2(cos(b*x + a), -sin(b*x
 + a) + 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 2*(3*(b*x + a)^2 - 6*(b*x +
a)*a + 2)*cos(6*b*x + 6*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(4*b*
x + 4*a) - 2*(3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*cos(2*b*x + 2*a) + (6*I*(b
*x + a)^2 - 12*I*(b*x + a)*a + 4*I)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 +
12*I*(b*x + a)*a - 4*I)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x +
a)*a - 4*I)*sin(2*b*x + 2*a) + 4)*arctan2(sin(b*x + a), cos(b*x + a) + 1) +
 (4*cos(6*b*x + 6*a) - 4*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 4*I*sin(6*
b*x + 6*a) - 4*I*sin(4*b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 4)*arctan2(sin(b
*x + a), cos(b*x + a) - 1) - (6*(b*x + a)^2 - 12*(b*x + a)*a + 6*((b*x + a)
^2 - 2*(b*x + a)*a)*cos(6*b*x + 6*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*cos(
4*b*x + 4*a) - 6*((b*x + a)^2 - 2*(b*x + a)*a)*cos(2*b*x + 2*a) + (6*I*(b*x
 + a)^2 - 12*I*(b*x + a)*a)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2 + 12*I*(b*
x + a)*a)*sin(4*b*x + 4*a) + (-6*I*(b*x + a)^2 + 12*I*(b*x + a)*a)*sin(2*b*
x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*I*(b*x + a)^2 - 8*
(b*x + a)*(3*I*a - 1) - 8*a)*cos(5*b*x + 5*a) - (-8*I*(b*x + a)^2 + 16*I*(b
*x + a)*a)*cos(3*b*x + 3*a) - (12*I*(b*x + a)^2 - 8*(b*x + a)*(3*I*a + 1) +
 8*a)*cos(b*x + a) + (8*cos(6*b*x + 6*a) - 8*cos(4*b*x + 4*a) - 8*cos(2*b*x
 + 2*a) + 8*I*sin(6*b*x + 6*a) - 8*I*sin(4*b*x + 4*a) - 8*I*sin(2*b*x + 2*a
) + 8)*dilog(I*e^(I*b*x + I*a)) - (8*cos(6*b*x + 6*a) - 8*cos(4*b*x + 4*a)
- 8*cos(2*b*x + 2*a) + 8*I*sin(6*b*x + 6*a) - 8*I*sin(4*b*x + 4*a) - 8*I*si
n(2*b*x + 2*a) + 8)*dilog(-I*e^(I*b*x + I*a)) + (12*b*x*cos(6*b*x + 6*a) -
12*b*x*cos(4*b*x + 4*a) - 12*b*x*cos(2*b*x + 2*a) + 12*I*b*x*sin(6*b*x + 6*
a) - 12*I*b*x*sin(4*b*x + 4*a) - 12*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(
-e^(I*b*x + I*a)) - (12*b*x*cos(6*b*x + 6*a) - 12*b*x*cos(4*b*x + 4*a) - 12
*b*x*cos(2*b*x + 2*a) + 12*I*b*x*sin(6*b*x + 6*a) - 12*I*b*x*sin(4*b*x + 4*
a) - 12*I*b*x*sin(2*b*x + 2*a) + 12*b*x)*dilog(e^(I*b*x + I*a)) - (-3*I*(b*
x + a)^2 + 6*I*(b*x + a)*a + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos
(6*b*x + 6*a) + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(4*b*x + 4*a)
+ (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos(2*b*x + 2*a) + (3*(b*x + a)
^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a +
 2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*a)
 - 2*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (3*I*(b
*x + a)^2 - 6*I*(b*x + a)*a + (3*I*(b*x + a)^2 - 6*I*(b*x + a)*a + 2*I)*cos
(6*b*x + 6*a) + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(4*b*x + 4*a)
 + (-3*I*(b*x + a)^2 + 6*I*(b*x + a)*a - 2*I)*cos(2*b*x + 2*a) - (3*(b*x +
a)^2 - 6*(b*x + a)*a + 2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a
```

```
 + 2)*sin(4*b*x + 4*a) + (3*(b*x + a)^2 - 6*(b*x + a)*a + 2)*sin(2*b*x + 2*
a) + 2*I)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-4*I
*b*x*cos(6*b*x + 6*a) + 4*I*b*x*cos(4*b*x + 4*a) + 4*I*b*x*cos(2*b*x + 2*a)
 + 4*b*x*sin(6*b*x + 6*a) - 4*b*x*sin(4*b*x + 4*a) - 4*b*x*sin(2*b*x + 2*a)
 - 4*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (4*
I*b*x*cos(6*b*x + 6*a) - 4*I*b*x*cos(4*b*x + 4*a) - 4*I*b*x*cos(2*b*x + 2*a
) - 4*b*x*sin(6*b*x + 6*a) + 4*b*x*sin(4*b*x + 4*a) + 4*b*x*sin(2*b*x + 2*a
) + 4*I*b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) - (-
12*I*cos(6*b*x + 6*a) + 12*I*cos(4*b*x + 4*a) + 12*I*cos(2*b*x + 2*a) + 12*
sin(6*b*x + 6*a) - 12*sin(4*b*x + 4*a) - 12*sin(2*b*x + 2*a) - 12*I)*polylo
g(3, -e^(I*b*x + I*a)) - (12*I*cos(6*b*x + 6*a) - 12*I*cos(4*b*x + 4*a) - 1
2*I*cos(2*b*x + 2*a) - 12*sin(6*b*x + 6*a) + 12*sin(4*b*x + 4*a) + 12*sin(2
*b*x + 2*a) + 12*I)*polylog(3, e^(I*b*x + I*a)) + (12*(b*x + a)^2 - (b*x +
a)*(24*a + 8*I) + 8*I*a)*sin(5*b*x + 5*a) - 8*((b*x + a)^2 - 2*(b*x + a)*a)
*sin(3*b*x + 3*a) + (12*(b*x + a)^2 - (b*x + a)*(24*a - 8*I) - 8*I*a)*sin(b
*x + a))/(-4*I*cos(6*b*x + 6*a) + 4*I*cos(4*b*x + 4*a) + 4*I*cos(2*b*x + 2*
a) + 4*sin(6*b*x + 6*a) - 4*sin(4*b*x + 4*a) - 4*sin(2*b*x + 2*a) - 4*I))/b
^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\cos(a+b\,x)^2\sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(x^2/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\csc^3(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*csc(b*x+a)**3*sec(b*x+a)**2,x)
```

```
[Out] Integral(x**2*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

3.287 $\int x \csc^3(a+bx) \sec^2(a+bx) \, dx$

Optimal. Leaf size=126

$$\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{Li}_2\left(e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{x\csc^2}{}$$

```
[Out] -3*x*arctanh(exp(I*(b*x+a)))/b-arctanh(sin(b*x+a))/b^2-1/2*csc(b*x+a)/b^2+3
/2*I*polylog(2,-exp(I*(b*x+a)))/b^2-3/2*I*polylog(2,exp(I*(b*x+a)))/b^2+3/2
*x*sec(b*x+a)/b-1/2*x*csc(b*x+a)^2*sec(b*x+a)/b
```

Rubi [A] time = 0.17, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {2622, 288, 321, 207, 4420, 6271, 12, 4183, 2279, 2391, 3770, 2621}

$$\frac{3i\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{2b^2}-\frac{3i\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{2b^2}-\frac{\csc(a+bx)}{2b^2}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{3x\tanh^{-1}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (-3*x*ArcTanh[E^(I*(a + b*x))])/b - ArcTanh[Sin[a + b*x]]/b^2 - Csc[a + b*x
]/(2*b^2) + (((3*I)/2)*PolyLog[2, -E^(I*(a + b*x))])/b^2 - (((3*I)/2)*PolyL
og[2, E^(I*(a + b*x))])/b^2 + (3*x*Sec[a + b*x])/(2*b) - (x*Csc[a + b*x]^2*
Sec[a + b*x])/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int x\csc^3(a+bx)\sec^2(a+bx)\,dx = -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}-\int$$

$$= -\frac{3x\tanh^{-1}(\cos(a+bx))}{2b}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}+\frac{\int}{}$$

$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(a+bx)}{2b}-\frac{\text{Sub}}{}$$

$$= -\frac{3\tanh^{-1}(\sin(a+bx))}{2b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}-\frac{x\csc^2(a+bx)\sec(}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3x\sec(a+bx)}{2b}$$

$$= -\frac{3x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}-\frac{\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{\csc(a+bx)}{2b^2}+\frac{3i\text{Li}_2\left(-e^{i(a+bx)}\right)}{2b^2}$$

Mathematica [B] time = 2.41, size = 282, normalized size = 2.24

$$12i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+12(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)-2\tan\left(\tfrac{1}{2}(a+bx)\right)-2\cot$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[a + b*x]^3*Sec[a + b*x]^2,x]
```

```
[Out] (8*b*x - 2*Cot[(a + b*x)/2] - b*x*Csc[(a + b*x)/2]^2 + 12*(a + b*x)*(Log[1
- E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + 8*Log[Cos[(a + b*x)/2] - S
in[(a + b*x)/2]] - 8*Log[Cos[(a + b*x)/2] + Sin[(a + b*x)/2]] - 12*a*Log[Ta
n[(a + b*x)/2]] + (12*I)*(PolyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a
 + b*x))]) + b*x*Sec[(a + b*x)/2]^2 + (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*
x)/2] - Sin[(a + b*x)/2]) - (8*b*x*Sin[(a + b*x)/2])/(Cos[(a + b*x)/2] + Si
n[(a + b*x)/2]) - 2*Tan[(a + b*x)/2])/(8*b^2)
```

fricas [B] time = 0.51, size = 527, normalized size = 4.18

$$\frac{6\,bx\cos\left(bx+a\right)^{2}-4\,bx+\left(-3i\,\cos\left(bx+a\right)^{3}+3i\,\cos\left(bx+a\right)\right)\mathrm{Li}_2\left(\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+\left(3i\,\cos\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(6*b*x*cos(b*x + a)^2 - 4*b*x + (-3*I*cos(b*x + a)^3 + 3*I*cos(b*x + a)
)*dilog(cos(b*x + a) + I*sin(b*x + a)) + (3*I*cos(b*x + a)^3 - 3*I*cos(b*x
+ a))*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*cos(b*x + a)^3 + 3*I*cos
(b*x + a))*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (3*I*cos(b*x + a)^3 - 3*
I*cos(b*x + a))*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*x*cos(b*x + a)
^3 - b*x*cos(b*x + a))*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*x*cos(
b*x + a)^3 - b*x*cos(b*x + a))*log(cos(b*x + a) - I*sin(b*x + a) + 1) - 3*(
a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x +
a) + 1/2) - 3*(a*cos(b*x + a)^3 - a*cos(b*x + a))*log(-1/2*cos(b*x + a) - 1
/2*I*sin(b*x + a) + 1/2) + 3*((b*x + a)*cos(b*x + a)^3 - (b*x + a)*cos(b*x
+ a))*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*((b*x + a)*cos(b*x + a)^3
 - (b*x + a)*cos(b*x + a))*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 2*(cos
(b*x + a)^3 - cos(b*x + a))*log(sin(b*x + a) + 1) + 2*(cos(b*x + a)^3 - cos
(b*x + a))*log(-sin(b*x + a) + 1) + 2*cos(b*x + a)*sin(b*x + a))/(b^2*cos(b
*x + a)^3 - b^2*cos(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\csc\left(bx+a\right)^{3}\sec\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x*csc(b*x + a)^3*sec(b*x + a)^2, x)
```

maple [A] time = 0.14, size = 182, normalized size = 1.44

$$\frac{3bx\,e^{5i(bx+a)} - 2bx\,e^{3i(bx+a)} - i e^{5i(bx+a)} + 3bx\,e^{i(bx+a)} + i e^{i(bx+a)}}{b^2\left(e^{2i(bx+a)} - 1\right)^2\left(1 + e^{2i(bx+a)}\right)} - \frac{3a\ln\left(e^{i(bx+a)} - 1\right)}{2b^2} + \frac{2i\arctan\left(e^{i(bx+a)}\right)}{b^2} + \frac{3i\,\mathrm{dilog}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(b*x+a)^3*sec(b*x+a)^2,x)
```

```
[Out] 1/b^2/(exp(2*I*(b*x+a))-1)^2/(1+exp(2*I*(b*x+a)))*(3*b*x*exp(5*I*(b*x+a))-2
*b*x*exp(3*I*(b*x+a))-I*exp(5*I*(b*x+a))+3*b*x*exp(I*(b*x+a))+I*exp(I*(b*x+
a)))-3/2/b^2*a*ln(exp(I*(b*x+a))-1)+2*I/b^2*arctan(exp(I*(b*x+a)))+3/2*I/b^
2*dilog(exp(I*(b*x+a)))+3/2*I/b^2*dilog(exp(I*(b*x+a))+1)-3/2/b*ln(exp(I*(b
*x+a))+1)*x
```

maxima [B] time = 0.69, size = 1184, normalized size = 9.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)^3*sec(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] (8*I*b*x*cos(3*b*x + 3*a) - 8*b*x*sin(3*b*x + 3*a) - (4*cos(6*b*x + 6*a) -
4*cos(4*b*x + 4*a) - 4*cos(2*b*x + 2*a) + 4*I*sin(6*b*x + 6*a) - 4*I*sin(4*
b*x + 4*a) - 4*I*sin(2*b*x + 2*a) + 4)*arctan2(2*(cos(b*x + 2*a)*cos(a) + s
in(b*x + 2*a)*sin(a))/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a
) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2), (cos(b*x + 2*a)
^2 - cos(a)^2 + sin(b*x + 2*a)^2 - sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 +
 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin
(a)^2)) - (6*b*x*cos(6*b*x + 6*a) - 6*b*x*cos(4*b*x + 4*a) - 6*b*x*cos(2*b*
x + 2*a) + 6*I*b*x*sin(6*b*x + 6*a) - 6*I*b*x*sin(4*b*x + 4*a) - 6*I*b*x*si
n(2*b*x + 2*a) + 6*b*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (6*b*x*co
s(6*b*x + 6*a) - 6*b*x*cos(4*b*x + 4*a) - 6*b*x*cos(2*b*x + 2*a) + 6*I*b*x*
sin(6*b*x + 6*a) - 6*I*b*x*sin(4*b*x + 4*a) - 6*I*b*x*sin(2*b*x + 2*a) + 6*
b*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 4*(3*I*b*x + 1)*cos(5*b*x +
 5*a) - 4*(3*I*b*x - 1)*cos(b*x + a) + (6*cos(6*b*x + 6*a) - 6*cos(4*b*x +
4*a) - 6*cos(2*b*x + 2*a) + 6*I*sin(6*b*x + 6*a) - 6*I*sin(4*b*x + 4*a) - 6
*I*sin(2*b*x + 2*a) + 6)*dilog(-e^(I*b*x + I*a)) - (6*cos(6*b*x + 6*a) - 6*
cos(4*b*x + 4*a) - 6*cos(2*b*x + 2*a) + 6*I*sin(6*b*x + 6*a) - 6*I*sin(4*b*
x + 4*a) - 6*I*sin(2*b*x + 2*a) + 6)*dilog(e^(I*b*x + I*a)) + (3*I*b*x*cos(
6*b*x + 6*a) - 3*I*b*x*cos(4*b*x + 4*a) - 3*I*b*x*cos(2*b*x + 2*a) - 3*b*x*
sin(6*b*x + 6*a) + 3*b*x*sin(4*b*x + 4*a) + 3*b*x*sin(2*b*x + 2*a) + 3*I*b*
x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (-3*I*b*x*co
```

```
s(6*b*x + 6*a) + 3*I*b*x*cos(4*b*x + 4*a) + 3*I*b*x*cos(2*b*x + 2*a) + 3*b*
x*sin(6*b*x + 6*a) - 3*b*x*sin(4*b*x + 4*a) - 3*b*x*sin(2*b*x + 2*a) - 3*I*
b*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-2*I*cos(
6*b*x + 6*a) + 2*I*cos(4*b*x + 4*a) + 2*I*cos(2*b*x + 2*a) + 2*sin(6*b*x +
6*a) - 2*sin(4*b*x + 4*a) - 2*sin(2*b*x + 2*a) - 2*I)*log((cos(b*x + 2*a)^2
 + cos(a)^2 - 2*cos(a)*sin(b*x + 2*a) + sin(b*x + 2*a)^2 + 2*cos(b*x + 2*a)
*sin(a) + sin(a)^2)/(cos(b*x + 2*a)^2 + cos(a)^2 + 2*cos(a)*sin(b*x + 2*a)
+ sin(b*x + 2*a)^2 - 2*cos(b*x + 2*a)*sin(a) + sin(a)^2)) + (12*b*x - 4*I)*
sin(5*b*x + 5*a) + (12*b*x + 4*I)*sin(b*x + a))/(-4*I*b^2*cos(6*b*x + 6*a)
+ 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(6*b*x + 6
*a) - 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2*b*x + 2*a) - 4*I*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\cos \left(a+b\,x\right)^2\,\sin \left(a+b\,x\right)^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(cos(a + b*x)^2*sin(a + b*x)^3),x)
[Out] int(x/(cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \csc^{3}{\left (a + bx \right)} \sec^{2}{\left (a + bx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(b*x+a)**3*sec(b*x+a)**2,x)
[Out] Integral(x*csc(a + b*x)**3*sec(a + b*x)**2, x)
```

3.288 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

Rubi [A] time = 0.50, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx = \int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Mathematica [A] time = 47.01, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{x},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x, x)
```

maple [A] time = 0.78, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right)}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x \cos(a+b\,x)^2 \sin(a+b\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(1/(x*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/x, x)
```

3.289 $\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

Rubi [A] time = 0.51, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx=\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Mathematica [A] time = 19.78, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^2)/x^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^2}{x^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^3 \sec(bx+a)^2}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^2/x^2, x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^3(bx+a)\right)\left(\sec^2(bx+a)\right)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

```
[Out] int(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^2/x^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{x^2 \cos(a+b\,x)^2 \sin(a+b\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3),x)
```

```
[Out] int(1/(x^2*cos(a + b*x)^2*sin(a + b*x)^3), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^2(a+bx)}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**2/x**2,x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**2/x**2, x)
```

3.290 $\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=25

$$\mathrm{Int}\left(\tan(a+bx)\sec^2(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

Rubi [A] time = 0.15, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Defer[Int][(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx = \int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Mathematica [A] time = 5.41, size = 0, normalized size = 0.00

$$\int (c+dx)^m \sec^2(a+bx)\tan(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] Integrate[(c + d*x)^m*Sec[a + b*x]^2*Tan[a + b*x], x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \sec(bx+a)^2 \tan(bx+a),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\sec^2(bx+a)\right) \tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
[Out] int((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \sec(bx+a)^2 \tan(bx+a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)^2*tan(b*x + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)\ (c+d\,x)^m}{\cos(a+b\,x)^2}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2,x)
[Out] int((tan(a + b*x)*(c + d*x)^m)/cos(a + b*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan(a+bx) \sec^2(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)*sec(a + b*x)**2, x)
```

3.291 $\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=139

$$-\frac{3d^4\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^5}+\frac{6id^3(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\tan(a+bx)}{b^2}+\frac{(c$$

```
[Out] 2*I*d*(d*x+c)^3/b^2-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*x
+c)*polylog(2,-exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b^5
+1/2*(d*x+c)^4*sec(b*x+a)^2/b-2*d*(d*x+c)^3*tan(b*x+a)/b^2
```

Rubi [A] time = 0.26, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4409, 4184, 3719, 2190, 2531, 2282, 6589}

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3d^4\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^5}-\frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{2d(c+dx)^3\text{ta}}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))]
)/b^3 + ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 - (3*d^4
*PolyLog[3, -E^((2*I)*(a + b*x))])/b^5 + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b)
 - (2*d*(c + d*x)^3*Tan[a + b*x])/b^2
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^4 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3\sec^2(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} + \frac{\left(6d^2\right)\int (c+dx)^2}{b^2}$$
$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4\sec^2(a+bx)}{2b} - \frac{2d(c+dx)^3\tan(a+bx)}{b^2} - \frac{(1}{}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^4\sec^2(a+b}{2b}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\text{Li}_2\left(-e^2\right.}{b^4}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\text{Li}_2\left(-e^2\right.}{b^4}$$
$$= \frac{2id(c+dx)^3}{b^2} - \frac{6d^2(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{6id^3(c+dx)\text{Li}_2\left(-e^2\right.}{b^4}$$

Mathematica [B] time = 6.56, size = 418, normalized size = 3.01

$$-\frac{6c^2d^2\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^3\left(\sin^2(a)+\cos^2(a)\right)} - \frac{2\sec(a)\sec(a+bx)\left(c^3d\sin(bx)+3c^2d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((-1/2*I)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)
*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
 (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^5*E^
(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log
[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^
2)) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi
 - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]]
)*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[C
ot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan
[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Si
n[a]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] +
 3*c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2
```

fricas [C] time = 0.57, size = 888, normalized size = 6.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 12*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 1
2*d^4*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*d^4*cos
(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 12*d^4*cos(b*x + a
)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (-12*I*b*d^4*x - 12*I*b*c*
d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (12*I*b*d^4*x +
12*I*b*c*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (12*I*b
*d^4*x + 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))
 + (-12*I*b*d^4*x - 12*I*b*c*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(co
s(b*x + a) + I*sin(b*x + a) + I) - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) - 6*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b
^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*cos(b*x + a)^2*log(-I*cos(b*x + a) + si
n(b*x + a) + 1) - 6*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + 2*a*b*c*d^3 - a^2*d^4)*c
os(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - 6*(b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)
 - 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + 3*b^3*c^2*d^2*x
 + b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)^2 \tan(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)^2*tan(b*x + a), x)
```

maple [B] time = 0.11, size = 489, normalized size = 3.52

$$\frac{2b\,d^4x^4\mathrm{e}^{2i(bx+a)}+8bc\,d^3x^3\mathrm{e}^{2i(bx+a)}+12b\,c^2d^2x^2\mathrm{e}^{2i(bx+a)}+8b\,c^3dx\,\mathrm{e}^{2i(bx+a)}-4id^4x^3\mathrm{e}^{2i(bx+a)}+2b\,c^4\mathrm{e}^{2i(bx+a)}-12ic}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^
2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a)
)+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2
*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^
2*x-2*I*c^3*d)/b^2/(1+exp(2*I*(b*x+a)))^2-6/b^3*d^2*c^2*ln(1+exp(2*I*(b*x+a
)))+12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))+12/b^5*d^4*a^2*ln(exp(I*(b*x+a)))+24*
I/b^3*d^3*c*a*x+6*I/b^4*d^3*c*polylog(2,-exp(2*I*(b*x+a)))+12*I/b^4*d^3*c*a
^2-12/b^3*d^3*c*ln(1+exp(2*I*(b*x+a)))*x-6/b^3*d^4*ln(1+exp(2*I*(b*x+a)))*x
^2+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x-3*d^4*polylog(3,-exp(2*I*(b*x
+a)))/b^5-24/b^4*d^3*c*a*ln(exp(I*(b*x+a)))-12*I/b^4*d^4*a^2*x+12*I/b^2*d^3
*c*x^2-8*I/b^5*d^4*a^3+4*I/b^2*d^4*x^3
```

maxima [B] time = 0.68, size = 3438, normalized size = 24.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^4*tan(b*x + a)^2 - 4*a*c^3*d*tan(b*x + a)^2/b + 6*a^2*c^2*d^2*tan(b*
x + a)^2/b^2 - 4*a^3*c*d^3*tan(b*x + a)^2/b^3 + a^4*d^4*tan(b*x + a)^2/b^4
+ 8*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(
b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a
)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*
sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c^3*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos
(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a
)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*
b*x + 2*a) + 1)*b) - 24*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2
*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x
 + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - co
s(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*c^2*d^2/((2*(2*c
os(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x +
2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2
*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + 24*(4*(b*x + a)*cos(2*b*x +
2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + s
in(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x
+ a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x
+ 2*a))*a^2*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x
 + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*
sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^3) - 8*
(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x
```

```
+ a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*co
s(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(
4*b*x + 4*a) - sin(2*b*x + 2*a))*a^3*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4
*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^
2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*
x + 2*a) + 1)*b^4) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*si
n(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*
x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x +
 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + s
in(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a
)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 +
 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a
)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*c^
2*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 +
4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x +
2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) - 12*(8*(b*x + a
)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*c
os(2*b*x + 2*a) + 4*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2
*a))*cos(4*b*x + 4*a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(
4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x
 + a)^2*sin(2*b*x + 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x
+ 4*a) - 4*(b*x + a)*sin(2*b*x + 2*a))*a*c*d^3/((2*(2*cos(2*b*x + 2*a) + 1)
*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*co
s(2*b*x + 2*a) + 1)*b^3) + 6*(8*(b*x + a)^2*cos(2*b*x + 2*a)^2 + 8*(b*x + a
)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4*((b*x + a)^2*co
s(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) - (2*(2*cos(2
*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)
^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x
 + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*
a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x + 2*a) - b*x - (b
*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a)*sin(2*b*x + 2*
a))*a^2*d^4/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a
)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*
b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^4) - 2*((6*(b
*x + a)^2*d^4 + 12*(b*c*d^3 - a*d^4)*(b*x + a) + 6*((b*x + a)^2*d^4 + 2*(b*
c*d^3 - a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^4 + 2*(b*c*d
^3 - a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^4 + (12*I*b*c*
d^3 - 12*I*a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^4 + (24
*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2
*a), cos(2*b*x + 2*a) + 1) - 4*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x
+ a)^2)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^4*d^4 + (8*I*b*c*d^3 - 4*(2*I*a +
 1)*d^4)*(b*x + a)^3 - 12*(b*c*d^3 - a*d^4)*(b*x + a)^2)*cos(2*b*x + 2*a) -
```

```
 (6*b*c*d^3 + 6*(b*x + a)*d^4 - 6*a*d^4 + 6*(b*c*d^3 + (b*x + a)*d^4 - a*d^
4)*cos(4*b*x + 4*a) + 12*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a)
 - (-6*I*b*c*d^3 - 6*I*(b*x + a)*d^4 + 6*I*a*d^4)*sin(4*b*x + 4*a) - (-12*I
*b*c*d^3 - 12*I*(b*x + a)*d^4 + 12*I*a*d^4)*sin(2*b*x + 2*a))*dilog(-e^(2*I
*b*x + 2*I*a)) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x +
a) + (-3*I*(b*x + a)^2*d^4 + (-6*I*b*c*d^3 + 6*I*a*d^4)*(b*x + a))*cos(4*b*
x + 4*a) + (-6*I*(b*x + a)^2*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a))*
cos(2*b*x + 2*a) + 3*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(
4*b*x + 4*a) + 6*((b*x + a)^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*
x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a)
+ 1) + (-3*I*d^4*cos(4*b*x + 4*a) - 6*I*d^4*cos(2*b*x + 2*a) + 3*d^4*sin(4*
b*x + 4*a) + 6*d^4*sin(2*b*x + 2*a) - 3*I*d^4)*polylog(3, -e^(2*I*b*x + 2*I
*a)) + (-4*I*(b*x + a)^3*d^4 + (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*si
n(4*b*x + 4*a) - (2*(b*x + a)^4*d^4 + (8*b*c*d^3 - (8*a - 4*I)*d^4)*(b*x +
a)^3 - (-12*I*b*c*d^3 + 12*I*a*d^4)*(b*x + a)^2)*sin(2*b*x + 2*a))/(-I*b^4*
cos(4*b*x + 4*a) - 2*I*b^4*cos(2*b*x + 2*a) + b^4*sin(4*b*x + 4*a) + 2*b^4*
sin(2*b*x + 2*a) - I*b^4))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan(a+b\,x)\,(c+d\,x)^4}{\cos(a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2,x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^4)/cos(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^4 \tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**4*tan(a + b*x)*sec(a + b*x)**2, x)
```

3.292 $\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=115

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id(c+dx)}{2b^2}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*po
lylog(2,-exp(2*I*(b*x+a)))/b^4+1/2*(d*x+c)^3*sec(b*x+a)^2/b-3/2*d*(d*x+c)^2
*tan(b*x+a)/b^2
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {4409, 4184, 3719, 2190, 2279, 2391}

$$\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\tan(a+bx)}{2b^2}+\frac{(c+dx)^3\sec^2(a+bx)}{2b}+\frac{3id}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b^3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((c + d*x)
^3*Sec[a + b*x]^2)/(2*b) - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\sec^2(a+bx)\,dx}{2b}$$

$$= \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{\left(3d^2\right)\int (c+dx)\,\mathrm{ta}}{b^2}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3\sec^2(a+bx)}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} - \frac{(6}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\sec^2(a+bx}{2b}$$

$$= \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)^3\sec^2(a+bx}{2b}$$

$$= \frac{3id(c+dx)^2}{2b^2} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{3id^3\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4} +$$

Mathematica [B] time = 6.38, size = 286, normalized size = 2.49

$$-\frac{3cd^2\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^3\left(\sin^2(a)+\cos^2(a)\right)}-\frac{3\sec(a)\sec(a+bx)\left(c^2d\sin(bx)+2cd^2x\,\mathrm{s}\right.}{2b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)^3*Sec[a + b*x]^2)/(2*b) - (3*c*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos
[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) - (3*d^
3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[C
ot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^(
(2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Si
n[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))
/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) -
 (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[
b*x]))/(2*b^2)
```

fricas [B] time = 0.53, size = 540, normalized size = 4.70

$$\frac{b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+b^3c^3-3i\,d^3\cos\left(bx+a\right)^2\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+3i\,d^3\cos\left(bx+a\right)^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 3*I*d^3*cos(
b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*di
log(I*cos(b*x + a) - sin(b*x + a)) + 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*
x + a) + sin(b*x + a)) - 3*I*d^3*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin
(b*x + a)) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b
*x + a) + I) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(
b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin
(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) + s
in(b*x + a) + 1) - 3*(b*d^3*x + a*d^3)*cos(b*x + a)^2*log(-I*cos(b*x + a) -
 sin(b*x + a) + 1) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) +
 I*sin(b*x + a) + I) - 3*(b*c*d^2 - a*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*
x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec (bx+a)^2 \tan (bx+a) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*tan(b*x + a), x)
```

maple [B] time = 0.10, size = 301, normalized size = 2.62

$$\frac{2b\,d^3x^3\mathrm{e}^{2i(bx+a)} - 3id^3x^2\mathrm{e}^{2i(bx+a)} + 6bc\,d^2x^2\mathrm{e}^{2i(bx+a)} - 6ic\,d^2x\,\mathrm{e}^{2i(bx+a)} + 6b\,c^2dx\,\mathrm{e}^{2i(bx+a)} - 3ic^2d\,\mathrm{e}^{2i(bx+a)} - 3id^3x^2}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] (2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x+a))+6*b*c*d^2*x^2*ex
p(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*x*exp(2*I*(b*x+a))-3*
I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(b*x+a))-6*I*c*d^2*x-3
*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))+6/b
^3*d^2*c*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+6*I/b^3*d^3*a*x+3*I/b^4*d^3*a^2
-3/b^3*d^3*ln(1+exp(2*I*(b*x+a)))*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4-6/b^4*d^3*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.63, size = 667, normalized size = 5.80

$$-\frac{6\,b^2c^2d + \left(6\,bd^3x + 6\,bcd^2 + 6\left(bd^3x + bcd^2\right)\cos\left(4\,bx + 4\,a\right) + 12\left(bd^3x + bcd^2\right)\cos\left(2\,bx + 2\,a\right) + \left(6i\,bd^3x + 6\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] -(6*b^2*c^2*d + (6*b*d^3*x + 6*b*c*d^2 + 6*(b*d^3*x + b*c*d^2)*cos(4*b*x +
4*a) + 12*(b*d^3*x + b*c*d^2)*cos(2*b*x + 2*a) + (6*I*b*d^3*x + 6*I*b*c*d^2
)*sin(4*b*x + 4*a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*sin(2*b*x + 2*a))*arctan
2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x)
*cos(4*b*x + 4*a) + (4*I*b^3*d^3*x^3 + 4*I*b^3*c^3 + 6*b^2*c^2*d + (12*I*b^
3*c*d^2 - 6*b^2*d^3)*x^2 + (12*I*b^3*c^2*d - 12*b^2*c*d^2)*x)*cos(2*b*x + 2
*a) - (3*d^3*cos(4*b*x + 4*a) + 6*d^3*cos(2*b*x + 2*a) + 3*I*d^3*sin(4*b*x
+ 4*a) + 6*I*d^3*sin(2*b*x + 2*a) + 3*d^3)*dilog(-e^(2*I*b*x + 2*I*a)) + (-
3*I*b*d^3*x - 3*I*b*c*d^2 + (-3*I*b*d^3*x - 3*I*b*c*d^2)*cos(4*b*x + 4*a) +
```

```
 (-6*I*b*d^3*x - 6*I*b*c*d^2)*cos(2*b*x + 2*a) + 3*(b*d^3*x + b*c*d^2)*sin(
4*b*x + 4*a) + 6*(b*d^3*x + b*c*d^2)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (-6*I*b^2*d^3*x^2 - 12*
I*b^2*c*d^2*x)*sin(4*b*x + 4*a) - (4*b^3*d^3*x^3 + 4*b^3*c^3 - 6*I*b^2*c^2*
d + 6*(2*b^3*c*d^2 + I*b^2*d^3)*x^2 + 12*(b^3*c^2*d + I*b^2*c*d^2)*x)*sin(2
*b*x + 2*a))/(-2*I*b^4*cos(4*b*x + 4*a) - 4*I*b^4*cos(2*b*x + 2*a) + 2*b^4*
sin(4*b*x + 4*a) + 4*b^4*sin(2*b*x + 2*a) - 2*I*b^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan (a+b\,x)\,\left(c+d\,x\right)^3}{\cos (a+b\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

```
[Out] int((tan(a + b*x)*(c + d*x)^3)/cos(a + b*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan (a+bx) \sec^2 (a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)*sec(a + b*x)**2, x)
```

3.293 $\int (c+dx)^2 \sec^2(a+bx) \tan(a+bx)\,dx$

Optimal. Leaf size=55

$$-\frac{d^2 \log(\cos(a+bx))}{b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2 \sec^2(a+bx)}{2b}$$

```
[Out] -d^2*ln(cos(b*x+a))/b^3+1/2*(d*x+c)^2*sec(b*x+a)^2/b-d*(d*x+c)*tan(b*x+a)/b
^2
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {4409, 4184, 3475}

$$-\frac{d(c+dx)\tan(a+bx)}{b^2} - \frac{d^2 \log(\cos(a+bx))}{b^3} + \frac{(c+dx)^2 \sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] -((d^2*Log[Cos[a + b*x]])/b^3) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (d*(c
 + d*x)*Tan[a + b*x])/b^2
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4184

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> -Sim
p[((c + d*x)^m*Cot[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Co
t[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^2\sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d\int (c+dx)\sec^2(a+bx)\,dx}{b}$$
$$= \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{d^2\int \tan(a+bx)\,dx}{b^2}$$
$$= -\frac{d^2\log(\cos(a+bx))}{b^3} + \frac{(c+dx)^2\sec^2(a+bx)}{2b} - \frac{d(c+dx)\tan(a+bx)}{b^2}$$

Mathematica [A] time = 0.51, size = 66, normalized size = 1.20

$$\frac{b^2(c+dx)^2\sec^2(a+bx) - 2bd\sec(a)\sin(bx)(c+dx)\sec(a+bx) - 2d^2(bx\tan(a)+\log(\cos(a+bx)))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (b^2*(c + d*x)^2*Sec[a + b*x]^2 - 2*b*d*(c + d*x)*Sec[a]*Sec[a + b*x]*Sin[b
*x] - 2*d^2*(Log[Cos[a + b*x]] + b*x*Tan[a]))/(2*b^3)
```

fricas [A] time = 0.45, size = 86, normalized size = 1.56

$$\frac{b^2d^2x^2 + 2\,b^2cdx - 2\,d^2\cos(bx+a)^2\log(-\cos(bx+a)) + b^2c^2 - 2\left(bd^2x + bcd\right)\cos(bx+a)\sin(bx+a)}{2\,b^3\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2*cos(b*x + a)^2*log(-cos(b*x + a)) +
b^2*c^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^
2)
```

giac [B] time = 1.81, size = 4474, normalized size = 81.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4*t
an(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d^2*x^2*tan
(1/2*b*x)^2*tan(1/2*a)^4 + b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^2*c*d*
```

```
x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b
^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^
4 - d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2
- 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2
*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*ta
n(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4
*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/
2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1
/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(t
an(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^2*d^2*x^
2*tan(1/2*b*x)^4 + 4*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*b^2*c^2*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d^2*x
^2*tan(1/2*a)^4 + 2*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b*c*d*tan(1/2*b
*x)^3*tan(1/2*a)^4 + 2*b^2*c*d*x*tan(1/2*b*x)^4 - 4*b*d^2*x*tan(1/2*b*x)^4*
tan(1/2*a) + 8*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*d^2*x*tan(1/2*b
*x)^3*tan(1/2*a)^2 + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b
*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan
(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*
tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a)
+ 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/
2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8
*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*t
an(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/
2*a)^2 - 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 8*d^2*log(4*(tan(1/2*b*x)
^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*
a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan
(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 +
 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b
*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*ta
n(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*
tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^
2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 2*b^2*c*d*x*tan(1/2*a)^4 - 4*b*d^2*x*
tan(1/2*b*x)*tan(1/2*a)^4 + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*ta
n(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8
+ 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*
b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(
1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2
*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a
)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a
) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 2*b^2*d^2*x^2*tan(1/2*b*x)^2 + b^2*c^2*tan(1/2*b*x)^4 - 4*b
*c*d*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^2*d^2*x^2*tan(1/2*a)^2 + 4*b^2*c^2*tan
(1/2*b*x)^2*tan(1/2*a)^2 - 24*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 24*b*c*d*
tan(1/2*b*x)^2*tan(1/2*a)^3 + b^2*c^2*tan(1/2*a)^4 - 4*b*c*d*tan(1/2*b*x)*t
an(1/2*a)^4 + 4*b^2*c*d*x*tan(1/2*b*x)^2 + 4*b*d^2*x*tan(1/2*b*x)^3 - d^2*l
```

og(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 24*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a) - 8*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*b^2*c*d*x*tan(1/2*a)^2 + 24*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^2 - 20*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)^2*tan(1/2*a)^2 + 4*b*d^2*x*tan(1/2*a)^3 - 8*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 - d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a)^4 + b^2*d^2*x^2 + 2*b^2*c^2*tan(1/2*b*x)^2 + 4*b*c*d*tan(1/2*b*x)^3 + 24*b*c*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b^2*c^2*tan(1/2*a)^2 + 24*b*c*d*tan(1/2*b*x)*tan(1/2*a)^2 + 4*b*c*d*tan(1/2*a)^3 + 2*b^2*c*d*x - 4*b*d^2*x*tan(1/2*b*x) + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x

```
)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(
1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*
tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3
 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1
/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1
/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*
x)^2 - 4*b*d^2*x*tan(1/2*a) + 8*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*
tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^
8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/
2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*ta
n(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 -
 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2
*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2
*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*b*x)
*tan(1/2*a) + 2*d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*a)^4 - 2*tan(1/2*b*x)^8*t
an(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(1/2*b*x)^8 + 8*tan(1/2*b*
x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 - 8*tan(1/2*b*x)^5*tan(1/2
*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*b*x)^5*tan(1/2*a) + 36*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*tan(1/2*b*x)^
4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^2*tan(1/2*a)^2 + 8*tan(1/
2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x)*tan(1/2*a) - 2*tan(1/2*
a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1))*tan(1/2*a)^2 + b^2*c^2 - 4*b
*c*d*tan(1/2*b*x) - 4*b*c*d*tan(1/2*a) - d^2*log(4*(tan(1/2*b*x)^8*tan(1/2*
a)^4 - 2*tan(1/2*b*x)^8*tan(1/2*a)^2 - 8*tan(1/2*b*x)^7*tan(1/2*a)^3 + tan(
1/2*b*x)^8 + 8*tan(1/2*b*x)^7*tan(1/2*a) + 16*tan(1/2*b*x)^6*tan(1/2*a)^2 -
 8*tan(1/2*b*x)^5*tan(1/2*a)^3 - 2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*tan(1/2*
b*x)^5*tan(1/2*a) + 36*tan(1/2*b*x)^4*tan(1/2*a)^2 + 8*tan(1/2*b*x)^3*tan(1
/2*a)^3 - 2*tan(1/2*b*x)^4 - 8*tan(1/2*b*x)^3*tan(1/2*a) + 16*tan(1/2*b*x)^
2*tan(1/2*a)^2 + 8*tan(1/2*b*x)*tan(1/2*a)^3 + tan(1/2*a)^4 - 8*tan(1/2*b*x
)*tan(1/2*a) - 2*tan(1/2*a)^2 + 1)/(tan(1/2*a)^4 + 2*tan(1/2*a)^2 + 1)))/(b
^3*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b^3*
tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^3*tan(1
/2*b*x)^4 + 8*b^3*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^3*tan(1/2*b*x)^2*tan(1/2
*a)^2 + 8*b^3*tan(1/2*b*x)*tan(1/2*a)^3 + b^3*tan(1/2*a)^4 - 2*b^3*tan(1/2*
b*x)^2 - 8*b^3*tan(1/2*b*x)*tan(1/2*a) - 2*b^3*tan(1/2*a)^2 + b^3)
```

maple [A] time = 0.03, size = 95, normalized size = 1.73

$$\frac{d^{2} x^{2}}{2 b \cos \left(b x+a\right)^{2}}-\frac{d^{2} \tan \left(b x+a\right) x}{b^{2}}-\frac{d^{2} \ln \left(\cos \left(b x+a\right)\right)}{b^{3}}+\frac{c d x}{b \cos \left(b x+a\right)^{2}}-\frac{c d \tan \left(b x+a\right)}{b^{2}}+\frac{c^{2}}{2 b \cos \left(b x+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 1/2/b*d^2/cos(b*x+a)^2*x^2-1/b^2*d^2*tan(b*x+a)*x-d^2*ln(cos(b*x+a))/b^3+1/
b*c*d/cos(b*x+a)^2*x-1/b^2*c*d*tan(b*x+a)+1/2/b*c^2/cos(b*x+a)^2
```

maxima [B] time = 0.52, size = 988, normalized size = 17.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c^2*tan(b*x + a)^2 - 2*a*c*d*tan(b*x + a)^2/b + a^2*d^2*tan(b*x + a)^2
/b^2 + 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 +
 (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*
x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a)
- 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*c*d/((2*(2*cos(2*b*x + 2*a) + 1)*
cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x +
4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos
(2*b*x + 2*a) + 1)*b) - 4*(4*(b*x + a)*cos(2*b*x + 2*a)^2 + 4*(b*x + a)*sin
(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + sin(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x + a)*sin(2*b*x + 2*a) -
cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x + 2*a))*a*d^2/((2*(2*cos
(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*
a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b
*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2) + (8*(b*x + a)^2*cos(2*b*x + 2*a
)^2 + 8*(b*x + a)^2*sin(2*b*x + 2*a)^2 + 4*(b*x + a)^2*cos(2*b*x + 2*a) + 4
*((b*x + a)^2*cos(2*b*x + 2*a) + (b*x + a)*sin(2*b*x + 2*a))*cos(4*b*x + 4*
a) - (2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2 + 4*
cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x + 2*
a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*log(cos(2*b*x + 2*a)^2
+ sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 4*((b*x + a)^2*sin(2*b*x +
 2*a) - b*x - (b*x + a)*cos(2*b*x + 2*a) - a)*sin(4*b*x + 4*a) - 4*(b*x + a
)*sin(2*b*x + 2*a))*d^2/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos
(4*b*x + 4*a)^2 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x +
 4*a)*sin(2*b*x + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b^2
))/b
```

mupad [B] time = 3.06, size = 150, normalized size = 2.73

$$-\frac{\frac{(c+d\,x)^2}{b}-\frac{\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,(c+d\,x)^2}{b}}{2\,\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+\mathrm{e}^{a\,4\mathrm{i}+b\,x\,4\mathrm{i}}+1}+\frac{d^2\,x\,2\mathrm{i}}{b^2}+\frac{b\,c^2+2\,b\,c\,d\,x-c\,d\,2\mathrm{i}+b\,d^2\,x^2-d^2\,x\,2\mathrm{i}}{b^2\,\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}-\frac{d^2\,\ln\left(\mathrm{e}^{a\,2\mathrm{i}}\,\mathrm{e}^{b\,x\,2\mathrm{i}}+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
```

```
[Out] (d^2*x*2i)/b^2 - ((c + d*x)^2/b - (exp(a*2i + b*x*2i)*(c + d*x)^2)/b)/(2*ex
p(a*2i + b*x*2i) + exp(a*4i + b*x*4i) + 1) + (b*c^2 - c*d*2i - d^2*x*2i + b
*d^2*x^2 + 2*b*c*d*x)/(b^2*(exp(a*2i + b*x*2i) + 1)) - (d^2*log(exp(a*2i)*e
xp(b*x*2i) + 1))/b^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan (a+bx) \sec^2 (a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)*sec(a + b*x)**2, x)
```

3.294 $\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx$

Optimal. Leaf size=35

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

```
[Out] 1/2*(d*x+c)*sec(b*x+a)^2/b-1/2*d*tan(b*x+a)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {4409, 3767, 8}

$$\frac{(c+dx)\sec^2(a+bx)}{2b}-\frac{d\tan(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] ((c + d*x)*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\sec^2(a+bx)\tan(a+bx)\,dx = \frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\int \sec^2(a+bx)\,dx}{2b}$$
$$= \frac{(c+dx)\sec^2(a+bx)}{2b} + \frac{d\,\text{Subst}(\int 1\,dx, x, -\tan(a+bx))}{2b^2}$$
$$= \frac{(c+dx)\sec^2(a+bx)}{2b} - \frac{d\tan(a+bx)}{2b^2}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.37

$$-\frac{d\tan(a+bx)}{2b^2} + \frac{c\sec^2(a+bx)}{2b} + \frac{dx\sec^2(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Tan[a + b*x],x]
```

```
[Out] (c*Sec[a + b*x]^2)/(2*b) + (d*x*Sec[a + b*x]^2)/(2*b) - (d*Tan[a + b*x])/(2
*b^2)
```

fricas [A] time = 0.41, size = 36, normalized size = 1.03

$$\frac{bdx - d\cos(bx+a)\sin(bx+a) + bc}{2\,b^2\cos(bx+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/2*(b*d*x - d*cos(b*x + a)*sin(b*x + a) + b*c)/(b^2*cos(b*x + a)^2)
```

giac [B] time = 1.84, size = 571, normalized size = 16.31

$$\frac{bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + bc\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^4 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^4\tan\left(\frac{1}{2}a\right)^2 + 2\,bdx\tan\left(\frac{1}{2}bx\right)^2\tan\left(\frac{1}{2}a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b*c*tan(1/2*b*x)^4*tan(1/2*a)^4 +
2*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^4 +
```

```
 2*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*d*tan(1/2*b*x)^4*tan(1/2*a)^3 + 2*b*
c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 2*d*tan(1/2*b*x)^3*tan(1/2*a)^4 + b*d*x*tan
(1/2*b*x)^4 + 4*b*d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + b*d*x*tan(1/2*a)^4 + b*
c*tan(1/2*b*x)^4 - 2*d*tan(1/2*b*x)^4*tan(1/2*a) + 4*b*c*tan(1/2*b*x)^2*tan
(1/2*a)^2 - 12*d*tan(1/2*b*x)^3*tan(1/2*a)^2 - 12*d*tan(1/2*b*x)^2*tan(1/2*
a)^3 + b*c*tan(1/2*a)^4 - 2*d*tan(1/2*b*x)*tan(1/2*a)^4 + 2*b*d*x*tan(1/2*b
*x)^2 + 2*b*d*x*tan(1/2*a)^2 + 2*b*c*tan(1/2*b*x)^2 + 2*d*tan(1/2*b*x)^3 +
12*d*tan(1/2*b*x)^2*tan(1/2*a) + 2*b*c*tan(1/2*a)^2 + 12*d*tan(1/2*b*x)*tan
(1/2*a)^2 + 2*d*tan(1/2*a)^3 + b*d*x + b*c - 2*d*tan(1/2*b*x) - 2*d*tan(1/2
*a))/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 - 2*b^2*tan(1/2*b*x)^4*tan(1/2*a)^2 -
 8*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^
2*tan(1/2*b*x)^4 + 8*b^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^2*tan(1/2*b*x)^2*
tan(1/2*a)^2 + 8*b^2*tan(1/2*b*x)*tan(1/2*a)^3 + b^2*tan(1/2*a)^4 - 2*b^2*t
an(1/2*b*x)^2 - 8*b^2*tan(1/2*b*x)*tan(1/2*a) - 2*b^2*tan(1/2*a)^2 + b^2)
```

maple [A] time = 0.03, size = 61, normalized size = 1.74

$$\frac{\frac{d\left(\frac{bx+a}{2\cos(bx+a)^2}-\frac{\tan(bx+a)}{2}\right)}{b}-\frac{da}{2b\cos(bx+a)^2}+\frac{c}{2\cos(bx+a)^2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x)
```

```
[Out] 1/b*(1/b*d*(1/2*(b*x+a)/cos(b*x+a)^2-1/2*tan(b*x+a))-1/2/b*d*a/cos(b*x+a)^2
+1/2*c/cos(b*x+a)^2)
```

maxima [B] time = 0.35, size = 283, normalized size = 8.09

$$\frac{c\tan(bx+a)^2-\frac{ad\tan(bx+a)^2}{b}+\frac{2\left(4\,(bx+a)\cos(2\,bx+2\,a)^2+4\,(bx+a)\sin(2\,bx+2\,a)^2+(2\,(bx+a)\cos(2\,bx+2\,a)+\sin(2\,bx+2\,a))\cos(4\,bx+4\,a)+2\,(\cdots}{\left(2\,(2\,\cos(2\,bx+2\,a)+1)\cos(4\,bx+4\,a)+\cos(4\,bx+4\,a)^2+4\,\cos(2\,bx+2\,a)^2+\sin(4\,bx+4\,a)^2\cdots\right.}}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*tan(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*(c*tan(b*x + a)^2 - a*d*tan(b*x + a)^2/b + 2*(4*(b*x + a)*cos(2*b*x + 2
*a)^2 + 4*(b*x + a)*sin(2*b*x + 2*a)^2 + (2*(b*x + a)*cos(2*b*x + 2*a) + si
n(2*b*x + 2*a))*cos(4*b*x + 4*a) + 2*(b*x + a)*cos(2*b*x + 2*a) + (2*(b*x +
 a)*sin(2*b*x + 2*a) - cos(2*b*x + 2*a) - 1)*sin(4*b*x + 4*a) - sin(2*b*x +
 2*a))*d/((2*(2*cos(2*b*x + 2*a) + 1)*cos(4*b*x + 4*a) + cos(4*b*x + 4*a)^2
 + 4*cos(2*b*x + 2*a)^2 + sin(4*b*x + 4*a)^2 + 4*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*sin(2*b*x + 2*a)^2 + 4*cos(2*b*x + 2*a) + 1)*b))/b
```

mupad [B] time = 2.19, size = 53, normalized size = 1.51

$$-\frac{d\,1\mathrm{i}+\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}\,\left(-b\,\left(2\,c+2\,d\,x\right)+d\,1\mathrm{i}\right)}{b^2\,{\left(\mathrm{e}^{a\,2\mathrm{i}+b\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

```
[Out] -(d*1i + exp(a*2i + b*x*2i)*(d*1i - b*(2*c + 2*d*x)))/(b^2*(exp(a*2i + b*x*
2i) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan(a+bx)\sec^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*tan(b*x+a),x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)*sec(a + b*x)**2, x)
```

3.295 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 7.24, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x),x]
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x), x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)^2\tan(bx+a)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)^2 \tan(bx+a)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(bx+a)\right) \tan(bx+a)}{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos(a+b\,x)^2\,\left(c+d\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x), x)
```

3.296 $\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.13, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx = \int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 10.39, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Tan[a + b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)^2\tan(bx+a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*tan(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)^2\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*tan(b*x + a)/(d*x + c)^2, x)
```

maple [A] time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(bx+a)\right)\tan(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*tan(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(a+b\,x)}{\cos(a+b\,x)^2\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
[Out] int(tan(a + b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(a+bx)\sec^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*tan(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)*sec(a + b*x)**2/(c + d*x)**2, x)
```

3.297 $\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\sec^3(a+bx)(c+dx)^m,x\right)-\text{Int}\left(\sec(a+bx)(c+dx)^m,x\right)$$

```
[Out] -Unintegrable((d*x+c)^m*sec(b*x+a),x)+Unintegrable((d*x+c)^m*sec(b*x+a)^3,x
)
```

Rubi [A] time = 0.08, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -Defer[Int][(c + d*x)^m*Sec[a + b*x], x] + Defer[Int][(c + d*x)^m*Sec[a + b
*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \sec(a+bx)\tan^2(a+bx)\,dx=-\int (c+dx)^m \sec(a+bx)\,dx+\int (c+dx)^m \sec^3(a+bx)\,dx$$

Mathematica [F] time = 180.02, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] $Aborted
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \sec(bx+a)\tan(bx+a)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sec(bx + a) \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

maple [A] time = 0.09, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sec(bx + a) \left(\tan^2(bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
[Out] int((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \sec(bx + a) \tan(bx + a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sec(b*x + a)*tan(b*x + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(a + b\,x)^2\,(c + d\,x)^m}{\cos(a + b\,x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x),x)
[Out] int((tan(a + b*x)^2*(c + d*x)^m)/cos(a + b*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \tan^2(a + bx) \sec(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*tan(a + b*x)**2*sec(a + b*x), x)
```

3.298 $\int (c+dx)^3 \sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=337

$$\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(ie^{i(a+bx)}\right)}{b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2}{}$$

```
[Out] -6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3+I*(d*x+c)^3*arctan(exp(I*(b*x+a
)))/b+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,
-I*exp(I*(b*x+a)))/b^2-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+3/2*I*d*(d*x
+c)^2*polylog(2,I*exp(I*(b*x+a)))/b^2+3*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x
+a)))/b^3-3*d^2*(d*x+c)*polylog(3,I*exp(I*(b*x+a)))/b^3+3*I*d^3*polylog(4,-
I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-3/2*d*(d*x+c)
^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] time = 0.41, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {4413, 4181, 2531, 6609, 2282, 6589, 4186, 2279, 2391}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{3d^2(c+dx)\text{PolyLog}\left(3,ie^{i(a+bx)}\right)}{b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{3id(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 + (I*(c + d*x)^3*ArcTan[
E^(I*(a + b*x))])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 - ((
(3*I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*P
olyLog[2, I*E^(I*(a + b*x))])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E
^(I*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (3*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b^3 + ((3*I)*d^3*PolyL
og[4, (-I)*E^(I*(a + b*x))])/b^4 - ((3*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))]
)/b^4 - (3*d*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Sec[a + b*x]*
Tan[a + b*x])/(2*b)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^3\sec(a+bx)\,dx + \int (c+dx)^3\sec^3(a+bx)\,dx$$

$$= \frac{2i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3d(c+dx)^2\sec(a+bx)}{2b^2} + \frac{(c+dx)^3\sec(}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3id(c+d}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{3id(c+d}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(\right.}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(\right.}{b}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} + \frac{i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{3id^3\mathrm{Li}_2\left(\right.}{b}$$

Mathematica [A] time = 3.40, size = 530, normalized size = 1.57

$$\frac{2ib^3c^3\tan^{-1}\left(e^{i(a+bx)}\right) - 3b^3c^2dx\log\left(1-ie^{i(a+bx)}\right) + 3b^3c^2dx\log\left(1+ie^{i(a+bx)}\right) - 3b^3cd^2x^2\log\left(1-ie^{i(a+bx)}\right) + 3b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] ((2*I)*b^3*c^3*ArcTan[E^(I*(a + b*x))] - (12*I)*b*c*d^2*ArcTan[E^(I*(a + b*
x))] - 3*b^3*c^2*d*x*Log[1 - I*E^(I*(a + b*x))] + 6*b*d^3*x*Log[1 - I*E^(I*
(a + b*x))] - 3*b^3*c*d^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[
1 - I*E^(I*(a + b*x))] + 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] - 6*b*d^3
*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))]
+ b^3*d^3*x^3*Log[1 + I*E^(I*(a + b*x))] - (3*I)*d*(-2*d^2 + b^2*(c + d*x)^
2)*PolyLog[2, (-I)*E^(I*(a + b*x))] + (3*I)*d*(-2*d^2 + b^2*(c + d*x)^2)*Po
lyLog[2, I*E^(I*(a + b*x))] + 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] +
6*b*d^3*x*PolyLog[3, (-I)*E^(I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a
 + b*x))] - 6*b*d^3*x*PolyLog[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4,
(-I)*E^(I*(a + b*x))] - (6*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))] + b^2*(c +
d*x)^2*Sec[a + b*x]*(-3*d + b*(c + d*x)*Tan[a + b*x]))/(2*b^4)
```

fricas [C] time = 0.64, size = 1311, normalized size = 3.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(-6*I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - 6*
I*d^3*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3*co
s(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x +
 a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b
^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) +
 sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d - 6*I*d
^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2
 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b
*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2
*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^3*c
^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*
log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2
- 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x
 + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*
x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a
)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2
 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^3*c^2*d - 2*b*d^3
)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 - 6*a)*d^3 + 3*(b^
3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*
d + 3*(a^2 - 2)*b*c*d^2 - (a^3 - 6*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a)
```

```
 - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*
cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog
(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*p
olylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) - 6*(b^2*d^3*x^2 + 2*b^2*c
*d^2*x + b^2*c^2*d)*cos(b*x + a) + 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3
*c^2*d*x + b^3*c^3)*sin(b*x + a))/(b^4*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\tan(bx+a)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*tan(b*x + a)^2, x)
```

maple [B] time = 0.21, size = 1127, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] 3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(4,-I*exp(I*(b*x+a)
))/b^4-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3*I*d^3*polylog(4,I*exp(I*(b
*x+a)))/b^4-3/2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2+3/2*I/b^2*d^3*po
lylog(2,I*exp(I*(b*x+a)))*x^2-I/b^4*d^3*a^3*arctan(exp(I*(b*x+a)))+6*I/b^4*
d^3*a*arctan(exp(I*(b*x+a)))-3/2*I/b^2*c^2*d*polylog(2,-I*exp(I*(b*x+a)))-6
*I/b^3*c*d^2*arctan(exp(I*(b*x+a)))+3/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+
a)))+1/2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))-3/b^3*d^3*polylog(3,I*exp(I*(b*
x+a)))*x-1/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3+1/2/b*d^3*ln(1+I*exp(I*(b*x+a
)))*x^3+3/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x-3/b^3*d^2*c*polylog(3,I*ex
p(I*(b*x+a)))+3/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))-1/2/b^4*a^3*d^3*ln(1
-I*exp(I*(b*x+a)))-3/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2+3/2/b*d^2*c*ln(1+
I*exp(I*(b*x+a)))*x^2-3/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x-3/2/b^2*c^2*d*ln
(1-I*exp(I*(b*x+a)))*a-3/2/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))+3/2/b*c^2*d
*ln(1+I*exp(I*(b*x+a)))*x+3/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a+3/2/b^3*a^
2*c*d^2*ln(1-I*exp(I*(b*x+a)))+I/b*c^3*arctan(exp(I*(b*x+a)))+3/b^3*d^3*ln(
1-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*d^3*ln(1+I*e
xp(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a-3*I/b^2*c*d^2*polylog(2
,-I*exp(I*(b*x+a)))*x+3*I/b^2*c*d^2*polylog(2,I*exp(I*(b*x+a)))*x+3*I/b^3*c
*d^2*a^2*arctan(exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))^2*(d^3*x^3*b*exp
(3*I*(b*x+a))+3*c*d^2*x^2*b*exp(3*I*(b*x+a))+3*c^2*d*x*b*exp(3*I*(b*x+a))-d
```

```
^3*x^3*b*exp(I*(b*x+a))+c^3*b*exp(3*I*(b*x+a))-3*c*d^2*x^2*b*exp(I*(b*x+a))
-3*I*d^3*x^2*exp(3*I*(b*x+a))-3*c^2*d*x*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(3*
I*(b*x+a))-c^3*b*exp(I*(b*x+a))-3*I*c^2*d*exp(3*I*(b*x+a))-3*I*d^3*x^2*exp(
I*(b*x+a))-6*I*c*d^2*x*exp(I*(b*x+a))-3*I*c^2*d*exp(I*(b*x+a)))-3*I/b^2*c^2
*d*a*arctan(exp(I*(b*x+a)))
```

maxima [B] time = 2.01, size = 3828, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*sin(b*x + a)
/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1))/b^3 - 4*((2*(b*x + a)^3*d^3 - 12*b*c*d^2 + 12*a*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^
3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*cos
(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d
^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*co
s(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*
b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I
*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^3 + 24*
I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^2 + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^3*d^3 - 12*b*
c*d^2 + 12*a*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(b^2*c^2*d - 2*a*b*c
*d^2 + (a^2 - 2)*d^3)*(b*x + a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
+ 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*
d^3)*(b*x + a))*cos(4*b*x + 4*a) + 4*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)
*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2 -
12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2 + (-6*I*b^2*c^2*d + 12*
I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-4*I*(b
*x + a)^3*d^3 + 24*I*b*c*d^2 - 24*I*a*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b
*x + a)^2 + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 + (-12*I*a^2 + 24*I)*d^3)*(b*
x + a))*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*
x + a)^3*d^3 - 12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (12*b*c*d^2
 - (12*a + 12*I)*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a + 24*I)*b*c*d^2 +
 12*(a^2 + 2*I*a)*d^3)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x + a)^3*d^3 + 1
2*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3 + (12*b*c*d^2 - (12*a - 12*I)
```

```
*d^3)*(b*x + a)^2 + (12*b^2*c^2*d - (24*a - 24*I)*b*c*d^2 + 12*(a^2 - 2*I*a
)*d^3)*(b*x + a))*cos(b*x + a) + (6*b^2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^
2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2
*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a
))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2
- 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*b^2*c^2*
d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 + (-6*I*a^2 + 12*I)*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I*
a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 + (-12*I*a^2 + 24*I)*d^3 + (-24*I*b*c*d^2
+ 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I*a)) - (6*b^
2*c^2*d - 12*a*b*c*d^2 + 6*(b*x + a)^2*d^3 + 6*(a^2 - 2)*d^3 + 12*(b*c*d^2
- a*d^3)*(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 -
2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d -
2*a*b*c*d^2 + (b*x + a)^2*d^3 + (a^2 - 2)*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x +
a))*cos(2*b*x + 2*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^
3 + (6*I*a^2 - 12*I)*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*sin(4*b*x
 + 4*a) + (12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + (12*I*a
^2 - 24*I)*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*d
ilog(-I*e^(I*b*x + I*a)) - (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 +
(-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c*d^2 +
(-3*I*a^2 + 6*I)*d^3)*(b*x + a) + (-I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a
*d^3 + (-3*I*b*c*d^2 + 3*I*a*d^3)*(b*x + a)^2 + (-3*I*b^2*c^2*d + 6*I*a*b*c
*d^2 + (-3*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-2*I*(b*x + a)^
3*d^3 + 12*I*b*c*d^2 - 12*I*a*d^3 + (-6*I*b*c*d^2 + 6*I*a*d^3)*(b*x + a)^2
+ (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 + (-6*I*a^2 + 12*I)*d^3)*(b*x + a))*cos(
2*b*x + 2*a) + ((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4
*b*x + 4*a) + 2*((b*x + a)^3*d^3 - 6*b*c*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3
)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x + a) + 1) - (
I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*(b*
x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x + a)
+ (I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (3*I*b*c*d^2 - 3*I*a*d^3)*
(b*x + a)^2 + (3*I*b^2*c^2*d - 6*I*a*b*c*d^2 + (3*I*a^2 - 6*I)*d^3)*(b*x +
a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (
6*I*b*c*d^2 - 6*I*a*d^3)*(b*x + a)^2 + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + (6
*I*a^2 - 12*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^3*d^3 - 6*b*c*
d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^
2 + (a^2 - 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x + a)^3*d^3 - 6*b*c
*d^2 + 6*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d
^2 + (a^2 - 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*sin(b*x + a) + 1) - (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b
*x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)
*polylog(4, I*e^(I*b*x + I*a)) + (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*
x + 2*a) + 12*I*d^3*sin(4*b*x + 4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*
```

```
polylog(4, -I*e^(I*b*x + I*a)) - (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I
*a*d^3 + (-12*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 12*I*a*d^3)*cos(4*b*x + 4*a)
 + (-24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*cos(2*b*x + 2*a) + 12*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a
)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)) - (12*I*b*c*
d^2 + 12*I*(b*x + a)*d^3 - 12*I*a*d^3 + (12*I*b*c*d^2 + 12*I*(b*x + a)*d^3
- 12*I*a*d^3)*cos(4*b*x + 4*a) + (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*
a*d^3)*cos(2*b*x + 2*a) - 12*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x +
4*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I
*e^(I*b*x + I*a)) - (4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12
*a^2*d^3 - 12*(-I*b*c*d^2 + (I*a - 1)*d^3)*(b*x + a)^2 + (12*I*b^2*c^2*d -
24*(I*a - 1)*b*c*d^2 + (12*I*a^2 - 24*a)*d^3)*(b*x + a))*sin(3*b*x + 3*a) -
 (-4*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 + (-12*I*
b*c*d^2 - 12*(-I*a - 1)*d^3)*(b*x + a)^2 + (-12*I*b^2*c^2*d - 24*(-I*a - 1)
*b*c*d^2 + (-12*I*a^2 - 24*a)*d^3)*(b*x + a))*sin(b*x + a))/(-4*I*b^3*cos(4
*b*x + 4*a) - 8*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(4*b*x + 4*a) + 8*b^3*sin
(2*b*x + 2*a) - 4*I*b^3))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \tan^2(a + bx) \sec(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*tan(a + b*x)**2*sec(a + b*x), x)
```

3.299 $\int (c+dx)^2 \sec(a+bx) \tan^2(a+bx)\,dx$

Optimal. Leaf size=193

$$\frac{d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^2}-$$

```
[Out] I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+d^2*arctanh(sin(b*x+a))/b^3-I*d*(d*x+c
)*polylog(2,-I*exp(I*(b*x+a)))/b^2+I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/
b^2+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3-d^2*polylog(3,I*exp(I*(b*x+a)))/b^
3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*sec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] time = 0.27, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {4413, 4181, 2531, 2282, 6589, 4186, 3770}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}-\frac{d^2\text{PolyLog}\left(3,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (d^2*ArcTanh[Sin[a + b*x]])/b^3
 - (I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 + (I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))])/b^2 + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b
^3 - (d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Sec[a + b*x])/b
^2 + ((c + d*x)^2*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)^2 \sec(a+bx)\,dx + \int (c+dx)^2 \sec^3(a+bx)\,dx$$
$$= \frac{2i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d(c+dx)\sec(a+bx)}{b^2} + \frac{(c+dx)^2 \sec(a}{2}$$
$$= \frac{i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \tanh^{-1}(\sin(a+bx))}{b^3} - \frac{2id(c+dx)\text{Li}_2\,(}{b^2}$$
$$= \frac{i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\text{Li}_2\left(-\right.}{b^2}$$
$$= \frac{i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\text{Li}_2\left(-\right.}{b^2}$$
$$= \frac{i(c+dx)^2 \tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{d^2 \tanh^{-1}(\sin(a+bx))}{b^3} - \frac{id(c+dx)\text{Li}_2\left(-\right.}{b^2}$$

Mathematica [B] time = 7.18, size = 526, normalized size = 2.73

$$\frac{ibc^2 \tan^{-1}\left(e^{i(a+bx)}\right) - id(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right) + id(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right) - bcdx\log\left(1-ie^{i(a+bx)}\right) + bcdx\log\left(1\right.}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*b*c^2*ArcTan[E^(I*(a + b*x))] - ((2*I)*d^2*ArcTan[E^(I*(a + b*x))])/b -
b*c*d*x*Log[1 - I*E^(I*(a + b*x))] - (b*d^2*x^2*Log[1 - I*E^(I*(a + b*x))])
/2 + b*c*d*x*Log[1 + I*E^(I*(a + b*x))] + (b*d^2*x^2*Log[1 + I*E^(I*(a + b*
x))])/2 - I*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x))] + I*d*(c + d*x)*Po
lyLog[2, I*E^(I*(a + b*x))] + (d^2*PolyLog[3, (-I)*E^(I*(a + b*x))])/b - (d
^2*PolyLog[3, I*E^(I*(a + b*x))])/b)/b^2 - (d*(c + d*x)*Sec[a])/b^2 + (c^2
+ 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])^2) + (-
(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - Sin[a/2])*(Cos[a/
2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2*x^2)/(4*b*(Cos[
a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2] + d^2*x*Sin[(b*
x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])
)
```

fricas [C] time = 0.55, size = 791, normalized size = 4.10

$$2\,d^2 \cos\left(bx+a\right)^2 \text{polylog}\left(3, i\,\cos\left(bx+a\right) + \sin\left(bx+a\right)\right) - 2\,d^2 \cos\left(bx+a\right)^2 \text{polylog}\left(3, i\,\cos\left(bx+a\right) - \sin\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 2*d^2
*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x +
 a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x + a)^2*pol
ylog(3, -I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x
 + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*co
s(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c
*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x -
2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (b^2*c^2
- 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d
^2)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(
b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x
+ 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) +
1) - (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
 + I*sin(b*x + a) + I) + (b^2*c^2 - 2*a*b*c*d + (a^2 - 2)*d^2)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 4*(b*d^2*x + b*c*d)*cos(b*x +
a) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(b*x + a))/(b^3*cos(b*x + a
)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \sec(bx+a)\tan(bx+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*tan(b*x + a)^2, x)
```

maple [B] time = 0.17, size = 584, normalized size = 3.03

$$-\frac{icd\,\mathrm{polylog}\left(2,-i\mathrm{e}^{i(bx+a)}\right)}{b^2}-\frac{cd\ln\left(1-i\mathrm{e}^{i(bx+a)}\right)a}{b^2}+\frac{d^2\,\mathrm{polylog}\left(3,-i\mathrm{e}^{i(bx+a)}\right)}{b^3}+\frac{cd\ln\left(1+i\mathrm{e}^{i(bx+a)}\right)a}{b^2}+\frac{d^2\ln\left(1+i\mathrm{e}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2*(d^2*x^2*b*exp(3*I*(b*x+a))+2*c*d*x*b*exp(3*I
*(b*x+a))+c^2*b*exp(3*I*(b*x+a))-d^2*x^2*b*exp(I*(b*x+a))-2*c*d*x*b*exp(I*(
b*x+a))-2*I*d^2*x*exp(3*I*(b*x+a))-c^2*b*exp(I*(b*x+a))-2*I*d*c*exp(3*I*(b*
x+a))-2*I*d^2*x*exp(I*(b*x+a))-2*I*c*d*exp(I*(b*x+a)))-1/b^2*c*d*ln(1-I*exp
(I*(b*x+a)))*a+d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+1/b^2*c*d*ln(1+I*exp(I*
(b*x+a)))*a+1/2/b*d^2*ln(1+I*exp(I*(b*x+a)))*x^2-1/2/b^3*a^2*d^2*ln(1+I*exp
(I*(b*x+a)))-1/2/b*d^2*ln(1-I*exp(I*(b*x+a)))*x^2-2*I/b^3*d^2*arctan(exp(I*
(b*x+a)))+I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x+I/b^2*c*d*polylog(2,I*exp
(I*(b*x+a)))-2*I/b^2*a*c*d*arctan(exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,-I*ex
p(I*(b*x+a)))+I/b^3*a^2*d^2*arctan(exp(I*(b*x+a)))+1/b*c*d*ln(1+I*exp(I*(b*
x+a)))*x-d^2*polylog(3,I*exp(I*(b*x+a)))/b^3-I/b^2*d^2*polylog(2,-I*exp(I*(
b*x+a)))*x+I/b*c^2*arctan(exp(I*(b*x+a)))+1/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x
+a)))-1/b*c*d*ln(1-I*exp(I*(b*x+a)))*x
```

maxima [B] time = 0.85, size = 1893, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - lo
g(sin(b*x + a) - 1)) - 2*a*c*d*(2*sin(b*x + a)/(sin(b*x + a)^2 - 1) + log(s
in(b*x + a) + 1) - log(sin(b*x + a) - 1))/b + a^2*d^2*(2*sin(b*x + a)/(sin(
b*x + a)^2 - 1) + log(sin(b*x + a) + 1) - log(sin(b*x + a) - 1))/b^2 - 4*((
2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^
2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*
d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x +
a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) -
 (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(
2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (2*(b*x + a)^2*d^2
+ 4*(b*c*d - a*d^2)*(b*x + a) - 4*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d
^2)*(b*x + a) - 2*d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a
*d^2)*(b*x + a) - 2*d^2)*cos(2*b*x + 2*a) - (-2*I*(b*x + a)^2*d^2 + (-4*I*b
*c*d + 4*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^2
*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) + 8*I*d^2)*sin(2*b*x + 2*a))*arct
an2(cos(b*x + a), -sin(b*x + a) + 1) - (4*(b*x + a)^2*d^2 - 8*I*b*c*d + 8*I
*a*d^2 + (8*b*c*d - (8*a + 8*I)*d^2)*(b*x + a))*cos(3*b*x + 3*a) + (4*(b*x
+ a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (8*b*c*d - (8*a - 8*I)*d^2)*(b*x + a))
*cos(b*x + a) + (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)
*d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*
x + 2*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(4*b*x + 4*a) -
(-8*I*b*c*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I
*b*x + I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*
d^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
```

```
 + 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8
*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b
*x + I*a)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I
*d^2 + (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) + 2*I*d^2)*
cos(4*b*x + 4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x +
a) + 4*I*d^2)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*s
in(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) -
 2*I*d^2 + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) - 2*I*d^2
)*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x +
a) - 4*I*d^2)*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x
+ a) - 2*d^2)*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*
x + a) - 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*s
in(b*x + a) + 1) - (-4*I*d^2*cos(4*b*x + 4*a) - 8*I*d^2*cos(2*b*x + 2*a) +
4*d^2*sin(4*b*x + 4*a) + 8*d^2*sin(2*b*x + 2*a) - 4*I*d^2)*polylog(3, I*e^(
I*b*x + I*a)) - (4*I*d^2*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^
2*sin(4*b*x + 4*a) - 8*d^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -I*e^(I*b
*x + I*a)) + 4*(-I*(b*x + a)^2*d^2 - 2*b*c*d + 2*a*d^2 + 2*(-I*b*c*d + (I*a
 - 1)*d^2)*(b*x + a))*sin(3*b*x + 3*a) - (-4*I*(b*x + a)^2*d^2 + 8*b*c*d -
8*a*d^2 + (-8*I*b*c*d - 8*(-I*a - 1)*d^2)*(b*x + a))*sin(b*x + a))/(-4*I*b^
2*cos(4*b*x + 4*a) - 8*I*b^2*cos(2*b*x + 2*a) + 4*b^2*sin(4*b*x + 4*a) + 8*
b^2*sin(2*b*x + 2*a) - 4*I*b^2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \tan^2(a+bx) \sec(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**2*sec(a + b*x), x)
```

3.300 $\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx$

Optimal. Leaf size=117

$$-\frac{id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+bx)\sec(a+bx)}{2b}$$

```
[Out] I*(d*x+c)*arctan(exp(I*(b*x+a)))/b-1/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/b^2
+1/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c)*s
ec(b*x+a)*tan(b*x+a)/b
```

Rubi [A] time = 0.13, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4413, 4181, 2279, 2391, 4185}

$$-\frac{id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}+\frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{(c+dx)\tan(a+}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] (I*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b - ((I/2)*d*PolyLog[2, (-I)*E^(I*(a
+ b*x))])/b^2 + ((I/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[a + b*
x])/(2*b^2) + ((c + d*x)*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\tan^2(a+bx)\,dx = -\int (c+dx)\sec(a+bx)\,dx + \int (c+dx)\sec^3(a+bx)\,dx$$
$$= \frac{2i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a}{2b}$$
$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\sec(a+bx)}{2b^2} + \frac{(c+dx)\sec(a+bx)\tan(a+}{2b}$$
$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{idLi_2\left(-ie^{i(a+bx)}\right)}{b^2} + \frac{idLi_2\left(ie^{i(a+bx)}\right)}{b^2} - \frac{d\sec(a}{2b}$$
$$= \frac{i(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{idLi_2\left(-ie^{i(a+bx)}\right)}{2b^2} + \frac{idLi_2\left(ie^{i(a+bx)}\right)}{2b^2} - \frac{d\sec(a}{2b}$$

Mathematica [B] time = 6.50, size = 555, normalized size = 4.74

$$-\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)}+\frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\sin\left(\frac{1}{2}(a+bx)\right)+\cos\left(\frac{1}{2}(a+bx)\right)\right)}-\frac{c\tanh^{-1}(\sin(a+bx))}{2b}+\frac{c\tan}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Tan[a + b*x]^2,x]
```

```
[Out] -1/2*(c*ArcTanh[Sin[a + b*x]])/b + (d*x*(a*Log[1 - Tan[(a + b*x)/2]] + I*Lo
g[1 + I*Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] - I*Log
```

```
[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)/2])] - I*Log[
1 + I*Tan[(a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + I*Log[1 -
 I*Tan[(a + b*x)/2]]*Log[(1/2 + I/2)*(1 + Tan[(a + b*x)/2])] - a*Log[1 + Ta
n[(a + b*x)/2]] - I*PolyLog[2, ((1 + I) - (1 - I)*Tan[(a + b*x)/2])/2] + I*
PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/2])] - I*PolyLog[2, ((1 + I) + (
1 - I)*Tan[(a + b*x)/2])/2] + I*PolyLog[2, ((1 - I) + (1 + I)*Tan[(a + b*x)
/2])/2]))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x)/2]] + I*Log[1 + I*Tan[(a + b*
x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])^2) - (d*Sin[(a
+ b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])) - (d*x)/(4*b*(Cos[
(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a +
 b*x)/2] + Sin[(a + b*x)/2])) + (c*Sec[a + b*x]*Tan[a + b*x])/(2*b)
```

fricas [B] time = 0.53, size = 435, normalized size = 3.72

$$\frac{i\,d\cos\left(bx+a\right)^{2}\operatorname{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)+i\,d\cos\left(bx+a\right)^{2}\operatorname{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx+a\right)\right)-i\,d\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + I*d*cos(b*x
+ a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - si
n(b*x + a)) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - (b*c - a*d)*cos(b*
x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*d*cos(b*x + a) + 2*(b*d*x +
b*c)*sin(b*x + a))/(b^2*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\sec\left(bx+a\right)\tan\left(bx+a\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*tan(b*x + a)^2, x)
```

maple [B] time = 0.12, size = 267, normalized size = 2.28

$$-\frac{i\left(bdx\,\mathrm{e}^{3i(bx+a)}-id\,\mathrm{e}^{3i(bx+a)}+cb\,\mathrm{e}^{3i(bx+a)}-bdx\,\mathrm{e}^{i(bx+a)}-id\,\mathrm{e}^{i(bx+a)}-cb\,\mathrm{e}^{i(bx+a)}\right)}{b^{2}\left(1+\mathrm{e}^{2i(bx+a)}\right)^{2}}+\frac{ic\arctan\left(\mathrm{e}^{i(bx+a)}\right)}{b}+\frac{d\ln\left(1+i\mathrm{e}\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2*(b*d*x*exp(3*I*(b*x+a))-I*d*exp(3*I*(b*x+a))+
c*b*exp(3*I*(b*x+a))-b*d*x*exp(I*(b*x+a))-I*d*exp(I*(b*x+a))-c*b*exp(I*(b*x
+a)))+I/b*c*arctan(exp(I*(b*x+a)))+1/2/b*d*ln(1+I*exp(I*(b*x+a)))*x+1/2/b^2
*d*ln(1+I*exp(I*(b*x+a)))*a-1/2/b*d*ln(1-I*exp(I*(b*x+a)))*x-1/2/b^2*d*ln(1
-I*exp(I*(b*x+a)))*a-1/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))+1/2*I/b^2*d*dilo
g(1-I*exp(I*(b*x+a)))-I/b^2*d*a*arctan(exp(I*(b*x+a)))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((tan(a + b*x)^2*(c + d*x))/cos(a + b*x),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan^{2}(a+bx)\sec(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*tan(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**2*sec(a + b*x), x)
```

3.301 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\sec^3(a+bx)}{c+dx},x\right)-\text{Int}\left(\frac{\sec(a+bx)}{c+dx},x\right)$$

```
[Out] -Unintegrable(sec(b*x+a)/(d*x+c),x)+Unintegrable(sec(b*x+a)^3/(d*x+c),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x), x] + Defer[Int][Sec[a + b*x]^3/(c + d*x
), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx=-\int \frac{\sec(a+bx)}{c+dx}\,dx+\int \frac{\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 26.39, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)^2}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c), x)
```

maple [A] time = 1.60, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\tan^2(bx+a)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x), x)
```

3.302 $\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=39

$$\text{Int}\left(\frac{\sec^3(a+bx)}{(c+dx)^2},x\right)-\text{Int}\left(\frac{\sec(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] -Unintegrable(sec(b*x+a)/(d*x+c)^2,x)+Unintegrable(sec(b*x+a)^3/(d*x+c)^2,x
)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] -Defer[Int][Sec[a + b*x]/(c + d*x)^2, x] + Defer[Int][Sec[a + b*x]^3/(c + d
*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx=-\int \frac{\sec(a+bx)}{(c+dx)^2}\,dx+\int \frac{\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 29.12, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Tan[a + b*x]^2)/(c + d*x)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\tan(bx+a)^2}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*tan(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\tan(bx+a)^2}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*tan(b*x + a)^2/(d*x + c)^2, x)
```

maple [A] time = 2.60, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\left(\tan^2(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan(a+b\,x)^2}{\cos(a+b\,x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(tan(a + b*x)^2/(cos(a + b*x)*(c + d*x)^2), x)
```

sympy [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(a+bx)\sec(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*tan(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**2*sec(a + b*x)/(c + d*x)**2, x)
```

3.303 $\int (c+dx)^m \tan^3(a+bx)\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\tan^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] Unintegrable((d*x+c)^m*tan(b*x+a)^3,x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.000, Rules used = {}

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Tan[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Tan[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \tan^3(a+bx)\,dx = \int (c+dx)^m \tan^3(a+bx)\,dx$$

Mathematica [A] time = 9.70, size = 0, normalized size = 0.00

$$\int (c+dx)^m \tan^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Tan[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Tan[a + b*x]^3, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \tan(bx+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="fricas")
[Out] integral((d*x + c)^m*tan(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \tan (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx + c)^m \left(\tan^3 (bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*tan(b*x+a)^3,x)
[Out] int((d*x+c)^m*tan(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \tan (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*tan(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*tan(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \tan (a + b\,x)^3 \,(c + d\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^m,x)
[Out] int(tan(a + b*x)^3*(c + d*x)^m, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \tan^3 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*tan(b*x+a)**3,x)
[Out] Integral((c + d*x)**m*tan(a + b*x)**3, x)
```

3.304 $\int (c + dx)^3 \tan^3(a + bx)\, dx$

Optimal. Leaf size=259

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3}-\frac{3id(c+dx}{}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-1/4*I*(d*x+c)^4/d-3*d^2*(d*x+c)*ln(1+
exp(2*I*(b*x+a)))/b^3+(d*x+c)^3*ln(1+exp(2*I*(b*x+a)))/b+3/2*I*d^3*polylog(
2,-exp(2*I*(b*x+a)))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2
+3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(
2*I*(b*x+a)))/b^4-3/2*d*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2
/b
```

Rubi [A] time = 0.36, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3720, 3719, 2190, 2279, 2391, 32, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Poly}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - ((I/4)*(c + d*x)^4)/d -
 (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^3 + ((c + d*x)^3*Log[1 +
E^((2*I)*(a + b*x))])/b + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/
b^4 - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d
^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (((3*I)/4)*d^3*Pol
yLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(2*b^2)
 + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\tan^3(a+bx)\,dx = \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2\tan^2(a+bx)\,dx}{2b} - \int (c+dx)^3\tan(a+b$$

$$= -\frac{i(c+dx)^4}{4d} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c}{1+e^{2i(}}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3d(c+dx)}{2}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{i(c+dx)^4}{4d} - \frac{3d^2(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b^3} + \frac{(c+dx)}{}$$

Mathematica [B] time = 6.89, size = 803, normalized size = 3.10

$$\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^3}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{3d\csc(a)\left(b^2e^{-i\tan^{-1}(\cot(a))}x^2 - \frac{\cot(a)\left(ibx\left(-2\tan^{-}\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Tan[a + b*x]^3,x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*
x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Poly
Log[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]
 - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + ((c + d*
x)^3*Sec[a + b*x]^2)/(2*b) + (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[
a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Co
s[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 +
 Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*
b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan
[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*
ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x
- ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos
[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[
a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]
] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)
*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^4*Sqrt[Csc[a]^
2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(c^2*d*Sin[b*x] + 2*c*d^
2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2
*x^2 + d^3*x^3)*Tan[a])/4
```

fricas [C] time = 0.45, size = 590, normalized size = 2.28

$$\frac{4\,b^3d^3x^3+12\,b^3cd^2x^2+12\,b^3c^2dx-3i\,d^3\mathrm{polylog}\left(4,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+3i\,d^3\mathrm{polylog}\left(4,\frac{\tan(bx+a)^2-2i\,\tan(bx\cdot}{\tan(bx+a)^2+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 3*I*d^3*polylog(4,
 (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 3*I*d^3*po
lylog(4, (tan(b*x + a)^2 - 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 4*
(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*tan(b*x + a)^2 +
(6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d - 6*I*d^3)*dilog(2*(I*t
an(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c
*d^2*x - 6*I*b^2*c^2*d + 6*I*d^3)*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*x +
a)^2 + 1) + 1) + 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3
*(b^3*c^2*d - b*d^3)*x)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) +
```

```
 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 3*b*c*d^2 + 3*(b^3*c^2*d - b*
d^3)*x)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 6*(b*d^3*x + b
*c*d^2)*polylog(3, (tan(b*x + a)^2 + 2*I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) + 6*(b*d^3*x + b*c*d^2)*polylog(3, (tan(b*x + a)^2 - 2*I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1)) - 12*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*
tan(b*x + a))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \tan (bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*tan(b*x + a)^3, x)
```

maple [B] time = 0.13, size = 720, normalized size = 2.78

$$\frac{3d^3 \operatorname{polylog}\left(3,-e^{2i(bx+a)}\right) x}{2b^3}+\frac{d^3 \ln\left(1+e^{2i(bx+a)}\right) x^3}{b}+\frac{6ic\, d^2a^2x}{b^2}-\frac{6ic^2dax}{b}+\frac{2d^3a^3 \ln\left(e^{i(bx+a)}\right)}{b^4}-\frac{3ia^4d^3}{2b^4}-ic\, d^2x^3-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*tan(b*x+a)^3,x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b*c^2*d*a*x+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/
b^4+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+I*c^3*x-3/b^3*d^2*c*ln(1+exp
(2*I*(b*x+a)))-3/b^3*d^3*ln(1+exp(2*I*(b*x+a)))*x+1/b*c^3*ln(1+exp(2*I*(b*x
+a)))+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))-3/2*I/b^4*a^4*d^3-I*c*d^2*x^3-3/2*I*
c^2*d*x^2+3/2/b^3*c*d^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,
-exp(2*I*(b*x+a)))*x+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*(b*x
+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^2*d*
x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2*I*(
b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2-1/4*I*d^3*x^4-2/b
*c^3*ln(exp(I*(b*x+a)))+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+6/b^3*d^2*c*ln(exp(
I*(b*x+a)))-6/b^4*d^3*a*ln(exp(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(2*I
*(b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6/b^3*c*d^2*a^2
*ln(exp(I*(b*x+a)))-3*I/b^2*c^2*d*a^2-2*I/b^3*a^3*d^3*x+4*I/b^3*c*d^2*a^3+6
/b^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-3*I/b^2*
polylog(2,-exp(2*I*(b*x+a)))*c*d^2*x+6*I/b^3*d^3*a*x+3/b*c^2*d*ln(1+exp(2*I
*(b*x+a)))*x+3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

maima [B] time = 1.19, size = 2405, normalized size = 9.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*tan(b*x+a)^3,x, algorithm="maxima")

[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b^3 + 2*(3*(b*x + a)^4*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a)^2 - (16*(b*x + a)^3*d^3 - 36*b*c*d^2 + 36*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a) + 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 8*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (16*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (32*I*(b*x + a)^3*d^3 - 72*I*b*c*d^2 + 72*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 - 72*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) + 3*((b*x + a)^4*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a)^3 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 2)*d^3)*(b*x + a)^2 - 24*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*(b*x + a)^4*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 + (24*b*c*d^2 - (24*a - 24*I)*d^3)*(b*x + a)^3 + (36*b^2*c^2*d - (72*a - 72*I)*b*c*d^2 + 36*(a^2 - 2*I*a - 1)*d^3)*(b*x + a)^2 - (-72*I*b^2*c^2*d - 72*(-2*I*a - 1)*b*c*d^2 + (-72*I*a^2 - 72*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 18*(a^2 - 1)*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 - 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 + (-18*I*a^2 + 18*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 48*I*(b*x + a)^2*d^3 + (-36*I*a^2 + 36*I)*d^3 + (-72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*I)*d^3)*(b*x + a) + (-8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 + 18*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-16*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 + 36*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 2*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(4*b*x + 4*a)

```
+ 4*(4*(b*x + a)^3*d^3 - 9*b*c*d^2 + 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a
)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 - 1)*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) -
 (12*d^3*cos(4*b*x + 4*a) + 24*d^3*cos(2*b*x + 2*a) + 12*I*d^3*sin(4*b*x +
4*a) + 24*I*d^3*sin(2*b*x + 2*a) + 12*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a))
 - (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*b*c*d^2 - 24*I
*(b*x + a)*d^3 + 18*I*a*d^3)*cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 48*I*(b*x
+ a)*d^3 + 36*I*a*d^3)*cos(2*b*x + 2*a) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 -
3*a*d^3)*sin(4*b*x + 4*a) + 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(
2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^4*d^3 + (-
12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a)^3 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2
+ (-18*I*a^2 + 36*I)*d^3)*(b*x + a)^2 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x +
a))*sin(4*b*x + 4*a) - (-6*I*(b*x + a)^4*d^3 - 36*I*b^2*c^2*d + 72*I*a*b*c*
d^2 - 36*I*a^2*d^3 + (-24*I*b*c*d^2 - 24*(-I*a - 1)*d^3)*(b*x + a)^3 + (-36
*I*b^2*c^2*d - 72*(-I*a - 1)*b*c*d^2 + (-36*I*a^2 - 72*a + 36*I)*d^3)*(b*x
+ a)^2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))/(-12*I*b^3*cos(4*b*x + 4*a) - 24*I*b^3*cos(2*b*x +
2*a) + 12*b^3*sin(4*b*x + 4*a) + 24*b^3*sin(2*b*x + 2*a) - 12*I*b^3))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan(a+bx)^3 (c+dx)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^3,x)
[Out] int(tan(a + b*x)^3*(c + d*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \tan^3(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*tan(b*x+a)**3,x)
[Out] Integral((c + d*x)**3*tan(a + b*x)**3, x)
```

3.305 $\int (c+dx)^2 \tan^3(a+bx)\,dx$

Optimal. Leaf size=169

$$\frac{d^2 \text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\log(\cos(a+bx))}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{d(c+dx)\tan(a+bx)}{b^2}+\frac{(c+dx)^2\log\left(1+e^{2i(a\cdot\right.}}{b}$$

[Out] c*d*x/b+1/2*d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-d^2*ln(cos(b*x+a))/b^3-I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*tan(b*x+a)^2/b

Rubi [A] time = 0.22, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3720, 3475, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{d(c+dx)\tan(a+bx)}{b^2}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{(c+}{}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^2*Tan[a + b*x]^3,x]

[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b - (d^2*Log[Cos[a + b*x]])/b^3 - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2)/(2*b)

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2\tan^3(a+bx)\,dx = \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\tan^2(a+bx)\,dx}{b} - \int (c+dx)^2\tan(a+bx)\,dx$$

$$= -\frac{i(c+dx)^3}{3d} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+d}{1+e^{2i(a+bx}}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{id(}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{id(}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{i(c+dx)^3}{3d} + \frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{id(}{}$$

Mathematica [B] time = 6.66, size = 454, normalized size = 2.69

$$-\frac{d^2\sec(a)(bx\sin(a)+\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx)))}{b^3\left(\sin^2(a)+\cos^2(a)\right)} + \frac{\sec(a)\sec(a+bx)\left(d^2(-x)\sin(bx)-cd\sin(b\right.}{b^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Tan[a + b*x]^3,x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I
*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*
Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (d^2
*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*
(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot
[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x -
 ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x
]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I
)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2
*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Si
n[b*x]))/b^2 - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

fricas [C] time = 0.45, size = 352, normalized size = 2.08

$$\frac{2\,b^2d^2x^2+4\,b^2cdx+d^2\mathrm{polylog}\left(3,\,\frac{\tan(bx+a)^2+2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+d^2\mathrm{polylog}\left(3,\,\frac{\tan(bx+a)^2-2i\,\tan(bx+a)-1}{\tan(bx+a)^2+1}\right)+2\left(b^2d^2x^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + d^2*polylog(3, (tan(b*x + a)^2 + 2*I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + d^2*polylog(3, (tan(b*x + a)^2 - 2*I
*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*tan(b*x + a)^2 + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(2*(I*tan(b*x + a)
- 1)/(tan(b*x + a)^2 + 1) + 1) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(2*(-I*tan
(b*x + a) - 1)/(tan(b*x + a)^2 + 1) + 1) + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 - d^2)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x + a)^2 + 1)) + 2*(b^2*d^
2*x^2 + 2*b^2*c*d*x + b^2*c^2 - d^2)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x
+ a)^2 + 1)) - 4*(b*d^2*x + b*c*d)*tan(b*x + a))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \tan(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*tan(b*x + a)^3, x)
```

maple [B] time = 0.10, size = 400, normalized size = 2.37

$$-\frac{id^2x^3}{3}-\frac{4icdax}{b}+\frac{4id^2a^3}{3b^3}+\frac{2b\,d^2x^2\mathrm{e}^{2i(bx+a)}-2id^2x\,\mathrm{e}^{2i(bx+a)}+4bcdx\,\mathrm{e}^{2i(bx+a)}-2icd\,\mathrm{e}^{2i(bx+a)}+2b\,c^2\mathrm{e}^{2i(bx+a)}-2id^2}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*tan(b*x+a)^3,x)
```

```
[Out] -1/3*I*d^2*x^3-4*I/b*c*d*a*x+4/3*I/b^3*d^2*a^3+2*(b*d^2*x^2*exp(2*I*(b*x+a)
)-I*d^2*x*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a))-I*c*d*exp(2*I*(b*x+a)
)+b*c^2*exp(2*I*(b*x+a))-I*d^2*x-I*c*d)/b^2/(1+exp(2*I*(b*x+a)))^2-I/b^2*c*
d*polylog(2,-exp(2*I*(b*x+a)))+I*c^2*x+2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+1/2
*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3-1/b^3*d^2*ln(1+exp(2*I*(b*x+a)))+2/b^
3*d^2*ln(exp(I*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))+1/b*c^2*ln(1+exp(
```

```
2*I*(b*x+a)))-2/b*c^2*ln(exp(I*(b*x+a)))+2*I/b^2*d^2*a^2*x-I*c*d*x^2+1/b*d^
2*ln(1+exp(2*I*(b*x+a)))*x^2-2*I/b^2*c*d*a^2+4/b^2*c*d*a*ln(exp(I*(b*x+a)))
-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x
```

maxima [B] time = 0.67, size = 1226, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1)) - 2*a*c*d*(1/(
sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2 - 1))/b + a^2*d^2*(1/(sin(b*x + a)
^2 - 1) - log(sin(b*x + a)^2 - 1))/b^2 + 2*(2*(b*x + a)^3*d^2 + 6*(b*c*d -
a*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (6*(b*x + a)^2*d^2 + 12*(b*c*d -
 a*d^2)*(b*x + a) - 6*d^2 + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a
) - d^2)*cos(4*b*x + 4*a) + 12*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x +
a) - d^2)*cos(2*b*x + 2*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^
2)*(b*x + a) - 6*I*d^2)*sin(4*b*x + 4*a) + (12*I*(b*x + a)^2*d^2 + (24*I*b*
c*d - 24*I*a*d^2)*(b*x + a) - 12*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x
 + 2*a), cos(2*b*x + 2*a) + 1) + 2*((b*x + a)^3*d^2 + 3*(b*c*d - a*d^2)*(b*
x + a)^2 - 6*(b*x + a)*d^2)*cos(4*b*x + 4*a) + (4*(b*x + a)^3*d^2 + (12*b*c
*d - (12*a - 12*I)*d^2)*(b*x + a)^2 + 12*b*c*d - 12*a*d^2 - (-24*I*b*c*d -
12*(-2*I*a - 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) + (6*b*c*d + 6*(b*x + a)*d
^2 - 6*a*d^2 + 6*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c
*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (-6*I*b*c*d - 6*I*(b*x + a)*
d^2 + 6*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*
I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) - (-3*I*(b*x + a)^2*
d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2 + (-3*I*(b*x + a)^2*d^2
+ (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) + 3*I*d^2)*cos(4*b*x + 4*a) + (-6*I*(b
*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) + 6*I*d^2)*cos(2*b*x +
 2*a) + 3*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(4*b*x +
 4*a) + 6*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) - d^2)*sin(2*b*x +
 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1
) - (-3*I*d^2*cos(4*b*x + 4*a) - 6*I*d^2*cos(2*b*x + 2*a) + 3*d^2*sin(4*b*x
 + 4*a) + 6*d^2*sin(2*b*x + 2*a) - 3*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)
) - (-2*I*(b*x + a)^3*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a)^2 + 12*I*(b*
x + a)*d^2)*sin(4*b*x + 4*a) - (-4*I*(b*x + a)^3*d^2 + (-12*I*b*c*d - 12*(-
I*a - 1)*d^2)*(b*x + a)^2 - 12*I*b*c*d + 12*I*a*d^2 + (24*b*c*d - (24*a - 1
2*I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^2*cos(4*b*x + 4*a) - 12*I*b^
2*cos(2*b*x + 2*a) + 6*b^2*sin(4*b*x + 4*a) + 12*b^2*sin(2*b*x + 2*a) - 6*I
*b^2))/b
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(a+b\,x)^3\,(c+d\,x)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \tan^3 (a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**2*tan(a + b*x)**3, x)
```

3.306 $\int (c+dx)\tan^3(a+bx)\,dx$

Optimal. Leaf size=108

$$-\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

```
[Out] 1/2*d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylo
g(2,-exp(2*I*(b*x+a)))/b^2-1/2*d*tan(b*x+a)/b^2+1/2*(d*x+c)*tan(b*x+a)^2/b
```

Rubi [A] time = 0.12, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3720, 3473, 8, 3719, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{dx}{2b}-\frac{i(c+dx)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x)
)])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a + b*x])/(
2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\tan^3(a+bx)\,dx = \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\int \tan^2(a+bx)\,dx}{2b} - \int (c+dx)\tan(a+bx)\,dx$$

$$= -\frac{i(c+dx)^2}{2d} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+bx)}{2b} + 2i\int \frac{e^{2i(a+bx)}(c+dx)}{1+e^{2i(a+bx)}}\,dx +$$

$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+}{2b}$$

$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c+dx)\tan^2(a+}{2b}$$

$$= \frac{dx}{2b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} - \frac{d\tan(a+bx)}{2b^2}$$

Mathematica [B] time = 6.15, size = 240, normalized size = 2.22

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))}-\frac{\cot(a)\left(i\text{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}(\cot\right.}\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Tan[a + b*x]^3,x]
```

```
[Out] (d*x*Sec[a + b*x]^2)/(2*b) + (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (C
ot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x
 - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b
*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2
*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[
a]^2*(Cos[a]^2 + Sin[a]^2)]) - (d*Sec[a]*Sec[a + b*x]*Sin[b*x])/(2*b^2) - (
d*x^2*Tan[a])/2 + (c*(2*Log[Cos[a + b*x]] + Tan[a + b*x]^2))/(2*b)
```

fricas [A] time = 0.43, size = 168, normalized size = 1.56

$$\frac{2\,bdx+2\,(bdx+bc)\tan(bx+a)^2+i\,d\text{Li}_2\left(\frac{2\,(i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)-i\,d\text{Li}_2\left(\frac{2\,(-i\,\tan(bx+a)-1)}{\tan(bx+a)^2+1}+1\right)+2\,(bdx+bc)\log\left(-\right.}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b*d*x + 2*(b*d*x + b*c)*tan(b*x + a)^2 + I*d*dilog(2*(I*tan(b*x + a)
 - 1)/(tan(b*x + a)^2 + 1) + 1) - I*d*dilog(2*(-I*tan(b*x + a) - 1)/(tan(b*
x + a)^2 + 1) + 1) + 2*(b*d*x + b*c)*log(-2*(I*tan(b*x + a) - 1)/(tan(b*x +
 a)^2 + 1)) + 2*(b*d*x + b*c)*log(-2*(-I*tan(b*x + a) - 1)/(tan(b*x + a)^2
+ 1)) - 2*d*tan(b*x + a))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\tan(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*tan(b*x + a)^3, x)
```

maple [A] time = 0.08, size = 183, normalized size = 1.69

$$-\frac{id\,x^2}{2}+icx+\frac{2bdx\,e^{2i(bx+a)}+2bc\,e^{2i(bx+a)}-id\,e^{2i(bx+a)}-id}{b^2\left(1+e^{2i(bx+a)}\right)^2}+\frac{c\ln\left(1+e^{2i(bx+a)}\right)}{b}-\frac{2c\ln\left(e^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-\frac{id\,a^2}{b^2}+\frac{d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*tan(b*x+a)^3,x)
```

```
[Out] -1/2*I*d*x^2+I*c*x+(2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp
(2*I*(b*x+a))-I*d)/b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c*ln(1+exp(2*I*(b*x+a)))-
2/b*c*ln(exp(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1+exp(2*I*(b*x+a)
))*x-1/2*I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.58, size = 519, normalized size = 4.81

$$-\frac{b^2dx^2+2\,b^2cx-(2\,bdx+2\,bc+2\,(bdx+bc)\cos(4\,bx+4\,a)+4\,(bdx+bc)\cos(2\,bx+2\,a)+(2i\,bdx+2i\,bc)\,s}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -(b^2*d*x^2 + 2*b^2*c*x - (2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*
a) + 4*(b*d*x + b*c)*cos(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4
*a) + (4*I*b*d*x + 4*I*b*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos
(2*b*x + 2*a) + 1) + (b^2*d*x^2 + 2*b^2*c*x)*cos(4*b*x + 4*a) + (2*b^2*d*x^
2 + 4*I*b*c + (4*b^2*c + 4*I*b*d)*x + 2*d)*cos(2*b*x + 2*a) + (d*cos(4*b*x
+ 4*a) + 2*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*
a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) - (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c
)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x + 2*a) + (b*d*x + b*c
)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)
^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) - (-I*b^2*d*x^2 - 2*I*b^2
*c*x)*sin(4*b*x + 4*a) - (-2*I*b^2*d*x^2 + 4*b*c - 4*(I*b^2*c - b*d)*x - 2*
I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x
 + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(a+b\,x\right)^3\,\left(c+d\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3*(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^3*(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\tan^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*tan(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*tan(a + b*x)**3, x)
```

3.307 $\int \frac{\tan^3(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tan^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/(d*x+c),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x),x]
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{c+dx} \, dx = \int \frac{\tan^3(a+bx)}{c+dx} \, dx$$

Mathematica [A] time = 6.53, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{c+dx} \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x),x]
[Out] Integrate[Tan[a + b*x]^3/(c + d*x), x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan\left(bx+a\right)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^3/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/(d*x + c), x)
```

maple [A] time = 1.76, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 - d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
```

```
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) + (d*cos(2*b*x + 2*a) + 2*(b*d*x + b*
c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*sin(2*b*x + 2*a))/(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x
 + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b
*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan\left(a+b\,x\right)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(tan(a + b*x)^3/(c + d*x), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x), x)
```

3.308 $\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\tan^3(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(tan(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[a + b*x]^3/(c + d*x)^2,x]
[Out] Defer[Int][Tan[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx = \int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Mathematica [A] time = 6.96, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[a + b*x]^3/(c + d*x)^2,x]
[Out] Integrate[Tan[a + b*x]^3/(c + d*x)^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(bx+a)^3}{d^2x^2 + 2\,cdx + c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(tan(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(bx+a)^3}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(b*x + a)^3/(d*x + c)^2, x)
```

maple [A] time = 2.73, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(bx+a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(tan(b*x+a)^3/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 3*d^2)*sin(2*b*x + 2*a)/(b^2
```

```
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
+ 2*(d*cos(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4
*a) + 2*d*sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*
c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2
*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2
*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\tan(a+bx)^3}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(a + b*x)^3/(c + d*x)^2,x)
```

```
[Out] int(tan(a + b*x)^3/(c + d*x)^2, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(tan(a + b*x)**3/(c + d*x)**2, x)
```

3.309 $\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\csc(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

Rubi [A] time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A] time = 13.37, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]*Sec[a + b*x]^3, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m \csc\left(bx+a\right)\sec\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

maple [A] time = 0.10, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\left(\sec^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
[Out] int((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)\sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)*sec(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3\,\sin(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.310 $\int (c+dx)^4 \csc(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=399

$$-\frac{3d^4\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^5}+\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}-\frac{3d^4\text{Li}_5\left(e^{2i(a+bx)}\right)}{2b^5}+\frac{6id^3(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a-\cdots}\right.}{b^4}$$

```
[Out] 2*I*d*(d*x+c)^3/b^2+1/2*(d*x+c)^4/b-2*(d*x+c)^4*arctanh(exp(2*I*(b*x+a)))/b
-6*d^2*(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b^3+6*I*d^3*(d*x+c)*polylog(2,-exp(
2*I*(b*x+a)))/b^4-2*I*d*(d*x+c)^3*polylog(2,exp(2*I*(b*x+a)))/b^2+3*I*d^3*(
d*x+c)*polylog(4,exp(2*I*(b*x+a)))/b^4-3*d^4*polylog(3,-exp(2*I*(b*x+a)))/b
^5-3*d^2*(d*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*d^2*(d*x+c)^2*polylog
(3,exp(2*I*(b*x+a)))/b^3+2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2-3
*I*d^3*(d*x+c)*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*d^4*polylog(5,-exp(2*I*
(b*x+a)))/b^5-3/2*d^4*polylog(5,exp(2*I*(b*x+a)))/b^5-2*d*(d*x+c)^3*tan(b*x
+a)/b^2+1/2*(d*x+c)^4*tan(b*x+a)^2/b
```

Rubi [A] time = 0.97, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 16, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.727$, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 32}

$$\frac{6id^3(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^4}-\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,e^{2i(a+bx)}\right)}{b^4}-\frac{3d^2\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] ((2*I)*d*(c + d*x)^3)/b^2 + (c + d*x)^4/(2*b) - (2*(c + d*x)^4*ArcTanh[E^((
2*I)*(a + b*x))])/b - (6*d^2*(c + d*x)^2*Log[1 + E^((2*I)*(a + b*x))])/b^3
+ ((6*I)*d^3*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + ((2*I)*d*(c
+ d*x)^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - ((2*I)*d*(c + d*x)^3*PolyL
og[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^4*PolyLog[3, -E^((2*I)*(a + b*x))])/
b^5 - (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c
+ d*x)^2*PolyLog[3, E^((2*I)*(a + b*x))])/b^3 - ((3*I)*d^3*(c + d*x)*PolyLo
g[4, -E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d^3*(c + d*x)*PolyLog[4, E^((2*I)*
(a + b*x))])/b^4 + (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (3*d^
4*PolyLog[5, E^((2*I)*(a + b*x))])/(2*b^5) - (2*d*(c + d*x)^3*Tan[a + b*x])
/b^2 + ((c + d*x)^4*Tan[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
```

```
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^4 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - (4d)\int (c+dx)^3$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - (4d)\int \frac{(c+dx)^3}{}$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3 (}{}$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - \frac{(2d)\int \left(2(c+dx\right.}{}$$

$$= \frac{(c+dx)^4 \log(\tan(a+bx))}{b} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} - \frac{(2d)\int (c+dx)^3 \mathrm{t}}{b}$$

$$= -\frac{2d(c+dx)^3 \tan(a+bx)}{b^2} + \frac{(c+dx)^4 \tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^4 \csc}{b}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2d(c+dx)^3 \tan(a+bx)}{b^2} + \frac{(c+dx)^4 \tan^2(a}{2b}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2}{}$$

$$= \frac{2id(c+dx)^3}{b^2} + \frac{(c+dx)^4}{2b} - \frac{2(c+dx)^4 \tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)^2}{}$$

Mathematica [B] time = 7.47, size = 2090, normalized size = 5.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -((c^2*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*
I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*L
```

og[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^3) - (c*d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a + b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/b^4 - (d^4*E^(I*a)*Csc[a]*((2*b^5*x^5)/E^((2*I)*a) + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 - E^((-I)*(a + b*x))] + (5*I)*b^4*(1 - E^((-2*I)*a))*x^4*Log[1 + E^((-I)*(a + b*x))] - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, -E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, -E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, -E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (20*(-1 + E^((2*I)*a))*(b^3*x^3*PolyLog[2, E^((-I)*(a + b*x))] - (3*I)*b^2*x^2*PolyLog[3, E^((-I)*(a + b*x))] - 6*b*x*PolyLog[4, E^((-I)*(a + b*x))] + (6*I)*PolyLog[5, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(10*b^5) + (x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4)*Csc[a]*Sec[a])/5 - ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) - ((I/2)*d^4*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^5*E^(I*a)) - (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + (d^4*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))])/b + (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - 6*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4, -E^((-2*I)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5)*Sec[a])/(20*E^(I*a)) + ((c + d*x)^4*Sec[a + b*x]^2)/(2*b) - (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (6*c^2*d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]^2)) + (c^4*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (6*c*d^3*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*L

```
og[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[
a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Co
t[a]]))])]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a
]^2)]) - (2*Sec[a]*Sec[a + b*x]*(c^3*d*Sin[b*x] + 3*c^2*d^2*x*Sin[b*x] + 3*
c*d^3*x^2*Sin[b*x] + d^4*x^3*Sin[b*x]))/b^2 - (2*c^3*d*Csc[a]*Sec[a]*(b^2*E
^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^(
(-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[
a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
 + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^
2]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C]   time = 1.05, size = 3308, normalized size = 8.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^
4*c^4 - 24*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) + I*sin(b*x + a)) - 2
4*d^4*cos(b*x + a)^2*polylog(5, cos(b*x + a) - I*sin(b*x + a)) + 24*d^4*cos
(b*x + a)^2*polylog(5, I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)
^2*polylog(5, I*cos(b*x + a) - sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylo
g(5, -I*cos(b*x + a) + sin(b*x + a)) + 24*d^4*cos(b*x + a)^2*polylog(5, -I*
cos(b*x + a) - sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x +
a) + I*sin(b*x + a)) - 24*d^4*cos(b*x + a)^2*polylog(5, -cos(b*x + a) - I*s
in(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x
- 4*I*b^3*c^3*d)*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (4*I
*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*cos
(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I
*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*c^2*d^2 + b*d^4)*
x)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 +
 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2 + b*
d^4)*x)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (4*I*b^3*d^4*
x^3 + 12*I*b^3*c*d^3*x^2 + 4*I*b^3*c^3*d + 12*I*b*c*d^3 + 12*I*(b^3*c^2*d^2
 + b*d^4)*x)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-4*I*b
^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 4*I*b^3*c^3*d - 12*I*b*c*d^3 - 12*I*(b^3*
c^2*d^2 + b*d^4)*x)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) +
(4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)
*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-4*I*b^3*d^4*x^3 -
 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*cos(b*x + a)^2*di
log(-cos(b*x + a) - I*sin(b*x + a)) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^
4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)^2*log(cos(b*x + a) +
I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 -
4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(cos(b*x + a)
```

```
+ I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2
+ 4*b^4*c^3*d*x + b^4*c^4)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a)
 + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*
b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x +
a) + I) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^
2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2*d^4)*x
^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin
(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*a^2*b^2
*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^2 + b^2
*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a
) - sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^3*d - 6*
a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^4*c^2*d^
2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*log(-I*cos
(b*x + a) + sin(b*x + a) + 1) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 4*a*b^3*c^
3*d - 6*a^2*b^2*c^2*d^2 + 4*(a^3 + 3*a)*b*c*d^3 - (a^4 + 6*a^2)*d^4 + 6*(b^
4*c^2*d^2 + b^2*d^4)*x^2 + 4*(b^4*c^3*d + 3*b^2*c*d^3)*x)*cos(b*x + a)^2*lo
g(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^
2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) +
 1/2*I*sin(b*x + a) + 1/2) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
 4*a^3*b*c*d^3 + a^4*d^4)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(
b*x + a) + 1/2) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^
4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*co
s(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c
^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)
*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*d^4*x^4 + 4*
b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b
^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) - I*
sin(b*x + a) + 1) - (b^4*c^4 - 4*a*b^3*c^3*d + 6*(a^2 + 1)*b^2*c^2*d^2 - 4*
(a^3 + 3*a)*b*c*d^3 + (a^4 + 6*a^2)*d^4)*cos(b*x + a)^2*log(-cos(b*x + a) -
 I*sin(b*x + a) + I) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2*polylog
(4, cos(b*x + a) + I*sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x
 + a)^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*
c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a)) + (-24*I*b*
d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x +
a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(4, -I*cos(b*x +
 a) + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*cos(b*x + a)^2*polylog(
4, -I*cos(b*x + a) - sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*cos(b*x
 + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b
*c*d^3)*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 12*(b^2
*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polylog(3, cos(b*x +
 a) + I*sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(
b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) +
 sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b
*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b
```

```
^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) +
 sin(b*x + a)) - 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2 + d^4)*cos(b
*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*
b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(
b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*cos(b*x + a)^2*p
olylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^
2 + 3*b^3*c^2*d^2*x + b^3*c^3*d)*cos(b*x + a)*sin(b*x + a))/(b^5*cos(b*x +
a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \csc(bx+a)\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*csc(b*x + a)*sec(b*x + a)^3, x)
```

maple [B] time = 0.25, size = 1729, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] 3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-3*d^4*polylog(3,-exp(2*I*(b*x+a)))
/b^5+1/b^5*d^4*a^4*ln(exp(I*(b*x+a))-1)+12/b^3*c^2*d^2*polylog(3,-exp(I*(b*
x+a)))+12/b^3*c^2*d^2*polylog(3,exp(I*(b*x+a)))-1/b^5*d^4*a^4*ln(1-exp(I*(b
*x+a)))+12/b^3*d^4*polylog(3,exp(I*(b*x+a)))*x^2+12/b^3*d^4*polylog(3,-exp(
I*(b*x+a)))*x^2-6/b^3*d^2*c^2*ln(1+exp(2*I*(b*x+a)))-6/b^3*d^4*ln(1+exp(2*I
*(b*x+a)))*x^2+4*I/b^2*d^4*x^3-8*I/b^5*d^4*a^3-3/b^3*c^2*d^2*polylog(3,-exp
(2*I*(b*x+a)))-3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+24*I/b^3*d^3*c*a*
x-24*d^4*polylog(5,-exp(I*(b*x+a)))/b^5-24*d^4*polylog(5,exp(I*(b*x+a)))/b^
5-12/b^3*d^3*c*ln(1+exp(2*I*(b*x+a)))*x+12*I/b^2*d^3*c*x^2+12*I/b^4*d^3*c*a
^2+6*I/b^4*d^4*polylog(2,-exp(2*I*(b*x+a)))*x-12*I/b^4*d^4*a^2*x+6*I/b^4*d^
3*c*polylog(2,-exp(2*I*(b*x+a)))+2*(b*d^4*x^4*exp(2*I*(b*x+a))+4*b*c*d^3*x^
3*exp(2*I*(b*x+a))+6*b*c^2*d^2*x^2*exp(2*I*(b*x+a))+4*b*c^3*d*x*exp(2*I*(b*
x+a))-2*I*d^4*x^3*exp(2*I*(b*x+a))+b*c^4*exp(2*I*(b*x+a))-6*I*c*d^3*x^2*exp
(2*I*(b*x+a))-6*I*c^2*d^2*x*exp(2*I*(b*x+a))-2*I*d^4*x^3-2*I*c^3*d*exp(2*I*
(b*x+a))-6*I*c*d^3*x^2-6*I*c^2*d^2*x-2*I*c^3*d)/b^2/(1+exp(2*I*(b*x+a)))^2-
4/b*c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+
a)))*x^2+6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-1/b*c^4*ln(1+exp(2*
I*(b*x+a)))+1/b*c^4*ln(exp(I*(b*x+a))+1)+1/b*c^4*ln(exp(I*(b*x+a))-1)+12/b^
5*d^4*a^2*ln(exp(I*(b*x+a)))+12/b^3*d^2*c^2*ln(exp(I*(b*x+a)))-1/b*d^4*ln(1
```

```
+exp(2*I*(b*x+a)))*x^4+4/b*c^3*d*ln(exp(I*(b*x+a))+1)*x+4/b*c^3*d*ln(1-exp(
I*(b*x+a)))*x+4/b^2*c^3*d*ln(1-exp(I*(b*x+a)))*a+6/b*c^2*d^2*ln(exp(I*(b*x+
a))+1)*x^2+24/b^3*c*d^3*polylog(3,-exp(I*(b*x+a)))*x-6/b^3*c^2*d^2*a^2*ln(1
-exp(I*(b*x+a)))+6/b*c^2*d^2*ln(1-exp(I*(b*x+a)))*x^2+24/b^3*c*d^3*polylog(
3,exp(I*(b*x+a)))*x+24*I/b^4*c*d^3*polylog(4,-exp(I*(b*x+a)))+24*I/b^4*c*d^
3*polylog(4,exp(I*(b*x+a)))-4*I/b^2*d^4*polylog(2,exp(I*(b*x+a)))*x^3+24*I/
b^4*d^4*polylog(4,exp(I*(b*x+a)))*x-4*I/b^2*d^4*polylog(2,-exp(I*(b*x+a)))*
x^3+24*I/b^4*d^4*polylog(4,-exp(I*(b*x+a)))*x-4*I/b^2*c^3*d*polylog(2,-exp(
I*(b*x+a)))-4*I/b^2*c^3*d*polylog(2,exp(I*(b*x+a)))-4/b^4*c*d^3*a^3*ln(exp(
I*(b*x+a))-1)+6/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a))-1)-4/b^2*c^3*d*a*ln(exp(I
*(b*x+a))-1)+1/b*d^4*ln(1-exp(I*(b*x+a)))*x^4+1/b*d^4*ln(exp(I*(b*x+a))+1)*
x^4-12*I/b^2*c*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-12*I/b^2*c^2*d^2*polylog(
2,-exp(I*(b*x+a)))*x-12*I/b^2*c^2*d^2*polylog(2,exp(I*(b*x+a)))*x-12*I/b^2*
c*d^3*polylog(2,exp(I*(b*x+a)))*x^2+4/b*c*d^3*ln(exp(I*(b*x+a))+1)*x^3+4/b*
c*d^3*ln(1-exp(I*(b*x+a)))*x^3+4/b^4*c*d^3*ln(1-exp(I*(b*x+a)))*a^3-24/b^4*
d^3*c*a*ln(exp(I*(b*x+a)))-4/b*c^3*d*ln(1+exp(2*I*(b*x+a)))*x-6/b^3*c*d^3*p
olylog(3,-exp(2*I*(b*x+a)))*x-6/b*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-3*I/b^
4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^4*polylog(2,-exp(2*I*(b*x+a)
))*x^3-3*I/b^4*c*d^3*polylog(4,-exp(2*I*(b*x+a)))+2*I/b^2*c^3*d*polylog(2,-
exp(2*I*(b*x+a)))
```

maxima [B] time = 6.05, size = 8770, normalized size = 21.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^4*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 4*a*c^3*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 6*a^2*c^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x +
 a)^2 - 1) - log(sin(b*x + a)^2))/b^2 - 4*a^3*c*d^3*(1/(sin(b*x + a)^2 - 1)
 + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + a^4*d^4*(1/(sin(b*x
 + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^4 + 2*(24*b
^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*a^3*d^4 + (12*(b*x + a)^4
*d^4 + 36*b^2*c^2*d^2 - 72*a*b*c*d^3 + 36*a^2*d^4 + 32*(b*c*d^3 - a*d^4)*(b
*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 24
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x
 + a) + 4*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8
*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d
^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a
^3 + 3*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 8*(3*(b*x + a)^4*d^4 + 9*b^2*c
^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*cos(2*b*x +
```

```
2*a) + (12*I*(b*x + a)^4*d^4 + 36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2
*d^4 + (32*I*b*c*d^3 - 32*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a
*b*c*d^3 + (36*I*a^2 + 36*I)*d^4)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^
2*c^2*d^2 + (72*I*a^2 + 72*I)*b*c*d^3 + (-24*I*a^3 - 72*I*a)*d^4)*(b*x + a)
)*sin(4*b*x + 4*a) + (24*I*(b*x + a)^4*d^4 + 72*I*b^2*c^2*d^2 - 144*I*a*b*c
*d^3 + 72*I*a^2*d^4 + (64*I*b*c*d^3 - 64*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c
^2*d^2 - 144*I*a*b*c*d^3 + (72*I*a^2 + 72*I)*d^4)*(b*x + a)^2 + (48*I*b^3*c
^3*d - 144*I*a*b^2*c^2*d^2 + (144*I*a^2 + 144*I)*b*c*d^3 + (-48*I*a^3 - 144
*I*a)*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x
 + 2*a) + 1) - (6*(b*x + a)^4*d^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*
c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*
d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a
)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*
cos(4*b*x + 4*a) + 12*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^
2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b
*x + a)^4*d^4 + (-24*I*b*c*d^3 + 24*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d
^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a
*b^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a)
 - (-12*I*(b*x + a)^4*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^3 + (-72
*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b*x + a)^2 + (-48*I*b^3*c
^3*d + 144*I*a*b^2*c^2*d^2 - 144*I*a^2*b*c*d^3 + 48*I*a^3*d^4)*(b*x + a))*s
in(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*(b*x + a)^4*d
^4 + 24*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*(b*x + a)^2 + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a) + 6*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b
^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^
2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a
)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3
*d^4)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 -
24*I*a*d^4)*(b*x + a)^3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4
)*(b*x + a)^2 + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 2
4*I*a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^4*d^4 + (48*I*b*
c*d^3 - 48*I*a*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*
I*a^2*d^4)*(b*x + a)^2 + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*
b*c*d^3 - 48*I*a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a),
-cos(b*x + a) + 1) - 24*((b*x + a)^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2
+ 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (12
*I*(b*x + a)^4*d^4 + 24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 - 24*
a^3*d^4 + (48*I*b*c*d^3 - 24*(2*I*a + 1)*d^4)*(b*x + a)^3 + (72*I*b^2*c^2*d
^2 - 72*(2*I*a + 1)*b*c*d^3 + (72*I*a^2 + 72*a)*d^4)*(b*x + a)^2 + (48*I*b^
3*c^3*d - 72*(2*I*a + 1)*b^2*c^2*d^2 + (144*I*a^2 + 144*a)*b*c*d^3 + (-48*I
*a^3 - 72*a^2)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (12*b^3*c^3*d - 36*a*b^2*
```

```
c^2*d^2 + 24*(b*x + a)^3*d^4 + 36*(a^2 + 1)*b*c*d^3 - 12*(a^3 + 3*a)*d^4 +
48*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 36*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1
)*d^4)*(b*x + a) + 12*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*
(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(
b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 24
*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 2*(b*x + a)^3*d^4 + 3*(a^2 + 1)*b*c*d^3 - (
a^3 + 3*a)*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c
*d^3 + (a^2 + 1)*d^4)*(b*x + a))*cos(2*b*x + 2*a) - (-12*I*b^3*c^3*d + 36*I
*a*b^2*c^2*d^2 - 24*I*(b*x + a)^3*d^4 + (-36*I*a^2 - 36*I)*b*c*d^3 + (12*I*
a^3 + 36*I*a)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a)^2 + (-36*I*b^2*c
^2*d^2 + 72*I*a*b*c*d^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a))*sin(4*b*x + 4*
a) - (-24*I*b^3*c^3*d + 72*I*a*b^2*c^2*d^2 - 48*I*(b*x + a)^3*d^4 + (-72*I*
a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4 + (-96*I*b*c*d^3 + 96*I*a*d^4
)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 + (-72*I*a^2 - 72*I)*d
^4)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (24*b^3*c^3*
d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 24*(b*x + a)^3*d^4 - 24*a^3*d^4 + 7
2*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*
(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d
^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d
^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a) + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2
 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a
)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(2*b*x + 2*a) +
 (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3
*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c
^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*(b*x + a))*sin(4*b*x + 4*a) + (48*
I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^
4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a)^2 + (144*I*b^2*c
^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)*(b*x + a))*sin(2*b*x + 2*a))*dilo
g(-e^(I*b*x + I*a)) + (24*b^3*c^3*d - 72*a*b^2*c^2*d^2 + 72*a^2*b*c*d^3 + 2
4*(b*x + a)^3*d^4 - 24*a^3*d^4 + 72*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 72*(b^2
*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a) + 24*(b^3*c^3*d - 3*a*b^2*c^2*d
^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*d^4 + 3*(b*c*d^3 - a*d^4)*(b*x +
 a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a))*cos(4*b*x + 4*a)
 + 48*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 + (b*x + a)^3*d^4 - a^3*
d^4 + 3*(b*c*d^3 - a*d^4)*(b*x + a)^2 + 3*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*
d^4)*(b*x + a))*cos(2*b*x + 2*a) + (24*I*b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 7
2*I*a^2*b*c*d^3 + 24*I*(b*x + a)^3*d^4 - 24*I*a^3*d^4 + (72*I*b*c*d^3 - 72*
I*a*d^4)*(b*x + a)^2 + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*a^2*d^4)*
(b*x + a))*sin(4*b*x + 4*a) + (48*I*b^3*c^3*d - 144*I*a*b^2*c^2*d^2 + 144*I
*a^2*b*c*d^3 + 48*I*(b*x + a)^3*d^4 - 48*I*a^3*d^4 + (144*I*b*c*d^3 - 144*I
*a*d^4)*(b*x + a)^2 + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*a^2*d^4)
*(b*x + a))*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-6*I*(b*x + a)^4*d^
4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*
I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1
8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2
```

```
- 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a) + (-6*I*(b*x + a)^4*d^
4 - 18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 18*I*a^2*d^4 + (-16*I*b*c*d^3 + 16*
I*a*d^4)*(b*x + a)^3 + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 + (-18*I*a^2 - 1
8*I)*d^4)*(b*x + a)^2 + (-12*I*b^3*c^3*d + 36*I*a*b^2*c^2*d^2 + (-36*I*a^2
- 36*I)*b*c*d^3 + (12*I*a^3 + 36*I*a)*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-
12*I*(b*x + a)^4*d^4 - 36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 36*I*a^2*d^4 + (
-32*I*b*c*d^3 + 32*I*a*d^4)*(b*x + a)^3 + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d
^3 + (-36*I*a^2 - 36*I)*d^4)*(b*x + a)^2 + (-24*I*b^3*c^3*d + 72*I*a*b^2*c^
2*d^2 + (-72*I*a^2 - 72*I)*b*c*d^3 + (24*I*a^3 + 72*I*a)*d^4)*(b*x + a))*co
s(2*b*x + 2*a) + 2*(3*(b*x + a)^4*d^4 + 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^
2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a
^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c
*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 4*(3*(b*x + a)^4*d^4
+ 9*b^2*c^2*d^2 - 18*a*b*c*d^3 + 9*a^2*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a)^
3 + 9*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (a^2 + 1)*d^4)*(b*x + a)^2 + 6*(b^3*c^3*
d - 3*a*b^2*c^2*d^2 + 3*(a^2 + 1)*b*c*d^3 - (a^3 + 3*a)*d^4)*(b*x + a))*sin
(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x +
2*a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3
+ (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^
3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a) +
 (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18*I*b^2
*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3*d - 3
6*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*cos(4*b*x +
 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^3 + (3
6*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*b^3*c^
3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a))*cos(
2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^
2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2
*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^
4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d
^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*co
s(b*x + a) + 1) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x +
 a)^3 + (18*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (1
2*I*b^3*c^3*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x
+ a) + (3*I*(b*x + a)^4*d^4 + (12*I*b*c*d^3 - 12*I*a*d^4)*(b*x + a)^3 + (18
*I*b^2*c^2*d^2 - 36*I*a*b*c*d^3 + 18*I*a^2*d^4)*(b*x + a)^2 + (12*I*b^3*c^3
*d - 36*I*a*b^2*c^2*d^2 + 36*I*a^2*b*c*d^3 - 12*I*a^3*d^4)*(b*x + a))*cos(4
*b*x + 4*a) + (6*I*(b*x + a)^4*d^4 + (24*I*b*c*d^3 - 24*I*a*d^4)*(b*x + a)^
3 + (36*I*b^2*c^2*d^2 - 72*I*a*b*c*d^3 + 36*I*a^2*d^4)*(b*x + a)^2 + (24*I*
b^3*c^3*d - 72*I*a*b^2*c^2*d^2 + 72*I*a^2*b*c*d^3 - 24*I*a^3*d^4)*(b*x + a)
)*cos(2*b*x + 2*a) - 3*((b*x + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 +
 6*(b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b
^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x
 + a)^4*d^4 + 4*(b*c*d^3 - a*d^4)*(b*x + a)^3 + 6*(b^2*c^2*d^2 - 2*a*b*c*d^
```

3 + a^2*d^4)*(b*x + a)^2 + 4*(b^3*c^3*d - 3*a*b^2*c^2*d^2 + 3*a^2*b*c*d^3 - a^3*d^4)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (18*I*d^4*cos(4*b*x + 4*a) + 36*I*d^4*cos(2*b*x + 2*a) - 18*d^4*sin(4*b*x + 4*a) - 36*d^4*sin(2*b*x + 2*a) + 18*I*d^4)*polylog(5, -e^(2*I*b*x + 2*I*a)) + (-144*I*d^4*cos(4*b*x + 4*a) - 288*I*d^4*cos(2*b*x + 2*a) + 144*d^4*sin(4*b*x + 4*a) + 288*d^4*sin(2*b*x + 2*a) - 144*I*d^4)*polylog(5, -e^(I*b*x + I*a)) + (-144*I*d^4*cos(4*b*x + 4*a) - 288*I*d^4*cos(2*b*x + 2*a) + 144*d^4*sin(4*b*x + 4*a) + 288*d^4*sin(2*b*x + 2*a) - 144*I*d^4)*polylog(5, e^(I*b*x + I*a)) + (24*b*c*d^3 + 36*(b*x + a)*d^4 - 24*a*d^4 + 12*(2*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*cos(4*b*x + 4*a) + 24*(2*b*c*d^3 + 3*(b*x + a)*d^4 - 2*a*d^4)*cos(2*b*x + 2*a) + (24*I*b*c*d^3 + 36*I*(b*x + a)*d^4 - 24*I*a*d^4)*sin(4*b*x + 4*a) + (48*I*b*c*d^3 + 72*I*(b*x + a)*d^4 - 48*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(2*I*b*x + 2*I*a)) - (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) + 288*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*sin(4*b*x + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x + a)*d^4 + 288*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, -e^(I*b*x + I*a)) - (144*b*c*d^3 + 144*(b*x + a)*d^4 - 144*a*d^4 + 144*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(4*b*x + 4*a) + 288*(b*c*d^3 + (b*x + a)*d^4 - a*d^4)*cos(2*b*x + 2*a) - (-144*I*b*c*d^3 - 144*I*(b*x + a)*d^4 + 144*I*a*d^4)*sin(4*b*x + 4*a) - (-288*I*b*c*d^3 - 288*I*(b*x + a)*d^4 + 288*I*a*d^4)*sin(2*b*x + 2*a))*polylog(4, e^(I*b*x + I*a)) + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a) + (-18*I*b^2*c^2*d^2 + 36*I*a*b*c*d^3 - 36*I*(b*x + a)^2*d^4 + (-18*I*a^2 - 18*I)*d^4 + (-48*I*b*c*d^3 + 48*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (-36*I*b^2*c^2*d^2 + 72*I*a*b*c*d^3 - 72*I*(b*x + a)^2*d^4 + (-36*I*a^2 - 36*I)*d^4 + (-96*I*b*c*d^3 + 96*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) + 6*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*(a^2 + 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) + 12*(3*b^2*c^2*d^2 - 6*a*b*c*d^3 + 6*(b*x + a)^2*d^4 + 3*(a^2 + 1)*d^4 + 8*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*(b*x + a)^2*d^4 + 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a) - 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 144*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a) + (72*I*b^2*c^2*d^2 - 144*I*a*b*c*d^3 + 72*I*(b*x + a)^2*d^4 + 72*I*a^2*d^4 + (144*I*b*c*d^3 - 144*I*a*d^4)*(b*x + a))*cos(4*b*x + 4*a) + (144*I*b^2*c^2*d^2 - 288*I*a*b*c*d^3 + 144*I*(b*x + a)^2*d^4 + 144*I*a^2*d^4 + (288*I*b*c*d^3 - 288*I*a*d^4)*(b*x + a))*cos(2*b*x + 2*a)

```
- 72*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x + a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 -
a*d^4)*(b*x + a))*sin(4*b*x + 4*a) - 144*(b^2*c^2*d^2 - 2*a*b*c*d^3 + (b*x
+ a)^2*d^4 + a^2*d^4 + 2*(b*c*d^3 - a*d^4)*(b*x + a))*sin(2*b*x + 2*a))*pol
ylog(3, e^(I*b*x + I*a)) + (-24*I*(b*x + a)^3*d^4 + (-72*I*b*c*d^3 + 72*I*a
*d^4)*(b*x + a)^2 + (-72*I*b^2*c^2*d^2 + 144*I*a*b*c*d^3 - 72*I*a^2*d^4)*(b
*x + a))*sin(4*b*x + 4*a) - (12*(b*x + a)^4*d^4 - 24*I*b^3*c^3*d + 72*I*a*b
^2*c^2*d^2 - 72*I*a^2*b*c*d^3 + 24*I*a^3*d^4 + (48*b*c*d^3 - (48*a - 24*I)*
d^4)*(b*x + a)^3 + (72*b^2*c^2*d^2 - (144*a - 72*I)*b*c*d^3 + 72*(a^2 - I*a
)*d^4)*(b*x + a)^2 + (48*b^3*c^3*d - (144*a - 72*I)*b^2*c^2*d^2 + 144*(a^2
- I*a)*b*c*d^3 - 24*(2*a^3 - 3*I*a^2)*d^4)*(b*x + a))*sin(2*b*x + 2*a))/(-6
*I*b^4*cos(4*b*x + 4*a) - 12*I*b^4*cos(2*b*x + 2*a) + 6*b^4*sin(4*b*x + 4*a
) + 12*b^4*sin(2*b*x + 2*a) - 6*I*b^4))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^4/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.311 $\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=325

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{Li}_3\left(e^{2i(a+\cdots}\right.}{2b^3}$$

```
[Out] 3/2*I*d*(d*x+c)^2/b^2+1/2*(d*x+c)^3/b-2*(d*x+c)^3*arctanh(exp(2*I*(b*x+a)))
/b-3*d^2*(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b^3+3/2*I*d^3*polylog(2,-exp(2*I*(b
*x+a)))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d*(d*x
+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*I*(b
*x+a)))/b^3+3/2*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/4*I*d^3*polyl
og(4,-exp(2*I*(b*x+a)))/b^4+3/4*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4-3/2*d
*(d*x+c)^2*tan(b*x+a)/b^2+1/2*(d*x+c)^3*tan(b*x+a)^2/b
```

Rubi [A] time = 0.64, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 18, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.818, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 6609, 2282, 6589, 3720, 3719, 2190, 2279, 2391, 32}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{2b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (((3*I)/2)*d*(c + d*x)^2)/b^2 + (c + d*x)^3/(2*b) - (2*(c + d*x)^3*ArcTanh[
E^((2*I)*(a + b*x))])/b - (3*d^2*(c + d*x)*Log[1 + E^((2*I)*(a + b*x))])/b^
3 + (((3*I)/2)*d^3*PolyLog[2, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/2)*d*(c
+ d*x)^2*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 - (((3*I)/2)*d*(c + d*x)^2*P
olyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)
*(a + b*x))])/(2*b^3) + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a + b*x))])/(
2*b^3) - (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3*I)/4)*
d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4 - (3*d*(c + d*x)^2*Tan[a + b*x])/(
2*b^2) + ((c + d*x)^3*Tan[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - (3d)\int (c+dx)$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - (3d)\int \frac{(c+dx}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int \left(2(c+d\right.}{}$$

$$= \frac{(c+dx)^3\log(\tan(a+bx))}{b} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} - \frac{(3d)\int (c+dx)^2}{2}$$

$$= -\frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^3 \mathrm{cs}}{b}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{3d(c+dx)^2\tan(a+bx)}{2b^2} + \frac{(c+dx)^3\tan^2(}{2b}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+dx}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+dx}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+dx}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+dx}{}$$

$$= \frac{3id(c+dx)^2}{2b^2} + \frac{(c+dx)^3}{2b} - \frac{2(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d^2(c+dx}{}$$

Mathematica [B] time = 6.91, size = 1486, normalized size = 4.57

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(c*d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2
*I)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*
Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I
```

```
)*(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 +
E^((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a
+ b*x))]))/E^((2*I)*a)))/b^3 - (d^3*E^(I*a)*Csc[a]*((b^4*x^4)/E^((2*I)*a) +
 (2*I)*b^3*(1 - E^((-2*I)*a))*x^3*Log[1 - E^((-I)*(a + b*x))] + (2*I)*b^3*(
1 - E^((-2*I)*a))*x^3*Log[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(
b^2*x^2*PolyLog[2, -E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-I)*(a
+ b*x))] - 2*PolyLog[4, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^((2
*I)*a))*(b^2*x^2*PolyLog[2, E^((-I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, E^((
-I)*(a + b*x))] - 2*PolyLog[4, E^((-I)*(a + b*x))]))/E^((2*I)*a)))/(4*b^4)
+ (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Csc[a]*Sec[a])/4 - ((I/4)*
c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(a + b*
x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (3*I)*(
1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I*a)) -
 (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*x^3*Log
[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*PolyLog[2,
-E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))] - Poly
Log[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + ((c + d*x)^3*Se
c[a + b*x]^2)/(2*b) - (c^3*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[
b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c*d^2*Sec[a]*(Cos[a]*Lo
g[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b^3*(Cos[a]^2 + Sin[a]
^2)) + (c^3*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[a] + Cos[a]*Sin[b*x]]*
Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan
[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*
b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))]
+ Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*Po
lyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2
*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*d^3*Csc[a]*((b^2*x^2)/E^(I*
ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a
]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[
a])/(2*b^4*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (3*Sec[a]*Sec[a + b*x]*(
c^2*d*Sin[b*x] + 2*c*d^2*x*Sin[b*x] + d^3*x^2*Sin[b*x]))/(2*b^2) - (3*c^2*d
*Csc[a]*Sec[a]*(b^2*E^(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[
a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*
I)*(b*x + ArcTan[Tan[a]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b
*x + ArcTan[Tan[a]]]] + I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan
[a])/Sqrt[1 + Tan[a]^2]))/(2*b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.83, size = 2260, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 + 6*I*d^3*cos(
b*x + a)^2*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 6*I*d^3*cos(b*x + a)
^2*polylog(4, cos(b*x + a) - I*sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polyl
og(4, I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, I*
cos(b*x + a) - sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x
 + a) + sin(b*x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) -
 sin(b*x + a)) - 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) + I*sin(b*
x + a)) + 6*I*d^3*cos(b*x + a)^2*polylog(4, -cos(b*x + a) - I*sin(b*x + a))
 + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d)*cos(b*x + a)^2*dilo
g(cos(b*x + a) + I*sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I
*b^2*c^2*d)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-3*I*b^2
*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2*dilog(
I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b
^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (
3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2
*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*
x - 3*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x
 + a)) + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*cos(b*x + a)^2
*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*
x - 3*I*b^2*c^2*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*x + a)) + (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2*log
(cos(b*x + a) + I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1
)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x +
a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x
 + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d +
 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I
*sin(b*x + a) + I) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d^3)*x)*cos(b*x
 + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b^3*d^3*x^3 + 3*b^3*c*d^2
*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(b^3*c^2*d + b*d
^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 3*a)*d^3 + 3*(
b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1
) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b^3*c^3 - 3*a*b^2*c^2*d +
3*a^2*b*c*d^2 - a^3*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b
*x + a) + 1/2) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c
^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*
x + a) + 1) - (b^3*c^3 - 3*a*b^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*
d^3)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^3*c^3 - 3*a*b
^2*c^2*d + 3*(a^2 + 1)*b*c*d^2 - (a^3 + 3*a)*d^3)*cos(b*x + a)^2*log(-cos(b
```

```
*x + a) - I*sin(b*x + a) + I) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylo
g(3, cos(b*x + a) + I*sin(b*x + a)) + 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, cos(b*x + a) - I*sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos(b*x +
 a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x + b*c*d^2)*cos
(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 6*(b*d^3*x + b*c*d^
2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 6*(b*d^3*x +
 b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 6*(b*
d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x + a))
+ 6*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b*x
 + a)) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x +
 a))/(b^4*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)\sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)*sec(b*x + a)^3, x)
```

maple [B] time = 0.16, size = 1115, normalized size = 3.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] -6*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x-6*I/b^2*c*d^2*polylog(2,-exp(I*(
b*x+a)))*x+6*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+3/2*I*d^3*polylog(2,-exp(2
*I*(b*x+a)))/b^4-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))-3/b^3*d^3*ln(1+exp(2*I*
(b*x+a)))*x-1/b*c^3*ln(1+exp(2*I*(b*x+a)))-1/b^4*d^3*a^3*ln(exp(I*(b*x+a))-
1)+6/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+6/b^3*c*d^2*polylog(3,exp(I*(b*x+
a)))+6/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+6/b^3*d^3*polylog(3,-exp(I*(b*x+
a)))*x+6*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-3/2/b^3*c*d^2*polylog(3,-exp(
2*I*(b*x+a)))-3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-3/4*I*d^3*polylog(
4,-exp(2*I*(b*x+a)))/b^4+(2*b*d^3*x^3*exp(2*I*(b*x+a))-3*I*d^3*x^2*exp(2*I*
(b*x+a))+6*b*c*d^2*x^2*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(2*I*(b*x+a))+6*b*c^
2*d*x*exp(2*I*(b*x+a))-3*I*c^2*d*exp(2*I*(b*x+a))-3*I*d^3*x^2+2*b*c^3*exp(2
*I*(b*x+a))-6*I*c*d^2*x-3*I*c^2*d)/b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c^3*ln(ex
p(I*(b*x+a))-1)+1/b*c^3*ln(exp(I*(b*x+a))+1)+3*I/b^2*c*d^2*polylog(2,-exp(2
*I*(b*x+a)))*x+3*I/b^2*d^3*x^2+3*I/b^4*d^3*a^2+6/b^3*d^2*c*ln(exp(I*(b*x+a)
))-6/b^4*d^3*a*ln(exp(I*(b*x+a)))+3/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)-3*I/
b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-3*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a))
```

```
)-3*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2-3*I/b^2*d^3*polylog(2,-exp(I*(b
*x+a)))*x^2+3/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+3/b*c^2*d*ln(1-exp(I*(b*x+a)))
*x+3/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-3/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+
3/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+3/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2-3/b^2
*c^2*d*a*ln(exp(I*(b*x+a))-1)+1/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+1/b^4*d^3*ln
(1-exp(I*(b*x+a)))*a^3+1/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-1/b*d^3*ln(1+exp(2*
I*(b*x+a)))*x^3+3/2*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2+6*I/b^3*d^3*
a*x+3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-3/b*c^2*d*ln(1+exp(2*I*(b*
x+a)))*x-3/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

maxima [B] time = 2.02, size = 4918, normalized size = 15.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 3*a*c^2*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log
(sin(b*x + a)^2))/b + 3*a^2*c*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a
)^2 - 1) - log(sin(b*x + a)^2))/b^2 - a^3*d^3*(1/(sin(b*x + a)^2 - 1) + log
(sin(b*x + a)^2 - 1) - log(sin(b*x + a)^2))/b^3 + 2*(18*b^2*c^2*d - 36*a*b*
c*d^2 + 18*a^2*d^3 + (8*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3 + 18*(b*c*d
^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x
 + a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)*(b
*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(4*b*
x + 4*a) + 4*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 9*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*cos(2
*b*x + 2*a) + (8*I*(b*x + a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (18*I*b*c*
d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^
2 + 18*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (16*I*(b*x + a)^3*d^3 + 36*I*b
*c*d^2 - 36*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c
^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 36*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))
*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (6*(b*x + a)^3*d^3 + 18*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x
 + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d
 - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^3*d^3
 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
(b*x + a))*cos(2*b*x + 2*a) - (-6*I*(b*x + a)^3*d^3 + (-18*I*b*c*d^2 + 18*I
*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3)*(b*
x + a))*sin(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 + (-36*I*b*c*d^2 + 36*I*a
*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3)*(b*x
+ a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*(b*x +
 a)^3*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3)*(b*x + a) + 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2
```

```
 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(
(b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*
d^2 + a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b
*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*
a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^
2 - 36*I*a*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*a^2*d
^3)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) -
 18*((b*x + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (1
2*I*(b*x + a)^3*d^3 + 18*b^2*c^2*d - 36*a*b*c*d^2 + 18*a^2*d^3 + (36*I*b*c*
d^2 - 18*(2*I*a + 1)*d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 36*(2*I*a + 1)*b*
c*d^2 + (36*I*a^2 + 36*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (9*b^2*c^2*d -
 18*a*b*c*d^2 + 12*(b*x + a)^2*d^3 + 9*(a^2 + 1)*d^3 + 18*(b*c*d^2 - a*d^3)
*(b*x + a) + 3*(3*b^2*c^2*d - 6*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)
*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6
*a*b*c*d^2 + 4*(b*x + a)^2*d^3 + 3*(a^2 + 1)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x
 + a))*cos(2*b*x + 2*a) - (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 - 12*I*(b*x + a)
^2*d^3 + (-9*I*a^2 - 9*I)*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a))*sin
(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I*(b*x + a)^2*d^3 +
(-18*I*a^2 - 18*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x
+ 2*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*
x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
- 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(4*b*x + 4*a) + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
+ 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I*
a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d^
3)*(b*x + a))*sin(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b
*x + a)^2*d^3 + 36*I*a^2*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(2
*b*x + 2*a))*dilog(-e^(I*b*x + I*a)) + (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b
*x + a)^2*d^3 + 18*a^2*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))
*cos(4*b*x + 4*a) + 36*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (18*I*b^2*c^2*d - 36*I
*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + 18*I*a^2*d^3 + (36*I*b*c*d^2 - 36*I*a*d
^3)*(b*x + a))*sin(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(
b*x + a)^2*d^3 + 36*I*a^2*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(
2*b*x + 2*a))*dilog(e^(I*b*x + I*a)) + (-4*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2
+ 9*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18
*I*a*b*c*d^2 + (-9*I*a^2 - 9*I)*d^3)*(b*x + a) + (-4*I*(b*x + a)^3*d^3 - 9*
I*b*c*d^2 + 9*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*
c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 9*I)*d^3)*(b*x + a))*cos(4*b*x + 4*a)
+ (-8*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-18*I*b*c*d^2 + 18*I
*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 18*I
)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d
^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 +
1)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 2*(4*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*
```

a*d^3 + 9*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(b^2*c^2*d - 2*a*b*c*d^2 + (a^2 + 1)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + 9*I*a^2*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*a^2*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 6*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (6*d^3*cos(4*b*x + 4*a) + 12*d^3*cos(2*b*x + 2*a) + 6*I*d^3*sin(4*b*x + 4*a) + 12*I*d^3*sin(2*b*x + 2*a) + 6*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) - (36*d^3*cos(4*b*x + 4*a) + 72*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) + 72*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*polylog(4, -e^(I*b*x + I*a)) - (36*d^3*cos(4*b*x + 4*a) + 72*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(4*b*x + 4*a) + 72*I*d^3*sin(2*b*x + 2*a) + 36*d^3)*polylog(4, e^(I*b*x + I*a)) + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3 + (-9*I*b*c*d^2 - 12*I*(b*x + a)*d^3 + 9*I*a*d^3)*cos(4*b*x + 4*a) + (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3)*cos(2*b*x + 2*a) + 3*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*x + a)*d^3 - 3*a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -e^(I*b*x + I*a)) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, e^(I*b*x + I*a)) + (-18*I*(b*x + a)^2*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (12*(b*x + a)^3*d^3 - 18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*a^2*d^3 + (36*b*c*d^2 - (36*a - 18*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a - 36*I)*b*c*d^2 + 36*(a^2 - I*a)*d^3)*(b*x + a))*sin(2*b*x + 2*a))/(-6*I*b^3*cos(4*b*x + 4*a) - 12*I*b

```
^3*cos(2*b*x + 2*a) + 6*b^3*sin(4*b*x + 4*a) + 12*b^3*sin(2*b*x + 2*a) - 6*
I*b^3))/b
```

mupad **[F(-1)]** time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)),x)
[Out] \text{Hanged}
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sec(b*x+a)**3,x)
[Out] Timed out
```

3.312 $\int (c+dx)^2 \csc(a+bx) \sec^3(a+bx)\,dx$

Optimal. Leaf size=201

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\log(\cos(a+bx))}{b^3}+\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-$$

```
[Out] c*d*x/b+1/2*d^2*x^2/b-2*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*ln(cos(b*
x+a))/b^3+I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*(d*x+c)*polylog(
2,exp(2*I*(b*x+a)))/b^2-1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/2*d^2*po
lylog(3,exp(2*I*(b*x+a)))/b^3-d*(d*x+c)*tan(b*x+a)/b^2+1/2*(d*x+c)^2*tan(b*
x+a)^2/b
```

Rubi [A] time = 0.41, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 13, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.591, Rules used = {2620, 14, 4420, 6741, 12, 6742, 2551, 4183, 2531, 2282, 6589, 3720, 3475}

$$\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{d^2\text{PolyLog}\left(3,e\right.}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (c*d*x)/b + (d^2*x^2)/(2*b) - (2*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/
b - (d^2*Log[Cos[a + b*x]])/b^3 + (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (I*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d^2*Po
lyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(2*b^3) - (d*(c + d*x)*Tan[a + b*x])/b^2 + ((c + d*x)^2*Tan[a + b*x]^2
)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2551

```
Int[Log[u_]*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[((a + b*x)^(m + 1
)*Log[u])/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[SimplifyIntegrand[((a +
 b*x)^(m + 1)*D[u, x])/u, x], x], x] /; FreeQ[{a, b, m}, x] && InverseFunct
ionFreeQ[u, x] && NeQ[m, -1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
```

```
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int (c+dx)\Big($$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - (2d)\int \frac{(c+dx)\big(}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\big(2\,\mathrm{lo}}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int \big(2(c+dx)\,\mathrm{lo}}{}$$

$$= \frac{(c+dx)^2\log(\tan(a+bx))}{b} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} - \frac{d\int (c+dx)\tan^2}{b}$$

$$= -\frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2\tan^2(a+bx)}{2b} + \frac{\int 2b(c+dx)^2\csc(2}{b}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c+dx)\tan(a+bx)}{b^2} + \frac{(c+dx)^2}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} - \frac{d(c}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(}{}$$

$$= \frac{cdx}{b} + \frac{d^2x^2}{2b} - \frac{2(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\log(\cos(a+bx))}{b^3} + \frac{id(}{}$$

<u>**Mathematica**</u> **[B]** time = 6.76, size = 875, normalized size = 4.35

$$-\frac{\sec(a)(\cos(a)\log(\cos(a)\cos(bx)-\sin(a)\sin(bx))+bx\sin(a))c^2}{b\left(\cos^2(a)+\sin^2(a)\right)} + \frac{\csc(a)(\log(\cos(bx)\sin(a)+\cos(a)\sin(bx))\,\mathrm{s}}{b\left(\cos^2(a)+\sin^2(a)\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] -1/6*(d^2*E^(I*a)*Csc[a]*((2*b^3*x^3)/E^((2*I)*a) + (3*I)*b^2*(1 - E^((-2*I
)*a))*x^2*Log[1 - E^((-I)*(a + b*x))] + (3*I)*b^2*(1 - E^((-2*I)*a))*x^2*Lo
```

```
g[1 + E^((-I)*(a + b*x))] - (6*(-1 + E^((2*I)*a))*(b*x*PolyLog[2, -E^((-I)*
(a + b*x))] - I*PolyLog[3, -E^((-I)*(a + b*x))]))/E^((2*I)*a) - (6*(-1 + E^
((2*I)*a))*(b*x*PolyLog[2, E^((-I)*(a + b*x))] - I*PolyLog[3, E^((-I)*(a +
b*x))]))/E^((2*I)*a)))/b^3 + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Csc[a]*Sec[a])/
3 - ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*
I)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
 - (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*
E^(I*a)) + ((c + d*x)^2*Sec[a + b*x]^2)/(2*b) - (c^2*Sec[a]*(Cos[a]*Log[Cos
[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) -
(d^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(
b^3*(Cos[a]^2 + Sin[a]^2)) + (c^2*Csc[a]*(-(b*x*Cos[a]) + Log[Cos[b*x]*Sin[
a] + Cos[a]*Sin[b*x]]*Sin[a]))/(b*(Cos[a]^2 + Sin[a]^2)) - (c*d*Csc[a]*((b^
2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*
Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x -
ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTa
n[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot
[a]^2])*Sec[a])/(b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + (Sec[a]*Sec[a
+ b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/b^2 - (c*d*Csc[a]*Sec[a]*(b^2*E^
(I*ArcTan[Tan[a]])*x^2 + ((I*b*x*(-Pi + 2*ArcTan[Tan[a]]) - Pi*Log[1 + E^((
-2*I)*b*x)] - 2*(b*x + ArcTan[Tan[a]])*Log[1 - E^((2*I)*(b*x + ArcTan[Tan[a
]]))] + Pi*Log[Cos[b*x]] + 2*ArcTan[Tan[a]]*Log[Sin[b*x + ArcTan[Tan[a]]]]
+ I*PolyLog[2, E^((2*I)*(b*x + ArcTan[Tan[a]]))])*Tan[a])/Sqrt[1 + Tan[a]^2
]))/(b^2*Sqrt[Sec[a]^2*(Cos[a]^2 + Sin[a]^2)])
```

fricas [C] time = 0.65, size = 1396, normalized size = 6.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x +
a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin
(b*x + a)) - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
 - 2*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) - 2*d^2*c
os(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) - 2*d^2*cos(b*x +
a)^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) + 2*d^2*cos(b*x + a)^2*polylog(3, -c
os(b*x + a) - I*sin(b*x + a)) + b^2*c^2 + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*
x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*c
os(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*
c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2
*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a)) + (2*I*b*d^2*
x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (-2*I
*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))
```

```
+ (2*I*b*d^2*x + 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
+ a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*s
in(b*x + a)) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos
(b*x + a) + I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(
b*x + a) + I) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)
 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*
cos(b*x + a) + sin(b*x + a) + 1) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d -
 a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*c^2
 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*
x + a) + 1/2) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*cos(b*x + a)^2*log(-1/2*cos
(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*
c*d - a^2*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b^
2*c^2 - 2*a*b*c*d + (a^2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin
(b*x + a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x
+ a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b^2*c^2 - 2*a*b*c*d + (a^
2 + 1)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 2*(b*d
^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)*sec(b*x + a)^3, x)
```

maple [B] time = 0.13, size = 614, normalized size = 3.05

$$\frac{2b\,d^2x^2\mathrm{e}^{2i(bx+a)}-2id^2x\,\mathrm{e}^{2i(bx+a)}+4bcdx\,\mathrm{e}^{2i(bx+a)}-2icd\,\mathrm{e}^{2i(bx+a)}+2b\,c^2\mathrm{e}^{2i(bx+a)}-2id^2x-2icd}{b^2\left(1+\mathrm{e}^{2i(bx+a)}\right)^2}+\frac{d^2a^2\ln\left(\mathrm{e}^{i(bx+a)}-\right.}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] -1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+1/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1
)+1/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-1/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+1/b*d
^2*ln(exp(I*(b*x+a))+1)*x^2+2*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+2*d^2*poly
```

```
log(3,exp(I*(b*x+a)))/b^3-1/b*c^2*ln(1+exp(2*I*(b*x+a)))+1/b*c^2*ln(exp(I*(
b*x+a))-1)+1/b*c^2*ln(exp(I*(b*x+a))+1)-1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+
2*(b*d^2*x^2*exp(2*I*(b*x+a))-I*d^2*x*exp(2*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b
*x+a))-I*c*d*exp(2*I*(b*x+a))+b*c^2*exp(2*I*(b*x+a))-I*d^2*x-I*c*d)/b^2/(1+
exp(2*I*(b*x+a)))^2+I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+I/b^2*c*d*poly
log(2,-exp(2*I*(b*x+a)))-2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+2/b*c*d*ln(1-exp(
I*(b*x+a)))*x+2/b^2*c*d*ln(1-exp(I*(b*x+a)))*a+2/b*c*d*ln(exp(I*(b*x+a))+1)
*x-2/b^2*c*d*a*ln(exp(I*(b*x+a))-1)-2*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*
x-2*I/b^2*d^2*polylog(2,exp(I*(b*x+a)))*x-2*I/b^2*c*d*polylog(2,-exp(I*(b*x
+a)))-2*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))-1/b^3*d^2*ln(1+exp(2*I*(b*x+a))
)+2/b^3*d^2*ln(exp(I*(b*x+a)))
```

maxima [B] time = 0.81, size = 2442, normalized size = 12.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(sin(b*x +
 a)^2)) - 2*a*c*d*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 - 1) - log(s
in(b*x + a)^2))/b + a^2*d^2*(1/(sin(b*x + a)^2 - 1) + log(sin(b*x + a)^2 -
1) - log(sin(b*x + a)^2))/b^2 - 2*(4*(b*x + a)*d^2*cos(4*b*x + 4*a) + 4*I*(
b*x + a)*d^2*sin(4*b*x + 4*a) - 4*b*c*d + 4*a*d^2 - (2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*
(b*x + a) + d^2)*cos(2*b*x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I
*a*d^2)*(b*x + a) + 2*I*d^2)*sin(4*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I
*b*c*d - 8*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(sin(2*b*
x + 2*a), cos(2*b*x + 2*a) + 1) + (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b
*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4*b*x + 4*a
) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (-
2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a))*sin(4*b*x + 4*a)
- (-4*I*(b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a))*sin(2*b*x + 2
*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*(b*x + a)^2*d^2 + 4*(b*c*
d - a*d^2)*(b*x + a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*co
s(4*b*x + 4*a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(2*b*
x + 2*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*sin(4*
b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*sin(
2*b*x + 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (4*I*(b*x + a)^2*d
^2 + 4*b*c*d - 4*a*d^2 + (8*I*b*c*d - 4*(2*I*a + 1)*d^2)*(b*x + a))*cos(2*b
*x + 2*a) + (2*b*c*d + 2*(b*x + a)*d^2 - 2*a*d^2 + 2*(b*c*d + (b*x + a)*d^2
 - a*d^2)*cos(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x +
2*a) - (-2*I*b*c*d - 2*I*(b*x + a)*d^2 + 2*I*a*d^2)*sin(4*b*x + 4*a) - (-4*
I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(2*I*b*
```

```
x + 2*I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d
^2 - a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8*
I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x
+ I*a)) - (4*b*c*d + 4*(b*x + a)*d^2 - 4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 -
 a*d^2)*cos(4*b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*
a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*sin(4*b*x + 4*a) + (8*I*b*
c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a
)) - (-I*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2 + (-I
*(b*x + a)^2*d^2 + (-2*I*b*c*d + 2*I*a*d^2)*(b*x + a) - I*d^2)*cos(4*b*x +
4*a) + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c*d + 4*I*a*d^2)*(b*x + a) - 2*I*d^2
)*cos(2*b*x + 2*a) + ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*
sin(4*b*x + 4*a) + 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a) + d^2)*
sin(2*b*x + 2*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x
 + 2*a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a) + (I*
(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x + a))*cos(4*b*x + 4*a) + (2*
I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (
(b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(4*b*x + 4*a) - 2*((b*x +
 a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)
^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - (I*(b*x + a)^2*d^2 + (2*I*b*c*d
 - 2*I*a*d^2)*(b*x + a) + (I*(b*x + a)^2*d^2 + (2*I*b*c*d - 2*I*a*d^2)*(b*x
 + a))*cos(4*b*x + 4*a) + (2*I*(b*x + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b
*x + a))*cos(2*b*x + 2*a) - ((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*sin(4*b*x + 4*a) - 2*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (-
I*d^2*cos(4*b*x + 4*a) - 2*I*d^2*cos(2*b*x + 2*a) + d^2*sin(4*b*x + 4*a) +
2*d^2*sin(2*b*x + 2*a) - I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) - (4*I*d^2
*cos(4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) - 8*d
^2*sin(2*b*x + 2*a) + 4*I*d^2)*polylog(3, -e^(I*b*x + I*a)) - (4*I*d^2*cos(
4*b*x + 4*a) + 8*I*d^2*cos(2*b*x + 2*a) - 4*d^2*sin(4*b*x + 4*a) - 8*d^2*si
n(2*b*x + 2*a) + 4*I*d^2)*polylog(3, e^(I*b*x + I*a)) + (4*(b*x + a)^2*d^2
- 4*I*b*c*d + 4*I*a*d^2 + (8*b*c*d - (8*a - 4*I)*d^2)*(b*x + a))*sin(2*b*x
+ 2*a))/(-2*I*b^2*cos(4*b*x + 4*a) - 4*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4
*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) - 2*I*b^2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)),x)
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \csc(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sec(b*x+a)**3,x)
[Out] Integral((c + d*x)**2*csc(a + b*x)*sec(a + b*x)**3, x)
```

3.313 $\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=139

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{c\tan^2(a+bx)}{2b}+\frac{c\log(\tan(a+bx))}{b}+\frac{dx\tan^2(a+bx)}{2b}-\frac{2dx\tan$$

```
[Out] 1/2*d*x/b-2*d*x*arctanh(exp(2*I*a+2*I*b*x))/b+c*ln(tan(b*x+a))/b+1/2*I*d*po
lylog(2,-exp(2*I*(b*x+a)))/b^2-1/2*I*d*polylog(2,exp(2*I*(b*x+a)))/b^2-1/2*
d*tan(b*x+a)/b^2+1/2*c*tan(b*x+a)^2/b+1/2*d*x*tan(b*x+a)^2/b
```

Rubi [A] time = 0.14, antiderivative size = 141, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2620, 14, 4420, 2548, 12, 4183, 2279, 2391, 3473, 8}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{2b^2}-\frac{d\tan(a+bx)}{2b^2}+\frac{(c+dx)\tan^2(a+bx)}{2b}+\frac{(c+dx)\log(\tan(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (d*x)/(2*b) - (2*d*x*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*x*Log[Tan[a + b*x
]])/b + ((c + d*x)*Log[Tan[a + b*x]])/b + ((I/2)*d*PolyLog[2, -E^((2*I)*(a
+ b*x))])/b^2 - ((I/2)*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d*Tan[a +
b*x])/(2*b^2) + ((c + d*x)*Tan[a + b*x]^2)/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2548

```
Int[Log[u_], x_Symbol] :> Simp[x*Log[u], x] - Int[SimplifyIntegrand[(x*D[u,
 x])/u, x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sec^3(a+bx)\,dx = \frac{(c+dx)\log(\tan(a+bx))}{b} + \frac{(c+dx)\tan^2(a+bx)}{2b} - d\int \Big(\frac{\log(\tan(a+b}{b}$$

$$= \frac{(c+dx)\log(\tan(a+bx))}{b} + \frac{(c+dx)\tan^2(a+bx)}{2b} - \frac{d\int \tan^2(a+bx)\,dx}{2b}$$

$$= -\frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\tan(a+bx))}{b} - \frac{d\tan(a+bx)}{2b^2} + \frac{(c}{}$$

$$= \frac{dx}{2b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\tan(a+bx))}{b} - \frac{d\tan(a+bx)}{2b^2} +$$

$$= \frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\tan(a}{b}$$

$$= \frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\tan(a}{b}$$

$$= \frac{dx}{2b} - \frac{2dx\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{dx\log(\tan(a+bx))}{b} + \frac{(c+dx)\log(\tan(a}{b}$$

Mathematica [A] time = 0.56, size = 212, normalized size = 1.53

$$\frac{d\left(\frac{1}{2}i\text{Li}_2\left(-e^{2i(a+bx)}\right)+\frac{1}{2}i(a+bx)^2-(a+bx)\log\left(1+e^{2i(a+bx)}\right)\right)}{b^2}+\frac{d\left((a+bx)\log\left(1-e^{2i(a+bx)}\right)-\frac{1}{2}i\left((a+bx)^2+\text{L}\right.\right.}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sec[a + b*x]^3,x]
```

```
[Out] (a*d*Log[Cos[a + b*x]])/b^2 - (a*d*Log[Sin[a + b*x]])/b^2 + (d*((I/2)*(a +
b*x)^2 - (a + b*x)*Log[1 + E^((2*I)*(a + b*x))] + (I/2)*PolyLog[2, -E^((2*I
)*(a + b*x))]))/b^2 + (d*((a + b*x)*Log[1 - E^((2*I)*(a + b*x))] - (I/2)*((
a + b*x)^2 + PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (d*x*Sec[a + b*x]^2)/
(2*b) - (c*(2*Log[Cos[a + b*x]] - 2*Log[Sin[a + b*x]] - Sec[a + b*x]^2))/(2
*b) - (d*Tan[a + b*x])/(2*b^2)
```

fricas [B] time = 0.56, size = 760, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(-I*d*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x + a)) + I*d*cos(b*x
 + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(I*c
os(b*x + a) + sin(b*x + a)) + I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin
(b*x + a)) + I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d
*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a)) + I*d*cos(b*x + a)^2*
dilog(-cos(b*x + a) + I*sin(b*x + a)) - I*d*cos(b*x + a)^2*dilog(-cos(b*x +
 a) - I*sin(b*x + a)) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) + I*s
in(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x
+ a) + I) + (b*d*x + b*c)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a)
+ 1) - (b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I) -
(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*
x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1) - (b*d*x + a
*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1) - (b*d*x + a*d)*
cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*cos(b*
x + a)^2*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) + (b*c - a*d)*co
s(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + (b*d*x + a
*d)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1) - (b*c - a*d)*co
s(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b*d*x + a*d)*cos(b*
x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - (b*c - a*d)*cos(b*x + a)
^2*log(-cos(b*x + a) - I*sin(b*x + a) + I) + b*d*x - d*cos(b*x + a)*sin(b*x
 + a) + b*c)/(b^2*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \csc (bx + a) \sec (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)*sec(b*x + a)^3, x)
```

maple [B] time = 0.11, size = 270, normalized size = 1.94

$$\frac{2bdx\,e^{2i(bx+a)} + 2bc\,e^{2i(bx+a)} - id\,e^{2i(bx+a)} - id}{b^2\left(1 + e^{2i(bx+a)}\right)^2} + \frac{c\ln\left(e^{i(bx+a)} - 1\right)}{b} - \frac{c\ln\left(1 + e^{2i(bx+a)}\right)}{b} + \frac{c\ln\left(e^{i(bx+a)} + 1\right)}{b} - \frac{d\ln\left(1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x)
```

```
[Out] (2*b*d*x*exp(2*I*(b*x+a))+2*b*c*exp(2*I*(b*x+a))-I*d*exp(2*I*(b*x+a))-I*d)/
b^2/(1+exp(2*I*(b*x+a)))^2+1/b*c*ln(exp(I*(b*x+a))-1)-1/b*c*ln(1+exp(2*I*(b
*x+a)))+1/b*c*ln(exp(I*(b*x+a))+1)-1/b*d*ln(1+exp(2*I*(b*x+a)))*x+1/2*I*d*p
olylog(2,-exp(2*I*(b*x+a)))/b^2+1/b*d*ln(exp(I*(b*x+a))+1)*x-I*d*polylog(2,
```

```
-exp(I*(b*x+a)))/b^2+1/b*d*ln(1-exp(I*(b*x+a)))*x+1/b^2*d*ln(1-exp(I*(b*x+a
)))*a-I*d*polylog(2,exp(I*(b*x+a)))/b^2-1/b^2*d*a*ln(exp(I*(b*x+a))-1)
```

maxima [B] time = 0.63, size = 1035, normalized size = 7.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos
(2*b*x + 2*a) + (2*I*b*d*x + 2*I*b*c)*sin(4*b*x + 4*a) + (4*I*b*d*x + 4*I*b
*c)*sin(2*b*x + 2*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2*
b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(4*b*x + 4*a) + 4*(b*d*x + b*c)*cos(2*b*
x + 2*a) - (-2*I*b*d*x - 2*I*b*c)*sin(4*b*x + 4*a) - (-4*I*b*d*x - 4*I*b*c)
*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(4*b
*x + 4*a) + 4*b*c*cos(2*b*x + 2*a) + 2*I*b*c*sin(4*b*x + 4*a) + 4*I*b*c*sin
(2*b*x + 2*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(4*b*x + 4*a) + 4*b*d*x*cos(2*b*x + 2*a) + 2*I*b*d*x*sin(4*b*x + 4*a) + 4
*I*b*d*x*sin(2*b*x + 2*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(2*b*x + 2*a) - (d*cos(4*b*x + 4*a) + 2
*d*cos(2*b*x + 2*a) + I*d*sin(4*b*x + 4*a) + 2*I*d*sin(2*b*x + 2*a) + d)*di
log(-e^(2*I*b*x + 2*I*a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) +
2*I*d*sin(4*b*x + 4*a) + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(-e^(I*b*x + I*
a)) + (2*d*cos(4*b*x + 4*a) + 4*d*cos(2*b*x + 2*a) + 2*I*d*sin(4*b*x + 4*a)
 + 4*I*d*sin(2*b*x + 2*a) + 2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c
 + (-I*b*d*x - I*b*c)*cos(4*b*x + 4*a) + (-2*I*b*d*x - 2*I*b*c)*cos(2*b*x +
 2*a) + (b*d*x + b*c)*sin(4*b*x + 4*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a))*
log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*
b*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*
cos(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b
*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b
*d*x + I*b*c + (I*b*d*x + I*b*c)*cos(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*c
os(2*b*x + 2*a) - (b*d*x + b*c)*sin(4*b*x + 4*a) - 2*(b*d*x + b*c)*sin(2*b*
x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b*
d*x + 4*b*c - 2*I*d)*sin(2*b*x + 2*a) + 2*d)/(-2*I*b^2*cos(4*b*x + 4*a) - 4
*I*b^2*cos(2*b*x + 2*a) + 2*b^2*sin(4*b*x + 4*a) + 4*b^2*sin(2*b*x + 2*a) -
 2*I*b^2)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)\csc(a + bx)\sec^3(a + bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)*sec(a + b*x)**3, x)
```

3.314 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 9.08, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\sec\left(bx+a\right)^{3}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)*sec(b*x + a)^3/(d*x + c), x)
```

maple [A] time = 3.35, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)\left(\sec^{3}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
(2*(b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b
*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*
b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*
b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 +
2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(2*(b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2 + d^2)*sin(2*b*x + 2*a)/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
```

```
b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cos(2*b*x + 2*a)), x) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*si
n(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*s
in(2*b*x + 2*a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2
 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x
^2 + 2*b^2*c*d*x + b^2*c^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x
+ c)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x
+ a) + c), x) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d
*x + c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + (d*cos(2
*b*x + 2*a) + 2*(b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + d*si
n(2*b*x + 2*a))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a)^2 + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x +
4*a)^2 + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(4*b*x + 4*a)*sin(2*b*x
 + 2*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(2*b*x + 2*a)^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+bx)^3 \sin(a+bx)\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c),x)
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x), x)
```

3.315 $\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=25

$$\text{Int}\left(\frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Defer[Int][(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 6.65, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2,x]
```

```
[Out] Integrate[(Csc[a + b*x]*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)\sec(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 5.62, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)\left(\sec^3(bx+a)\right)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] (4*(b*d*x + b*c)*cos(2*b*x + 2*a)^2 + 4*(b*d*x + b*c)*sin(2*b*x + 2*a)^2 +
2*((b*d*x + b*c)*cos(2*b*x + 2*a) - d*sin(2*b*x + 2*a))*cos(4*b*x + 4*a) +
2*(b*d*x + b*c)*cos(2*b*x + 2*a) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b
^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c
^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*inte
grate(2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 3*d^2)*sin(2*b*x + 2*a)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
```

```
(b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^
4)*cos(2*b*x + 2*a)^2 + (b^2*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2
+ 4*b^2*c^3*d*x + b^2*c^4)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^4*x^4 + 4*b^2*c*d^
3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4)*cos(2*b*x + 2*a)), x)
- (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 +
 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2
*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 +
4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a
)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2
*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
s(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2
*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2 + 2*c
*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)
*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + (b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3 + 3*b^
2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a)^2 + 4*(b^2*d^3*x^3
+ 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a)^2 + (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)^2 + 4*(b^
2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4*b*x + 4*a)*sin
(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^
2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b
*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cos(2*b*x + 2*a))*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x
+ (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(
b*x + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + 2*(d*cos
(2*b*x + 2*a) + (b*d*x + b*c)*sin(2*b*x + 2*a) + d)*sin(4*b*x + 4*a) + 2*d*
sin(2*b*x + 2*a))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
+ (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(4*b*x + 4*a
)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x
 + 2*a)^2 + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sin(4
*b*x + 4*a)^2 + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)
*sin(4*b*x + 4*a)*sin(2*b*x + 2*a) + 4*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*sin(2*b*x + 2*a)^2 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3 + 2*(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d
*x + b^2*c^3)*cos(2*b*x + 2*a))*cos(4*b*x + 4*a) + 4*(b^2*d^3*x^3 + 3*b^2*c
*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cos(2*b*x + 2*a))
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos(a+bx)^3 \sin(a+bx) (c+dx)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)*sec(a + b*x)**3/(c + d*x)**2, x)
```

3.316 $\int (c+dx)^m \csc^2(a+bx) \sec^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\csc^2(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

Rubi [A] time = 0.23, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx = \int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A] time = 30.51, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^2*Sec[a + b*x]^3, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

maple [A] time = 0.14, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\csc^2(bx+a)\right) \left(\sec^3(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
[Out] int((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{\cos(a+b\,x)^3 \sin(a+b\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2),x)
[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.317 $\int (c+dx)^3 \csc^2(a+bx) \sec^3(a+bx)\,dx$

Optimal. Leaf size=486

$$\frac{3id^3\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^4}-\frac{6d^3\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(e^{i(a+bx)}\right)}{b^4}-\frac{9id^3\text{Li}_4\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{9id^3\text{Li}_4\left(ie^{i(a-}\right.}{b^4}$$

```
[Out] -6*I*d^2*(d*x+c)*polylog(2,exp(I*(b*x+a)))/b^3-3*I*(d*x+c)^3*arctan(exp(I*(
b*x+a)))/b-6*d*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b^2-3/2*(d*x+c)^3*csc(b*x+
a)/b-3*I*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4+6*I*d^2*(d*x+c)*polylog(2,-exp
(I*(b*x+a)))/b^3+3*I*d^3*polylog(2,-I*exp(I*(b*x+a)))/b^4+9/2*I*d*(d*x+c)^2
*polylog(2,-I*exp(I*(b*x+a)))/b^2-9/2*I*d*(d*x+c)^2*polylog(2,I*exp(I*(b*x+
a)))/b^2+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*
x+a)))/b^4-9*d^2*(d*x+c)*polylog(3,-I*exp(I*(b*x+a)))/b^3+9*d^2*(d*x+c)*pol
ylog(3,I*exp(I*(b*x+a)))/b^3+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4-9*I*d^3*po
lylog(4,-I*exp(I*(b*x+a)))/b^4-6*I*d^2*(d*x+c)*arctan(exp(I*(b*x+a)))/b^3-3
/2*d*(d*x+c)^2*sec(b*x+a)/b^2+1/2*(d*x+c)^3*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A] time = 1.21, antiderivative size = 486, normalized size of antiderivative = 1.00, number of steps used = 44, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 6609, 2282, 6589, 4183, 2622, 6741, 2279, 2391}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^3}-\frac{9d^2(c+dx)\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^3}+\frac{9d^2(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-6*I)*d^2*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^3 - ((3*I)*(c + d*x)^3*Arc
Tan[E^(I*(a + b*x))])/b - (6*d*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b^2 -
(3*(c + d*x)^3*Csc[a + b*x])/(2*b) + ((6*I)*d^2*(c + d*x)*PolyLog[2, -E^(I*
(a + b*x))])/b^3 + ((3*I)*d^3*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^4 + (((9*
I)/2)*d*(c + d*x)^2*PolyLog[2, (-I)*E^(I*(a + b*x))])/b^2 - ((3*I)*d^3*Poly
Log[2, I*E^(I*(a + b*x))])/b^4 - (((9*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(I
*(a + b*x))])/b^2 - ((6*I)*d^2*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^3 -
 (6*d^3*PolyLog[3, -E^(I*(a + b*x))])/b^4 - (9*d^2*(c + d*x)*PolyLog[3, (-I
)*E^(I*(a + b*x))])/b^3 + (9*d^2*(c + d*x)*PolyLog[3, I*E^(I*(a + b*x))])/b
^3 + (6*d^3*PolyLog[3, E^(I*(a + b*x))])/b^4 - ((9*I)*d^3*PolyLog[4, (-I)*E
^(I*(a + b*x))])/b^4 + ((9*I)*d^3*PolyLog[4, I*E^(I*(a + b*x))])/b^4 - (3*d
*(c + d*x)^2*Sec[a + b*x])/(2*b^2) + ((c + d*x)^3*Csc[a + b*x]*Sec[a + b*x]
^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6741

```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3}{}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3}{}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3}{}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3}{}$$

$$= \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^3\csc(a+bx)}{2b} + \frac{(c+dx)^3}{}$$

$$= \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} -$$

$$= \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} + \frac{3(c+dx)^3\tanh^{-1}(\sin(a+bx))}{2b} -$$

$$= -\frac{9d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} -$$

$$= -\frac{9d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+dx)^2\tanh^{-1}(\cos(a+bx))}{2b^2} -$$

$$= -\frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{3d(c+}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{9d(c+}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+}{}$$

$$= -\frac{6id^2(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^3} - \frac{3i(c+dx)^3\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d(c+}{}$$

Mathematica [A] time = 7.90, size = 819, normalized size = 1.69

$$-\frac{\csc(a+bx)\left(bc^3+3b\cos(2a+2bx)c^3+3bdxc^2+9bdx\cos(2a+2bx)c^2+3d\sin(2a+2bx)c^2+3bd^2x^2c+9bd^2\right.}{4b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (3*d*((c + d*x)^2*Log[1 - E^(I*(a + b*x))] - (c + d*x)^2*Log[1 + E^(I*(a +
b*x))] + ((2*I)*d*(b*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] + I*d*PolyLog[3
, -E^(I*(a + b*x))]))/b^2 + (2*d*((-I)*b*(c + d*x)*PolyLog[2, E^(I*(a + b*x
))] + d*PolyLog[3, E^(I*(a + b*x))]))/b^2))/b^2 - (3*((2*I)*b^3*c^3*ArcTan[
E^(I*(a + b*x))] + (4*I)*b*c*d^2*ArcTan[E^(I*(a + b*x))] - 3*b^3*c^2*d*x*Lo
g[1 - I*E^(I*(a + b*x))] - 2*b*d^3*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^3*c*d
^2*x^2*Log[1 - I*E^(I*(a + b*x))] - b^3*d^3*x^3*Log[1 - I*E^(I*(a + b*x))]
+ 3*b^3*c^2*d*x*Log[1 + I*E^(I*(a + b*x))] + 2*b*d^3*x*Log[1 + I*E^(I*(a +
b*x))] + 3*b^3*c*d^2*x^2*Log[1 + I*E^(I*(a + b*x))] + b^3*d^3*x^3*Log[1 + I
*E^(I*(a + b*x))] - I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(I*(a
 + b*x))] + I*d*(2*d^2 + 3*b^2*(c + d*x)^2)*PolyLog[2, I*E^(I*(a + b*x))] +
 6*b*c*d^2*PolyLog[3, (-I)*E^(I*(a + b*x))] + 6*b*d^3*x*PolyLog[3, (-I)*E^(
I*(a + b*x))] - 6*b*c*d^2*PolyLog[3, I*E^(I*(a + b*x))] - 6*b*d^3*x*PolyLog
[3, I*E^(I*(a + b*x))] + (6*I)*d^3*PolyLog[4, (-I)*E^(I*(a + b*x))] - (6*I)
*d^3*PolyLog[4, I*E^(I*(a + b*x))]))/(2*b^4) - (Csc[a + b*x]*Sec[a + b*x]^2
*(b*c^3 + 3*b*c^2*d*x + 3*b*c*d^2*x^2 + b*d^3*x^3 + 3*b*c^3*Cos[2*a + 2*b*x
] + 9*b*c^2*d*x*Cos[2*a + 2*b*x] + 9*b*c*d^2*x^2*Cos[2*a + 2*b*x] + 3*b*d^3
*x^3*Cos[2*a + 2*b*x] + 3*c^2*d*Sin[2*a + 2*b*x] + 6*c*d^2*x*Sin[2*a + 2*b*
x] + 3*d^3*x^2*Sin[2*a + 2*b*x]))/(4*b^2)
```

fricas [C] time = 0.84, size = 2218, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 18*I*d^3*cos(b*x + a
)^2*polylog(4, I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + 18*I*d^3*cos(b
*x + a)^2*polylog(4, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*I*d^3
*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) - 1
8*I*d^3*cos(b*x + a)^2*polylog(4, -I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
 a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) + I*sin(b*x + a))*sin(b
*x + a) + 12*d^3*cos(b*x + a)^2*polylog(3, cos(b*x + a) - I*sin(b*x + a))*s
in(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
```

```
a))*sin(b*x + a) - 12*d^3*cos(b*x + a)^2*polylog(3, -cos(b*x + a) - I*sin(b
*x + a))*sin(b*x + a) + 2*b^3*c^3 + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*x
+ a)^2*dilog(cos(b*x + a) + I*sin(b*x + a))*sin(b*x + a) + (12*I*b*d^3*x +
12*I*b*c*d^2)*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x + a))*sin(b*x +
 a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b
*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-9*I*b^2*d^3
*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d - 6*I*d^3)*cos(b*x + a)^2*dilog(I*c
os(b*x + a) - sin(b*x + a))*sin(b*x + a) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^
2*x + 9*I*b^2*c^2*d + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b
*x + a))*sin(b*x + a) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d
 + 6*I*d^3)*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x +
a) + (-12*I*b*d^3*x - 12*I*b*c*d^2)*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*
sin(b*x + a))*sin(b*x + a) + (12*I*b*d^3*x + 12*I*b*c*d^2)*cos(b*x + a)^2*d
ilog(-cos(b*x + a) - I*sin(b*x + a))*sin(b*x + a) - 6*(b^2*d^3*x^2 + 2*b^2*
c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*
sin(b*x + a) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*
a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a)
- 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)^2*log(cos(b*x +
a) - I*sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2
 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*
x + a) + I)*sin(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d
 - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a
)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*
c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) + 1)*s
in(b*x + a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*
d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*d^3)*x)*cos(b*x + a)^2*log(-I*co
s(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*
x^2 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + (a^3 + 2*a)*d^3 + (3*b^3*c^2*d + 2*b*
d^3)*x)*cos(b*x + a)^2*log(-I*cos(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a)
 + 6*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x +
a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*s
in(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b
*x + a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + 3*(b^3*c^3
 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*lo
g(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 6*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(
b*x + a) + 1)*sin(b*x + a) - 3*(b^3*c^3 - 3*a*b^2*c^2*d + (3*a^2 + 2)*b*c*d
^2 - (a^3 + 2*a)*d^3)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + I
)*sin(b*x + a) - 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, I*cos(b*x
 + a) + sin(b*x + a))*sin(b*x + a) + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*
polylog(3, I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 18*(b*d^3*x + b*c*
d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a)
 + 18*(b*d^3*x + b*c*d^2)*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a) - sin(b
```

```
*x + a))*sin(b*x + a) - 6*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x +
b^3*c^3)*cos(b*x + a)^2 - 6*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b
*x + a)*sin(b*x + a))/(b^4*cos(b*x + a)^2*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

maple [B] time = 0.56, size = 1629, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -9*I*d^3*polylog(4,-I*exp(I*(b*x+a)))/b^4+3*I*d^3*polylog(2,-I*exp(I*(b*x+a
)))/b^4+9*I*d^3*polylog(4,I*exp(I*(b*x+a)))/b^4-6*d^3*polylog(3,-exp(I*(b*x
+a)))/b^4+6*d^3*polylog(3,exp(I*(b*x+a)))/b^4+3/b^2*c^2*d*ln(exp(I*(b*x+a))
-1)-3/b^2*c^2*d*ln(exp(I*(b*x+a))+1)+3/b^4*d^3*a^2*ln(exp(I*(b*x+a))-1)-3*I
*d^3*polylog(2,I*exp(I*(b*x+a)))/b^4-3/b^2*d^3*ln(exp(I*(b*x+a))+1)*x^2+3/b
^2*d^3*ln(1-exp(I*(b*x+a)))*x^2+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^2-9*I/b^3*
a^2*c*d^2*arctan(exp(I*(b*x+a)))+9*I/b^2*c*d^2*polylog(2,-I*exp(I*(b*x+a)))
*x+9*I/b^2*a*c^2*d*arctan(exp(I*(b*x+a)))-9*I/b^2*c*d^2*polylog(2,I*exp(I*(
b*x+a)))*x+6*I/b^4*d^3*a*arctan(exp(I*(b*x+a)))-6*I/b^3*c*d^2*arctan(exp(I*
(b*x+a)))-3/2/b^4*a^3*d^3*ln(1+I*exp(I*(b*x+a)))+9/b^3*d^3*polylog(3,I*exp(
I*(b*x+a)))*x+3/2/b*d^3*ln(1-I*exp(I*(b*x+a)))*x^3-3/2/b*d^3*ln(1+I*exp(I*(
b*x+a)))*x^3-9/b^3*d^3*polylog(3,-I*exp(I*(b*x+a)))*x+9/b^3*d^2*c*polylog(3
,I*exp(I*(b*x+a)))-9/b^3*d^2*c*polylog(3,-I*exp(I*(b*x+a)))+3/2/b^4*a^3*d^3
*ln(1-I*exp(I*(b*x+a)))+9/2/b*d^2*c*ln(1-I*exp(I*(b*x+a)))*x^2-9/2/b*d^2*c*
ln(1+I*exp(I*(b*x+a)))*x^2+9/2/b*c^2*d*ln(1-I*exp(I*(b*x+a)))*x+9/2/b^2*c^2
*d*ln(1-I*exp(I*(b*x+a)))*a+9/2/b^3*a^2*c*d^2*ln(1+I*exp(I*(b*x+a)))-9/2/b*
c^2*d*ln(1+I*exp(I*(b*x+a)))*x-9/2/b^2*c^2*d*ln(1+I*exp(I*(b*x+a)))*a-9/2/b
^3*a^2*c*d^2*ln(1-I*exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(
b*x+a))-1)*(3*d^3*x^3*b*exp(5*I*(b*x+a))+9*c*d^2*x^2*b*exp(5*I*(b*x+a))+9*c
^2*d*x*b*exp(5*I*(b*x+a))+2*d^3*x^3*b*exp(3*I*(b*x+a))+3*c^3*b*exp(5*I*(b*x
+a))+6*c*d^2*x^2*b*exp(3*I*(b*x+a))+6*I*c*d^2*x*exp(I*(b*x+a))+6*c^2*d*x*b*
exp(3*I*(b*x+a))+3*d^3*x^3*b*exp(I*(b*x+a))-6*I*c*d^2*x*exp(5*I*(b*x+a))+2*
c^3*b*exp(3*I*(b*x+a))+9*c*d^2*x^2*b*exp(I*(b*x+a))-3*I*d^3*x^2*exp(5*I*(b*
x+a))+9*c^2*d*x*b*exp(I*(b*x+a))+3*c^3*b*exp(I*(b*x+a))+3*I*c^2*d*exp(I*(b*
```

```
x+a))+3*I*d^3*x^2*exp(I*(b*x+a))-3*I*c^2*d*exp(5*I*(b*x+a)))+3/b^3*d^3*ln(1
-I*exp(I*(b*x+a)))*x+3/b^4*d^3*ln(1-I*exp(I*(b*x+a)))*a-3/b^3*d^3*ln(1+I*ex
p(I*(b*x+a)))*x-3/b^4*d^3*ln(1+I*exp(I*(b*x+a)))*a-6/b^3*c*d^2*a*ln(exp(I*(
b*x+a))-1)+6/b^3*d^3*ln(1-exp(I*(b*x+a)))*a*x+6*I/b^3*d^2*c*dilog(exp(I*(b*
x+a))+1)-6*I/b^4*d^3*a*dilog(exp(I*(b*x+a))+1)+6*I/b^4*d^3*polylog(2,-exp(I
*(b*x+a)))*a-6*I/b^4*d^3*polylog(2,exp(I*(b*x+a)))*a+6*I/b^3*dilog(exp(I*(b
*x+a)))*c*d^2-6*I/b^4*dilog(exp(I*(b*x+a)))*d^3*a-3*I/b*c^3*arctan(exp(I*(b
*x+a)))-6*I/b^3*d^3*polylog(2,exp(I*(b*x+a)))*x-6/b^2*d^2*c*ln(exp(I*(b*x+a
))+1)*x+6*I/b^3*d^3*polylog(2,-exp(I*(b*x+a)))*x+3*I/b^4*a^3*d^3*arctan(exp
(I*(b*x+a)))-9/2*I/b^2*c^2*d*polylog(2,I*exp(I*(b*x+a)))+9/2*I/b^2*c^2*d*po
lylog(2,-I*exp(I*(b*x+a)))-9/2*I/b^2*d^3*polylog(2,I*exp(I*(b*x+a)))*x^2+9/
2*I/b^2*d^3*polylog(2,-I*exp(I*(b*x+a)))*x^2
```

maxima [B] time = 6.46, size = 8032, normalized size = 16.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 3*a*c^2*d*(2*(3*sin(b*x + a
)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(
sin(b*x + a) - 1))/b + 3*a^2*c*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^
3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2
- a^3*d^3*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^3 - 4*((6*(b*x + a)^3*d^3 +
 12*b*c*d^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d
- 6*a*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2
 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 +
 (3*a^2 + 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*
d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b
*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3
 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (
-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(6
*b*x + 6*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*
c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*
I*a^2 - 12*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*
I*b*c*d^2 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^
2*c^2*d - 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*
a))*arctan2(cos(b*x + a), sin(b*x + a) + 1) + (6*(b*x + a)^3*d^3 + 12*b*c*d
^2 - 12*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 6*(3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a*d^
```

```
3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^2 +
 2)*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*((b*x + a)^3*d^3 + 2*b*c*d^2 - 2*a
*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3*a^
2 + 2)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*((b*x + a)^3*d^3 + 2*b*c*d^2 -
2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 + (3
*a^2 + 2)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b
*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2
*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 12*I)*d^3)*(b*x + a))*sin(6*b*x + 6*
a) + (-6*I*(b*x + a)^3*d^3 - 12*I*b*c*d^2 + 12*I*a*d^3 + (-18*I*b*c*d^2 + 1
8*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 - 1
2*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (6*I*(b*x + a)^3*d^3 + 12*I*b*c*d^2
 - 12*I*a*d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d -
 36*I*a*b*c*d^2 + (18*I*a^2 + 12*I)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*arcta
n2(cos(b*x + a), -sin(b*x + a) + 1) + (12*b^2*c^2*d - 24*a*b*c*d^2 + 12*(b*
x + a)^2*d^3 + 12*a^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*(b^2*c^2*d
 - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3
 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*(b^2*c^2*d - 2*a*b*c
*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x
 + 2*a) + (-12*I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a
^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-12*I*
b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*(b*x + a)^2*d^3 - 12*I*a^2*d^3 + (-24*I*b
*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (12*I*b^2*c^2*d - 24*I*a
*b*c*d^2 + 12*I*(b*x + a)^2*d^3 + 12*I*a^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3
)*(b*x + a))*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (1
2*b^2*c^2*d - 24*a*b*c*d^2 + 12*a^2*d^3 - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2
*d^3)*cos(6*b*x + 6*a) - 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(4*b*x +
 4*a) + 12*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(2*b*x + 2*a) - (12*I*b^2
*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*sin(6*b*x + 6*a) - (12*I*b^2*c^2*d
 - 24*I*a*b*c*d^2 + 12*I*a^2*d^3)*sin(4*b*x + 4*a) - (-12*I*b^2*c^2*d + 24*I
*a*b*c*d^2 - 12*I*a^2*d^3)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x
 + a) - 1) + (12*(b*x + a)^2*d^3 + 24*(b*c*d^2 - a*d^3)*(b*x + a) - 12*((b*x
 + a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 12*((b*x +
a)^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 12*((b*x + a)^
2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*a) + (-12*I*(b*x + a)^
2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-12*I*(
b*x + a)^2*d^3 + (-24*I*b*c*d^2 + 24*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) +
 (12*I*(b*x + a)^2*d^3 + (24*I*b*c*d^2 - 24*I*a*d^3)*(b*x + a))*sin(2*b*x +
 2*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*(b*x + a)^3*d^3 - 12*
I*b^2*c^2*d + 24*I*a*b*c*d^2 - 12*I*a^2*d^3 + (36*b*c*d^2 - (36*a + 12*I)*d
^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a + 24*I)*b*c*d^2 + 12*(3*a^2 + 2*I*a
)*d^3)*(b*x + a))*cos(5*b*x + 5*a) - 8*((b*x + a)^3*d^3 + 3*(b*c*d^2 - a*d^
3)*(b*x + a)^2 + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*(b*x + a))*cos(3*b*x
 + 3*a) - (12*(b*x + a)^3*d^3 + 12*I*b^2*c^2*d - 24*I*a*b*c*d^2 + 12*I*a^2*
d^3 + (36*b*c*d^2 - (36*a - 12*I)*d^3)*(b*x + a)^2 + (36*b^2*c^2*d - (72*a
```

```
- 24*I)*b*c*d^2 + 12*(3*a^2 - 2*I*a)*d^3)*(b*x + a))*cos(b*x + a) + (18*b^2
*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^3 + 36*(b*c*d^
2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (
3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6*a) - 6*(3*b^2
*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a
*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x +
 a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(2*b*x + 2*
a) + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2
- 12*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + (-
18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I)
*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + (18*I*b^2
*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^2 + 12*I)*d^3 + (3
6*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x + I
*a)) - (18*b^2*c^2*d - 36*a*b*c*d^2 + 18*(b*x + a)^2*d^3 + 6*(3*a^2 + 2)*d^
3 + 36*(b*c*d^2 - a*d^3)*(b*x + a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x
+ a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(6*b*x + 6
*a) - 6*(3*b^2*c^2*d - 6*a*b*c*d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 +
6*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + 6*(3*b^2*c^2*d - 6*a*b*c*
d^2 + 3*(b*x + a)^2*d^3 + (3*a^2 + 2)*d^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a))*
cos(2*b*x + 2*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3
+ (18*I*a^2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(6*b*x
+ 6*a) - (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + 18*I*(b*x + a)^2*d^3 + (18*I*a^
2 + 12*I)*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a) - (
-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 18*I*(b*x + a)^2*d^3 + (-18*I*a^2 - 12*I
)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*dilog(-I*
e^(I*b*x + I*a)) - (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2
+ (b*x + a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a
*d^3)*cos(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2
*a) - (24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(6*b*x + 6*a) - (
24*I*b*c*d^2 + 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(4*b*x + 4*a) - (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(-e^(I*b*x
 + I*a)) + (24*b*c*d^2 + 24*(b*x + a)*d^3 - 24*a*d^3 - 24*(b*c*d^2 + (b*x +
 a)*d^3 - a*d^3)*cos(6*b*x + 6*a) - 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*co
s(4*b*x + 4*a) + 24*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*cos(2*b*x + 2*a) + (-
24*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(6*b*x + 6*a) + (-24*I*b
*c*d^2 - 24*I*(b*x + a)*d^3 + 24*I*a*d^3)*sin(4*b*x + 4*a) + (24*I*b*c*d^2
+ 24*I*(b*x + a)*d^3 - 24*I*a*d^3)*sin(2*b*x + 2*a))*dilog(e^(I*b*x + I*a))
 + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (
-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6
*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*c
os(6*b*x + 6*a) + (6*I*b^2*c^2*d - 12*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6
*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (-6*
I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 6*(b^2*c^2*d - 2*a*b*c*d
^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x +
```

```
6*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2
 - a*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x +
 a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(
cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (6*I*b^2*c^2*d - 12
*I*a*b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d
^3)*(b*x + a) + (-6*I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*
I*a^2*d^3 + (-12*I*b*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-6*
I*b^2*c^2*d + 12*I*a*b*c*d^2 - 6*I*(b*x + a)^2*d^3 - 6*I*a^2*d^3 + (-12*I*b
*c*d^2 + 12*I*a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (6*I*b^2*c^2*d - 12*I*a*
b*c*d^2 + 6*I*(b*x + a)^2*d^3 + 6*I*a^2*d^3 + (12*I*b*c*d^2 - 12*I*a*d^3)*(
b*x + a))*cos(2*b*x + 2*a) + 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 +
 a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 6*(b^2*c^2*d -
 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 + 2*(b*c*d^2 - a*d^3)*(b*x + a))*s
in(4*b*x + 4*a) - 6*(b^2*c^2*d - 2*a*b*c*d^2 + (b*x + a)^2*d^3 + a^2*d^3 +
2*(b*c*d^2 - a*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b
*x + a)^2 - 2*cos(b*x + a) + 1) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*
a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c
*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2
 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 1
8*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (-3*I*(
b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x
+ a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*
b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a
^2 + 6*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) + 3*((b*x + a)^3*d^3 + 2*b*c*d^2
 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^2 +
 (3*a^2 + 2)*d^3)*(b*x + a))*sin(6*b*x + 6*a) + 3*((b*x + a)^3*d^3 + 2*b*c*
d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d^
2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) - 3*((b*x + a)^3*d^3 + 2*b
*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c
*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + s
in(b*x + a)^2 + 2*sin(b*x + a) + 1) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 +
 6*I*a*d^3 + (-9*I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*
I*a*b*c*d^2 + (-9*I*a^2 - 6*I)*d^3)*(b*x + a) + (3*I*(b*x + a)^3*d^3 + 6*I*
b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*x + a)^2 + (9*I*b^2*c^2*
d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (3*
I*(b*x + a)^3*d^3 + 6*I*b*c*d^2 - 6*I*a*d^3 + (9*I*b*c*d^2 - 9*I*a*d^3)*(b*
x + a)^2 + (9*I*b^2*c^2*d - 18*I*a*b*c*d^2 + (9*I*a^2 + 6*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^3*d^3 - 6*I*b*c*d^2 + 6*I*a*d^3 + (-9*
I*b*c*d^2 + 9*I*a*d^3)*(b*x + a)^2 + (-9*I*b^2*c^2*d + 18*I*a*b*c*d^2 + (-9
*I*a^2 - 6*I)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - 3*((b*x + a)^3*d^3 + 2*b*c
*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*c*d
^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(6*b*x + 6*a) - 3*((b*x + a)^3*d^3 + 2*
b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a*b*
c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(4*b*x + 4*a) + 3*((b*x + a)^3*d^3 +
```

```
 2*b*c*d^2 - 2*a*d^3 + 3*(b*c*d^2 - a*d^3)*(b*x + a)^2 + (3*b^2*c^2*d - 6*a
*b*c*d^2 + (3*a^2 + 2)*d^3)*(b*x + a))*sin(2*b*x + 2*a))*log(cos(b*x + a)^2
 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (36*d^3*cos(6*b*x + 6*a) + 36*d^3
*cos(4*b*x + 4*a) - 36*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) + 3
6*I*d^3*sin(4*b*x + 4*a) - 36*I*d^3*sin(2*b*x + 2*a) - 36*d^3)*polylog(4, I
*e^(I*b*x + I*a)) - (36*d^3*cos(6*b*x + 6*a) + 36*d^3*cos(4*b*x + 4*a) - 36
*d^3*cos(2*b*x + 2*a) + 36*I*d^3*sin(6*b*x + 6*a) + 36*I*d^3*sin(4*b*x + 4*
a) - 36*I*d^3*sin(2*b*x + 2*a) - 36*d^3)*polylog(4, -I*e^(I*b*x + I*a)) + (
36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 36*I*a*d^3 + (-36*I*b*c*d^2 - 36*I*(b*x
 + a)*d^3 + 36*I*a*d^3)*cos(6*b*x + 6*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*
d^3 + 36*I*a*d^3)*cos(4*b*x + 4*a) + (36*I*b*c*d^2 + 36*I*(b*x + a)*d^3 - 3
6*I*a*d^3)*cos(2*b*x + 2*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(6*b*
x + 6*a) + 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) - 36*(b*c*
d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, I*e^(I*b*x + I*a)
) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^3 + 36*I*a*d^3 + (36*I*b*c*d^2 + 36*I
*(b*x + a)*d^3 - 36*I*a*d^3)*cos(6*b*x + 6*a) + (36*I*b*c*d^2 + 36*I*(b*x +
 a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + (-36*I*b*c*d^2 - 36*I*(b*x + a)*d^
3 + 36*I*a*d^3)*cos(2*b*x + 2*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin
(6*b*x + 6*a) - 36*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a) + 36*
(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(2*b*x + 2*a))*polylog(3, -I*e^(I*b*x
+ I*a)) + (24*I*d^3*cos(6*b*x + 6*a) + 24*I*d^3*cos(4*b*x + 4*a) - 24*I*d^3
*cos(2*b*x + 2*a) - 24*d^3*sin(6*b*x + 6*a) - 24*d^3*sin(4*b*x + 4*a) + 24*
d^3*sin(2*b*x + 2*a) - 24*I*d^3)*polylog(3, -e^(I*b*x + I*a)) + (-24*I*d^3*
cos(6*b*x + 6*a) - 24*I*d^3*cos(4*b*x + 4*a) + 24*I*d^3*cos(2*b*x + 2*a) +
24*d^3*sin(6*b*x + 6*a) + 24*d^3*sin(4*b*x + 4*a) - 24*d^3*sin(2*b*x + 2*a)
 + 24*I*d^3)*polylog(3, e^(I*b*x + I*a)) + (-12*I*(b*x + a)^3*d^3 - 12*b^2*
c^2*d + 24*a*b*c*d^2 - 12*a^2*d^3 - 12*(3*I*b*c*d^2 + (-3*I*a + 1)*d^3)*(b*
x + a)^2 + (-36*I*b^2*c^2*d - 24*(-3*I*a + 1)*b*c*d^2 + (-36*I*a^2 + 24*a)*
d^3)*(b*x + a))*sin(5*b*x + 5*a) + (-8*I*(b*x + a)^3*d^3 + (-24*I*b*c*d^2 +
 24*I*a*d^3)*(b*x + a)^2 + (-24*I*b^2*c^2*d + 48*I*a*b*c*d^2 - 24*I*a^2*d^3
)*(b*x + a))*sin(3*b*x + 3*a) + (-12*I*(b*x + a)^3*d^3 + 12*b^2*c^2*d - 24*
a*b*c*d^2 + 12*a^2*d^3 + (-36*I*b*c*d^2 - 12*(-3*I*a - 1)*d^3)*(b*x + a)^2
+ (-36*I*b^2*c^2*d - 24*(-3*I*a - 1)*b*c*d^2 + (-36*I*a^2 - 24*a)*d^3)*(b*x
 + a))*sin(b*x + a))/(-4*I*b^3*cos(6*b*x + 6*a) - 4*I*b^3*cos(4*b*x + 4*a)
+ 4*I*b^3*cos(2*b*x + 2*a) + 4*b^3*sin(6*b*x + 6*a) + 4*b^3*sin(4*b*x + 4*a
) - 4*b^3*sin(2*b*x + 2*a) + 4*I*b^3))/b
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.318 $\int (c+dx)^2 \csc^2(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=341

$$\frac{2id^2\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(e^{i(a+bx)}\right)}{b^3}-\frac{3d^2\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^3}+\frac{3d^2\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^3}+\frac{d^2\tanh^{-1}(\sin(a+bx))}{b^3}+\frac{3id(c+d\ldots}{\ldots}$$

```
[Out] -3*I*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b+2*d^2*x*arctanh(exp(I*(b*x+a)))/b^2
-6*d*(d*x+c)*arctanh(exp(I*(b*x+a)))/b^2-d^2*x*arctanh(cos(b*x+a))/b^2+d*(d
*x+c)*arctanh(cos(b*x+a))/b^2+d^2*arctanh(sin(b*x+a))/b^3-3/2*(d*x+c)^2*csc
(b*x+a)/b+2*I*d^2*polylog(2,-exp(I*(b*x+a)))/b^3+3*I*d*(d*x+c)*polylog(2,-I
*exp(I*(b*x+a)))/b^2-3*I*d*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^2-2*I*d^2*
polylog(2,exp(I*(b*x+a)))/b^3-3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*
polylog(3,I*exp(I*(b*x+a)))/b^3-d*(d*x+c)*sec(b*x+a)/b^2+1/2*(d*x+c)^2*csc(
b*x+a)*sec(b*x+a)^2/b
```

Rubi [A] time = 0.65, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 19, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.792, Rules used = {2621, 288, 321, 207, 4420, 6688, 12, 6742, 6273, 4181, 2531, 2282, 6589, 4183, 2279, 2391, 2622, 6271, 3770}

$$\frac{3id(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^2}-\frac{3id(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^2}+\frac{2id^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\ldots}{b^3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b + (2*d^2*x*ArcTanh[E^(I*(a +
 b*x))])/b^2 - (6*d*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b^2 - (d^2*x*ArcTan
h[Cos[a + b*x]])/b^2 + (d*(c + d*x)*ArcTanh[Cos[a + b*x]])/b^2 + (d^2*ArcTa
nh[Sin[a + b*x]])/b^3 - (3*(c + d*x)^2*Csc[a + b*x])/(2*b) + ((2*I)*d^2*Pol
yLog[2, -E^(I*(a + b*x))])/b^3 + ((3*I)*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a
 + b*x))])/b^2 - ((3*I)*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - ((
2*I)*d^2*PolyLog[2, E^(I*(a + b*x))])/b^3 - (3*d^2*PolyLog[3, (-I)*E^(I*(a
+ b*x))])/b^3 + (3*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - (d*(c + d*x)*Se
c[a + b*x])/b^2 + ((c + d*x)^2*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rule 6273

```
Int[((a_.) + ArcTanh[u_]*(b_.))*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*(a + b*ArcTanh[u]))/(d*(m + 1)), x] - Dist[b/(d*(m +
1)), Int[SimplifyIntegrand[((c + d*x)^(m + 1)*D[u, x])/(1 - u^2), x], x], x
] /; FreeQ[{a, b, c, d, m}, x] && NeQ[m, -1] && InverseFunctionFreeQ[u, x]
&&  !FunctionOfQ[(c + d*x)^(m + 1), u, x] && FalseQ[PowerVariableExpn[u, m
+ 1, x]]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6688

```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2}{}$$

$$= \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)^2\csc(a+bx)}{2b} + \frac{(c+dx)^2}{}$$

$$= \frac{d(c+dx)\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - 3$$

$$= \frac{d(c+dx)\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{3(c+dx)^2\tanh^{-1}(\sin(a+bx))}{2b} - 3$$

$$= -\frac{6d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2x\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c+dx)}{}$$

$$= -\frac{6d(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{d^2x\tanh^{-1}(\cos(a+bx))}{b^2} + \frac{d(c+dx)}{}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{ta}}{b}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{ta}}{b}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{ta}}{b}$$

$$= -\frac{3i(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{2d^2x\tanh^{-1}\left(e^{i(a+bx)}\right)}{b^2} - \frac{6d(c+dx)\,\mathrm{ta}}{b}$$

Mathematica [B] time = 7.53, size = 889, normalized size = 2.61

$$\frac{2\left(\frac{\left(\left(bx+\tan^{-1}(\tan(a))\right)\left(\log\left(1-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\log\left(1+e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)+i\left(\mathrm{Li}_2\left(-e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)-\mathrm{Li}_2\left(e^{i\left(bx+\tan^{-1}(\tan(a))\right)}\right)\right)\right)\sec(a)}{\sqrt{\tan^2(a)+1}} - \frac{2}{}\right.}{b^3}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*((6*I)*b^2*c^2*ArcTan[E^(I*(a + b*x))] + (4*I)*d^2*ArcTan[E^(I*(a + b*
x))] - 6*b^2*c*d*x*Log[1 - I*E^(I*(a + b*x))] - 3*b^2*d^2*x^2*Log[1 - I*E^(
I*(a + b*x))] + 6*b^2*c*d*x*Log[1 + I*E^(I*(a + b*x))] + 3*b^2*d^2*x^2*Log[
1 + I*E^(I*(a + b*x))] - (6*I)*b*d*(c + d*x)*PolyLog[2, (-I)*E^(I*(a + b*x)
)] + (6*I)*b*d*(c + d*x)*PolyLog[2, I*E^(I*(a + b*x))] + 6*d^2*PolyLog[3, (
-I)*E^(I*(a + b*x))] - 6*d^2*PolyLog[3, I*E^(I*(a + b*x))])/b^3 - ((c + d*x
)*Csc[a]*Sec[a]*(b*c*Cos[a] + b*d*x*Cos[a] + d*Sin[a]))/b^2 + ((4*I)*c*d*Ar
cTan[(I*Cos[a] - I*Sin[a]*Tan[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/(b^2*Sq
rt[Cos[a]^2 + Sin[a]^2]) + (Sec[a/2]*Sec[a/2 + (b*x)/2]*(-(c^2*Sin[(b*x)/2]
) - 2*c*d*x*Sin[(b*x)/2] - d^2*x^2*Sin[(b*x)/2]))/(2*b) + (Csc[a/2]*Csc[a/2
 + (b*x)/2]*(c^2*Sin[(b*x)/2] + 2*c*d*x*Sin[(b*x)/2] + d^2*x^2*Sin[(b*x)/2]
))/(2*b) + (c^2 + 2*c*d*x + d^2*x^2)/(4*b*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (
b*x)/2])^2) + (-(c*d*Sin[(b*x)/2]) - d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] - S
in[a/2])*(Cos[a/2 + (b*x)/2] - Sin[a/2 + (b*x)/2])) + (-c^2 - 2*c*d*x - d^2
*x^2)/(4*b*(Cos[a/2 + (b*x)/2] + Sin[a/2 + (b*x)/2])^2) + (c*d*Sin[(b*x)/2]
 + d^2*x*Sin[(b*x)/2])/(b^2*(Cos[a/2] + Sin[a/2])*(Cos[a/2 + (b*x)/2] + Sin
[a/2 + (b*x)/2])) + (2*d^2*((-2*ArcTan[Tan[a]]*ArcTanh[(-Cos[a] + Sin[a]*Ta
n[(b*x)/2])/Sqrt[Cos[a]^2 + Sin[a]^2]])/Sqrt[Cos[a]^2 + Sin[a]^2] + (((b*x
+ ArcTan[Tan[a]])*(Log[1 - E^(I*(b*x + ArcTan[Tan[a]]))] - Log[1 + E^(I*(b*
x + ArcTan[Tan[a]]))]) + I*(PolyLog[2, -E^(I*(b*x + ArcTan[Tan[a]]))] - Pol
yLog[2, E^(I*(b*x + ArcTan[Tan[a]]))]))*Sec[a])/Sqrt[1 + Tan[a]^2]))/b^3
```

fricas [C] time = 0.65, size = 1362, normalized size = 3.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^2*d^2*x^2 - 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) + I*sin(b*x
+ a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(cos(b*x + a) - I*sin(b*x
+ a))*sin(b*x + a) - 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) + I*sin(b*x
 + a))*sin(b*x + a) + 4*I*d^2*cos(b*x + a)^2*dilog(-cos(b*x + a) - I*sin(b*
x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) + sin
(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, I*cos(b*x + a) -
sin(b*x + a))*sin(b*x + a) - 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x + a
) + sin(b*x + a))*sin(b*x + a) + 6*d^2*cos(b*x + a)^2*polylog(3, -I*cos(b*x
 + a) - sin(b*x + a))*sin(b*x + a) + 4*b^2*c*d*x + (-6*I*b*d^2*x - 6*I*b*c*
d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) + (-6*I
*b*d^2*x - 6*I*b*c*d)*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*s
in(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*cos(b*x + a)^2*dilog(-I*cos(b*x + a
```

```
) + sin(b*x + a))*sin(b*x + a) + (6*I*b*d^2*x + 6*I*b*c*d)*cos(b*x + a)^2*d
ilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos
(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + 1)*sin(b*x + a) + (3*b^2*c^
2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*
x + a) + I)*sin(b*x + a) - 4*(b*d^2*x + b*c*d)*cos(b*x + a)^2*log(cos(b*x +
 a) - I*sin(b*x + a) + 1)*sin(b*x + a) - (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 +
2)*d^2)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a)
+ 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*
cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*
x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x + a) +
 1)*sin(b*x + a) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(
b*x + a)^2*log(-I*cos(b*x + a) + sin(b*x + a) + 1)*sin(b*x + a) - 3*(b^2*d^
2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2*log(-I*cos(b*x +
a) - sin(b*x + a) + 1)*sin(b*x + a) + 4*(b*c*d - a*d^2)*cos(b*x + a)^2*log(
-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2)*sin(b*x + a) + 4*(b*c*d - a*d
^2)*cos(b*x + a)^2*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2)*sin(b*
x + a) + 4*(b*d^2*x + a*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) + I*sin(b*x +
 a) + 1)*sin(b*x + a) + (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x +
 a)^2*log(-cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x + a) + 4*(b*d^2*x + a
*d^2)*cos(b*x + a)^2*log(-cos(b*x + a) - I*sin(b*x + a) + 1)*sin(b*x + a) -
 (3*b^2*c^2 - 6*a*b*c*d + (3*a^2 + 2)*d^2)*cos(b*x + a)^2*log(-cos(b*x + a)
 - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b^2*c^2 - 6*(b^2*d^2*x^2 + 2*b^2*c*
d*x + b^2*c^2)*cos(b*x + a)^2 - 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x +
a))/(b^3*cos(b*x + a)^2*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^2 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

maple [B] time = 0.32, size = 770, normalized size = 2.26

$$-\frac{3cd\ln\left(1+ie^{i(bx+a)}\right)x}{b}-\frac{3icd\,\mathrm{polylog}\left(2,ie^{i(bx+a)}\right)}{b^2}-\frac{3id^2\,\mathrm{polylog}\left(2,ie^{i(bx+a)}\right)x}{b^2}+\frac{3cd\ln\left(1-ie^{i(bx+a)}\right)a}{b^2}+\frac{3cd\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -3*d^2*polylog(3,-I*exp(I*(b*x+a)))/b^3+3*d^2*polylog(3,I*exp(I*(b*x+a)))/b
^3-2/b^2*d^2*ln(exp(I*(b*x+a))+1)*x+2*I/b^3*d^2*dilog(exp(I*(b*x+a))+1)+2/b
```

```
^2*d*c*ln(exp(I*(b*x+a))-1)-2/b^2*d*c*ln(exp(I*(b*x+a))+1)-2/b^3*d^2*a*ln(e
xp(I*(b*x+a))-1)-3*I/b^3*a^2*d^2*arctan(exp(I*(b*x+a)))+3*I/b^2*d^2*polylog
(2,-I*exp(I*(b*x+a)))*x-3*I/b^2*c*d*polylog(2,I*exp(I*(b*x+a)))+3*I/b^2*c*d
*polylog(2,-I*exp(I*(b*x+a)))-3*I/b^2*d^2*polylog(2,I*exp(I*(b*x+a)))*x-3/b
*c*d*ln(1+I*exp(I*(b*x+a)))*x+3/b^2*c*d*ln(1-I*exp(I*(b*x+a)))*a+3/b*c*d*ln
(1-I*exp(I*(b*x+a)))*x-3/b^2*c*d*ln(1+I*exp(I*(b*x+a)))*a+6*I/b^2*a*c*d*arc
tan(exp(I*(b*x+a)))+2*I/b^3*dilog(exp(I*(b*x+a)))*d^2-3/2/b*d^2*ln(1+I*exp(
I*(b*x+a)))*x^2+3/2/b^3*a^2*d^2*ln(1+I*exp(I*(b*x+a)))+3/2/b*d^2*ln(1-I*exp
(I*(b*x+a)))*x^2-3/2/b^3*a^2*d^2*ln(1-I*exp(I*(b*x+a)))-3*I/b*c^2*arctan(ex
p(I*(b*x+a)))-2*I/b^3*d^2*arctan(exp(I*(b*x+a)))-I/b^2/(1+exp(2*I*(b*x+a)))
^2/(exp(2*I*(b*x+a))-1)*(3*d^2*x^2*b*exp(5*I*(b*x+a))+6*c*d*x*b*exp(5*I*(b*
x+a))+3*c^2*b*exp(5*I*(b*x+a))+2*d^2*x^2*b*exp(3*I*(b*x+a))+4*c*d*x*b*exp(3
*I*(b*x+a))-2*I*d^2*x*exp(5*I*(b*x+a))+2*c^2*b*exp(3*I*(b*x+a))+3*d^2*x^2*b
*exp(I*(b*x+a))-2*I*c*d*exp(5*I*(b*x+a))+6*c*d*x*b*exp(I*(b*x+a))+3*c^2*b*e
xp(I*(b*x+a))+2*I*d^2*x*exp(I*(b*x+a))+2*I*d*c*exp(I*(b*x+a)))
```

maxima [B] time = 1.62, size = 3819, normalized size = 11.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log
(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1)) - 2*a*c*d*(2*(3*sin(b*x + a)^
2 - 2)/(sin(b*x + a)^3 - sin(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(si
n(b*x + a) - 1))/b + a^2*d^2*(2*(3*sin(b*x + a)^2 - 2)/(sin(b*x + a)^3 - si
n(b*x + a)) - 3*log(sin(b*x + a) + 1) + 3*log(sin(b*x + a) - 1))/b^2 - 4*((
6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b*x + a)^2
*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - 2*(3*(b*x +
a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) + 2*(3*(b*
x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*a) + (-6*
I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(6*b
*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) -
4*I*d^2)*sin(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 12*I*a*d^2
)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), sin(b*x + a)
 + 1) + (6*(b*x + a)^2*d^2 + 12*(b*c*d - a*d^2)*(b*x + a) + 4*d^2 - 2*(3*(b
*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(6*b*x + 6*a) - 2*(
3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(4*b*x + 4*a) +
 2*(3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*cos(2*b*x + 2*
a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*x + a) - 4*I*d^2
)*sin(6*b*x + 6*a) + (-6*I*(b*x + a)^2*d^2 + (-12*I*b*c*d + 12*I*a*d^2)*(b*
x + a) - 4*I*d^2)*sin(4*b*x + 4*a) + (6*I*(b*x + a)^2*d^2 + (12*I*b*c*d - 1
2*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(2*b*x + 2*a))*arctan2(cos(b*x + a), -si
n(b*x + a) + 1) + (8*b*c*d + 8*(b*x + a)*d^2 - 8*a*d^2 - 8*(b*c*d + (b*x +
```

```
a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*
b*x + 4*a) + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) + (-8*I*b*c
*d - 8*I*(b*x + a)*d^2 + 8*I*a*d^2)*sin(6*b*x + 6*a) + (-8*I*b*c*d - 8*I*(b
*x + a)*d^2 + 8*I*a*d^2)*sin(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2
- 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (8
*b*c*d - 8*a*d^2 - 8*(b*c*d - a*d^2)*cos(6*b*x + 6*a) - 8*(b*c*d - a*d^2)*c
os(4*b*x + 4*a) + 8*(b*c*d - a*d^2)*cos(2*b*x + 2*a) - (8*I*b*c*d - 8*I*a*d
^2)*sin(6*b*x + 6*a) - (8*I*b*c*d - 8*I*a*d^2)*sin(4*b*x + 4*a) - (-8*I*b*c
*d + 8*I*a*d^2)*sin(2*b*x + 2*a))*arctan2(sin(b*x + a), cos(b*x + a) - 1) -
 (8*(b*x + a)*d^2*cos(6*b*x + 6*a) + 8*(b*x + a)*d^2*cos(4*b*x + 4*a) - 8*(
b*x + a)*d^2*cos(2*b*x + 2*a) + 8*I*(b*x + a)*d^2*sin(6*b*x + 6*a) + 8*I*(b
*x + a)*d^2*sin(4*b*x + 4*a) - 8*I*(b*x + a)*d^2*sin(2*b*x + 2*a) - 8*(b*x
+ a)*d^2)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - (12*(b*x + a)^2*d^2 -
8*I*b*c*d + 8*I*a*d^2 + (24*b*c*d - (24*a + 8*I)*d^2)*(b*x + a))*cos(5*b*x
+ 5*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(3*b*x + 3*a)
 - (12*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (24*b*c*d - (24*a - 8*I)*d
^2)*(b*x + a))*cos(b*x + a) + (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 - 12*
(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^
2 - a*d^2)*cos(4*b*x + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x
+ 2*a) + (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(6*b*x + 6*a) +
 (-12*I*b*c*d - 12*I*(b*x + a)*d^2 + 12*I*a*d^2)*sin(4*b*x + 4*a) + (12*I*b
*c*d + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(I*e^(I*b*x
+ I*a)) - (12*b*c*d + 12*(b*x + a)*d^2 - 12*a*d^2 - 12*(b*c*d + (b*x + a)*d
^2 - a*d^2)*cos(6*b*x + 6*a) - 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(4*b*x
 + 4*a) + 12*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(2*b*x + 2*a) - (12*I*b*c*d
 + 12*I*(b*x + a)*d^2 - 12*I*a*d^2)*sin(6*b*x + 6*a) - (12*I*b*c*d + 12*I*(
b*x + a)*d^2 - 12*I*a*d^2)*sin(4*b*x + 4*a) - (-12*I*b*c*d - 12*I*(b*x + a)
*d^2 + 12*I*a*d^2)*sin(2*b*x + 2*a))*dilog(-I*e^(I*b*x + I*a)) + (8*d^2*cos
(6*b*x + 6*a) + 8*d^2*cos(4*b*x + 4*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*s
in(6*b*x + 6*a) + 8*I*d^2*sin(4*b*x + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) - 8*d
^2)*dilog(-e^(I*b*x + I*a)) - (8*d^2*cos(6*b*x + 6*a) + 8*d^2*cos(4*b*x + 4
*a) - 8*d^2*cos(2*b*x + 2*a) + 8*I*d^2*sin(6*b*x + 6*a) + 8*I*d^2*sin(4*b*x
 + 4*a) - 8*I*d^2*sin(2*b*x + 2*a) - 8*d^2)*dilog(e^(I*b*x + I*a)) + (-4*I*
b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2 + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*
I*a*d^2)*cos(6*b*x + 6*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos
(4*b*x + 4*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2)*cos(2*b*x + 2*
a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) - 4*(b*c*d + (b*x +
 a)*d^2 - a*d^2)*sin(4*b*x + 4*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(2
*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (4
*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2 + (-4*I*b*c*d - 4*I*(b*x + a)*d^2
+ 4*I*a*d^2)*cos(6*b*x + 6*a) + (-4*I*b*c*d - 4*I*(b*x + a)*d^2 + 4*I*a*d^2
)*cos(4*b*x + 4*a) + (4*I*b*c*d + 4*I*(b*x + a)*d^2 - 4*I*a*d^2)*cos(2*b*x
+ 2*a) + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*sin(6*b*x + 6*a) + 4*(b*c*d + (b
*x + a)*d^2 - a*d^2)*sin(4*b*x + 4*a) - 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*s
in(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1)
```

```
+ (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x + a) + 2*I*d^2 + (-3*
I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(6*b*x
 + 6*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*
d^2)*cos(4*b*x + 4*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x
 + a) + 2*I*d^2)*cos(2*b*x + 2*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2)*
(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) + (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d^2
)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) - (3*(b*x + a)^2*d^2 + 6*(b*c*d - a*d
^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*x + a)^2 + sin(b*x + a)^
2 + 2*sin(b*x + a) + 1) + (-3*I*(b*x + a)^2*d^2 + (-6*I*b*c*d + 6*I*a*d^2)*
(b*x + a) - 2*I*d^2 + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a*d^2)*(b*x +
 a) + 2*I*d^2)*cos(6*b*x + 6*a) + (3*I*(b*x + a)^2*d^2 + (6*I*b*c*d - 6*I*a
*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (-3*I*(b*x + a)^2*d^2 + (-6*I
*b*c*d + 6*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(2*b*x + 2*a) - (3*(b*x + a)^2*
d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(6*b*x + 6*a) - (3*(b*x + a)^
2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(4*b*x + 4*a) + (3*(b*x + a
)^2*d^2 + 6*(b*c*d - a*d^2)*(b*x + a) + 2*d^2)*sin(2*b*x + 2*a))*log(cos(b*
x + a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + (-12*I*d^2*cos(6*b*x + 6*
a) - 12*I*d^2*cos(4*b*x + 4*a) + 12*I*d^2*cos(2*b*x + 2*a) + 12*d^2*sin(6*b
*x + 6*a) + 12*d^2*sin(4*b*x + 4*a) - 12*d^2*sin(2*b*x + 2*a) + 12*I*d^2)*p
olylog(3, I*e^(I*b*x + I*a)) + (12*I*d^2*cos(6*b*x + 6*a) + 12*I*d^2*cos(4*
b*x + 4*a) - 12*I*d^2*cos(2*b*x + 2*a) - 12*d^2*sin(6*b*x + 6*a) - 12*d^2*s
in(4*b*x + 4*a) + 12*d^2*sin(2*b*x + 2*a) - 12*I*d^2)*polylog(3, -I*e^(I*b*
x + I*a)) - 4*(3*I*(b*x + a)^2*d^2 + 2*b*c*d - 2*a*d^2 + 2*(3*I*b*c*d + (-3
*I*a + 1)*d^2)*(b*x + a))*sin(5*b*x + 5*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I
*b*c*d + 16*I*a*d^2)*(b*x + a))*sin(3*b*x + 3*a) + (-12*I*(b*x + a)^2*d^2 +
 8*b*c*d - 8*a*d^2 + (-24*I*b*c*d - 8*(-3*I*a - 1)*d^2)*(b*x + a))*sin(b*x
+ a))/(-4*I*b^2*cos(6*b*x + 6*a) - 4*I*b^2*cos(4*b*x + 4*a) + 4*I*b^2*cos(2
*b*x + 2*a) + 4*b^2*sin(6*b*x + 6*a) + 4*b^2*sin(4*b*x + 4*a) - 4*b^2*sin(2
*b*x + 2*a) + 4*I*b^2))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

[Out] Timed out

3.319 $\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=162

$$\frac{3id\text{Li}_2\left(-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{Li}_2\left(ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+bx)}{2b}+\frac{(c+dx)\csc(\cdots}{\cdots}$$

```
[Out] -3*I*d*x*arctan(exp(I*(b*x+a)))/b-d*arctanh(cos(b*x+a))/b^2+3/2*c*arctanh(s
in(b*x+a))/b-3/2*(d*x+c)*csc(b*x+a)/b+3/2*I*d*polylog(2,-I*exp(I*(b*x+a)))/
b^2-3/2*I*d*polylog(2,I*exp(I*(b*x+a)))/b^2-1/2*d*sec(b*x+a)/b^2+1/2*(d*x+c
)*csc(b*x+a)*sec(b*x+a)^2/b
```

Rubi [A] time = 0.20, antiderivative size = 182, normalized size of antiderivative = 1.12, number of steps used = 13, number of rules used = 12, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.546, Rules used = {2621, 288, 321, 207, 4420, 6271, 12, 4181, 2279, 2391, 3770, 2622}

$$\frac{3id\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{2b^2}-\frac{3id\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{2b^2}-\frac{d\sec(a+bx)}{2b^2}-\frac{d\tanh^{-1}(\cos(a+bx))}{b^2}-\frac{3(c+dx)\csc(a+bx)}{2b}\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] ((-3*I)*d*x*ArcTan[E^(I*(a + b*x))])/b - (d*ArcTanh[Cos[a + b*x]])/b^2 - (3
*d*x*ArcTanh[Sin[a + b*x]])/(2*b) + (3*(c + d*x)*ArcTanh[Sin[a + b*x]])/(2*
b) - (3*(c + d*x)*Csc[a + b*x])/(2*b) + (((3*I)/2)*d*PolyLog[2, (-I)*E^(I*(
a + b*x))])/b^2 - (((3*I)/2)*d*PolyLog[2, I*E^(I*(a + b*x))])/b^2 - (d*Sec[
a + b*x])/(2*b^2) + ((c + d*x)*Csc[a + b*x]*Sec[a + b*x]^2)/(2*b)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
```

```
/; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
```

```
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4420

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Module[{u = IntHide[Csc[a + b*x]^n*Sec[a + b*
x]^p, x]}, Dist[(c + d*x)^m, u, x] - Dist[d*m, Int[(c + d*x)^(m - 1)*u, x],
 x]] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p] && GtQ[m, 0] && NeQ[n, p]
```

Rule 6271

```
Int[ArcTanh[u_], x_Symbol] :> Simp[x*ArcTanh[u], x] - Int[SimplifyIntegrand
[(x*D[u, x])/(1 - u^2), x], x] /; InverseFunctionFreeQ[u, x]
```

Rubi steps

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx = \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx)\csc(a}{}$$

$$= \frac{3(c+dx)\tanh^{-1}(\sin(a+bx))}{2b} - \frac{3(c+dx)\csc(a+bx)}{2b} + \frac{(c+dx)\csc(a}{}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx)\tanh^{-}}{2l}$$

$$= -\frac{3d\tanh^{-1}(\cos(a+bx))}{2b^2} - \frac{3dx\tanh^{-1}(\sin(a+bx))}{2b} + \frac{3(c+dx)\tanh^{-}}{2l}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a+b}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a+b}{2b}$$

$$= -\frac{3idx\tan^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{d\tanh^{-1}(\cos(a+bx))}{b^2} - \frac{3dx\tanh^{-1}(\sin(a+b}{2b}$$

Mathematica [C] time = 6.59, size = 669, normalized size = 4.13

$$\frac{d\log\left(\sin\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{d\log\left(\cos\left(\frac{1}{2}(a+bx)\right)\right)}{b^2} - \frac{d\sin\left(\frac{1}{2}(a+bx)\right)}{2b^2\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+bx)\right)\right)} + \frac{d\sin\left(\frac{1}{2}(a\right.}{2b^2\left(\sin\left(\frac{1}{2}(a+bx)\right)\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sec[a + b*x]^3,x]
```

```
[Out] (d*(a*Cos[(a + b*x)/2] - (a + b*x)*Cos[(a + b*x)/2])*Csc[(a + b*x)/2])/(2*b
^2) - (c*Csc[a + b*x]*Hypergeometric2F1[-1/2, 2, 1/2, Sin[a + b*x]^2])/b -
(d*Log[Cos[(a + b*x)/2]])/b^2 + (d*Log[Sin[(a + b*x)/2]])/b^2 - (3*d*x*(a*L
og[1 - Tan[(a + b*x)/2]] - a*Log[1 + Tan[(a + b*x)/2]] - I*(Log[1 + I*Tan[(
a + b*x)/2]]*Log[(1/2 - I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, ((1 + I)
- (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(1/2 +
 I/2)*(1 + Tan[(a + b*x)/2])] + PolyLog[2, (-1/2 - I/2)*(I + Tan[(a + b*x)/
2])]) - I*(Log[1 - I*Tan[(a + b*x)/2]]*Log[(-1/2 + I/2)*(-1 + Tan[(a + b*x)
/2])] + PolyLog[2, ((1 + I) + (1 - I)*Tan[(a + b*x)/2])/2]) + I*(Log[1 + I*
Tan[(a + b*x)/2]]*Log[(-1/2 - I/2)*(-1 + Tan[(a + b*x)/2])] + PolyLog[2, ((
1 - I) + (1 + I)*Tan[(a + b*x)/2])/2])))/(2*b*(a - I*Log[1 - I*Tan[(a + b*x
)/2]] + I*Log[1 + I*Tan[(a + b*x)/2]])) + (d*x)/(4*b*(Cos[(a + b*x)/2] - Si
n[(a + b*x)/2])^2) - (d*Sin[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] - Sin[(a
 + b*x)/2])) - (d*x)/(4*b*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])^2) + (d*Sin
[(a + b*x)/2])/(2*b^2*(Cos[(a + b*x)/2] + Sin[(a + b*x)/2])) + (d*Sec[(a +
b*x)/2]*(a*Sin[(a + b*x)/2] - (a + b*x)*Sin[(a + b*x)/2]))/(2*b^2)
```

fricas [B] time = 0.54, size = 592, normalized size = 3.65

$$\frac{-3i\,d\cos\left(bx+a\right)^2\,\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)+\sin\left(bx+a\right)\right)\sin\left(bx+a\right)-3i\,d\cos\left(bx+a\right)^2\,\mathrm{Li}_2\left(i\,\cos\left(bx+a\right)-\sin\left(bx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(-3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a
) - 3*I*d*cos(b*x + a)^2*dilog(I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a)
+ 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) + sin(b*x + a))*sin(b*x + a) +
 3*I*d*cos(b*x + a)^2*dilog(-I*cos(b*x + a) - sin(b*x + a))*sin(b*x + a) +
3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) + I*sin(b*x + a) + I)*sin(b*x
 + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(cos(b*x + a) - I*sin(b*x + a) + I)
*sin(b*x + a) - 2*d*cos(b*x + a)^2*log(1/2*cos(b*x + a) + 1/2)*sin(b*x + a)
 + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) + sin(b*x + a) + 1)*si
n(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(I*cos(b*x + a) - sin(b*x +
a) + 1)*sin(b*x + a) + 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1)*sin(b*x + a) - 3*(b*d*x + a*d)*cos(b*x + a)^2*log(-I*cos
(b*x + a) - sin(b*x + a) + 1)*sin(b*x + a) + 2*d*cos(b*x + a)^2*log(-1/2*co
s(b*x + a) + 1/2)*sin(b*x + a) + 3*(b*c - a*d)*cos(b*x + a)^2*log(-cos(b*x
+ a) + I*sin(b*x + a) + I)*sin(b*x + a) - 3*(b*c - a*d)*cos(b*x + a)^2*log(
-cos(b*x + a) - I*sin(b*x + a) + I)*sin(b*x + a) + 2*b*d*x - 6*(b*d*x + b*c
```

```
)*cos(b*x + a)^2 - 2*d*cos(b*x + a)*sin(b*x + a) + 2*b*c)/(b^2*cos(b*x + a)
^2*sin(b*x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc(bx+a)^2\sec(bx+a)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sec(b*x + a)^3, x)
```

maple [B] time = 0.16, size = 344, normalized size = 2.12

$$-\frac{i\left(3bdx\,e^{5i(bx+a)}+3cb\,e^{5i(bx+a)}+2bdx\,e^{3i(bx+a)}+2cb\,e^{3i(bx+a)}-id\,e^{5i(bx+a)}+3bdx\,e^{i(bx+a)}+3cb\,e^{i(bx+a)}+id\,e^{i(bx+}\right.}{b^2\left(1+e^{2i(bx+a)}\right)^2\left(e^{2i(bx+a)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x)
```

```
[Out] -I/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)*(3*b*d*x*exp(5*I*(b*x+a)
)+3*c*b*exp(5*I*(b*x+a))+2*b*d*x*exp(3*I*(b*x+a))+2*c*b*exp(3*I*(b*x+a))-I*
d*exp(5*I*(b*x+a))+3*b*d*x*exp(I*(b*x+a))+3*c*b*exp(I*(b*x+a))+I*d*exp(I*(b
*x+a)))-3*I/b*c*arctan(exp(I*(b*x+a)))+3*I/b^2*d*a*arctan(exp(I*(b*x+a)))+d
/b^2*ln(exp(I*(b*x+a))-1)-d/b^2*ln(exp(I*(b*x+a))+1)-3/2*I/b^2*d*dilog(1-I*
exp(I*(b*x+a)))+3/2/b*d*ln(1-I*exp(I*(b*x+a)))*x+3/2/b^2*d*ln(1-I*exp(I*(b*
x+a)))*a+3/2*I/b^2*d*dilog(1+I*exp(I*(b*x+a)))-3/2/b*d*ln(1+I*exp(I*(b*x+a)
))*x-3/2/b^2*d*ln(1+I*exp(I*(b*x+a)))*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\csc^2(a+bx)\sec^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sec(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*csc(a + b*x)**2*sec(a + b*x)**3, x)
```

3.320 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

Rubi [A] time = 0.19, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx=\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Mathematica [A] time = 20.74, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x), x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d*x + c), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 3.25, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\left(\sec^{3}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(a+bx\right)\sec^{3}\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x), x)
```

3.321 $\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=27

$$\mathrm{Int}\left(\frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] CannotIntegrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.20, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Defer[Int][(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx=\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 25.88, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\csc\left(bx+a\right)^2\sec\left(bx+a\right)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 4.29, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\left(\sec^{3}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^3\,\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^2*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(a+bx\right)\sec^{3}\left(a+bx\right)}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**2*sec(a + b*x)**3/(c + d*x)**2, x)
```

3.322 $\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=27

$$\text{Int}\left(\csc^3(a+bx)\sec^3(a+bx)(c+dx)^m,x\right)$$

```
[Out] CannotIntegrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

Rubi [A] time = 0.26, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] Defer[Int][(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

Rubi steps

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx=\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Mathematica [A] time = 32.37, size = 0, normalized size = 0.00

$$\int (c+dx)^m \csc^3(a+bx)\sec^3(a+bx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
[Out] Integrate[(c + d*x)^m*Csc[a + b*x]^3*Sec[a + b*x]^3, x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left((dx+c)^m\csc\left(bx+a\right)^3\sec\left(bx+a\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

[Out] integral((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)

maple [A] time = 0.12, size = 0, normalized size = 0.00

$$\int (dx + c)^m \left(\csc^3(bx + a)\right) \left(\sec^3(bx + a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)

[Out] int((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^m \csc(bx + a)^3 \sec(bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")

[Out] integrate((d*x + c)^m*csc(b*x + a)^3*sec(b*x + a)^3, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c + d\,x)^m}{\cos(a + b\,x)^3 \sin(a + b\,x)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3),x)

[Out] int((c + d*x)^m/(cos(a + b*x)^3*sin(a + b*x)^3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.323 $\int (c+dx)^3 \csc^3(a+bx) \sec^3(a+bx) \, dx$

Optimal. Leaf size=318

$$\frac{3id^3\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_2\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{2b^4}+\frac{3id^3\text{Li}_4\left(e^{2i(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+-$$

```
[Out] -6*d^2*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b^3-4*(d*x+c)^3*arctanh(exp(2*I*(b
*x+a)))/b-3*d*(d*x+c)^2*csc(2*b*x+2*a)/b^2-2*(d*x+c)^3*cot(2*b*x+2*a)*csc(2
*b*x+2*a)/b+3/2*I*d^3*polylog(2,-exp(2*I*(b*x+a)))/b^4+3*I*d*(d*x+c)^2*poly
log(2,-exp(2*I*(b*x+a)))/b^2-3/2*I*d^3*polylog(2,exp(2*I*(b*x+a)))/b^4-3*I*
d*(d*x+c)^2*polylog(2,exp(2*I*(b*x+a)))/b^2-3*d^2*(d*x+c)*polylog(3,-exp(2*
I*(b*x+a)))/b^3+3*d^2*(d*x+c)*polylog(3,exp(2*I*(b*x+a)))/b^3-3/2*I*d^3*pol
ylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(4,exp(2*I*(b*x+a)))/b^4
```

Rubi [A] time = 0.32, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4419, 4186, 4183, 2279, 2391, 2531, 6609, 2282, 6589}

$$-\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3d^2(c+dx)\text{PolyLog}\left(3,e^{2i(a+bx)}\right)}{b^3}+\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b^3 - (4*(c + d*x)^3*ArcTan
h[E^((2*I)*(a + b*x))])/b - (3*d*(c + d*x)^2*Csc[2*a + 2*b*x])/b^2 - (2*(c
+ d*x)^3*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (((3*I)/2)*d^3*PolyLog[2, -
E^((2*I)*(a + b*x))])/b^4 + ((3*I)*d*(c + d*x)^2*PolyLog[2, -E^((2*I)*(a +
b*x))])/b^2 - (((3*I)/2)*d^3*PolyLog[2, E^((2*I)*(a + b*x))])/b^4 - ((3*I)*
d*(c + d*x)^2*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (3*d^2*(c + d*x)*PolyL
og[3, -E^((2*I)*(a + b*x))])/b^3 + (3*d^2*(c + d*x)*PolyLog[3, E^((2*I)*(a
+ b*x))])/b^3 - (((3*I)/2)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 + (((3
*I)/2)*d^3*PolyLog[4, E^((2*I)*(a + b*x))])/b^4
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
```

```
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^3 \csc^3(2a+2bx)\,dx$$

$$= -\frac{3d(c+dx)^2\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^3\cot(2a+2bx)\csc(2a+2bx}{b}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

$$= -\frac{6d^2(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b^3} - \frac{4(c+dx)^3\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{3d(c}{}$$

Mathematica [A] time = 8.63, size = 483, normalized size = 1.52

$$-\frac{8b^3c^3\tanh^{-1}\left(e^{2i(a+bx)}\right) - 12b^3c^2dx\log\left(1-e^{2i(a+bx)}\right) + 12b^3c^2dx\log\left(1+e^{2i(a+bx)}\right) - 12b^3cd^2x^2\log\left(1-e^{2i(a+b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(8*b^3*c^3*ArcTanh[E^((2*I)*(a + b*x))] + 12*b*c*d^2*ArcTanh[E^((2*I)*
(a + b*x))] + 2*b^2*(c + d*x)^2*(3*d + 2*b*(c + d*x)*Cot[2*(a + b*x)])*Csc[
2*(a + b*x)] - 12*b^3*c^2*d*x*Log[1 - E^((2*I)*(a + b*x))] - 6*b*d^3*x*Log[
```

```
1 - E^((2*I)*(a + b*x))] - 12*b^3*c*d^2*x^2*Log[1 - E^((2*I)*(a + b*x))] -
4*b^3*d^3*x^3*Log[1 - E^((2*I)*(a + b*x))] + 12*b^3*c^2*d*x*Log[1 + E^((2*I
)*(a + b*x))] + 6*b*d^3*x*Log[1 + E^((2*I)*(a + b*x))] + 12*b^3*c*d^2*x^2*L
og[1 + E^((2*I)*(a + b*x))] + 4*b^3*d^3*x^3*Log[1 + E^((2*I)*(a + b*x))] -
(3*I)*d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, -E^((2*I)*(a + b*x))] + (3*I)*
d*(d^2 + 2*b^2*(c + d*x)^2)*PolyLog[2, E^((2*I)*(a + b*x))] + 6*b*c*d^2*Pol
yLog[3, -E^((2*I)*(a + b*x))] + 6*b*d^3*x*PolyLog[3, -E^((2*I)*(a + b*x))]
- 6*b*c*d^2*PolyLog[3, E^((2*I)*(a + b*x))] - 6*b*d^3*x*PolyLog[3, E^((2*I)
*(a + b*x))] + (3*I)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))] - (3*I)*d^3*PolyL
og[4, E^((2*I)*(a + b*x))])/b^4
```

fricas [C] time = 0.95, size = 4193, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3 - 2*(b^3*d^3*
x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(b*x + a)^2 - 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d)*cos(b*x + a)*sin(b*x + a) - ((-6*I*b^2*d
^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^4 + (6*I*
b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*d
ilog(cos(b*x + a) + I*sin(b*x + a)) - ((6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x
+ 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*
d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(cos(b*x + a) - I*sin
(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^
3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3
*I*d^3)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a)) - ((6*I*b^2*d^
3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*
b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*d
ilog(I*cos(b*x + a) - sin(b*x + a)) - ((6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x
+ 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*I*b^2*d^3*x^2 - 12*I*b^2*c*
d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + si
n(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d
^3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d +
3*I*d^3)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - ((6*I*b^2*
d^3*x^2 + 12*I*b^2*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^4 + (-6*
I*b^2*d^3*x^2 - 12*I*b^2*c*d^2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^2)
*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((-6*I*b^2*d^3*x^2 - 12*I*b^2*c*d^
2*x - 6*I*b^2*c^2*d - 3*I*d^3)*cos(b*x + a)^4 + (6*I*b^2*d^3*x^2 + 12*I*b^2
*c*d^2*x + 6*I*b^2*c^2*d + 3*I*d^3)*cos(b*x + a)^2)*dilog(-cos(b*x + a) - I
*sin(b*x + a)) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2
+ 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*
x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*lo
```

```
g(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a
^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*
c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^2)*log(cos(
b*x + a) + I*sin(b*x + a) + I) - ((2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*
c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^
3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + ((2*b^3*c^3 - 6*a*b^
2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4 - (2*b^
3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x
+ a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + ((2*b^3*d^3*x^3 + 6*b^3*c*
d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*
d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c
^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b
*x + a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*b^3
*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c
^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^
2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*co
s(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + ((2*b^3*d^3*x^3 + 6*
b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^
3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a
*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)
*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + ((2*b^3*d^3*x^3
+ 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(
2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 +
 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3
)*x)*cos(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((2*b^3*c^3
- 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x + a)^4
 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*
cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - ((2*b^3
*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*cos(b*x +
 a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*
d^3)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) - ((
2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 +
3*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b
^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3
*c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1)
+ ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*
cos(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^
3 + 3*a)*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - ((2
*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3
*a)*d^3 + 3*(2*b^3*c^2*d + b*d^3)*x)*cos(b*x + a)^4 - (2*b^3*d^3*x^3 + 6*b^
3*c*d^2*x^2 + 6*a*b^2*c^2*d - 6*a^2*b*c*d^2 + (2*a^3 + 3*a)*d^3 + 3*(2*b^3*
c^2*d + b*d^3)*x)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) +
 ((2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3 + 3*a)*d^3)*c
os(b*x + a)^4 - (2*b^3*c^3 - 6*a*b^2*c^2*d + 3*(2*a^2 + 1)*b*c*d^2 - (2*a^3
```

```
 + 3*a)*d^3)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - (12*
I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x + a)^2)*polylog(4, cos(b*x + a) + I
*sin(b*x + a)) - (-12*I*d^3*cos(b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polyl
og(4, cos(b*x + a) - I*sin(b*x + a)) - (12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*
cos(b*x + a)^2)*polylog(4, I*cos(b*x + a) + sin(b*x + a)) - (-12*I*d^3*cos(
b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, I*cos(b*x + a) - sin(b*x +
 a)) - (-12*I*d^3*cos(b*x + a)^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, -I*c
os(b*x + a) + sin(b*x + a)) - (12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x +
 a)^2)*polylog(4, -I*cos(b*x + a) - sin(b*x + a)) - (-12*I*d^3*cos(b*x + a)
^4 + 12*I*d^3*cos(b*x + a)^2)*polylog(4, -cos(b*x + a) + I*sin(b*x + a)) -
(12*I*d^3*cos(b*x + a)^4 - 12*I*d^3*cos(b*x + a)^2)*polylog(4, -cos(b*x + a
) - I*sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b
*c*d^2)*cos(b*x + a)^2)*polylog(3, cos(b*x + a) + I*sin(b*x + a)) - 12*((b*
d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polyl
og(3, cos(b*x + a) - I*sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)
^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin(b*
x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(
b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*((b*d^3*x + b*c*
d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3
*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)) -
12*((b*d^3*x + b*c*d^2)*cos(b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2
)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 12*((b*d^3*x + b*c*d^2)*cos(
b*x + a)^4 - (b*d^3*x + b*c*d^2)*cos(b*x + a)^2)*polylog(3, -cos(b*x + a) -
 I*sin(b*x + a)))/(b^4*cos(b*x + a)^4 - b^4*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

maple [B] time = 0.22, size = 1329, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] -3/2*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4+3/2*I*d^3*polylog(2,-exp(2*I*(b
*x+a)))/b^4-3/b^3*d^2*c*ln(1+exp(2*I*(b*x+a)))-3/b^3*d^3*ln(1+exp(2*I*(b*x+
```

```
a)))*x-2/b*c^3*ln(1+exp(2*I*(b*x+a)))-2/b^4*d^3*a^3*ln(exp(I*(b*x+a))-1)+12
/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+12/b^3*c*d^2*polylog(3,exp(I*(b*x+a))
)+12/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x+12/b^3*d^3*polylog(3,-exp(I*(b*x+a
)))*x-3*I*d^3*polylog(2,exp(I*(b*x+a)))/b^4-3/b^3*c*d^2*polylog(3,-exp(2*I*
(b*x+a)))-3/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+2/b*c^3*ln(exp(I*(b*x+a)
)-1)+2/b*c^3*ln(exp(I*(b*x+a))+1)+3/b^3*d^2*c*ln(exp(I*(b*x+a))-1)+3/b^3*d^
2*c*ln(exp(I*(b*x+a))+1)+3/b^3*d^3*ln(exp(I*(b*x+a))+1)*x+3/b^3*d^3*ln(1-ex
p(I*(b*x+a)))*x+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a-3/b^4*d^3*a*ln(exp(I*(b*x+
a))-1)-3*I/b^4*d^3*polylog(2,-exp(I*(b*x+a)))+2/b^2/(1+exp(2*I*(b*x+a)))^2/
(exp(2*I*(b*x+a))-1)^2*(2*d^3*x^3*b*exp(6*I*(b*x+a))+6*c*d^2*x^2*b*exp(6*I*
(b*x+a))+6*c^2*d*x*b*exp(6*I*(b*x+a))+2*c^3*b*exp(6*I*(b*x+a))-3*I*d^3*x^2*
exp(6*I*(b*x+a))+2*b*d^3*x^3*exp(2*I*(b*x+a))-6*I*c*d^2*x*exp(6*I*(b*x+a))+
6*b*c*d^2*x^2*exp(2*I*(b*x+a))-3*I*c^2*d*exp(6*I*(b*x+a))+6*b*c^2*d*x*exp(2
*I*(b*x+a))+2*b*c^3*exp(2*I*(b*x+a))+3*I*d^3*x^2*exp(2*I*(b*x+a))+6*I*c*d^2
*x*exp(2*I*(b*x+a))+3*I*c^2*d*exp(2*I*(b*x+a)))+12*I/b^4*d^3*polylog(4,exp(
I*(b*x+a)))+12*I/b^4*d^3*polylog(4,-exp(I*(b*x+a)))-6*I/b^2*d^3*polylog(2,e
xp(I*(b*x+a)))*x^2-6*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2+3*I/b^2*c^2*d
*polylog(2,-exp(2*I*(b*x+a)))+3*I/b^2*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2-
6*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))-6*I/b^2*c^2*d*polylog(2,-exp(I*(b*x
+a)))+6/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))-1)+6/b*c^2*d*ln(exp(I*(b*x+a))+1)*x
+6/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+6/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-6/b^3*
c*d^2*a^2*ln(1-exp(I*(b*x+a)))+6/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2+6/b*c*d^2
*ln(exp(I*(b*x+a))+1)*x^2-6/b^2*c^2*d*a*ln(exp(I*(b*x+a))-1)+2/b*d^3*ln(1-e
xp(I*(b*x+a)))*x^3+2/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3+2/b*d^3*ln(exp(I*(b*x
+a))+1)*x^3-2/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-6/b*c^2*d*ln(1+exp(2*I*(b*x+
a)))*x-6/b*c*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+6*I/b^2*polylog(2,-exp(2*I*(b*x
+a)))*c*d^2*x-12*I/b^2*polylog(2,-exp(I*(b*x+a)))*c*d^2*x-12*I/b^2*polylog(
2,exp(I*(b*x+a)))*c*d^2*x
```

maxima [B] time = 3.73, size = 5610, normalized size = 17.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 3*a*c^2*d*((2*sin(b*x + a)^
2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*lo
g(sin(b*x + a)^2))/b + 3*a^2*c*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4
- sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2
- a^3*d^3*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^3 + 2*((16*(b*x + a)^3*d^3
+ 18*b*c*d^2 - 18*a*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*
d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 2*(8*(b*x + a)^3*d^3 + 9*b*c
```

```
*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*
c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 4*(8*(b*x + a)^3*d^3
 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d
 - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (16*I*(b*x +
 a)^3*d^3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x +
a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))
*sin(8*b*x + 8*a) + (-32*I*(b*x + a)^3*d^3 - 36*I*b*c*d^2 + 36*I*a*d^3 + (-
72*I*b*c*d^2 + 72*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2
 + (-72*I*a^2 - 36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(2*b*x +
 2*a), cos(2*b*x + 2*a) + 1) - (12*(b*x + a)^3*d^3 + 18*b*c*d^2 - 18*a*d^3
+ 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2
 + 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*
d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*
(b*x + a))*cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 +
 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 +
1)*d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-12*I*(b*x + a)^3*d^3 - 18*I*b*c*d^2
 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2 + (-36*I*b^2*c^2*d
 + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a))*sin(8*b*x + 8*a) - (
24*I*(b*x + a)^3*d^3 + 36*I*b*c*d^2 - 36*I*a*d^3 + (72*I*b*c*d^2 - 72*I*a*d
^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 144*I*a*b*c*d^2 + (72*I*a^2 + 36*I)*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (1
8*b*c*d^2 - 18*a*d^3 + 18*(b*c*d^2 - a*d^3)*cos(8*b*x + 8*a) - 36*(b*c*d^2
- a*d^3)*cos(4*b*x + 4*a) - (-18*I*b*c*d^2 + 18*I*a*d^3)*sin(8*b*x + 8*a) -
 (36*I*b*c*d^2 - 36*I*a*d^3)*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*
x + a) - 1) + (12*(b*x + a)^3*d^3 + 36*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 18*(
2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a) + 6*(2*(b*x + a)^3*d
^3 + 6*(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^
2 + 1)*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 12*(2*(b*x + a)^3*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b
*x + a))*cos(4*b*x + 4*a) + (12*I*(b*x + a)^3*d^3 + (36*I*b*c*d^2 - 36*I*a*
d^3)*(b*x + a)^2 + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3
)*(b*x + a))*sin(8*b*x + 8*a) + (-24*I*(b*x + a)^3*d^3 + (-72*I*b*c*d^2 + 7
2*I*a*d^3)*(b*x + a)^2 + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 + (-72*I*a^2 -
36*I)*d^3)*(b*x + a))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a)
 + 1) + (24*I*(b*x + a)^3*d^3 + 36*b^2*c^2*d - 72*a*b*c*d^2 + 36*a^2*d^3 -
36*(-2*I*b*c*d^2 + (2*I*a - 1)*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I
*a - 1)*b*c*d^2 + (72*I*a^2 - 72*a)*d^3)*(b*x + a))*cos(6*b*x + 6*a) + (24*
I*(b*x + a)^3*d^3 - 36*b^2*c^2*d + 72*a*b*c*d^2 - 36*a^2*d^3 + (72*I*b*c*d^
2 - 36*(2*I*a + 1)*d^3)*(b*x + a)^2 + (72*I*b^2*c^2*d - 72*(2*I*a + 1)*b*c*
d^2 + (72*I*a^2 + 72*a)*d^3)*(b*x + a))*cos(2*b*x + 2*a) - (18*b^2*c^2*d -
36*a*b*c*d^2 + 24*(b*x + a)^2*d^3 + 9*(2*a^2 + 1)*d^3 + 36*(b*c*d^2 - a*d^3
)*(b*x + a) + 3*(6*b^2*c^2*d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2
+ 1)*d^3 + 12*(b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 6*(6*b^2*c^2*
d - 12*a*b*c*d^2 + 8*(b*x + a)^2*d^3 + 3*(2*a^2 + 1)*d^3 + 12*(b*c*d^2 - a*
d^3)*(b*x + a))*cos(4*b*x + 4*a) - (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 - 24*I
```

```
*(b*x + a)^2*d^3 + (-18*I*a^2 - 9*I)*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*
x + a))*sin(8*b*x + 8*a) - (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 48*I*(b*x + a
)^2*d^3 + (36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*si
n(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (36*b^2*c^2*d - 72*a*b*c*d^2
+ 36*(b*x + a)^2*d^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3)*(b*x + a)
+ 18*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(
b*c*d^2 - a*d^3)*(b*x + a))*cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^
2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*co
s(4*b*x + 4*a) + (36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 +
(36*I*a^2 + 18*I)*d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(8*b*x +
8*a) + (-72*I*b^2*c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a
^2 - 36*I)*d^3 + (-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a)
)*dilog(-e^(I*b*x + I*a)) + (36*b^2*c^2*d - 72*a*b*c*d^2 + 36*(b*x + a)^2*d
^3 + 18*(2*a^2 + 1)*d^3 + 72*(b*c*d^2 - a*d^3)*(b*x + a) + 18*(2*b^2*c^2*d
- 4*a*b*c*d^2 + 2*(b*x + a)^2*d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(
b*x + a))*cos(8*b*x + 8*a) - 36*(2*b^2*c^2*d - 4*a*b*c*d^2 + 2*(b*x + a)^2*
d^3 + (2*a^2 + 1)*d^3 + 4*(b*c*d^2 - a*d^3)*(b*x + a))*cos(4*b*x + 4*a) + (
36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + 36*I*(b*x + a)^2*d^3 + (36*I*a^2 + 18*I)*
d^3 + (72*I*b*c*d^2 - 72*I*a*d^3)*(b*x + a))*sin(8*b*x + 8*a) + (-72*I*b^2*
c^2*d + 144*I*a*b*c*d^2 - 72*I*(b*x + a)^2*d^3 + (-72*I*a^2 - 36*I)*d^3 + (
-144*I*b*c*d^2 + 144*I*a*d^3)*(b*x + a))*sin(4*b*x + 4*a))*dilog(e^(I*b*x +
 I*a)) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-18*I*b*c*d^2 +
 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2 + (-18*I*a^2 -
 9*I)*d^3)*(b*x + a) + (-8*I*(b*x + a)^3*d^3 - 9*I*b*c*d^2 + 9*I*a*d^3 + (-
18*I*b*c*d^2 + 18*I*a*d^3)*(b*x + a)^2 + (-18*I*b^2*c^2*d + 36*I*a*b*c*d^2
+ (-18*I*a^2 - 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (16*I*(b*x + a)^3*d^
3 + 18*I*b*c*d^2 - 18*I*a*d^3 + (36*I*b*c*d^2 - 36*I*a*d^3)*(b*x + a)^2 + (
36*I*b^2*c^2*d - 72*I*a*b*c*d^2 + (36*I*a^2 + 18*I)*d^3)*(b*x + a))*cos(4*b
*x + 4*a) + (8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2 - a*d^3)
*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x + a))*s
in(8*b*x + 8*a) - 2*(8*(b*x + a)^3*d^3 + 9*b*c*d^2 - 9*a*d^3 + 18*(b*c*d^2
- a*d^3)*(b*x + a)^2 + 9*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x
 + a))*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*co
s(2*b*x + 2*a) + 1) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*
I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (1
8*I*a^2 + 9*I)*d^3)*(b*x + a) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*
d^3 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*
c*d^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (-12*I*(b*x + a
)^3*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a
)^2 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a)
)*cos(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^
2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b
*x + a))*sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*
(b*c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*
d^3)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*c
```

```
os(b*x + a) + 1) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3 + (18*I*b
*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d^2 + (18*I
*a^2 + 9*I)*d^3)*(b*x + a) + (6*I*(b*x + a)^3*d^3 + 9*I*b*c*d^2 - 9*I*a*d^3
 + (18*I*b*c*d^2 - 18*I*a*d^3)*(b*x + a)^2 + (18*I*b^2*c^2*d - 36*I*a*b*c*d
^2 + (18*I*a^2 + 9*I)*d^3)*(b*x + a))*cos(8*b*x + 8*a) + (-12*I*(b*x + a)^3
*d^3 - 18*I*b*c*d^2 + 18*I*a*d^3 + (-36*I*b*c*d^2 + 36*I*a*d^3)*(b*x + a)^2
 + (-36*I*b^2*c^2*d + 72*I*a*b*c*d^2 + (-36*I*a^2 - 18*I)*d^3)*(b*x + a))*c
os(4*b*x + 4*a) - 3*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*c*d^2 -
 a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3)*(b*x
+ a))*sin(8*b*x + 8*a) + 6*(2*(b*x + a)^3*d^3 + 3*b*c*d^2 - 3*a*d^3 + 6*(b*
c*d^2 - a*d^3)*(b*x + a)^2 + 3*(2*b^2*c^2*d - 4*a*b*c*d^2 + (2*a^2 + 1)*d^3
)*(b*x + a))*sin(4*b*x + 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(
b*x + a) + 1) + (12*d^3*cos(8*b*x + 8*a) - 24*d^3*cos(4*b*x + 4*a) + 12*I*d
^3*sin(8*b*x + 8*a) - 24*I*d^3*sin(4*b*x + 4*a) + 12*d^3)*polylog(4, -e^(2*
I*b*x + 2*I*a)) - (72*d^3*cos(8*b*x + 8*a) - 144*d^3*cos(4*b*x + 4*a) + 72*
I*d^3*sin(8*b*x + 8*a) - 144*I*d^3*sin(4*b*x + 4*a) + 72*d^3)*polylog(4, -e
^(I*b*x + I*a)) - (72*d^3*cos(8*b*x + 8*a) - 144*d^3*cos(4*b*x + 4*a) + 72*
I*d^3*sin(8*b*x + 8*a) - 144*I*d^3*sin(4*b*x + 4*a) + 72*d^3)*polylog(4, e^
(I*b*x + I*a)) + (-18*I*b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3 + (-18*I*
b*c*d^2 - 24*I*(b*x + a)*d^3 + 18*I*a*d^3)*cos(8*b*x + 8*a) + (36*I*b*c*d^2
 + 48*I*(b*x + a)*d^3 - 36*I*a*d^3)*cos(4*b*x + 4*a) + 6*(3*b*c*d^2 + 4*(b*
x + a)*d^3 - 3*a*d^3)*sin(8*b*x + 8*a) - 12*(3*b*c*d^2 + 4*(b*x + a)*d^3 -
3*a*d^3)*sin(4*b*x + 4*a))*polylog(3, -e^(2*I*b*x + 2*I*a)) + (72*I*b*c*d^2
 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3 + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 7
2*I*a*d^3)*cos(8*b*x + 8*a) + (-144*I*b*c*d^2 - 144*I*(b*x + a)*d^3 + 144*I
*a*d^3)*cos(4*b*x + 4*a) - 72*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(8*b*x +
 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d^3)*sin(4*b*x + 4*a))*polylog(3,
-e^(I*b*x + I*a)) + (72*I*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3 + (72*I
*b*c*d^2 + 72*I*(b*x + a)*d^3 - 72*I*a*d^3)*cos(8*b*x + 8*a) + (-144*I*b*c*
d^2 - 144*I*(b*x + a)*d^3 + 144*I*a*d^3)*cos(4*b*x + 4*a) - 72*(b*c*d^2 + (
b*x + a)*d^3 - a*d^3)*sin(8*b*x + 8*a) + 144*(b*c*d^2 + (b*x + a)*d^3 - a*d
^3)*sin(4*b*x + 4*a))*polylog(3, e^(I*b*x + I*a)) - (24*(b*x + a)^3*d^3 - 3
6*I*b^2*c^2*d + 72*I*a*b*c*d^2 - 36*I*a^2*d^3 + (72*b*c*d^2 - (72*a + 36*I)
*d^3)*(b*x + a)^2 + (72*b^2*c^2*d - (144*a + 72*I)*b*c*d^2 + 72*(a^2 + I*a)
*d^3)*(b*x + a))*sin(6*b*x + 6*a) - (24*(b*x + a)^3*d^3 + 36*I*b^2*c^2*d -
72*I*a*b*c*d^2 + 36*I*a^2*d^3 + (72*b*c*d^2 - (72*a - 36*I)*d^3)*(b*x + a)^
2 + (72*b^2*c^2*d - (144*a - 72*I)*b*c*d^2 + 72*(a^2 - I*a)*d^3)*(b*x + a))
*sin(2*b*x + 2*a))/(-6*I*b^3*cos(8*b*x + 8*a) + 12*I*b^3*cos(4*b*x + 4*a) +
 6*b^3*sin(8*b*x + 8*a) - 12*b^3*sin(4*b*x + 4*a) - 6*I*b^3))/b
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.324 $\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=190

$$-\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{Li}_3\left(e^{2i(a+bx)}\right)}{b^3}-\frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3}+\frac{2id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{Li}_2\left(e^{2i(\cdot}\right.}{b^2}$$

```
[Out] -4*(d*x+c)^2*arctanh(exp(2*I*(b*x+a)))/b-d^2*arctanh(cos(2*b*x+2*a))/b^3-2*
d*(d*x+c)*csc(2*b*x+2*a)/b^2-2*(d*x+c)^2*cot(2*b*x+2*a)*csc(2*b*x+2*a)/b+2*
I*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2-2*I*d*(d*x+c)*polylog(2,exp(2*
I*(b*x+a)))/b^2-d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+d^2*polylog(3,exp(2*I*
(b*x+a)))/b^3
```

Rubi [A] time = 0.21, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {4419, 4186, 3770, 4183, 2531, 2282, 6589}

$$\frac{2id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{2id(c+dx)\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{d^2\text{PolyLog}\left(3,\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-4*(c + d*x)^2*ArcTanh[E^((2*I)*(a + b*x))])/b - (d^2*ArcTanh[Cos[2*a + 2*
b*x]])/b^3 - (2*d*(c + d*x)*Csc[2*a + 2*b*x])/b^2 - (2*(c + d*x)^2*Cot[2*a
+ 2*b*x]*Csc[2*a + 2*b*x])/b + ((2*I)*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a +
 b*x))])/b^2 - ((2*I)*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))])/b^2 - (d
^2*PolyLog[3, -E^((2*I)*(a + b*x))])/b^3 + (d^2*PolyLog[3, E^((2*I)*(a + b*
x))])/b^3
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4186

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> -Simp[(b^2*(c + d*x)^m*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n -
 1)), x] + (Dist[(b^2*d^2*m*(m - 1))/(f^2*(n - 1)*(n - 2)), Int[(c + d*x)^(
m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[(b^2*(n - 2))/(n - 1), Int[(
c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[(b^2*d*m*(c + d*x)^(m -
1)*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /; FreeQ[{b, c, d,
e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^3(a+bx)\sec^3(a+bx)\,dx = 8\int (c+dx)^2 \csc^3(2a+2bx)\,dx$$

$$= -\frac{2d(c+dx)\csc(2a+2bx)}{b^2} - \frac{2(c+dx)^2\cot(2a+2bx)\csc(2a+2bx)}{b} +$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d$$

$$= -\frac{4(c+dx)^2\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b} - \frac{d^2\tanh^{-1}(\cos(2a+2bx))}{b^3} - \frac{2d(c+d$$

Mathematica [B] time = 8.18, size = 381, normalized size = 2.01

$$8\left(\frac{\csc(a)\csc(a+bx)\left(cd\sin(bx)+d^2x\sin(bx)\right)}{8b^2} + \frac{\sec(a)\sec(a+bx)\left(d^2(-x)\sin(bx)-cd\sin(bx)\right)}{8b^2} - \frac{d\csc(2a)(c\right.$$
$$4b^2$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] 8*(-1/4*(d*(c + d*x)*Csc[2*a])/b^2 + ((-c^2 - 2*c*d*x - d^2*x^2)*Csc[a + b*
x]^2)/(16*b) - (4*b^2*c^2*ArcTanh[E^((2*I)*(a + b*x))] + 2*d^2*ArcTanh[E^((
2*I)*(a + b*x))] - 4*b^2*c*d*x*Log[1 - E^((2*I)*(a + b*x))] - 2*b^2*d^2*x^2
*Log[1 - E^((2*I)*(a + b*x))] + 4*b^2*c*d*x*Log[1 + E^((2*I)*(a + b*x))] +
2*b^2*d^2*x^2*Log[1 + E^((2*I)*(a + b*x))] - (2*I)*b*d*(c + d*x)*PolyLog[2,
 -E^((2*I)*(a + b*x))] + (2*I)*b*d*(c + d*x)*PolyLog[2, E^((2*I)*(a + b*x))
] + d^2*PolyLog[3, -E^((2*I)*(a + b*x))] - d^2*PolyLog[3, E^((2*I)*(a + b*x
))])/(8*b^3) + ((c^2 + 2*c*d*x + d^2*x^2)*Sec[a + b*x]^2)/(16*b) + (Sec[a]*
Sec[a + b*x]*(-(c*d*Sin[b*x]) - d^2*x*Sin[b*x]))/(8*b^2) + (Csc[a]*Csc[a +
b*x]*(c*d*Sin[b*x] + d^2*x*Sin[b*x]))/(8*b^2))
```

fricas [C] time = 0.70, size = 2387, normalized size = 12.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*cos(b*x + a)^2 - 2*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) - (
(-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*cos(b
*x + a)^2)*dilog(cos(b*x + a) + I*sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*c*d
)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^2)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 + (4*I
*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilog(I*cos(b*x + a) + sin(b*x + a))
- ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*co
s(b*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - ((4*I*b*d^2*x + 4*I*b*
c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*c
os(b*x + a) + sin(b*x + a)) - ((-4*I*b*d^2*x - 4*I*b*c*d)*cos(b*x + a)^4 +
(4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x +
 a)) - ((4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^4 + (-4*I*b*d^2*x - 4*I*b*c*
d)*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x + a)) - ((-4*I*b*d^2*x -
 4*I*b*c*d)*cos(b*x + a)^4 + (4*I*b*d^2*x + 4*I*b*c*d)*cos(b*x + a)^2)*dilo
g(-cos(b*x + a) - I*sin(b*x + a)) - ((2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*
cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*
b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 +
 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I) - ((2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^4 - (2*b^2*d^2*x^2 + 4
*b^2*c*d*x + 2*b^2*c^2 + d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*
b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(cos(b*x + a) - I
*sin(b*x + a) + I) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
os(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d
*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a
*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) +
2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*
d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-I*cos(b*x
 + a) + sin(b*x + a) + 1) + 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*co
s(b*x + a)^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) - ((2*b^2*c^2 - 4*a*b
*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 +
1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) -
 ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4
*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) - 1/2*I*s
in(b*x + a) + 1/2) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*c
os(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x +
 a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d +
(2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)
*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I) - 2*((b^2*d^2*x^2
+ 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^4 - (b^2*d^2*x^2 + 2*b^2*
c*d*x + 2*a*b*c*d - a^2*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x
```

```
+ a) + 1) + ((2*b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^4 - (2*
b^2*c^2 - 4*a*b*c*d + (2*a^2 + 1)*d^2)*cos(b*x + a)^2)*log(-cos(b*x + a) -
I*sin(b*x + a) + I) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3
, cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)
^2)*polylog(3, cos(b*x + a) - I*sin(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2
*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 4*(d^2*cos(b*x
 + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 4
*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -I*cos(b*x + a) + sin
(b*x + a)) + 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*polylog(3, -I*cos(
b*x + a) - sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*x + a)^2)*poly
log(3, -cos(b*x + a) + I*sin(b*x + a)) - 4*(d^2*cos(b*x + a)^4 - d^2*cos(b*
x + a)^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)))/(b^3*cos(b*x + a)^4 -
 b^3*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^3 \sec(bx+a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

maple [B] time = 0.16, size = 716, normalized size = 3.77

$$\frac{2d^2a^2\ln\left(e^{i(bx+a)}-1\right)}{b^3}+\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)x^2}{b}-\frac{2d^2\ln\left(1-e^{i(bx+a)}\right)a^2}{b^3}+\frac{2d^2\ln\left(e^{i(bx+a)}+1\right)x^2}{b}-\frac{2c^2\ln\left(1+e^{2i(bx+a)}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] -d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+2/b^3*d^2*a^2*ln(exp(I*(b*x+a))-1)+2/
b*d^2*ln(1-exp(I*(b*x+a)))*x^2-2/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^2+2/b*d^2*l
n(exp(I*(b*x+a))+1)*x^2+4*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+4*d^2*polylog(
3,exp(I*(b*x+a)))/b^3-2/b*c^2*ln(1+exp(2*I*(b*x+a)))+2/b*c^2*ln(exp(I*(b*x+
a))-1)+2/b*c^2*ln(exp(I*(b*x+a))+1)-2/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2+4/b^
2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)^2*(d^2*x^2*b*exp(6*I*(b*x+a))
+2*c*d*x*b*exp(6*I*(b*x+a))+c^2*b*exp(6*I*(b*x+a))-I*d^2*x*exp(6*I*(b*x+a))
+b*d^2*x^2*exp(2*I*(b*x+a))-I*c*d*exp(6*I*(b*x+a))+2*b*c*d*x*exp(2*I*(b*x+a
))+b*c^2*exp(2*I*(b*x+a))+I*d^2*x*exp(2*I*(b*x+a))+I*c*d*exp(2*I*(b*x+a)))-
4/b*c*d*ln(1+exp(2*I*(b*x+a)))*x+4/b*c*d*ln(1-exp(I*(b*x+a)))*x+4/b^2*c*d*l
n(1-exp(I*(b*x+a)))*a+4/b*c*d*ln(exp(I*(b*x+a))+1)*x-4/b^2*c*d*a*ln(exp(I*(
b*x+a))-1)-1/b^3*d^2*ln(1+exp(2*I*(b*x+a)))+1/b^3*d^2*ln(exp(I*(b*x+a))+1)+
```

```
1/b^3*d^2*ln(exp(I*(b*x+a))-1)+2*I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x-4
*I/b^2*polylog(2,-exp(I*(b*x+a)))*d^2*x-4*I/b^2*polylog(2,exp(I*(b*x+a)))*d
^2*x+2*I/b^2*c*d*polylog(2,-exp(2*I*(b*x+a)))-4*I/b^2*c*d*polylog(2,-exp(I*
(b*x+a)))-4*I/b^2*c*d*polylog(2,exp(I*(b*x+a)))
```

maxima [B] time = 1.10, size = 2722, normalized size = 14.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(c^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log
(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2)) - 2*a*c*d*((2*sin(b*x + a)^2
- 1)/(sin(b*x + a)^4 - sin(b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(
sin(b*x + a)^2))/b + a^2*d^2*((2*sin(b*x + a)^2 - 1)/(sin(b*x + a)^4 - sin(
b*x + a)^2) + 2*log(sin(b*x + a)^2 - 1) - 2*log(sin(b*x + a)^2))/b^2 + 2*((
4*(b*x + a)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*
d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 4*(2*(b*x + a)^
2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) + (4*I*(b*x + a
)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*sin(8*b*x + 8*a) + (
-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a) - 4*I*d^2)*sin(
4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (4*(b*x + a
)^2*d^2 + 8*(b*c*d - a*d^2)*(b*x + a) + 2*d^2 + 2*(2*(b*x + a)^2*d^2 + 4*(b
*c*d - a*d^2)*(b*x + a) + d^2)*cos(8*b*x + 8*a) - 4*(2*(b*x + a)^2*d^2 + 4*
(b*c*d - a*d^2)*(b*x + a) + d^2)*cos(4*b*x + 4*a) - (-4*I*(b*x + a)^2*d^2 +
 (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*sin(8*b*x + 8*a) - (8*I*(b*x
 + a)^2*d^2 + (16*I*b*c*d - 16*I*a*d^2)*(b*x + a) + 4*I*d^2)*sin(4*b*x + 4*
a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*d^2*cos(8*b*x + 8*a) - 4*d
^2*cos(4*b*x + 4*a) + 2*I*d^2*sin(8*b*x + 8*a) - 4*I*d^2*sin(4*b*x + 4*a) +
 2*d^2)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (4*(b*x + a)^2*d^2 + 8*(b
*c*d - a*d^2)*(b*x + a) + 4*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))
*cos(8*b*x + 8*a) - 8*((b*x + a)^2*d^2 + 2*(b*c*d - a*d^2)*(b*x + a))*cos(4
*b*x + 4*a) + (4*I*(b*x + a)^2*d^2 + (8*I*b*c*d - 8*I*a*d^2)*(b*x + a))*sin
(8*b*x + 8*a) + (-8*I*(b*x + a)^2*d^2 + (-16*I*b*c*d + 16*I*a*d^2)*(b*x + a
))*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(-I*(b*x
+ a)^2*d^2 - b*c*d + a*d^2 + (-2*I*b*c*d + (2*I*a - 1)*d^2)*(b*x + a))*cos(
6*b*x + 6*a) + (8*I*(b*x + a)^2*d^2 - 8*b*c*d + 8*a*d^2 + (16*I*b*c*d - 8*(
2*I*a + 1)*d^2)*(b*x + a))*cos(2*b*x + 2*a) - (4*b*c*d + 4*(b*x + a)*d^2 -
4*a*d^2 + 4*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 8*(b*c*d + (
b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) - (-4*I*b*c*d - 4*I*(b*x + a)*d^2 +
4*I*a*d^2)*sin(8*b*x + 8*a) - (8*I*b*c*d + 8*I*(b*x + a)*d^2 - 8*I*a*d^2)*s
in(4*b*x + 4*a))*dilog(-e^(2*I*b*x + 2*I*a)) + (8*b*c*d + 8*(b*x + a)*d^2 -
 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 16*(b*c*d +
 (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2 -
```

```
 8*I*a*d^2)*sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16*I*a*d
^2)*sin(4*b*x + 4*a))*dilog(-e^(I*b*x + I*a)) + (8*b*c*d + 8*(b*x + a)*d^2
- 8*a*d^2 + 8*(b*c*d + (b*x + a)*d^2 - a*d^2)*cos(8*b*x + 8*a) - 16*(b*c*d
+ (b*x + a)*d^2 - a*d^2)*cos(4*b*x + 4*a) + (8*I*b*c*d + 8*I*(b*x + a)*d^2
- 8*I*a*d^2)*sin(8*b*x + 8*a) + (-16*I*b*c*d - 16*I*(b*x + a)*d^2 + 16*I*a*
d^2)*sin(4*b*x + 4*a))*dilog(e^(I*b*x + I*a)) + (-2*I*(b*x + a)^2*d^2 + (-4
*I*b*c*d + 4*I*a*d^2)*(b*x + a) - I*d^2 + (-2*I*(b*x + a)^2*d^2 + (-4*I*b*c
*d + 4*I*a*d^2)*(b*x + a) - I*d^2)*cos(8*b*x + 8*a) + (4*I*(b*x + a)^2*d^2
+ (8*I*b*c*d - 8*I*a*d^2)*(b*x + a) + 2*I*d^2)*cos(4*b*x + 4*a) + (2*(b*x +
 a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*a) - 2*(2*(b*x
 + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x + 4*a))*log(cos(
2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (2*I*(b*x +
 a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)^2*d^
2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2)*cos(8*b*x + 8*a) + (-4*I*(b*
x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x + 4*
a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x + 8*
a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x +
4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (2*I*(b*x
 + a)^2*d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2 + (2*I*(b*x + a)^2*
d^2 + (4*I*b*c*d - 4*I*a*d^2)*(b*x + a) + I*d^2)*cos(8*b*x + 8*a) + (-4*I*(
b*x + a)^2*d^2 + (-8*I*b*c*d + 8*I*a*d^2)*(b*x + a) - 2*I*d^2)*cos(4*b*x +
4*a) - (2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(8*b*x +
8*a) + 2*(2*(b*x + a)^2*d^2 + 4*(b*c*d - a*d^2)*(b*x + a) + d^2)*sin(4*b*x
+ 4*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + (-2*I*d
^2*cos(8*b*x + 8*a) + 4*I*d^2*cos(4*b*x + 4*a) + 2*d^2*sin(8*b*x + 8*a) - 4
*d^2*sin(4*b*x + 4*a) - 2*I*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + (8*I*d^
2*cos(8*b*x + 8*a) - 16*I*d^2*cos(4*b*x + 4*a) - 8*d^2*sin(8*b*x + 8*a) + 1
6*d^2*sin(4*b*x + 4*a) + 8*I*d^2)*polylog(3, -e^(I*b*x + I*a)) + (8*I*d^2*c
os(8*b*x + 8*a) - 16*I*d^2*cos(4*b*x + 4*a) - 8*d^2*sin(8*b*x + 8*a) + 16*d
^2*sin(4*b*x + 4*a) + 8*I*d^2)*polylog(3, e^(I*b*x + I*a)) - (8*(b*x + a)^2
*d^2 - 8*I*b*c*d + 8*I*a*d^2 + (16*b*c*d - (16*a + 8*I)*d^2)*(b*x + a))*sin
(6*b*x + 6*a) - (8*(b*x + a)^2*d^2 + 8*I*b*c*d - 8*I*a*d^2 + (16*b*c*d - (1
6*a - 8*I)*d^2)*(b*x + a))*sin(2*b*x + 2*a))/(-2*I*b^2*cos(8*b*x + 8*a) + 4
*I*b^2*cos(4*b*x + 4*a) + 2*b^2*sin(8*b*x + 8*a) - 4*b^2*sin(4*b*x + 4*a) -
 2*I*b^2))/b
```

mupad **[F(-1)]** time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**3*sec(b*x+a)**3,x)
[Out] Timed out
```

3.325 $\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx$

Optimal. Leaf size=110

$$\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{Li}_2\left(e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\cot(2a+2bx)\csc(2a}{b}$$

```
[Out] -4*(d*x+c)*arctanh(exp(2*I*(b*x+a)))/b-d*csc(2*b*x+2*a)/b^2-2*(d*x+c)*cot(2
*b*x+2*a)*csc(2*b*x+2*a)/b+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2-I*d*polylog
(2,exp(2*I*(b*x+a)))/b^2
```

Rubi [A] time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {4419, 4185, 4183, 2279, 2391}

$$\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-\frac{id\text{PolyLog}\left(2,e^{2i(a+bx)}\right)}{b^2}-\frac{d\csc(2a+2bx)}{b^2}-\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{2(c+dx)\cot(2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] (-4*(c + d*x)*ArcTanh[E^((2*I)*(a + b*x))])/b - (d*Csc[2*a + 2*b*x])/b^2 -
(2*(c + d*x)*Cot[2*a + 2*b*x]*Csc[2*a + 2*b*x])/b + (I*d*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 - (I*d*PolyLog[2, E^((2*I)*(a + b*x))])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4419

```
Int[Csc[(a_.) + (b_.)*(x_)]^(n_.)*((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Dist[2^n, Int[(c + d*x)^m*Csc[2*a + 2*b*x]^n,
 x], x] /; FreeQ[{a, b, c, d, m}, x] && IntegerQ[n] && RationalQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\csc^3(a+bx)\sec^3(a+bx)\,dx &= 8\int (c+dx)\csc^3(2a+2bx)\,dx \\
&= -\frac{d\csc(2a+2bx)}{b^2}-\frac{2(c+dx)\cot(2a+2bx)\csc(2a+2bx)}{b}+4\int (c+ \\
&= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\csc(2a+2bx)}{b^2}-\frac{2(c+dx)\cot(2a+ }{b} \\
&= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\csc(2a+2bx)}{b^2}-\frac{2(c+dx)\cot(2a+ }{b} \\
&= -\frac{4(c+dx)\tanh^{-1}\left(e^{2i(a+bx)}\right)}{b}-\frac{d\csc(2a+2bx)}{b^2}-\frac{2(c+dx)\cot(2a+ }{b}
\end{aligned}
$$

Mathematica [B] time = 2.09, size = 236, normalized size = 2.15

$$
\frac{d\left(i\left(\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)-\mathrm{Li}_2\left(e^{2i(a+bx)}\right)\right)+2(a+bx)\left(\log\left(1-e^{2i(a+bx)}\right)-\log\left(1+e^{2i(a+bx)}\right)\right)\right)}{b^2}-\frac{d\tan(a+bx)}{2b^2}-\frac{d\cot}{2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^3*Sec[a + b*x]^3,x]
```

```
[Out] -1/2*(d*Cot[a + b*x])/b^2 - (c*Csc[a + b*x]^2)/(2*b) + (d*(2*a - 2*(a + b*x
))*Csc[a + b*x]^2)/(4*b^2) - (2*c*Log[Cos[a + b*x]])/b + (2*c*Log[Sin[a + b
*x]])/b - (2*a*d*Log[Tan[a + b*x]])/b^2 + (d*(2*(a + b*x)*(Log[1 - E^((2*I)
*(a + b*x))] - Log[1 + E^((2*I)*(a + b*x))]) + I*(PolyLog[2, -E^((2*I)*(a +
 b*x))] - PolyLog[2, E^((2*I)*(a + b*x))])))/b^2 + (c*Sec[a + b*x]^2)/(2*b)
```

```
 + (d*(-2*a + 2*(a + b*x))*Sec[a + b*x]^2)/(4*b^2) - (d*Tan[a + b*x])/(2*b^
2)
```

fricas [B] time = 0.60, size = 1193, normalized size = 10.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(b*d*x - 2*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*sin(b*x + a)
+ b*c - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(cos(b*x + a) +
 I*sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2*I*d*cos(b*x + a)^2)*dilog(cos(
b*x + a) - I*sin(b*x + a)) - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)
*dilog(I*cos(b*x + a) + sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2*I*d*cos(b
*x + a)^2)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (2*I*d*cos(b*x + a)^4 - 2
*I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (-2*I*d*cos(b*
x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - (2
*I*d*cos(b*x + a)^4 - 2*I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a) + I*sin(b*x
 + a)) - (-2*I*d*cos(b*x + a)^4 + 2*I*d*cos(b*x + a)^2)*dilog(-cos(b*x + a)
 - I*sin(b*x + a)) - 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*
x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x +
 a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(cos(b*x + a) + I*sin(b*x + a) + I)
- 2*((b*d*x + b*c)*cos(b*x + a)^4 - (b*d*x + b*c)*cos(b*x + a)^2)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)
*cos(b*x + a)^2)*log(cos(b*x + a) - I*sin(b*x + a) + I) + 2*((b*d*x + a*d)*
cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(I*cos(b*x + a) + sin(b*x
 + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2
)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + 2*((b*d*x + a*d)*cos(b*x + a)^4
- (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 2
*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)*log(-I*cos(b
*x + a) - sin(b*x + a) + 1) - 2*((b*c - a*d)*cos(b*x + a)^4 - (b*c - a*d)*c
os(b*x + a)^2)*log(-1/2*cos(b*x + a) + 1/2*I*sin(b*x + a) + 1/2) - 2*((b*c
- a*d)*cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-1/2*cos(b*x + a) -
 1/2*I*sin(b*x + a) + 1/2) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d
)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 2*((b*c - a*d)*
cos(b*x + a)^4 - (b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) + I*sin(b*x
+ a) + I) - 2*((b*d*x + a*d)*cos(b*x + a)^4 - (b*d*x + a*d)*cos(b*x + a)^2)
*log(-cos(b*x + a) - I*sin(b*x + a) + 1) + 2*((b*c - a*d)*cos(b*x + a)^4 -
(b*c - a*d)*cos(b*x + a)^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/(b^2*c
os(b*x + a)^4 - b^2*cos(b*x + a)^2)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \csc (bx + a)^3 \sec (bx + a)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^3*sec(b*x + a)^3, x)
```

maple [B] time = 0.17, size = 325, normalized size = 2.95

$$\frac{4bdx\,e^{6i(bx+a)}+4cb\,e^{6i(bx+a)}-2id\,e^{6i(bx+a)}+4bdx\,e^{2i(bx+a)}+4bc\,e^{2i(bx+a)}+2id\,e^{2i(bx+a)}}{b^2\left(1+e^{2i(bx+a)}\right)^2\left(e^{2i(bx+a)}-1\right)^2}+\frac{2c\ln\left(e^{i(bx+a)}-1\right)}{b}-\frac{2c\,l}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x)
```

```
[Out] 2/b^2/(1+exp(2*I*(b*x+a)))^2/(exp(2*I*(b*x+a))-1)^2*(2*b*d*x*exp(6*I*(b*x+a
))+2*c*b*exp(6*I*(b*x+a))-I*d*exp(6*I*(b*x+a))+2*b*d*x*exp(2*I*(b*x+a))+2*b
*c*exp(2*I*(b*x+a))+I*d*exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))-1)-2/b*c*
ln(1+exp(2*I*(b*x+a)))+2/b*c*ln(exp(I*(b*x+a))+1)-2/b*d*ln(1+exp(2*I*(b*x+a
)))*x+I*d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b*d*ln(exp(I*(b*x+a))+1)*x-2*I
*d*polylog(2,-exp(I*(b*x+a)))/b^2+2/b*d*ln(1-exp(I*(b*x+a)))*x+2/b^2*d*ln(1
-exp(I*(b*x+a)))*a-2*I/b^2*d*polylog(2,exp(I*(b*x+a)))-2/b^2*d*a*ln(exp(I*(
b*x+a))-1)
```

maxima [B] time = 0.75, size = 1078, normalized size = 9.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^3*sec(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -((2*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*cos
(4*b*x + 4*a) + (2*I*b*d*x + 2*I*b*c)*sin(8*b*x + 8*a) + (-4*I*b*d*x - 4*I*
b*c)*sin(4*b*x + 4*a))*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - (2
*b*d*x + 2*b*c + 2*(b*d*x + b*c)*cos(8*b*x + 8*a) - 4*(b*d*x + b*c)*cos(4*b
*x + 4*a) - (-2*I*b*d*x - 2*I*b*c)*sin(8*b*x + 8*a) - (4*I*b*d*x + 4*I*b*c)
*sin(4*b*x + 4*a))*arctan2(sin(b*x + a), cos(b*x + a) + 1) - (2*b*c*cos(8*b
*x + 8*a) - 4*b*c*cos(4*b*x + 4*a) + 2*I*b*c*sin(8*b*x + 8*a) - 4*I*b*c*sin
(4*b*x + 4*a) + 2*b*c)*arctan2(sin(b*x + a), cos(b*x + a) - 1) + (2*b*d*x*c
os(8*b*x + 8*a) - 4*b*d*x*cos(4*b*x + 4*a) + 2*I*b*d*x*sin(8*b*x + 8*a) - 4
*I*b*d*x*sin(4*b*x + 4*a) + 2*b*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) +
1) + (4*I*b*d*x + 4*I*b*c + 2*d)*cos(6*b*x + 6*a) + (4*I*b*d*x + 4*I*b*c -
2*d)*cos(2*b*x + 2*a) - (d*cos(8*b*x + 8*a) - 2*d*cos(4*b*x + 4*a) + I*d*si
n(8*b*x + 8*a) - 2*I*d*sin(4*b*x + 4*a) + d)*dilog(-e^(2*I*b*x + 2*I*a)) +
(2*d*cos(8*b*x + 8*a) - 4*d*cos(4*b*x + 4*a) + 2*I*d*sin(8*b*x + 8*a) - 4*I
```

```
*d*sin(4*b*x + 4*a) + 2*d)*dilog(-e^(I*b*x + I*a)) + (2*d*cos(8*b*x + 8*a)
- 4*d*cos(4*b*x + 4*a) + 2*I*d*sin(8*b*x + 8*a) - 4*I*d*sin(4*b*x + 4*a) +
2*d)*dilog(e^(I*b*x + I*a)) + (-I*b*d*x - I*b*c + (-I*b*d*x - I*b*c)*cos(8*
b*x + 8*a) + (2*I*b*d*x + 2*I*b*c)*cos(4*b*x + 4*a) + (b*d*x + b*c)*sin(8*b
*x + 8*a) - 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(2*b*x + 2*a)^2 + sin(
2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*
b*c)*cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*cos(4*b*x + 4*a) - (b*d*x +
b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 + 2*cos(b*x + a) + 1) + (I*b*d*x + I*b*c + (I*b*d*x + I*
b*c)*cos(8*b*x + 8*a) + (-2*I*b*d*x - 2*I*b*c)*cos(4*b*x + 4*a) - (b*d*x +
b*c)*sin(8*b*x + 8*a) + 2*(b*d*x + b*c)*sin(4*b*x + 4*a))*log(cos(b*x + a)^
2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) - (4*b*d*x + 4*b*c - 2*I*d)*sin(6*
b*x + 6*a) - (4*b*d*x + 4*b*c + 2*I*d)*sin(2*b*x + 2*a))/(-I*b^2*cos(8*b*x
+ 8*a) + 2*I*b^2*cos(4*b*x + 4*a) + b^2*sin(8*b*x + 8*a) - 2*b^2*sin(4*b*x
+ 4*a) - I*b^2)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(cos(a + b*x)^3*sin(a + b*x)^3),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**3*sec(b*x+a)**3,x)
```

```
[Out] Timed out
```

3.326 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=24

$$8\text{Int}\left(\frac{\csc^3(2a+2bx)}{c+dx},x\right)$$

```
[Out] 8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c),x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx = 8\int \frac{\csc^3(2a+2bx)}{c+dx}\,dx$$

Mathematica [A] time = 28.70, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x),x]
```

```
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x), x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^3}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{3}\sec\left(bx+a\right)^{3}}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^3*sec(b*x + a)^3/(d*x + c), x)
```

maple [A] time = 1.05, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\left(\sec^{3}\left(bx+a\right)\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c),x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\cos\left(a+b\,x\right)^{3}\,\sin\left(a+b\,x\right)^{3}\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c),x)
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x), x)
```

3.327 $\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=24

$$8\text{Int}\left(\frac{\csc^3(2a+2bx)}{(c+dx)^2},x\right)$$

```
[Out] 8*Unintegrable(csc(2*b*x+2*a)^3/(d*x+c)^2,x)
```

Rubi [A] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] 8*Defer[Int][Csc[2*a + 2*b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx = 8\int \frac{\csc^3(2a+2bx)}{(c+dx)^2}\,dx$$

Mathematica [A] time = 32.47, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(a+bx)\sec^3(a+bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^3*Sec[a + b*x]^3)/(c + d*x)^2, x]
```

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(bx+a)^3\sec(bx+a)^3}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^3*sec(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
[Out] Timed out
```

maple [A] time = 3.79, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{3}\left(bx+a\right)\right)\left(\sec^{3}\left(bx+a\right)\right)}{\left(dx+c\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
[Out] int(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^3*sec(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\cos\left(a+b\,x\right)}^3\,{\sin\left(a+b\,x\right)}^3\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2),x)
[Out] int(1/(cos(a + b*x)^3*sin(a + b*x)^3*(c + d*x)^2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(a+bx\right)}\sec^{3}{\left(a+bx\right)}}{\left(c+dx\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**3*sec(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(csc(a + b*x)**3*sec(a + b*x)**3/(c + d*x)**2, x)
```

3.328 $\int x \cos^{\frac{5}{2}}(a+bx) \sin(a+bx)\,dx$

Optimal. Leaf size=83

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] -2/7*x*cos(b*x+a)^(7/2)/b+20/147*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1
/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+4/49*cos(b*x+a)^(5/2)*sin(b
*x+a)/b^2+20/147*sin(b*x+a)*cos(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2641}

$$\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{4\sin(a+bx)\cos^{\frac{5}{2}}(a+bx)}{49b^2}+\frac{20\sin(a+bx)\sqrt{\cos(a+bx)}}{147b^2}-\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(7/2))/(7*b) + (20*EllipticF[(a + b*x)/2, 2])/(147*b^2)
+ (20*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(147*b^2) + (4*Cos[a + b*x]^(5/2)*Si
n[a + b*x])/(49*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\cos^{\frac{7}{2}}(a+bx)\,dx}{7b}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx)}{49b^2}+\frac{10\int\cos^{\frac{3}{2}}(a+bx)\,dx}{49b}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2}+\frac{4\cos^{\frac{5}{2}}(a+bx)\sin(a+bx}{49b^2}\\ &= -\frac{2x\cos^{\frac{7}{2}}(a+bx)}{7b}+\frac{20F\left(\frac{1}{2}(a+bx)\middle|2\right)}{147b^2}+\frac{20\sqrt{\cos(a+bx)}\,\sin(a+bx)}{147b^2}+\frac{4\cos}{}\end{aligned}$$

Mathematica [A] time = 0.33, size = 73, normalized size = 0.88

$$\frac{40F\left(\frac{1}{2}(a+bx)\middle|2\right)+\sqrt{\cos(a+bx)}\,(46\sin(a+bx)+6\sin(3(a+bx))-63bx\cos(a+bx)-21bx\cos(3(a+bx)))}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (40*EllipticF[(a + b*x)/2, 2] + Sqrt[Cos[a + b*x]]*(-63*b*x*Cos[a + b*x] -
21*b*x*Cos[3*(a + b*x)] + 46*Sin[a + b*x] + 6*Sin[3*(a + b*x)]))/(294*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)^{\frac{5}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\left(\cos^{\frac{5}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
[Out] int(x*cos(b*x+a)^(5/2)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)^{\frac{5}{2}}\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)^(5/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\cos\left(a+b\,x\right)}^{5/2}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(5/2)*sin(a + b*x),x)
[Out] int(x*cos(a + b*x)^(5/2)*sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(5/2)*sin(b*x+a),x)
[Out] Timed out
```

3.329 $\int x \cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -2/5*x*cos(b*x+a)^(5/2)/b+12/25*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/
2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+4/25*cos(b*x+a)^(3/2)*sin(b*
x+a)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\sin(a+bx)\cos^{\frac{3}{2}}(a+bx)}{25b^2}-\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(5/2))/(5*b) + (12*EllipticE[(a + b*x)/2, 2])/(25*b^2) +
 (4*Cos[a + b*x]^(3/2)*Sin[a + b*x])/(25*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int\cos^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}+\frac{6\int\sqrt{\cos(a+bx)}\,dx}{25b}$$
$$= -\frac{2x\cos^{\frac{5}{2}}(a+bx)}{5b}+\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}+\frac{4\cos^{\frac{3}{2}}(a+bx)\sin(a+bx)}{25b^2}$$

Mathematica [A] time = 0.37, size = 51, normalized size = 0.85

$$-\frac{2\left(\cos^{\frac{3}{2}}(a+bx)(5bx\cos(a+bx)-2\sin(a+bx))-6E\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (-2*(-6*EllipticE[(a + b*x)/2, 2] + Cos[a + b*x]^(3/2)*(5*b*x*Cos[a + b*x]
- 2*Sin[a + b*x])))/(25*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x \left(\cos^{\frac{3}{2}}\left(bx+a\right)\right) \sin \left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
[Out] int(x*cos(b*x+a)^(3/2)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos\left(bx+a\right)^{\frac{3}{2}} \sin\left(bx+a\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)^(3/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x \cos\left(a+b\,x\right)^{3/2} \sin\left(a+b\,x\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(3/2)*sin(a + b*x),x)
[Out] int(x*cos(a + b*x)^(3/2)*sin(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)**(3/2)*sin(b*x+a),x)
[Out] Timed out
```

3.330 $\int x\sqrt{\cos(a+bx)}\,\sin(a+bx)\,dx$

Optimal. Leaf size=60

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] -2/3*x*cos(b*x+a)^(3/2)/b+4/9*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*
a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+4/9*sin(b*x+a)*cos(b*x+a)^(1/2
)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2635, 2641}

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sin(a+bx)\sqrt{\cos(a+bx)}}{9b^2}-\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x*Cos[a + b*x]^(3/2))/(3*b) + (4*EllipticF[(a + b*x)/2, 2])/(9*b^2) + (
4*Sqrt[Cos[a + b*x]]*Sin[a + b*x])/(9*b^2)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\sqrt{\cos(a+bx)}\sin(a+bx)\,dx &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int\cos^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}+\frac{2\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{9b}\\ &= -\frac{2x\cos^{\frac{3}{2}}(a+bx)}{3b}+\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}+\frac{4\sqrt{\cos(a+bx)}\sin(a+bx)}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.15, size = 52, normalized size = 0.87

$$\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)+2\sqrt{\cos(a+bx)}\,(2\sin(a+bx)-3bx\cos(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Cos[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (4*EllipticF[(a + b*x)/2, 2] + 2*Sqrt[Cos[a + b*x]]*(-3*b*x*Cos[a + b*x] +
2*Sin[a + b*x]))/(9*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cos(bx+a)}\,\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x\sin\left(bx+a\right)\left(\sqrt{\cos}\ \left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
[Out] int(x*sin(b*x+a)*cos(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\cos\left(bx+a\right)}\,\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(cos(b*x + a))*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sqrt{\cos\left(a+b\,x\right)}\,\sin\left(a+b\,x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)^(1/2)*sin(a + b*x),x)
[Out] int(x*cos(a + b*x)^(1/2)*sin(a + b*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin\left(a+bx\right)\sqrt{\cos\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*cos(b*x+a)**(1/2),x)
[Out] Integral(x*sin(a + b*x)*sqrt(cos(a + b*x)), x)
```

3.331 $\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx$

Optimal. Leaf size=33

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

```
[Out] 4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2
*a),2^(1/2))/b^2-2*x*cos(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3444, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x\sqrt{\cos(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (-2*x*Sqrt[Cos[a + b*x]])/b + (4*EllipticE[(a + b*x)/2, 2])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\sqrt{\cos(a+bx)}}\,dx &= -\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{2\int\sqrt{\cos(a+bx)}\,dx}{b}\\ &= -\frac{2x\sqrt{\cos(a+bx)}}{b}+\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}\end{aligned}$$

Mathematica [B] time = 1.76, size = 181, normalized size = 5.48

$$\frac{4\cos^2\left(\frac{1}{2}(a+bx)\right)^{3/2}\sqrt{\frac{\cos(a+bx)}{(\cos(a+bx)+1)^2}}\sqrt{\frac{1}{\cos(a+bx)+1}}\left(\left(2\tan\left(\frac{1}{2}(a+bx)\right)-bx\right)\sqrt{\cos(a+bx)\sec^2\left(\frac{1}{2}(a+bx)\right)}-2\ldots\right.}{b^2\sqrt{\frac{\cos(a\ldots}{\cos(a+\ldots}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Cos[a + b*x]],x]
```

```
[Out] (4*(Cos[(a + b*x)/2]^2)^(3/2)*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])^2]*Sqrt[
(1 + Cos[a + b*x])^(-1)]*(2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Se
c[(a + b*x)/2]^2] - 2*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Sec[(a +
 b*x)/2]^2] + Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^2]*(-(b*x) + 2*Tan[(a + b*
x)/2])))/(b^2*Sqrt[Cos[a + b*x]/(1 + Cos[a + b*x])])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\sqrt{\cos(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

maple [C] time = 0.16, size = 310, normalized size = 9.39

$$-\frac{(bx+2i)\left(1+e^{2i(bx+a)}\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{\left(1+e^{2i(bx+a)}\right)e^{-i(bx+a)}}}-\frac{2i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{\left(1+e^{2i(bx+a)}\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{ie^{i(bx+a)}}\left(-2i\,\mathrm{EllipticE}\left(\ldots\right.\right.}{\sqrt{e^{3i(bx+a)}+\ldots}}\right.}{b^2\sqrt{\left(1+\ldots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(1/2),x)
```

```
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)/((exp(I*(b*x+a))^2+1)/exp(I*(b*
x+a)))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)/((exp(I*(b*x
+a))^2+1)/exp(I*(b*x+a)))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx + a)}{\sqrt{\cos(bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(cos(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\, \sin(a + b\, x)}{\sqrt{\cos(a + b\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(1/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a + bx)}{\sqrt{\cos(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sqrt(cos(a + b*x)), x)
```

3.332 $\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=33

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

```
[Out] -4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2+2*x/b/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3444, 2641}

$$\frac{2x}{b\sqrt{\cos(a+bx)}}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p + 1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{b}$$
$$= \frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Mathematica [A] time = 0.16, size = 33, normalized size = 1.00

$$\frac{2x}{b\sqrt{\cos(a+bx)}} - \frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(3/2),x]
[Out] (2*x)/(b*Sqrt[Cos[a + b*x]]) - (4*EllipticF[(a + b*x)/2, 2])/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\sin(a+b\,x)}{{\cos(a+b\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(3/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/cos(a + b*x)**(3/2), x)
```

3.333 $\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=60

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*x/b/cos(b*x+a)^(3/2)+4/3*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a
)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))/b^2-4/3*sin(b*x+a)/b^2/cos(b*x+a)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2639}

$$\frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}-\frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}+\frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(3*b*Cos[a + b*x]^(3/2)) + (4*EllipticE[(a + b*x)/2, 2])/(3*b^2) - (4
*Sin[a + b*x])/(3*b^2*Sqrt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\cos^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{3b}$$
$$= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}} + \frac{2\int \sqrt{\cos(a+bx)}\,dx}{3b}$$
$$= \frac{2x}{3b\cos^{\frac{3}{2}}(a+bx)} + \frac{4E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2} - \frac{4\sin(a+bx)}{3b^2\sqrt{\cos(a+bx)}}$$

Mathematica [A] time = 0.19, size = 54, normalized size = 0.90

$$\frac{2\left(-\sin(2(a+bx))+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+bx\right)}{3b^2\cos^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(5/2),x]
```

```
[Out] (2*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2] - Sin[2*(a + b*x)]
))/(3*b^2*Cos[a + b*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\cos(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x \sin(a+b\,x)}{\cos(a+b\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(5/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(5/2),x)
```

```
[Out] Timed out
```

3.334 $\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=60

$$-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/cos(b*x+a)^(5/2)-4/15*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*
a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))/b^2-4/15*sin(b*x+a)/b^2/cos(b*x+a)
^(3/2)
```

Rubi [A] time = 0.04, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2641}

$$-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}+\frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (2*x)/(5*b*Cos[a + b*x]^(5/2)) - (4*EllipticF[(a + b*x)/2, 2])/(15*b^2) - (
4*Sin[a + b*x])/(15*b^2*Cos[a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\cos^{\frac{7}{2}}(a+bx)}\,dx &= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{2\int \frac{1}{\cos^{\frac{5}{2}}(a+bx)}\,dx}{5b}\\ &= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}-\frac{2\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{15b}\\ &= \frac{2x}{5b\cos^{\frac{5}{2}}(a+bx)}-\frac{4F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}-\frac{4\sin(a+bx)}{15b^2\cos^{\frac{3}{2}}(a+bx)}\end{aligned}$$

Mathematica [A] time = 0.23, size = 53, normalized size = 0.88

$$-\frac{2\left(\sin(2(a+bx))+2\cos^{\frac{5}{2}}(a+bx)F\left(\frac{1}{2}(a+bx)\middle|2\right)-3bx\right)}{15b^2\cos^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(7/2),x]
```

```
[Out] (-2*(-3*b*x + 2*Cos[a + b*x]^(5/2)*EllipticF[(a + b*x)/2, 2] + Sin[2*(a + b
*x)]))/(15*b^2*Cos[a + b*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x \sin \left(b x +a \right)}{\cos \left(b x +a \right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

```
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin \left(b x + a\right)}{\cos \left(b x + a\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\sin\left(a+b\,x\right)}{{\cos\left(a+b\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(7/2),x)
```

```
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.335 $\int \frac{x \sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\, dx$

Optimal. Leaf size=83

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

```
[Out] 2/7*x/b/cos(b*x+a)^(7/2)+12/35*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2
*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))/b^2-4/35*sin(b*x+a)/b^2/cos(b*x+a
)^(5/2)-12/35*sin(b*x+a)/b^2/cos(b*x+a)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3444, 2636, 2639}

$$\frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (2*x)/(7*b*Cos[a + b*x]^(7/2)) + (12*EllipticE[(a + b*x)/2, 2])/(35*b^2) -
(4*Sin[a + b*x])/(35*b^2*Cos[a + b*x]^(5/2)) - (12*Sin[a + b*x])/(35*b^2*Sq
rt[Cos[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3444

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]^(p_.)*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(
n_.)], x_Symbol] :> -Simp[(x^(m - n + 1)*Cos[a + b*x^n]^(p + 1))/(b*n*(p +
```

```
1)), x] + Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Cos[a + b*x^n]^(p +
 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\cos^{\frac{9}{2}}(a+bx)}\,dx &= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\cos^{\frac{7}{2}}(a+bx)}\,dx}{7b}\\ &= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{6\int \frac{1}{\cos^{\frac{3}{2}}(a+bx)}\,dx}{35b}\\ &= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}} + \frac{6\int \sqrt{\cos(a+bx)}\,dx}{35b}\\ &= \frac{2x}{7b\cos^{\frac{7}{2}}(a+bx)} + \frac{12E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2} - \frac{4\sin(a+bx)}{35b^2\cos^{\frac{5}{2}}(a+bx)} - \frac{12\sin(a+bx)}{35b^2\sqrt{\cos(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.26, size = 65, normalized size = 0.78

$$\frac{-10\sin(2(a+bx)) - 3\sin(4(a+bx)) + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) + 20bx}{70b^2\cos^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Cos[a + b*x]^(9/2),x]
```

```
[Out] (20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2, 2] - 10*Sin[2*(a + b
*x)] - 3*Sin[4*(a + b*x)])/(70*b^2*Cos[a + b*x]^(7/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
[Out] int(x*sin(b*x+a)/cos(b*x+a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sin(bx+a)}{\cos(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*sin(b*x + a)/cos(b*x + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(a+b\,x)}{\cos(a+b\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/cos(a + b*x)^(9/2),x)
[Out] int((x*sin(a + b*x))/cos(a + b*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/cos(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.336 $\int x \sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=103

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\,\mathrm{se}}{}$$

```
[Out] 2/7*x*sec(b*x+a)^(7/2)/b-4/35*sec(b*x+a)^(5/2)*sin(b*x+a)/b^2-12/35*sin(b*x
+a)*sec(b*x+a)^(1/2)/b^2+12/35*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2
*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)
/b^2
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4\sin(a+bx)\sec^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\sin(a+bx)\sqrt{\sec(a+bx)}}{35b^2}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{35b^2}+\frac{2x\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

```
[Out] (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(35*b^
2) + (2*x*Sec[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/
(35*b^2) - (4*Sec[a + b*x]^(5/2)*Sin[a + b*x])/(35*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sec^{\frac{9}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sec^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2} - \frac{6\int \sec^{\frac{3}{2}}(a+bx)\,dx}{35b}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\,\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2}$$
$$= \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec(a+bx)}\,\sin(a+bx)}{35b^2} - \frac{4\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)}{35b^2}$$
$$= \frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{35b^2} + \frac{2x\sec^{\frac{7}{2}}(a+bx)}{7b} - \frac{12\sqrt{\sec}}{}$$

Mathematica [A] time = 0.28, size = 65, normalized size = 0.63

$$\frac{\sec^{\frac{7}{2}}(a+bx)\left(-10\sin(2(a+bx)) - 3\sin(4(a+bx)) + 24\cos^{\frac{7}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right) + 20bx\right)}{70b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(9/2)*Sin[a + b*x],x]
```

```
[Out] (Sec[a + b*x]^(7/2)*(20*b*x + 24*Cos[a + b*x]^(7/2)*EllipticE[(a + b*x)/2,
2] - 10*Sin[2*(a + b*x)] - 3*Sin[4*(a + b*x)]))/(70*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{9}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x \left(\sec^{\frac{9}{2}}(bx+a)\right) \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

```
[Out] int(x*sec(b*x+a)^(9/2)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{9}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(9/2)*sin(b*x+a),x, algorithm="maxima")
```

```
[Out] integrate(x*sec(b*x + a)^(9/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sin(a+b\,x) \left(\frac{1}{\cos(a+b\,x)}\right)^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2),x)
```

```
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(9/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(9/2)*sin(b*x+a),x)
```

```
[Out] Timed out
```

3.337 $\int x \sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] 2/5*x*sec(b*x+a)^(5/2)/b-4/15*sec(b*x+a)^(3/2)*sin(b*x+a)/b^2-4/15*(cos(1/2
*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2
))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2641}

$$-\frac{4\sin(a+bx)\sec^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (-4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(15*b^
2) + (2*x*Sec[a + b*x]^(5/2))/(5*b) - (4*Sec[a + b*x]^(3/2)*Sin[a + b*x])/(
15*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}\int x\sec^{\frac{7}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{2\int\sec^{\frac{5}{2}}(a+bx)\,dx}{5b}\\ &= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}-\frac{2\int\sqrt{\sec(a+bx)}\,dx}{15b}\\ &= \frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)}{15b^2}-\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx}\right.}{15b}\\ &= -\frac{4\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{15b^2}+\frac{2x\sec^{\frac{5}{2}}(a+bx)}{5b}-\frac{4\sec^{\frac{3}{2}}(}{}\end{aligned}$$

Mathematica [A] time = 0.23, size = 61, normalized size = 0.76

$$\frac{2\sqrt{\sec(a+bx)}\left(-2\tan(a+bx)+3bx\sec^2(a+bx)-2\sqrt{\cos(a+bx)}F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(7/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sqrt[Sec[a + b*x]]*(-2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 3*
b*x*Sec[a + b*x]^2 - 2*Tan[a + b*x]))/(15*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec\left(bx+a\right)^{\frac{7}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{7}{2}}\left(bx+a\right)\right)\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
[Out] int(x*sec(b*x+a)^(7/2)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec\left(bx+a\right)^{\frac{7}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(7/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(7/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sin\left(a+b\,x\right)\,{\left(\frac{1}{\cos\left(a+b\,x\right)}\right)}^{7/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(7/2)*sin(b*x+a),x)
[Out] Timed out
```

3.338 $\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=80

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 2/3*x*sec(b*x+a)^(3/2)/b-4/3*sin(b*x+a)*sec(b*x+a)^(1/2)/b^2+4/3*(cos(1/2*b
*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1/2))
*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3768, 3771, 2639}

$$-\frac{4\sin(a+bx)\sqrt{\sec(a+bx)}}{3b^2}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2, 2]*Sqrt[Sec[a + b*x]])/(3*b^2)
 + (2*x*Sec[a + b*x]^(3/2))/(3*b) - (4*Sqrt[Sec[a + b*x]]*Sin[a + b*x])/(3*
b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}\int x\sec^{\frac{5}{2}}(a+bx)\sin(a+bx)\,dx &= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\sec^{\frac{3}{2}}(a+bx)\,dx}{3b}\\ &= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\,\sin(a+bx)}{3b^2}+\frac{2\int\frac{1}{\sqrt{\sec(a+bx)}}\,dx}{3b}\\ &= \frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+bx)}\,\sin(a+bx)}{3b^2}+\frac{\left(2\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right.}{3b}\\ &= \frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{3b^2}+\frac{2x\sec^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\sec(a+}}{}\end{aligned}$$

Mathematica [A] time = 0.18, size = 54, normalized size = 0.68

$$\frac{2\sec^{\frac{3}{2}}(a+bx)\left(-\sin(2(a+bx))+2\cos^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}(a+bx)\middle|2\right)+bx\right)}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(5/2)*Sin[a + b*x],x]
```

```
[Out] (2*Sec[a + b*x]^(3/2)*(b*x + 2*Cos[a + b*x]^(3/2)*EllipticE[(a + b*x)/2, 2]
 - Sin[2*(a + b*x)]))/(3*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{5}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="giac")
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x\left(\sec^{\frac{5}{2}}(bx+a)\right)\sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
[Out] int(x*sec(b*x+a)^(5/2)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{5}{2}} \sin(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(5/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(5/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sin(a+b\,x)\,{\left(\frac{1}{\cos(a+b\,x)}\right)}^{5/2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(5/2)*sin(b*x+a),x)
[Out] Timed out
```

3.339 $\int x \sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

```
[Out] 2*x*sec(b*x+a)^(1/2)/b-4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*El
lipticF(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4212, 3771, 2641}

$$\frac{2x\sqrt{\sec(a+bx)}}{b}-\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*x*Sqrt[Sec[a + b*x]])/b - (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2
]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sec^{\frac{3}{2}}(a+bx)\sin(a+bx)\,dx = \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{2\int\sqrt{\sec(a+bx)}\,dx}{b}$$

$$= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int\frac{1}{\sqrt{\cos(a+bx)}}\,dx}{b}$$

$$= \frac{2x\sqrt{\sec(a+bx)}}{b} - \frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

Mathematica [A] time = 0.13, size = 42, normalized size = 0.79

$$\frac{2\sqrt{\sec(a+bx)}\left(bx-2\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sec[a + b*x]^(3/2)*Sin[a + b*x],x]
```

```
[Out] (2*(b*x - 2*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2])*Sqrt[Sec[a + b*x]
])/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sec\left(bx+a\right)^{\frac{3}{2}}\sin\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x \left(\sec^{\frac{3}{2}}(bx+a)\right)\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
[Out] int(x*sec(b*x+a)^(3/2)*sin(b*x+a),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sec(bx+a)^{\frac{3}{2}} \sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)^(3/2)*sin(b*x+a),x, algorithm="maxima")
[Out] integrate(x*sec(b*x + a)^(3/2)*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sin(a+b\,x)\,\left(\frac{1}{\cos(a+b\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sec(b*x+a)**(3/2)*sin(b*x+a),x)
[Out] Timed out
```

3.340 $\int x\sqrt{\sec(a+bx)}\sin(a+bx)\,dx$

Optimal. Leaf size=53

$$\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

```
[Out] -2*x/b/sec(b*x+a)^(1/2)+4*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*E
llipticE(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4212, 3771, 2639}

$$\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (-2*x)/(b*Sqrt[Sec[a + b*x]]) + (4*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x)/2
, 2]*Sqrt[Sec[a + b*x]])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int x\sqrt{\sec(a+bx)}\,\sin(a+bx)\,dx = -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{\left(2\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,dx}{b}$$
$$= -\frac{2x}{b\sqrt{\sec(a+bx)}} + \frac{4\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{b^2}$$

Mathematica [B] time = 2.15, size = 132, normalized size = 2.49

$$\frac{2\left(2\tan\left(\frac{1}{2}(a+bx)\right) - \frac{2\sec^2\left(\frac{1}{2}(a+bx)\right)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}} + \frac{2\sec^2\left(\frac{1}{2}(a+bx)\right)E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(a+bx)\right)\right)\middle|-1\right)}{\sqrt{\cos(a+bx)\sec^4\left(\frac{1}{2}(a+bx)\right)}} - bx\right)}{b^2\sqrt{\sec(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[Sec[a + b*x]]*Sin[a + b*x],x]
```

```
[Out] (2*(-(b*x) + (2*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sec[(a + b*x)/2]^2)
/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] - (2*EllipticF[ArcSin[Tan[(a + b*x)/
2]], -1]*Sec[(a + b*x)/2]^2)/Sqrt[Cos[a + b*x]*Sec[(a + b*x)/2]^4] + 2*Tan[
(a + b*x)/2]))/(b^2*Sqrt[Sec[a + b*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sec(bx+a)}\,\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="giac")
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

maple [C] time = 0.08, size = 310, normalized size = 5.85

$$-\frac{(bx+2i)\left(1+e^{2i(bx+a)}\right)\sqrt{2}\,\sqrt{\frac{e^{i(bx+a)}}{1+e^{2i(bx+a)}}}\,e^{-i(bx+a)}}{b^2}-\frac{2i\left(-\frac{2\left(1+e^{2i(bx+a)}\right)}{\sqrt{\left(1+e^{2i(bx+a)}\right)e^{i(bx+a)}}}+\frac{i\sqrt{-i\left(e^{i(bx+a)}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{i(bx+a)}-i\right)}\,\sqrt{ie^{i(bx+a}}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)*sec(b*x+a)^(1/2),x)
[Out] -(b*x+2*I)*(exp(I*(b*x+a))^2+1)/b^2*2^(1/2)*(exp(I*(b*x+a))/(exp(I*(b*x+a))
^2+1))^(1/2)/exp(I*(b*x+a))-2*I/b^2*(-2*(exp(I*(b*x+a))^2+1)/((exp(I*(b*x+a
))^2+1)*exp(I*(b*x+a)))^(1/2)+I*(-I*(exp(I*(b*x+a))+I))^(1/2)*2^(1/2)*(I*(e
xp(I*(b*x+a))-I))^(1/2)*(I*exp(I*(b*x+a)))^(1/2)/(exp(I*(b*x+a))^3+exp(I*(b
*x+a)))^(1/2)*(-2*I*EllipticE((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))+I*
EllipticF((-I*(exp(I*(b*x+a))+I))^(1/2),1/2*2^(1/2))))*2^(1/2)*(exp(I*(b*x+
a))/(exp(I*(b*x+a))^2+1))^(1/2)*((exp(I*(b*x+a))^2+1)*exp(I*(b*x+a)))^(1/2)
/exp(I*(b*x+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sec(bx+a)}\,\sin(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*sqrt(sec(b*x + a))*sin(b*x + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\sin(a+b\,x)\,\sqrt{\frac{1}{\cos(a+b\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2),x)
[Out] int(x*sin(a + b*x)*(1/cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin(a+bx)\sqrt{\sec(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)*sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)*sqrt(sec(a + b*x)), x)
```

3.341 $\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx$

Optimal. Leaf size=80

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

```
[Out] -2/3*x/b/sec(b*x+a)^(3/2)+4/9*sin(b*x+a)/b^2/sec(b*x+a)^(1/2)+4/9*(cos(1/2*
b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2)
)*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{4\sqrt{\cos(a+bx)}\,\sqrt{\sec(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)}{9b^2}-\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (-2*x)/(3*b*Sec[a + b*x]^(3/2)) + (4*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)
/2, 2]*Sqrt[Sec[a + b*x]])/(9*b^2) + (4*Sin[a + b*x])/(9*b^2*Sqrt[Sec[a + b
*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\sin(a+bx)}{\sqrt{\sec(a+bx)}}\,dx &= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{2\int \sqrt{\sec(a+bx)}\,dx}{9b}\\
&= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}+\frac{\left(2\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \frac{1}{\sqrt{\cos(a+bx)}}\,dx}{9b}\\
&= -\frac{2x}{3b\sec^{\frac{3}{2}}(a+bx)}+\frac{4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{9b^2}+\frac{4\sin(a+bx)}{9b^2\sqrt{\sec(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.24, size = 63, normalized size = 0.79

$$\frac{\sqrt{\sec(a+bx)}\left(2\sin(2(a+bx))-6bx\cos^2(a+bx)+4\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sqrt[Sec[a + b*x]],x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-6*b*x*Cos[a + b*x]^2 + 4*Sqrt[Cos[a + b*x]]*EllipticF
[(a + b*x)/2, 2] + 2*Sin[2*(a + b*x)]))/(9*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx+a)}{\sqrt{\sec(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sqrt(sec(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(a+b\,x)}{\sqrt{\frac{1}{\cos(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2),x)
```

```
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a + bx)}{\sqrt{\sec(a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sqrt(sec(a + b*x)), x)
```

3.342 $\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=80

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

```
[Out] -2/5*x/b/sec(b*x+a)^(5/2)+4/25*sin(b*x+a)/b^2/sec(b*x+a)^(3/2)+12/25*(cos(1
/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+1/2*a)*EllipticE(sin(1/2*b*x+1/2*a),2^(1
/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2639}

$$\frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}+\frac{12\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)}{25b^2}-\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (-2*x)/(5*b*Sec[a + b*x]^(5/2)) + (12*Sqrt[Cos[a + b*x]]*EllipticE[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(25*b^2) + (4*Sin[a + b*x])/(25*b^2*Sec[a + b*x
]^(3/2))
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)} + \frac{6\int \frac{1}{\sqrt{\sec(a+bx)}}\,dx}{25b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)} + \frac{\left(6\sqrt{\cos(a+bx)}\sqrt{\sec(a+bx)}\right)\int \sqrt{\cos(a+bx)}\,d}{25b}$$

$$= -\frac{2x}{5b\sec^{\frac{5}{2}}(a+bx)} + \frac{12\sqrt{\cos(a+bx)}\,E\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{25b^2} + \frac{4\sin(a+bx)}{25b^2\sec^{\frac{3}{2}}(a+bx)}$$

Mathematica [B] time = 8.10, size = 212, normalized size = 2.65

$$\cos^2\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec(a+bx)}\left(\left(5(a+bx)-12\tan\left(\frac{1}{2}(a+bx)\right)-5a\right)\left(\tan^2\left(\frac{1}{2}(a+bx)\right)-1\right)-12\sqrt{\cos(a+bx)\,s}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-1/10*(x*Cos[a + b*x]) - (x*Cos[3*(a + b*x)])/10 + Sin
[a + b*x]/(25*b) + Sin[3*(a + b*x)]/(25*b)))/b + (Cos[(a + b*x)/2]^2*Sqrt[S
ec[a + b*x]]*(12*EllipticE[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a + b*x]*
Sec[(a + b*x)/2]^4] - 12*EllipticF[ArcSin[Tan[(a + b*x)/2]], -1]*Sqrt[Cos[a
 + b*x]*Sec[(a + b*x)/2]^4] + (-5*a + 5*(a + b*x) - 12*Tan[(a + b*x)/2])*(-
1 + Tan[(a + b*x)/2]^2)))/(25*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin (bx + a)}{\sec (bx + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x \sin (bx + a)}{\sec (bx + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin (bx + a)}{\sec (bx + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin (a+b\,x)}{{\left(\frac{1}{\cos (a+b\,x)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2),x)
```

```
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a + bx)}{\sec^{\frac{3}{2}}(a + bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(3/2), x)
```

3.343 $\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)} \, dx$

Optimal. Leaf size=103

$$\frac{4 \sin(a+bx)}{49 b^2 \sec^{\frac{5}{2}}(a+bx)}+\frac{20 \sin(a+bx)}{147 b^2 \sqrt{\sec(a+bx)}}+\frac{20 \sqrt{\cos(a+bx)} \sqrt{\sec(a+bx)} F\left(\frac{1}{2}(a+bx)\middle| 2\right)}{147 b^2}-\frac{2x}{7 b \sec^{\frac{7}{2}}(a+bx)}$$

```
[Out] -2/7*x/b/sec(b*x+a)^(7/2)+4/49*sin(b*x+a)/b^2/sec(b*x+a)^(5/2)+20/147*sin(b
*x+a)/b^2/sec(b*x+a)^(1/2)+20/147*(cos(1/2*b*x+1/2*a)^2)^(1/2)/cos(1/2*b*x+
1/2*a)*EllipticF(sin(1/2*b*x+1/2*a),2^(1/2))*cos(b*x+a)^(1/2)*sec(b*x+a)^(1
/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4212, 3769, 3771, 2641}

$$\frac{4 \sin(a+bx)}{49 b^2 \sec^{\frac{5}{2}}(a+bx)}+\frac{20 \sin(a+bx)}{147 b^2 \sqrt{\sec(a+bx)}}+\frac{20 \sqrt{\cos(a+bx)} \sqrt{\sec(a+bx)} F\left(\frac{1}{2}(a+bx)\middle| 2\right)}{147 b^2}-\frac{2x}{7 b \sec^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (-2*x)/(7*b*Sec[a + b*x]^(7/2)) + (20*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x
)/2, 2]*Sqrt[Sec[a + b*x]])/(147*b^2) + (4*Sin[a + b*x])/(49*b^2*Sec[a + b*
x]^(5/2)) + (20*Sin[a + b*x])/(147*b^2*Sqrt[Sec[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4212

```
Int[(x_)^(m_.)*Sec[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*Sin[(a_.) + (b_.)*(x_)^(n
_.)], x_Symbol] :> Simp[(x^(m - n + 1)*Sec[a + b*x^n]^(p - 1))/(b*n*(p - 1)
), x] - Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Sec[a + b*x^n]^(p - 1
), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
1]
```

Rubi steps

$$\int \frac{x\sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sec^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{10\int \frac{1}{\sec^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}} + \frac{10\int \sqrt{\sec(a+bx)}\,dx}{147b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} + \frac{20\sin(a+bx)}{147b^2\sqrt{\sec(a+bx)}} + \frac{\left(10\sqrt{\cos(a+bx)}\sqrt{\sec(a+\right.}{147b}$$

$$= -\frac{2x}{7b\sec^{\frac{7}{2}}(a+bx)} + \frac{20\sqrt{\cos(a+bx)}\,F\left(\frac{1}{2}(a+bx)\middle|2\right)\sqrt{\sec(a+bx)}}{147b^2} + \frac{4\sin(a+bx)}{49b^2\sec^{\frac{5}{2}}(a+bx)} +$$

Mathematica [A] time = 0.34, size = 89, normalized size = 0.86

$$\frac{\sqrt{\sec(a+bx)}\left(52\sin(2(a+bx))+6\sin(4(a+bx))-84bx\cos(2(a+bx))-21bx\cos(4(a+bx))+80\sqrt{\cos(a+b\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sin[a + b*x])/Sec[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[a + b*x]]*(-63*b*x - 84*b*x*Cos[2*(a + b*x)] - 21*b*x*Cos[4*(a +
b*x)] + 80*Sqrt[Cos[a + b*x]]*EllipticF[(a + b*x)/2, 2] + 52*Sin[2*(a + b*x
)] + 6*Sin[4*(a + b*x)]))/(588*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

```
[Out] int(x*sin(b*x+a)/sec(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(bx + a)}{\sec(bx + a)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sin(b*x + a)/sec(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sin(a + b\,x)}{{\left(\frac{1}{\cos(a+b\,x)}\right)}^{5/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2),x)
```

```
[Out] int((x*sin(a + b*x))/(1/cos(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sin(a+bx)}{\sec^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*sin(b*x+a)/sec(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*sin(a + b*x)/sec(a + b*x)**(5/2), x)
```

3.344 $\int x \cos(a+bx) \sin^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=88

$$-\frac{20F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\,\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

```
[Out] 20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipt
icF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/49*cos(b*x+a)*sin(b*x+a)^(5/2)
/b^2+2/7*x*sin(b*x+a)^(7/2)/b+20/147*cos(b*x+a)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2641}

$$-\frac{20F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{4\sin^{\frac{5}{2}}(a+bx)\cos(a+bx)}{49b^2}+\frac{20\sqrt{\sin(a+bx)}\,\cos(a+bx)}{147b^2}+\frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (-20*EllipticF[(a - Pi/2 + b*x)/2, 2])/(147*b^2) + (20*Cos[a + b*x]*Sqrt[Si
n[a + b*x]])/(147*b^2) + (4*Cos[a + b*x]*Sin[a + b*x]^(5/2))/(49*b^2) + (2*
x*Sin[a + b*x]^(7/2))/(7*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)\,dx = \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{2\int \sin^{\frac{7}{2}}(a+bx)\,dx}{7b}$$
$$= \frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b} - \frac{10\int \sin^{\frac{3}{2}}(a+bx)\,dx}{49b}$$
$$= \frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2} + \frac{4\cos(a+bx)\sin^{\frac{5}{2}}(a+bx)}{49b^2} + \frac{2x\sin^{\frac{7}{2}}(a+bx)}{7b}$$
$$= -\frac{20F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{147b^2} + \frac{20\cos(a+bx)\sqrt{\sin(a+bx)}}{147b^2} + \frac{4\cos(a+bx)\sin^{\frac{5}{2}}}{49b^2}$$

Mathematica [A] time = 0.54, size = 67, normalized size = 0.76

$$\frac{40F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+\sqrt{\sin(a+bx)}\left(84bx\sin^3(a+bx)+46\cos(a+bx)-6\cos(3(a+bx))\right)}{294b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(5/2),x]
```

```
[Out] (40*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + Sqrt[Sin[a + b*x]]*(46*Cos[a + b*
x] - 6*Cos[3*(a + b*x)] + 84*b*x*Sin[a + b*x]^3))/(294*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x \cos(bx+a) \left(\sin^{\frac{5}{2}}(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \cos(bx+a) \sin(bx+a)^{\frac{5}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \cos(a+b\,x) \sin(a+b\,x)^{5/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(5/2),x)
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(5/2),x)
[Out] Timed out
```

3.345 $\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx$

Optimal. Leaf size=65

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] 12/25*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+4/25*cos(b*x+a)*sin(b*x+a)^(3/2)/
b^2+2/5*x*sin(b*x+a)^(5/2)/b
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{4\sin^{\frac{3}{2}}(a+bx)\cos(a+bx)}{25b^2}+\frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(25*b^2) + (4*Cos[a + b*x]*Sin[a + b
*x]^(3/2))/(25*b^2) + (2*x*Sin[a + b*x]^(5/2))/(5*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)\,dx = \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{2\int \sin^{\frac{5}{2}}(a+bx)\,dx}{5b}$$
$$= \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b} - \frac{6\int \sqrt{\sin(a+bx)}\,dx}{25b}$$
$$= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{25b^2} + \frac{4\cos(a+bx)\sin^{\frac{3}{2}}(a+bx)}{25b^2} + \frac{2x\sin^{\frac{5}{2}}(a+bx)}{5b}$$

Mathematica [C] time = 0.90, size = 108, normalized size = 1.66

$$\frac{\sqrt{\sin(a+bx)}\left(4\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+2\sin(2(a+bx))-5bx\cos\right.}{25b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sin[a + b*x]^(3/2),x]
```

```
[Out] (Sqrt[Sin[a + b*x]]*(5*b*x - 5*b*x*Cos[2*(a + b*x)] + 2*Sin[2*(a + b*x)] -
12*Tan[(a + b*x)/2] + 4*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^
2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(25*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sin(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\left(\sin^{\frac{3}{2}}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\sin\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*sin(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,{\sin\left(a+b\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(3/2),x)
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sin^{\frac{3}{2}}\left(a+bx\right)\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(3/2),x)
[Out] Integral(x*sin(a + b*x)**(3/2)*cos(a + b*x), x)
```

3.346 $\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] 4/9*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2/3*x*sin(b*x+a)^(3/2)/b+4/9*cos(b*
x+a)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2635, 2641}

$$-\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{4\sqrt{\sin(a+bx)}\cos(a+bx)}{9b^2}+\frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(9*b^2) + (4*Cos[a + b*x]*Sqrt[Sin[a
+ b*x]])/(9*b^2) + (2*x*Sin[a + b*x]^(3/2))/(3*b)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int x\cos(a+bx)\sqrt{\sin(a+bx)}\,dx = \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int\sin^{\frac{3}{2}}(a+bx)\,dx}{3b}$$
$$= \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2} + \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b} - \frac{2\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{9b}$$
$$= -\frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{9b^2} + \frac{4\cos(a+bx)\sqrt{\sin(a+bx)}}{9b^2} + \frac{2x\sin^{\frac{3}{2}}(a+bx)}{3b}$$

Mathematica [A] time = 0.19, size = 56, normalized size = 0.86

$$\frac{4F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+2\sqrt{\sin(a+bx)}\,(3bx\sin(a+bx)+2\cos(a+bx))}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Sin[a + b*x]],x]
```

```
[Out] (4*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] + 2*Sqrt[Sin[a + b*x]]*(2*Cos[a + b*
x] + 3*b*x*Sin[a + b*x]))/(9*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sqrt{\sin(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\left(\sqrt{\sin}\,\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
[Out] int(x*cos(b*x+a)*sin(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\sqrt{\sin\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)^(1/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*sqrt(sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,\sqrt{\sin\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*sin(a + b*x)^(1/2),x)
[Out] int(x*cos(a + b*x)*sin(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{\sin\left(a+bx\right)}\cos\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*sin(b*x+a)**(1/2),x)
[Out] Integral(x*sqrt(sin(a + b*x))*cos(a + b*x), x)
```

3.347 $\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx$

Optimal. Leaf size=38

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] 4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(c
os(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2+2*x*sin(b*x+a)^(1/2)/b
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3443, 2639}

$$\frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/b^2 + (2*x*Sqrt[Sin[a + b*x]])/b
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sqrt{\sin(a+bx)}}\,dx &= \frac{2x\sqrt{\sin(a+bx)}}{b}-\frac{2\int \sqrt{\sin(a+bx)}\,dx}{b}\\ &= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2}+\frac{2x\sqrt{\sin(a+bx)}}{b}\end{aligned}$$

Mathematica [C] time = 1.16, size = 86, normalized size = 2.26

$$\frac{2\sqrt{\sin(a+bx)}\left(2\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)-6\tan\left(\frac{1}{2}(a+bx)\right)+3bx\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Sin[a + b*x]],x]
```

```
[Out] (2*Sqrt[Sin[a + b*x]]*(3*b*x - 6*Tan[(a + b*x)/2] + 2*Hypergeometric2F1[1/2
, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec[(a + b*x)/2]^2]*Tan[(a + b*x)/2])
)/(3*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\sin(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

maple [C] time = 0.12, size = 308, normalized size = 8.11

$$-\frac{i(bx+2i)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\,e^{-i(bx+a)}}{b^2\sqrt{-i\left(e^{2i(bx+a)}-1\right)e^{-i(bx+a)}}}-\frac{2\left(\frac{2i\left(i-ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(i-ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\,\sqrt{-2e^{i(bx+a)}+2}\,\sqrt{-e^{i(bx+a)}}\left(-2\,\mathrm{EllipticE}\left(\sqrt{e^{i(bx+a)}}\right.\right.}{\sqrt{-ie^{3i(bx+a)}+ie^{i(bx+a)}}}\right.}{b^2\sqrt{-i\left(e^{2i(bx+a)}\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(1/2),x)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(
I*(b*x+a)))^(1/2)/exp(I*(b*x+a))-2/b^2*(2*I*(I-I*exp(I*(b*x+a))^2)/(exp(I*(
b*x+a))*(I-I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*(b
*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(-I*exp(I*(b*x+a))^3+I*exp(I*(b*x+a
)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((ex
p(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)/(-I*(exp(I*(b*x+a))^2-1)/exp(I
*(b*x+a)))^(1/2)*(-I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+
a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos (bx + a)}{\sqrt{\sin (bx + a)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(sin(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\, \cos (a + b\, x)}{\sqrt{\sin (a + b\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(1/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos (a + bx)}{\sqrt{\sin (a + bx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(sin(a + b*x)), x)
```

3.348 $\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=38

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

```
[Out] -4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(
cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2*x/b/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {3443, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x}{b\sqrt{\sin(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/b^2 - (2*x)/(b*Sqrt[Sin[a + b*x]])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{3}{2}}(a+bx)}\,dx = -\frac{2x}{b\sqrt{\sin(a+bx)}} + \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}$$
$$= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{b^2} - \frac{2x}{b\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.18, size = 37, normalized size = 0.97

$$\frac{2\left(-\frac{bx}{\sqrt{\sin(a+bx)}} - 2F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(3/2),x]
[Out] (2*(-2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2] - (b*x)/Sqrt[Sin[a + b*x]]))/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(3/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(a+bx\right)}{\sin^{\frac{3}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(3/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(3/2), x)
```

3.349 $\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

```
[Out] 4/3*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE
(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/3*x/b/sin(b*x+a)^(3/2)-4/3*cos(b*
x+a)/b^2/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-4*EllipticE[(a - Pi/2 + b*x)/2, 2])/(3*b^2) - (2*x)/(3*b*Sin[a + b*x]^(3/
2)) - (4*Cos[a + b*x])/(3*b^2*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{x\cos(a+bx)}{\sin^{\frac{5}{2}}(a+bx)}\,dx &= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}+\frac{2\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{3b}\\
&= -\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}-\frac{2\int\sqrt{\sin(a+bx)}\,dx}{3b}\\
&= -\frac{4E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{3b^2}-\frac{2x}{3b\sin^{\frac{3}{2}}(a+bx)}-\frac{4\cos(a+bx)}{3b^2\sqrt{\sin(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 56, normalized size = 0.86

$$-\frac{2\left(\sin(2(a+bx))-2\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{3b^2\sin^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(5/2),x]
```

```
[Out] (-2*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(3/2) + Sin[2
*(a + b*x)]))/(3*b^2*Sin[a + b*x]^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\sin\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(5/2),x)
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(a+bx\right)}{\sin^{\frac{5}{2}}\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(5/2),x)
[Out] Integral(x*cos(a + b*x)/sin(a + b*x)**(5/2), x)
```

3.350 $\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx$

Optimal. Leaf size=65

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

```
[Out] -4/15*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/5*x/b/sin(b*x+a)^(5/2)-4/15*cos
(b*x+a)/b^2/sin(b*x+a)^(3/2)
```

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2641}

$$\frac{4F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}-\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (4*EllipticF[(a - Pi/2 + b*x)/2, 2])/(15*b^2) - (2*x)/(5*b*Sin[a + b*x]^(5/
2)) - (4*Cos[a + b*x])/(15*b^2*Sin[a + b*x]^(3/2))
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
```

```
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{7}{2}}(a+bx)}\,dx = -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= -\frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)} + \frac{2\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{15b}$$

$$= \frac{4F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{15b^2} - \frac{2x}{5b\sin^{\frac{5}{2}}(a+bx)} - \frac{4\cos(a+bx)}{15b^2\sin^{\frac{3}{2}}(a+bx)}$$

Mathematica [A] time = 0.23, size = 57, normalized size = 0.88

$$-\frac{2\left(\sin(2(a+bx)) + 2\sin^{\frac{5}{2}}(a+bx)F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right) + 3bx\right)}{15b^2\sin^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(7/2),x]
```

```
[Out] (-2*(3*b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(5/2) + Sin
[2*(a + b*x)]))/(15*b^2*Sin[a + b*x]^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

```
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \cos(bx+a)}{\sin(bx+a)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(7/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\sin\left(a+b\,x\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(7/2),x)
```

```
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(7/2),x)
```

```
[Out] Timed out
```

3.351 $\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx$

Optimal. Leaf size=88

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

```
[Out] 12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*Ellipti
cE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))/b^2-2/7*x/b/sin(b*x+a)^(7/2)-4/35*cos
(b*x+a)/b^2/sin(b*x+a)^(5/2)-12/35*cos(b*x+a)/b^2/sin(b*x+a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3443, 2636, 2639}

$$-\frac{12E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-\frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)}-\frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}-\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-12*EllipticE[(a - Pi/2 + b*x)/2, 2])/(35*b^2) - (2*x)/(7*b*Sin[a + b*x]^(
7/2)) - (4*Cos[a + b*x])/(35*b^2*Sin[a + b*x]^(5/2)) - (12*Cos[a + b*x])/(3
5*b^2*Sqrt[Sin[a + b*x]])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3443

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sin[(a_.) + (b_.)*(x_)^(n_.)]^
(p_.), x_Symbol] :> Simp[(x^(m - n + 1)*Sin[a + b*x^n]^(p + 1))/(b*n*(p + 1
```

```
)), x] - Dist[(m - n + 1)/(b*n*(p + 1)), Int[x^(m - n)*Sin[a + b*x^n]^(p +
1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\sin^{\frac{9}{2}}(a+bx)}\,dx = -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} + \frac{2\int \frac{1}{\sin^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} + \frac{6\int \frac{1}{\sin^{\frac{3}{2}}(a+bx)}\,dx}{35b}$$

$$= -\frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}} - \frac{6\int \sqrt{\sin(a+bx)}\,dx}{35b}$$

$$= -\frac{12E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)}{35b^2} - \frac{2x}{7b\sin^{\frac{7}{2}}(a+bx)} - \frac{4\cos(a+bx)}{35b^2\sin^{\frac{5}{2}}(a+bx)} - \frac{12\cos(a+bx)}{35b^2\sqrt{\sin(a+bx)}}$$

Mathematica [A] time = 0.29, size = 73, normalized size = 0.83

$$-\frac{2\left(\sin(2(a+bx)) + 6\sin^3(a+bx)\cos(a+bx) - 6\sin^{\frac{7}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right) + 5bx\right)}{35b^2\sin^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sin[a + b*x]^(9/2),x]
```

```
[Out] (-2*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE[(-2*a + Pi - 2*b*x
)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2*Sin[a + b*x]^(7/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
[Out] int(x*cos(b*x+a)/sin(b*x+a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sin(bx+a)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)^(9/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)/sin(b*x + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos(a+b\,x)}{{\sin(a+b\,x)}^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/sin(a + b*x)^(9/2),x)
[Out] int((x*cos(a + b*x))/sin(a + b*x)^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/sin(b*x+a)**(9/2),x)
```

```
[Out] Timed out
```

3.352 $\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx$

Optimal. Leaf size=108

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-$$

```
[Out] -4/35*cos(b*x+a)*csc(b*x+a)^(5/2)/b^2-2/7*x*csc(b*x+a)^(7/2)/b-12/35*cos(b*
x+a)*csc(b*x+a)^(1/2)/b^2+12/35*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2
*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^
(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2639}

$$-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{35b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-12*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(35*b^2) - (4*Cos[a + b*x]*Csc[a + b*
x]^(5/2))/(35*b^2) - (2*x*Csc[a + b*x]^(7/2))/(7*b) - (12*Sqrt[Csc[a + b*x]
]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(35*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{9}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}+\frac{2\int\csc^{\frac{7}{2}}(a+bx)\,dx}{7b}$$

$$= -\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+bx)}{7b}+\frac{6\int\csc^{\frac{3}{2}}(a+bx)\,dx}{35b}$$

$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+b}{7b}$$

$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+b}{7b}$$

$$= -\frac{12\cos(a+bx)\sqrt{\csc(a+bx)}}{35b^2}-\frac{4\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)}{35b^2}-\frac{2x\csc^{\frac{7}{2}}(a+b}{7b}$$

Mathematica [A] time = 0.27, size = 73, normalized size = 0.68

$$-\frac{2\csc^{\frac{7}{2}}(a+bx)\left(\sin(2(a+bx))+6\sin^3(a+bx)\cos(a+bx)-6\sin^{\frac{7}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+5bx\right)}{35b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(9/2),x]
```

```
[Out] (-2*Csc[a + b*x]^(7/2)*(5*b*x + 6*Cos[a + b*x]*Sin[a + b*x]^3 - 6*EllipticE
[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a + b*x]^(7/2) + Sin[2*(a + b*x)]))/(35*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{9}{2}}(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

```
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(9/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{9}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(9/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(9/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos(a+b\,x)\,{\left(\frac{1}{\sin(a+b\,x)}\right)}^{9/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(9/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(9/2),x)
[Out] Timed out
```

3.353 $\int x \cos(a+bx) \csc^{\frac{7}{2}}(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

```
[Out] -4/15*cos(b*x+a)*csc(b*x+a)^(3/2)/b^2-2/5*x*csc(b*x+a)^(5/2)/b-4/15*(sin(1/
2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2641}

$$-\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}+\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Csc[a + b*x]^(3/2))/(15*b^2) - (2*x*Csc[a + b*x]^(5/2))/(5
*b) + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(15*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{7}{2}}(a+bx)\,dx = -\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int \csc^{\frac{5}{2}}(a+bx)\,dx}{5b}$$

$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{2\int \sqrt{\csc(a+bx)}\,dx}{15b}$$

$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+}\right.}{15b}$$

$$= -\frac{4\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)}{15b^2}-\frac{2x\csc^{\frac{5}{2}}(a+bx)}{5b}+\frac{4\sqrt{\csc(a+bx)}F\left(\frac{1}{2}\left(a-\frac{\pi}{2}\right.\right.}{15b}$$

Mathematica [A] time = 0.29, size = 65, normalized size = 0.76

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\cot(a+bx)+3bx\csc^2(a+bx)+2\sqrt{\sin(a+bx)}F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{15b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(7/2),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(2*Cot[a + b*x] + 3*b*x*Csc[a + b*x]^2 + 2*EllipticF
[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]]))/(15*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{7}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(7/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{7}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(7/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(7/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\cos(a+b\,x)\left(\frac{1}{\sin(a+b\,x)}\right)^{7/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(7/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(7/2),x)
[Out] Timed out
```

3.354 $\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx$

Optimal. Leaf size=85

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

```
[Out] -2/3*x*csc(b*x+a)^(3/2)/b-4/3*cos(b*x+a)*csc(b*x+a)^(1/2)/b^2+4/3*(sin(1/2*
a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/
4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3768, 3771, 2639}

$$-\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-4*Cos[a + b*x]*Sqrt[Csc[a + b*x]])/(3*b^2) - (2*x*Csc[a + b*x]^(3/2))/(3*
b) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*
x]])/(3*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$
\begin{aligned}
\int x\cos(a+bx)\csc^{\frac{5}{2}}(a+bx)\,dx &= -\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}+\frac{2\int\csc^{\frac{3}{2}}(a+bx)\,dx}{3b}\\
&= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{2\int\frac{1}{\sqrt{\csc(a+bx)}}\,dx}{3b}\\
&= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{\left(2\sqrt{\csc(a+bx)}\,\sqrt{\sin(a+b\cdots}{3b}\\
&= -\frac{4\cos(a+bx)\sqrt{\csc(a+bx)}}{3b^2}-\frac{2x\csc^{\frac{3}{2}}(a+bx)}{3b}-\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}\cdots}{3b^2}
\end{aligned}
$$

Mathematica [A] time = 0.18, size = 56, normalized size = 0.66

$$
-\frac{2\csc^{\frac{3}{2}}(a+bx)\left(\sin(2(a+bx))-2\sin^{\frac{3}{2}}(a+bx)E\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{3b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(5/2),x]
```

```
[Out] (-2*Csc[a + b*x]^(3/2)*(b*x - 2*EllipticE[(-2*a + Pi - 2*b*x)/4, 2]*Sin[a +
 b*x]^(3/2) + Sin[2*(a + b*x)]))/(3*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="giac")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\left(\csc^{\frac{5}{2}}(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(5/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\cos(a+b\,x)\,{\left(\frac{1}{\sin(a+b\,x)}\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(5/2),x)
[Out] Timed out
```

3.355 $\int x \cos(a+bx) \csc^{\frac{3}{2}}(a+bx)\, dx$

Optimal. Leaf size=58

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

```
[Out] -2*x*csc(b*x+a)^(1/2)/b-4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4
*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*
sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4213, 3771, 2641}

$$\frac{4\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}-\frac{2x\sqrt{\csc(a+bx)}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*x*Sqrt[Csc[a + b*x]])/b + (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b
*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\csc^{\frac{3}{2}}(a+bx)\,dx = -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{2\int\sqrt{\csc(a+bx)}\,dx}{b}$$
$$= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\frac{1}{\sqrt{\sin(a+bx)}}\,dx}{b}$$
$$= -\frac{2x\sqrt{\csc(a+bx)}}{b}+\frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

Mathematica [A] time = 0.14, size = 46, normalized size = 0.79

$$-\frac{2\sqrt{\csc(a+bx)}\left(2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)+bx\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Csc[a + b*x]^(3/2),x]
```

```
[Out] (-2*Sqrt[Csc[a + b*x]]*(b*x + 2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Si
n[a + b*x]]))/b^2
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\csc(bx+a)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\left(\csc^{\frac{3}{2}}\left(bx+a\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
[Out] int(x*cos(b*x+a)*csc(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\csc\left(bx+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x*cos(b*x + a)*csc(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2),x)
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(3/2),x)
[Out] Timed out
```

3.356 $\int x \cos(a+bx)\sqrt{\csc(a+bx)}\,dx$

Optimal. Leaf size=58

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

```
[Out] 2*x/b/csc(b*x+a)^(1/2)+4*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*
Pi+1/2*b*x)*EllipticE(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*s
in(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4213, 3771, 2639}

$$\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(b*Sqrt[Csc[a + b*x]]) - (4*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 +
b*x)/2, 2]*Sqrt[Sin[a + b*x]])/b^2
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int x\cos(a+bx)\sqrt{\csc(a+bx)}\,dx=\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{2\int\frac{1}{\sqrt{\csc(a+bx)}}\,dx}{b}$$
$$=\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,dx}{b}$$
$$=\frac{2x}{b\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{b^2}$$

Mathematica [C] time = 0.72, size = 106, normalized size = 1.83

$$\frac{4\sin\left(\frac{1}{2}(a+bx)\right)\cos\left(\frac{1}{2}(a+bx)\right)\sqrt{\csc(a+bx)}\left(2\tan\left(\frac{1}{2}(a+bx)\right)\sqrt{\sec^2\left(\frac{1}{2}(a+bx)\right)}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+b\right.\right.\right.}{3b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[a + b*x]*Sqrt[Csc[a + b*x]],x]
```

```
[Out] (4*Cos[(a + b*x)/2]*Sqrt[Csc[a + b*x]]*Sin[(a + b*x)/2]*(3*b*x - 6*Tan[(a +
 b*x)/2] + 2*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2]*Sqrt[Sec
[(a + b*x)/2]^2]*Tan[(a + b*x)/2]))/(3*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(bx+a)\sqrt{\csc(bx+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

maple [C] time = 0.12, size = 308, normalized size = 5.31

$$-\frac{i\left(bx+2i\right)\left(e^{2i(bx+a)}-1\right)\sqrt{2}\,\sqrt{\frac{ie^{i(bx+a)}}{e^{2i(bx+a)}-1}}\;e^{-i(bx+a)}}{b^{2}}-\frac{2\left(-\frac{2i\left(-i+ie^{2i(bx+a)}\right)}{\sqrt{e^{i(bx+a)}\left(-i+ie^{2i(bx+a)}\right)}}-\frac{\sqrt{e^{i(bx+a)}+1}\;\sqrt{-2\,e^{i(bx+a)}+2}\;\sqrt{-e^{i(bx+a)}}\;\left(-2\right.}{\sqrt{i}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)*csc(b*x+a)^(1/2),x)
```

```
[Out] -I*(b*x+2*I)*(exp(I*(b*x+a))^2-1)/b^2*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)/exp(I*(b*x+a))-2/b^2*(-2*I*(-I+I*exp(I*(b*x+a))^2)/(exp(I*
(b*x+a))*(-I+I*exp(I*(b*x+a))^2))^(1/2)-(exp(I*(b*x+a))+1)^(1/2)*(-2*exp(I*
(b*x+a))+2)^(1/2)*(-exp(I*(b*x+a)))^(1/2)/(I*exp(I*(b*x+a))^3-I*exp(I*(b*x+
a)))^(1/2)*(-2*EllipticE((exp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))+EllipticF((e
xp(I*(b*x+a))+1)^(1/2),1/2*2^(1/2))))*2^(1/2)*(I*exp(I*(b*x+a))/(exp(I*(b*x
+a))^2-1))^(1/2)*(I*(exp(I*(b*x+a))^2-1)*exp(I*(b*x+a)))^(1/2)/exp(I*(b*x+a
))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(bx+a\right)\sqrt{\csc\left(bx+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)*sqrt(csc(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int x\,\cos\left(a+b\,x\right)\,\sqrt{\frac{1}{\sin\left(a+b\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int(x*cos(a + b*x)*(1/sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(a+bx\right)\sqrt{\csc\left(a+bx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)*csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)*sqrt(csc(a + b*x)), x)
```

3.357 $\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$

Optimal. Leaf size=85

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

```
[Out] 2/3*x/b/csc(b*x+a)^(3/2)+4/9*cos(b*x+a)/b^2/csc(b*x+a)^(1/2)+4/9*(sin(1/2*a
+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4
*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}}-\frac{4\sqrt{\sin(a+bx)}\,\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{9b^2}+\frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*x)/(3*b*Csc[a + b*x]^(3/2)) + (4*Cos[a + b*x])/(9*b^2*Sqrt[Csc[a + b*x]]
) - (4*Sqrt[Csc[a + b*x]]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x
]])/(9*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\begin{aligned}\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{3b} \\ &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{2\int \sqrt{\csc(a+bx)}\,dx}{9b} \\ &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{\left(2\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int \frac{1}{\sqrt{\sin(a+bx)}}\,dx}{9b} \\ &= \frac{2x}{3b\csc^{\frac{3}{2}}(a+bx)} + \frac{4\cos(a+bx)}{9b^2\sqrt{\csc(a+bx)}} - \frac{4\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{9b^2}\end{aligned}$$

Mathematica [A] time = 0.23, size = 65, normalized size = 0.76

$$\frac{2\sqrt{\csc(a+bx)}\left(3bx\sin^2(a+bx)+\sin(2(a+bx))+2\sqrt{\sin(a+bx)}\,F\left(\frac{1}{4}(-2a-2bx+\pi)\middle|2\right)\right)}{9b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Sqrt[Csc[a + b*x]],x]
```

```
[Out] (2*Sqrt[Csc[a + b*x]]*(2*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b
*x]] + 3*b*x*Sin[a + b*x]^2 + Sin[2*(a + b*x)]))/(9*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\sqrt{\csc(bx+a)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/sqrt(csc(b*x + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos(a+b\,x)}{\sqrt{\frac{1}{\sin(a+b\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2),x)
```

```
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\sqrt{\csc(a+bx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(1/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/sqrt(csc(a + b*x)), x)
```

3.358 $\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$

Optimal. Leaf size=85

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

```
[Out] 2/5*x/b/csc(b*x+a)^(5/2)+4/25*cos(b*x+a)/b^2/csc(b*x+a)^(3/2)+12/25*(sin(1/
2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/2*a+1/4*Pi+1/2*b*x)*EllipticE(cos(1/2*a+
1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.04, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2639}

$$\frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)}-\frac{12\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{25b^2}+\frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] (2*x)/(5*b*Csc[a + b*x]^(5/2)) + (4*Cos[a + b*x])/(25*b^2*Csc[a + b*x]^(3/2
)) - (12*Sqrt[Csc[a + b*x]]*EllipticE[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b
*x]])/(25*b^2)
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx = \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{5}{2}}(a+bx)}\,dx}{5b}$$

$$= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{6\int \frac{1}{\sqrt{\csc(a+bx)}}\,dx}{25b}$$

$$= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{\left(6\sqrt{\csc(a+bx)}\sqrt{\sin(a+bx)}\right)\int\sqrt{\sin(a+bx)}\,dx}{25b}$$

$$= \frac{2x}{5b\csc^{\frac{5}{2}}(a+bx)} + \frac{4\cos(a+bx)}{25b^2\csc^{\frac{3}{2}}(a+bx)} - \frac{12\sqrt{\csc(a+bx)}\,E\left(\frac{1}{2}\left(a-\frac{\pi}{2}+bx\right)\middle|2\right)\sqrt{\sin(a+bx)}}{25b^2}$$

Mathematica [C] time = 0.98, size = 114, normalized size = 1.34

$$\frac{\tan\left(\frac{1}{2}(a+bx)\right)\left(4\sqrt{2}\sqrt{\frac{1}{\cos(a+bx)+1}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\tan^2\left(\frac{1}{2}(a+bx)\right)\right)+10bx\sin(a+bx)+5bx\sin(2(a+bx))+4\,\mathrm{c}}{25b^2\sqrt{\csc(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(3/2),x]
```

```
[Out] ((-10 + 4*Cos[a + b*x] + 2*Cos[2*(a + b*x)] + 4*Sqrt[2]*Sqrt[(1 + Cos[a + b
*x])^(-1)]*Hypergeometric2F1[1/2, 3/4, 7/4, -Tan[(a + b*x)/2]^2] + 10*b*x*S
in[a + b*x] + 5*b*x*Sin[2*(a + b*x)])*Tan[(a + b*x)/2])/(25*b^2*Sqrt[Csc[a
+ b*x]])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(bx+a)}{\csc(bx+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos(a+b\,x)}{\left(\frac{1}{\sin(a+b\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2),x)
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{3}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(3/2),x)
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(3/2), x)
```

3.359 $\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$

Optimal. Leaf size=108

$$\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

```
[Out] 2/7*x/b/csc(b*x+a)^(7/2)+4/49*cos(b*x+a)/b^2/csc(b*x+a)^(5/2)+20/147*cos(b*
x+a)/b^2/csc(b*x+a)^(1/2)+20/147*(sin(1/2*a+1/4*Pi+1/2*b*x)^2)^(1/2)/sin(1/
2*a+1/4*Pi+1/2*b*x)*EllipticF(cos(1/2*a+1/4*Pi+1/2*b*x),2^(1/2))*csc(b*x+a)
^(1/2)*sin(b*x+a)^(1/2)/b^2
```

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {4213, 3769, 3771, 2641}

$$\frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)}+\frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}}-\frac{20\sqrt{\sin(a+bx)}\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a+bx-\frac{\pi}{2}\right)\middle|2\right)}{147b^2}+\frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (2*x)/(7*b*Csc[a + b*x]^(7/2)) + (4*Cos[a + b*x])/(49*b^2*Csc[a + b*x]^(5/2
)) + (20*Cos[a + b*x])/(147*b^2*Sqrt[Csc[a + b*x]]) - (20*Sqrt[Csc[a + b*x]
]*EllipticF[(a - Pi/2 + b*x)/2, 2]*Sqrt[Sin[a + b*x]])/(147*b^2)
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3769

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4213

```
Int[Cos[(a_.) + (b_.)*(x_)^(n_.)]*Csc[(a_.) + (b_.)*(x_)^(n_.)]^(p_)*(x_)^(
m_.), x_Symbol] :> -Simp[(x^(m - n + 1)*Csc[a + b*x^n]^(p - 1))/(b*n*(p - 1
)), x] + Dist[(m - n + 1)/(b*n*(p - 1)), Int[x^(m - n)*Csc[a + b*x^n]^(p -
1), x], x] /; FreeQ[{a, b, p}, x] && IntegerQ[n] && GeQ[m - n, 0] && NeQ[p,
 1]
```

Rubi steps

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx = \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} - \frac{2\int \frac{1}{\csc^{\frac{7}{2}}(a+bx)}\,dx}{7b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} - \frac{10\int \frac{1}{\csc^{\frac{3}{2}}(a+bx)}\,dx}{49b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{10\int\sqrt{\csc(a+bx)}\,dx}{147b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{\left(10\sqrt{\csc(a+bx)}\sqrt{\sin(a+b}\right.}{147b}$$

$$= \frac{2x}{7b\csc^{\frac{7}{2}}(a+bx)} + \frac{4\cos(a+bx)}{49b^2\csc^{\frac{5}{2}}(a+bx)} + \frac{20\cos(a+bx)}{147b^2\sqrt{\csc(a+bx)}} - \frac{20\sqrt{\csc(a+bx)}\,F\left(\frac{1}{2}\left(a-\frac{\pi}{2}\right.\right.}{147b^2}$$

Mathematica [A] time = 0.39, size = 93, normalized size = 0.86

$$\frac{\sqrt{\csc(a+bx)}\left(52\sin(2(a+bx)) - 6\sin(4(a+bx)) - 84bx\cos(2(a+bx)) + 21bx\cos(4(a+bx)) + 80\sqrt{\sin(a+bx}\right.}{588b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Cos[a + b*x])/Csc[a + b*x]^(5/2),x]
```

```
[Out] (Sqrt[Csc[a + b*x]]*(63*b*x - 84*b*x*Cos[2*(a + b*x)] + 21*b*x*Cos[4*(a + b
*x)] + 80*EllipticF[(-2*a + Pi - 2*b*x)/4, 2]*Sqrt[Sin[a + b*x]] + 52*Sin[2
*(a + b*x)] - 6*Sin[4*(a + b*x)]))/(588*b^2)
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

```
[Out] int(x*cos(b*x+a)/csc(b*x+a)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos\left(bx+a\right)}{\csc\left(bx+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(x*cos(b*x + a)/csc(b*x + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\cos\left(a+b\,x\right)}{{\left(\frac{1}{\sin\left(a+b\,x\right)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2),x)
```

```
[Out] int((x*cos(a + b*x))/(1/sin(a + b*x))^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\cos(a+bx)}{\csc^{\frac{5}{2}}(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(b*x+a)/csc(b*x+a)**(5/2),x)
```

```
[Out] Integral(x*cos(a + b*x)/csc(a + b*x)**(5/2), x)
```

3.360 $\int x \csc(x) \sin(3x)\, dx$

Optimal. Leaf size=31

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3\cos^2(x)}{4} + 2x\sin(x)\cos(x)$$

```
[Out] 1/2*x^2+3/4*cos(x)^2+2*x*cos(x)*sin(x)-1/4*sin(x)^2
```

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4431, 3310, 30}

$$\frac{x^2}{2} - \frac{\sin^2(x)}{4} + \frac{3\cos^2(x)}{4} + 2x\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Csc[x]*Sin[3*x],x]
```

```
[Out] x^2/2 + (3*Cos[x]^2)/4 + 2*x*Cos[x]*Sin[x] - Sin[x]^2/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int x\csc(x)\sin(3x)\,dx = \int \left(3x\cos^2(x) - x\sin^2(x)\right)\,dx$$
$$= 3\int x\cos^2(x)\,dx - \int x\sin^2(x)\,dx$$
$$= \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4} - \frac{\int x\,dx}{2} + \frac{3\int x\,dx}{2}$$
$$= \frac{x^2}{2} + \frac{3\cos^2(x)}{4} + 2x\cos(x)\sin(x) - \frac{\sin^2(x)}{4}$$

Mathematica [A] time = 0.02, size = 22, normalized size = 0.71

$$\frac{x^2}{2} + x\sin(2x) + \frac{1}{2}\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Csc[x]*Sin[3*x],x]
[Out] x^2/2 + Cos[2*x]/2 + x*Sin[2*x]
```

fricas [A] time = 0.45, size = 17, normalized size = 0.55

$$2\,x\cos(x)\sin(x) + \frac{1}{2}\,x^2 + \cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="fricas")
[Out] 2*x*cos(x)*sin(x) + 1/2*x^2 + cos(x)^2
```

giac [A] time = 0.15, size = 18, normalized size = 0.58

$$\frac{1}{2}\,x^2 + x\sin\left(2\,x\right) + \frac{1}{2}\,\cos\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="giac")
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

maple [A] time = 0.06, size = 26, normalized size = 0.84

$$4x\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) - \frac{3x^2}{2} - \left(\sin^2(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*csc(x)*sin(3*x),x)
```

```
[Out] 4*x*(1/2*cos(x)*sin(x)+1/2*x)-3/2*x^2-sin(x)^2
```

maxima [A] time = 0.32, size = 18, normalized size = 0.58

$$\frac{1}{2}x^2 + x\sin(2x) + \frac{1}{2}\cos(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + x*sin(2*x) + 1/2*cos(2*x)
```

mupad [B] time = 0.09, size = 18, normalized size = 0.58

$$\frac{\cos(2x)}{2} + x\sin(2x) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*sin(3*x))/sin(x),x)
```

```
[Out] cos(2*x)/2 + x*sin(2*x) + x^2/2
```

sympy [A] time = 1.88, size = 37, normalized size = 1.19

$$-x^2\sin^2(x) - x^2\cos^2(x) + \frac{3x^2}{2} + 2x\sin(x)\cos(x) + \cos^2(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*csc(x)*sin(3*x),x)
```

```
[Out] -x**2*sin(x)**2 - x**2*cos(x)**2 + 3*x**2/2 + 2*x*sin(x)*cos(x) + cos(x)**2
```

3.361 $\int (c+dx)^4 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=131

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3d\cos$$

```
[Out] 3/2*d^4*x-d*(d*x+c)^3+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(x)^2+3*d*(d*x+c)^
3*cos(x)^2+3*d^4*cos(x)*sin(x)-6*d^2*(d*x+c)^2*cos(x)*sin(x)+2*(d*x+c)^4*co
s(x)*sin(x)+3/2*d^3*(d*x+c)*sin(x)^2-d*(d*x+c)^3*sin(x)^2
```

Rubi [A] time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3}{2}d^3\sin^2(x)(c+dx)-\frac{9}{2}d^3\cos^2(x)(c+dx)-6d^2\sin(x)\cos(x)(c+dx)^2+\frac{(c+dx)^5}{5d}-d(c+dx)^3-d\sin^2(x)(c+dx)^3+3d\cos$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] (3*d^4*x)/2 - d*(c + d*x)^3 + (c + d*x)^5/(5*d) - (9*d^3*(c + d*x)*Cos[x]^2
)/2 + 3*d*(c + d*x)^3*Cos[x]^2 + 3*d^4*Cos[x]*Sin[x] - 6*d^2*(c + d*x)^2*Co
s[x]*Sin[x] + 2*(c + d*x)^4*Cos[x]*Sin[x] + (3*d^3*(c + d*x)*Sin[x]^2)/2 -
d*(c + d*x)^3*Sin[x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^4\cos^2(x)-(c+dx)^4\sin^2(x)\right)\,dx\\
&= 3\int (c+dx)^4\cos^2(x)\,dx-\int (c+dx)^4\sin^2(x)\,dx\\
&= 3d(c+dx)^3\cos^2(x)+2(c+dx)^4\cos(x)\sin(x)-d(c+dx)^3\sin^2(x)-\frac{1}{2}\int (c+d\\
&= \frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)-6d^2(c+dx)^2\cos(x)\sin\\
&= -d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d^4\cos(\\
&= \frac{3d^4x}{2}-d(c+dx)^3+\frac{(c+dx)^5}{5d}-\frac{9}{2}d^3(c+dx)\cos^2(x)+3d(c+dx)^3\cos^2(x)+3d
\end{aligned}$$

Mathematica [A] time = 0.22, size = 154, normalized size = 1.18

$$c^4x+2c^3dx^2+2c^2d^2x^3+d\cos(2x)\left(2c^3+6c^2dx+3cd^2\left(2x^2-1\right)+d^3x\left(2x^2-3\right)\right)+\frac{1}{2}\sin(2x)\left(2c^4+8c^3dx+6c^2d\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[x]*Sin[3*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + d*(2*c^3 +
6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d^2*(-1 + 2*x^2))*Cos[2*x] + ((2*c^4 +
 8*c^3*d*x + 4*c*d^3*x*(-3 + 2*x^2) + 6*c^2*d^2*(-1 + 2*x^2) + d^4*(3 - 6*x
^2 + 2*x^4))*Sin[2*x])/2
```

fricas [A] time = 0.45, size = 200, normalized size = 1.53

$$\frac{1}{5}d^4x^5+cd^3x^4+2\left(c^2d^2-d^4\right)x^3+2\left(c^3d-3\,cd^3\right)x^2+2\left(2\,d^4x^3+6\,cd^3x^2+2\,c^3d-3\,cd^3+3\left(2\,c^2d^2-d^4\right)x\right)\cos(x)^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*(c^2*d^2 - d^4)*x^3 + 2*(c^3*d - 3*c*d^3)*x^2 +
 2*(2*d^4*x^3 + 6*c*d^3*x^2 + 2*c^3*d - 3*c*d^3 + 3*(2*c^2*d^2 - d^4)*x)*co
s(x)^2 + (2*d^4*x^4 + 8*c*d^3*x^3 + 2*c^4 - 6*c^2*d^2 + 3*d^4 + 6*(2*c^2*d^
2 - d^4)*x^2 + 4*(2*c^3*d - 3*c*d^3)*x)*cos(x)*sin(x) + (c^4 - 6*c^2*d^2 +
3*d^4)*x
```

giac [A] time = 2.64, size = 167, normalized size = 1.27

$$\frac{1}{5}d^4x^5+cd^3x^4+2\,c^2d^2x^3+2\,c^3dx^2+c^4x+\left(2\,d^4x^3+6\,cd^3x^2+6\,c^2d^2x-3\,d^4x+2\,c^3d-3\,cd^3\right)\cos\left(2\,x\right)+\frac{1}{2}\left(2\,d^4x^4+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/5*d^4*x^5 + c*d^3*x^4 + 2*c^2*d^2*x^3 + 2*c^3*d*x^2 + c^4*x + (2*d^4*x^3
+ 6*c*d^3*x^2 + 6*c^2*d^2*x - 3*d^4*x + 2*c^3*d - 3*c*d^3)*cos(2*x) + 1/2*(
2*d^4*x^4 + 8*c*d^3*x^3 + 12*c^2*d^2*x^2 - 6*d^4*x^2 + 8*c^3*d*x - 12*c*d^3
*x + 2*c^4 - 6*c^2*d^2 + 3*d^4)*sin(2*x)
```

maple [B] time = 0.08, size = 260, normalized size = 1.98

$$4d^4\left(x^4\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+x^3\left(\cos^2(x)\right)-3x^2\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{3x\left(\cos^2(x)\right)}{2}+\frac{3\cos(x)\sin(x)}{4}+\frac{3x}{4}+x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^4*(x^4*(1/2*cos(x)*sin(x)+1/2*x)+x^3*cos(x)^2-3*x^2*(1/2*cos(x)*sin(x)+
1/2*x)-3/2*x*cos(x)^2+3/4*cos(x)*sin(x)+3/4*x+x^3-2/5*x^5)+16*d^3*c*(x^3*(1
/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin(x)+1/2*x)+3/
8*x^2+3/8*sin(x)^2-3/8*x^4)+24*c^2*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x
*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)-1/5*d^4*x^5+16*c^3*d*(x*(1/2*cos
(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-c*d^3*x^4+4*c^4*(1/2*cos(x)*sin(x)+
1/2*x)-2*c^2*d^2*x^3-2*c^3*d*x^2-c^4*x
```

maxima [A] time = 0.35, size = 146, normalized size = 1.11

$$2\left(x^2+2\,x\sin\left(2\,x\right)+\cos\left(2\,x\right)\right)c^3d+\left(2\,x^3+6\,x\cos\left(2\,x\right)+3\left(2\,x^2-1\right)\sin\left(2\,x\right)\right)c^2d^2+\left(x^4+3\left(2\,x^2-1\right)\cos\left(2\,x\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^3*d + (2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*c^2*d^2 + (x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 - 3*x)*si
n(2*x))*c*d^3 + 1/10*(2*x^5 + 10*(2*x^3 - 3*x)*cos(2*x) + 5*(2*x^4 - 6*x^2
+ 3)*sin(2*x))*d^4 + c^4*(x + sin(2*x))
```

mupad [B] time = 2.26, size = 212, normalized size = 1.62

$$c^4\,\sin\left(2\,x\right)+\frac{3\,d^4\,\sin\left(2\,x\right)}{2}+c^4\,x+\frac{d^4\,x^5}{5}-3\,c^2\,d^2\,\sin\left(2\,x\right)+2\,d^4\,x^3\,\cos\left(2\,x\right)-3\,d^4\,x^2\,\sin\left(2\,x\right)+d^4\,x^4\,\sin\left(2\,x\right)+2\,c^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^4)/sin(x),x)
```

```
[Out] c^4*sin(2*x) + (3*d^4*sin(2*x))/2 + c^4*x + (d^4*x^5)/5 - 3*c^2*d^2*sin(2*x
) + 2*d^4*x^3*cos(2*x) - 3*d^4*x^2*sin(2*x) + d^4*x^4*sin(2*x) + 2*c^3*d*x^
2 + c*d^3*x^4 + 2*c^2*d^2*x^3 - 3*c*d^3*cos(2*x) + 2*c^3*d*cos(2*x) - 3*d^4
*x*cos(2*x) + 6*c^2*d^2*x^2*sin(2*x) - 6*c*d^3*x*sin(2*x) + 4*c^3*d*x*sin(2
*x) + 6*c^2*d^2*x*cos(2*x) + 6*c*d^3*x^2*cos(2*x) + 4*c*d^3*x^3*sin(2*x)
```

sympy [B] time = 25.35, size = 440, normalized size = 3.36

$$c^4x+c^4\sin\left(2x\right)-4c^3dx^2\sin^2\left(x\right)-4c^3dx^2\cos^2\left(x\right)+6c^3dx^2+8c^3dx\sin\left(x\right)\cos\left(x\right)+4c^3d\cos^2\left(x\right)-4c^2d^2x^3\sin^2\left(x\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(x)*sin(3*x),x)
```

```
[Out] c**4*x + c**4*sin(2*x) - 4*c**3*d*x**2*sin(x)**2 - 4*c**3*d*x**2*cos(x)**2
+ 6*c**3*d*x**2 + 8*c**3*d*x*sin(x)*cos(x) + 4*c**3*d*cos(x)**2 - 4*c**2*d*
*2*x**3*sin(x)**2 - 4*c**2*d**2*x**3*cos(x)**2 + 6*c**2*d**2*x**3 + 12*c**2
*d**2*x**2*sin(x)*cos(x) - 6*c**2*d**2*x*sin(x)**2 + 6*c**2*d**2*x*cos(x)**
2 - 6*c**2*d**2*sin(x)*cos(x) - 2*c*d**3*x**4*sin(x)**2 - 2*c*d**3*x**4*cos
(x)**2 + 3*c*d**3*x**4 + 8*c*d**3*x**3*sin(x)*cos(x) - 6*c*d**3*x**2*sin(x)
**2 + 6*c*d**3*x**2*cos(x)**2 - 12*c*d**3*x*sin(x)*cos(x) - 6*c*d**3*cos(x)
**2 - 2*d**4*x**5*sin(x)**2/5 - 2*d**4*x**5*cos(x)**2/5 + 3*d**4*x**5/5 + 2
*d**4*x**4*sin(x)*cos(x) - 2*d**4*x**3*sin(x)**2 + 2*d**4*x**3*cos(x)**2 -
6*d**4*x**2*sin(x)*cos(x) + 3*d**4*x*sin(x)**2 - 3*d**4*x*cos(x)**2 + 3*d**
4*sin(x)*cos(x)
```

3.362 $\int (c+dx)^3 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=115

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3-\frac{3d}{}$$

```
[Out] -3/2*c*d^2*x-3/4*d^3*x^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(x)^2+9/4*d*(d*x+c)^2*c
os(x)^2-3*d^2*(d*x+c)*cos(x)*sin(x)+2*(d*x+c)^3*cos(x)*sin(x)+3/8*d^3*sin(x
)^2-3/4*d*(d*x+c)^2*sin(x)^2
```

Rubi [A] time = 0.14, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3}{2}cd^2x-3d^2\sin(x)\cos(x)(c+dx)+\frac{(c+dx)^4}{4d}-\frac{3}{4}d\sin^2(x)(c+dx)^2+\frac{9}{4}d\cos^2(x)(c+dx)^2+2\sin(x)\cos(x)(c+dx)^3-\frac{3d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (-3*c*d^2*x)/2 - (3*d^3*x^2)/4 + (c + d*x)^4/(4*d) - (9*d^3*Cos[x]^2)/8 + (
9*d*(c + d*x)^2*Cos[x]^2)/4 - 3*d^2*(c + d*x)*Cos[x]*Sin[x] + 2*(c + d*x)^3
*Cos[x]*Sin[x] + (3*d^3*Sin[x]^2)/8 - (3*d*(c + d*x)^2*Sin[x]^2)/4
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^3\cos^2(x)-(c+dx)^3\sin^2(x)\right)\,dx\\ &= 3\int (c+dx)^3\cos^2(x)\,dx-\int (c+dx)^3\sin^2(x)\,dx\\ &= \frac{9}{4}d(c+dx)^2\cos^2(x)+2(c+dx)^3\cos(x)\sin(x)-\frac{3}{4}d(c+dx)^2\sin^2(x)-\frac{1}{2}\int (c+\\ &= \frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+dx)\cos(x)\sin(x)+2(c\\ &= -\frac{3}{2}cd^2x-\frac{3d^3x^2}{4}+\frac{(c+dx)^4}{4d}-\frac{9}{8}d^3\cos^2(x)+\frac{9}{4}d(c+dx)^2\cos^2(x)-3d^2(c+dx\end{aligned}$$

Mathematica [A] time = 0.16, size = 109, normalized size = 0.95

$$\frac{1}{4}\left(3d\cos(2x)\left(2c^2+4cdx+d^2\left(2x^2-1\right)\right)+2\sin(2x)\left(2c^3+6c^2dx+3cd^2\left(2x^2-1\right)+d^3x\left(2x^2-3\right)\right)+x\left(4c^3+\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[x]*Sin[3*x],x]
```

```
[Out] (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(2*c^2 + 4*c*d*x + d^2
*(-1 + 2*x^2))*Cos[2*x] + 2*(2*c^3 + 6*c^2*d*x + d^3*x*(-3 + 2*x^2) + 3*c*d
^2*(-1 + 2*x^2))*Sin[2*x])/4
```

fricas [A] time = 0.47, size = 127, normalized size = 1.10

$$\frac{1}{4}d^3x^4+cd^2x^3+\frac{3}{2}\left(c^2d-d^3\right)x^2+\frac{3}{2}\left(2d^3x^2+4cd^2x+2c^2d-d^3\right)\cos(x)^2+\left(2d^3x^3+6cd^2x^2+2c^3-3cd^2+3\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*(c^2*d - d^3)*x^2 + 3/2*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(x)^2 + (2*d^3*x^3 + 6*c*d^2*x^2 + 2*c^3 - 3*c*d^2 +
3*(2*c^2*d - d^3)*x)*cos(x)*sin(x) + (c^3 - 3*c*d^2)*x
```

giac [A] time = 0.29, size = 112, normalized size = 0.97

$$\frac{1}{4}d^3x^4+cd^2x^3+\frac{3}{2}c^2dx^2+c^3x+\frac{3}{4}\left(2d^3x^2+4cd^2x+2c^2d-d^3\right)\cos\left(2x\right)+\frac{1}{2}\left(2d^3x^3+6cd^2x^2+6c^2dx-3d^3x+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + c^3*x + 3/4*(2*d^3*x^2 + 4*c*d^2*
x + 2*c^2*d - d^3)*cos(2*x) + 1/2*(2*d^3*x^3 + 6*c*d^2*x^2 + 6*c^2*d*x - 3*
d^3*x + 2*c^3 - 3*c*d^2)*sin(2*x)
```

maple [A] time = 0.06, size = 179, normalized size = 1.56

$$4d^3\left(x^3\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)+\frac{3x^2\left(\cos^2(x)\right)}{4}-\frac{3x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)}{2}+\frac{3x^2}{8}+\frac{3\left(\sin^2(x)\right)}{8}-\frac{3x^4}{8}\right)+12c\,d^2\left(x^2\left(\frac{\cos(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^3*(x^3*(1/2*cos(x)*sin(x)+1/2*x)+3/4*x^2*cos(x)^2-3/2*x*(1/2*cos(x)*sin
(x)+1/2*x)+3/8*x^2+3/8*sin(x)^2-3/8*x^4)+12*c*d^2*(x^2*(1/2*cos(x)*sin(x)+1
/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x-1/3*x^3)+12*c^2*d*(x*(1/2*cos(
x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)-1/4*d^3*x^4+4*c^3*(1/2*cos(x)*sin(x)
+1/2*x)-c*d^2*x^3-3/2*c^2*d*x^2-c^3*x
```

maxima [A] time = 0.34, size = 101, normalized size = 0.88

$$\frac{3}{2}\left(x^2+2x\sin\left(2x\right)+\cos\left(2x\right)\right)c^2d+\frac{1}{2}\left(2x^3+6x\cos\left(2x\right)+3\left(2x^2-1\right)\sin\left(2x\right)\right)cd^2+\frac{1}{4}\left(x^4+3\left(2x^2-1\right)\cos(2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 3/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*c^2*d + 1/2*(2*x^3 + 6*x*cos(2*x) + 3*(
2*x^2 - 1)*sin(2*x))*c*d^2 + 1/4*(x^4 + 3*(2*x^2 - 1)*cos(2*x) + 2*(2*x^3 -
 3*x)*sin(2*x))*d^3 + c^3*(x + sin(2*x))
```

mupad [B] time = 0.34, size = 136, normalized size = 1.18

$$c^3\sin\left(2x\right)-\frac{3d^3\cos\left(2x\right)}{4}+c^3x+\frac{d^3x^4}{4}+\frac{3d^3x^2\cos\left(2x\right)}{2}+d^3x^3\sin\left(2x\right)+\frac{3c^2dx^2}{2}+c\,d^2x^3+\frac{3c^2d\cos\left(2x\right)}{2}-\frac{3c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^3)/sin(x),x)
```

```
[Out] c^3*sin(2*x) - (3*d^3*cos(2*x))/4 + c^3*x + (d^3*x^4)/4 + (3*d^3*x^2*cos(2*
x))/2 + d^3*x^3*sin(2*x) + (3*c^2*d*x^2)/2 + c*d^2*x^3 + (3*c^2*d*cos(2*x))
/2 - (3*c*d^2*sin(2*x))/2 - (3*d^3*x*sin(2*x))/2 + 3*c*d^2*x*cos(2*x) + 3*c
^2*d*x*sin(2*x) + 3*c*d^2*x^2*sin(2*x)
```

sympy [B] time = 13.05, size = 289, normalized size = 2.51

$$c^3x+c^3\sin(2x)-3c^2dx^2\sin^2(x)-3c^2dx^2\cos^2(x)+\frac{9c^2dx^2}{2}+6c^2dx\sin(x)\cos(x)+3c^2d\cos^2(x)-2cd^2x^3\sin^2(x)-2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(x)*sin(3*x),x)
```

```
[Out] c**3*x + c**3*sin(2*x) - 3*c**2*d*x**2*sin(x)**2 - 3*c**2*d*x**2*cos(x)**2
+ 9*c**2*d*x**2/2 + 6*c**2*d*x*sin(x)*cos(x) + 3*c**2*d*cos(x)**2 - 2*c*d**
2*x**3*sin(x)**2 - 2*c*d**2*x**3*cos(x)**2 + 3*c*d**2*x**3 + 6*c*d**2*x**2*
sin(x)*cos(x) - 3*c*d**2*x*sin(x)**2 + 3*c*d**2*x*cos(x)**2 - 3*c*d**2*sin(
x)*cos(x) - d**3*x**4*sin(x)**2/2 - d**3*x**4*cos(x)**2/2 + 3*d**3*x**4/4 +
 2*d**3*x**3*sin(x)*cos(x) - 3*d**3*x**2*sin(x)**2/2 + 3*d**3*x**2*cos(x)**
2/2 - 3*d**3*x*sin(x)*cos(x) - 3*d**3*cos(x)**2/2
```

3.363 $\int (c+dx)^2 \csc(x)\sin(3x)\,dx$

Optimal. Leaf size=73

$$\frac{(c+dx)^3}{3d}-\frac{1}{2}d\sin^2(x)(c+dx)+\frac{3}{2}d\cos^2(x)(c+dx)+2\sin(x)\cos(x)(c+dx)^2-\frac{d^2x}{2}-d^2\sin(x)\cos(x)$$

```
[Out] -1/2*d^2*x+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(x)^2-d^2*cos(x)*sin(x)+2*(d*x+
c)^2*cos(x)*sin(x)-1/2*d*(d*x+c)*sin(x)^2
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{(c+dx)^3}{3d}-\frac{1}{2}d\sin^2(x)(c+dx)+\frac{3}{2}d\cos^2(x)(c+dx)+2\sin(x)\cos(x)(c+dx)^2-\frac{d^2x}{2}-d^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] -(d^2*x)/2 + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[x]^2)/2 - d^2*Cos[x]*Si
n[x] + 2*(c + d*x)^2*Cos[x]*Sin[x] - (d*(c + d*x)*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)^2\cos^2(x)-(c+dx)^2\sin^2(x)\right)\,dx\\ &= 3\int (c+dx)^2\cos^2(x)\,dx-\int (c+dx)^2\sin^2(x)\,dx\\ &= \frac{3}{2}d(c+dx)\cos^2(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}d(c+dx)\sin^2(x)-\frac{1}{2}\int (c+d\\ &= \frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\sin(x)-\frac{1}{2}\\ &= -\frac{d^2x}{2}+\frac{(c+dx)^3}{3d}+\frac{3}{2}d(c+dx)\cos^2(x)-d^2\cos(x)\sin(x)+2(c+dx)^2\cos(x)\,\mathrm{si}\end{aligned}$$

Mathematica [A] time = 0.11, size = 60, normalized size = 0.82

$$\sin(x)\cos(x)\left(2c^2+4cdx+d^2\left(2x^2-1\right)\right)+c^2x+cdx^2+d\cos(2x)(c+dx)+\frac{d^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[x]*Sin[3*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + d*(c + d*x)*Cos[2*x] + (2*c^2 + 4*c*d*x + d
^2*(-1 + 2*x^2))*Cos[x]*Sin[x]
```

fricas [A] time = 0.45, size = 70, normalized size = 0.96

$$\frac{1}{3}d^2x^3+cdx^2+2\left(d^2x+cd\right)\cos(x)^2+\left(2\,d^2x^2+4\,cdx+2\,c^2-d^2\right)\cos(x)\sin(x)+\left(c^2-d^2\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="fricas")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + 2*(d^2*x + c*d)*cos(x)^2 + (2*d^2*x^2 + 4*c*d*x + 2
*c^2 - d^2)*cos(x)*sin(x) + (c^2 - d^2)*x
```

giac [A] time = 0.16, size = 64, normalized size = 0.88

$$\frac{1}{3}d^2x^3 + cdx^2 + c^2x + \left(d^2x + cd\right)\cos(2x) + \frac{1}{2}\left(2d^2x^2 + 4cdx + 2c^2 - d^2\right)\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="giac")
```

```
[Out] 1/3*d^2*x^3 + c*d*x^2 + c^2*x + (d^2*x + c*d)*cos(2*x) + 1/2*(2*d^2*x^2 + 4
*c*d*x + 2*c^2 - d^2)*sin(2*x)
```

maple [A] time = 0.06, size = 107, normalized size = 1.47

$$4d^2\left(x^2\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) + \frac{x\left(\cos^2(x)\right)}{2} - \frac{\cos(x)\sin(x)}{4} - \frac{x}{4} - \frac{x^3}{3}\right) + 8cd\left(x\left(\frac{\cos(x)\sin(x)}{2} + \frac{x}{2}\right) - \frac{x^2}{4} - \frac{\left(\sin^2(x\right.}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(x)*sin(3*x),x)
```

```
[Out] 4*d^2*(x^2*(1/2*cos(x)*sin(x)+1/2*x)+1/2*x*cos(x)^2-1/4*cos(x)*sin(x)-1/4*x
-1/3*x^3)+8*c*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c^2*(1
/2*cos(x)*sin(x)+1/2*x)-1/3*d^2*x^3-c*d*x^2-c^2*x
```

maxima [A] time = 0.33, size = 60, normalized size = 0.82

$$\left(x^2 + 2x\sin(2x) + \cos(2x)\right)cd + \frac{1}{6}\left(2x^3 + 6x\cos(2x) + 3\left(2x^2 - 1\right)\sin(2x)\right)d^2 + c^2(x + \sin(2x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] (x^2 + 2*x*sin(2*x) + cos(2*x))*c*d + 1/6*(2*x^3 + 6*x*cos(2*x) + 3*(2*x^2
- 1)*sin(2*x))*d^2 + c^2*(x + sin(2*x))
```

mupad [B] time = 1.82, size = 73, normalized size = 1.00

$$c^2\sin(2x) - \frac{d^2\sin(2x)}{2} + c^2x + \frac{d^2x^3}{3} + d^2x^2\sin(2x) + c\,d\cos(2x) + d^2x\cos(2x) + c\,d\,x^2 + 2\,c\,d\,x\sin(2x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x)^2)/sin(x),x)
```

```
[Out] c^2*sin(2*x) - (d^2*sin(2*x))/2 + c^2*x + (d^2*x^3)/3 + d^2*x^2*sin(2*x) +
c*d*cos(2*x) + d^2*x*cos(2*x) + c*d*x^2 + 2*c*d*x*sin(2*x)
```

sympy [B] time = 6.97, size = 155, normalized size = 2.12

$$c^2x+c^2\sin(2x)-2cdx^2\sin^2(x)-2cdx^2\cos^2(x)+3cdx^2+4cdx\sin(x)\cos(x)+2cd\cos^2(x)-\frac{2d^2x^3\sin^2(x)}{3}-\frac{2d^2x^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(x)*sin(3*x),x)
```

```
[Out] c**2*x + c**2*sin(2*x) - 2*c*d*x**2*sin(x)**2 - 2*c*d*x**2*cos(x)**2 + 3*c*
d*x**2 + 4*c*d*x*sin(x)*cos(x) + 2*c*d*cos(x)**2 - 2*d**2*x**3*sin(x)**2/3
- 2*d**2*x**3*cos(x)**2/3 + d**2*x**3 + 2*d**2*x**2*sin(x)*cos(x) - d**2*x*
sin(x)**2 + d**2*x*cos(x)**2 - d**2*sin(x)*cos(x)
```

3.364 $\int (c+dx)\csc(x)\sin(3x)\,dx$

Optimal. Leaf size=41

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

```
[Out] c*x+1/2*d*x^2+3/4*d*cos(x)^2+2*(d*x+c)*cos(x)*sin(x)-1/4*d*sin(x)^2
```

Rubi [A] time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4431, 3310}

$$2\sin(x)\cos(x)(c+dx)+cx+\frac{dx^2}{2}-\frac{1}{4}d\sin^2(x)+\frac{3}{4}d\cos^2(x)$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[x]*Sin[3*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[x]^2)/4 + 2*(c + d*x)*Cos[x]*Sin[x] - (d*Sin[x]^
2)/4
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\csc(x)\sin(3x)\,dx &= \int \left(3(c+dx)\cos^2(x)-(c+dx)\sin^2(x)\right)\,dx\\
&= 3\int (c+dx)\cos^2(x)\,dx-\int (c+dx)\sin^2(x)\,dx\\
&= \frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)-\frac{1}{2}\int (c+dx)\,dx+\frac{3}{2}\int (c+d\\
&= cx+\frac{dx^2}{2}+\frac{3}{4}d\cos^2(x)+2(c+dx)\cos(x)\sin(x)-\frac{1}{4}d\sin^2(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 34, normalized size = 0.83

$$cx+c\sin(2x)+\frac{dx^2}{2}+dx\sin(2x)+\frac{1}{2}d\cos(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[x]*Sin[3*x],x]
[Out] c*x + (d*x^2)/2 + (d*Cos[2*x])/2 + c*Sin[2*x] + d*x*Sin[2*x]
```

fricas [A] time = 0.44, size = 27, normalized size = 0.66

$$\frac{1}{2}\,dx^2+d\cos(x)^2+2\,(dx+c)\cos(x)\sin(x)+cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="fricas")
[Out] 1/2*d*x^2 + d*cos(x)^2 + 2*(d*x + c)*cos(x)*sin(x) + c*x
```

giac [A] time = 1.51, size = 27, normalized size = 0.66

$$\frac{1}{2}\,dx^2+cx+\frac{1}{2}\,d\cos(2\,x)+(dx+c)\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="giac")
[Out] 1/2*d*x^2 + c*x + 1/2*d*cos(2*x) + (d*x + c)*sin(2*x)
```

maple [A] time = 0.06, size = 52, normalized size = 1.27

$$4d\left(x\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{x^2}{4}-\frac{\left(\sin^2(x)\right)}{4}\right)+4c\left(\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)-\frac{d\,x^2}{2}-cx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(x)*sin(3*x),x)
```

```
[Out] 4*d*(x*(1/2*cos(x)*sin(x)+1/2*x)-1/4*x^2-1/4*sin(x)^2)+4*c*(1/2*cos(x)*sin(
x)+1/2*x)-1/2*d*x^2-c*x
```

maxima [A] time = 0.34, size = 27, normalized size = 0.66

$$\frac{1}{2}\left(x^2 + 2\,x\sin\left(2\,x\right) + \cos\left(2\,x\right)\right)d + c(x + \sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x, algorithm="maxima")
```

```
[Out] 1/2*(x^2 + 2*x*sin(2*x) + cos(2*x))*d + c*(x + sin(2*x))
```

mupad [B] time = 1.72, size = 30, normalized size = 0.73

$$c\,\sin\left(2\,x\right) + c\,x + \frac{d\,x^2}{2} + \frac{d\,\cos\left(2\,x\right)}{2} + d\,x\,\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*x)*(c + d*x))/sin(x),x)
```

```
[Out] c*sin(2*x) + c*x + (d*x^2)/2 + (d*cos(2*x))/2 + d*x*sin(2*x)
```

sympy [A] time = 3.85, size = 56, normalized size = 1.37

$$cx + c\sin\left(2x\right) - dx^2\sin^2\left(x\right) - dx^2\cos^2\left(x\right) + \frac{3dx^2}{2} + 2dx\sin\left(x\right)\cos\left(x\right) + d\cos^2\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(x)*sin(3*x),x)
```

```
[Out] c*x + c*sin(2*x) - d*x**2*sin(x)**2 - d*x**2*cos(x)**2 + 3*d*x**2/2 + 2*d*x
*sin(x)*cos(x) + d*cos(x)**2
```

3.365 $\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx$

Optimal. Leaf size=57

$$\frac{2\cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

Rubi [A] time = 0.25, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2\cos\left(\frac{2c}{d}\right)\mathrm{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x])/d + Log[c + d*x]/d + (2*Sin[(2*
c)/d]*SinIntegral[(2*c)/d + 2*x])/d
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(x)}{c+dx}-\frac{\sin^2(x)}{c+dx}\right)dx\\
&= 3\int \frac{\cos^2(x)}{c+dx}\,dx-\int \frac{\sin^2(x)}{c+dx}\,dx\\
&= 3\int \left(\frac{1}{2(c+dx)}+\frac{\cos(2x)}{2(c+dx)}\right)dx-\int \left(\frac{1}{2(c+dx)}-\frac{\cos(2x)}{2(c+dx)}\right)dx\\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\int \frac{\cos(2x)}{c+dx}\,dx+\frac{3}{2}\int \frac{\cos(2x)}{c+dx}\,dx\\
&= \frac{\log(c+dx)}{d}+\frac{1}{2}\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx+\frac{1}{2}\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx+\frac{1}{2}\,\mathrm{s}\\
&= \frac{2\cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2c}{d}+2x\right)}{d}+\frac{\log(c+dx)}{d}+\frac{2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2c}{d}+2x\right)}{d}
\end{aligned}
$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.86

$$
\frac{2\cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(2\left(\frac{c}{d}+x\right)\right)+2\sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(2\left(\frac{c}{d}+x\right)\right)+\log(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x),x]
```

```
[Out] (2*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + Log[c + d*x] + 2*Sin[(2*c)/d]*Si
nIntegral[2*(c/d + x)])/d
```

fricas [A] time = 0.44, size = 62, normalized size = 1.09

$$
\frac{\left(\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+\mathrm{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)+2\,\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+\log\left(dx+c\right)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(d*x + c)/d) + cos_integral(-2*(d*x + c)/d))*cos(2*c/d) + 
2*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + log(d*x + c))/d
```

giac [A] time = 2.95, size = 51, normalized size = 0.89

$$\frac{2 \cos\left(\frac{2c}{d}\right)\mathrm{Ci}\left(\frac{2(dx+c)}{d}\right) + 2 \sin\left(\frac{2c}{d}\right)\mathrm{Si}\left(\frac{2(dx+c)}{d}\right) + \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="giac")
```

```
[Out] (2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 2*sin(2*c/d)*sin_integral(2*(d*
x + c)/d) + log(d*x + c))/d
```

maple [A] time = 0.06, size = 58, normalized size = 1.02

$$\frac{2\,\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d} + \frac{\ln(dx+c)}{d} + \frac{2\,\mathrm{Si}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] 2*Ci(2*c/d+2*x)*cos(2*c/d)/d+ln(d*x+c)/d+2*Si(2*c/d+2*x)*sin(2*c/d)/d
```

maxima [C] time = 0.38, size = 95, normalized size = 1.67

$$-\frac{\left(E_1\left(\frac{2i\,dx+2i\,c}{d}\right) + E_1\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2c}{d}\right) - \left(-i\,E_1\left(\frac{2i\,dx+2i\,c}{d}\right) + i\,E_1\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2c}{d}\right) - \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + exp_integral_e(1, -(2*I*d*x + 2
*I*c)/d))*cos(2*c/d) - (-I*exp_integral_e(1, (2*I*d*x + 2*I*c)/d) + I*exp_i
ntegral_e(1, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d) - log(d*x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin(3\,x)}{\sin(x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(3x)\csc(x)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c),x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x), x)
```

3.366 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx$

Optimal. Leaf size=78

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

```
[Out] -3*cos(x)^2/d/(d*x+c)-4*cos(2*c/d)*Si(2*c/d+2*x)/d^2+4*Ci(2*c/d+2*x)*sin(2*
c/d)/d^2+sin(x)^2/d/(d*x+c)
```

Rubi [A] time = 0.24, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^2}-\frac{4\cos\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{3\cos^2(x)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

```
[Out] (-3*Cos[x]^2)/(d*(c + d*x)) + (4*CosIntegral[(2*c)/d + 2*x]*Sin[(2*c)/d])/d
^2 + Sin[x]^2/(d*(c + d*x)) - (4*Cos[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d
^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{(c+dx)^2}\,dx &= \int \left(\frac{3\cos^2(x)}{(c+dx)^2}-\frac{\sin^2(x)}{(c+dx)^2}\right)dx\\
&= 3\int \frac{\cos^2(x)}{(c+dx)^2}\,dx-\int \frac{\sin^2(x)}{(c+dx)^2}\,dx\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{2\int \frac{\sin(2x)}{2(c+dx)}\,dx}{d}+\frac{6\int -\frac{\sin(2x)}{2(c+dx)}\,dx}{d}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{\int \frac{\sin(2x)}{c+dx}\,dx}{d}-\frac{3\int \frac{\sin(2x)}{c+dx}\,dx}{d}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{\sin^2(x)}{d(c+dx)}-\frac{\cos\left(\frac{2c}{d}\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d}-\frac{\left(3\cos\left(\frac{2c}{d}\right)\right)\int \frac{\sin\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d}+\frac{\sin\left(\frac{2c}{d}\right.}{}\\
&= -\frac{3\cos^2(x)}{d(c+dx)}+\frac{4\operatorname{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d^2}+\frac{\sin^2(x)}{d(c+dx)}-\frac{4\cos\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2c}{d}+2x\right)}{d^2}
\end{aligned}
$$

Mathematica [A] time = 0.14, size = 61, normalized size = 0.78

$$\frac{4\sin\left(\frac{2c}{d}\right)\text{Ci}\left(2\left(\frac{c}{d}+x\right)\right)-4\cos\left(\frac{2c}{d}\right)\text{Si}\left(2\left(\frac{c}{d}+x\right)\right)-\frac{d(2\cos(2x)+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^2,x]
```

```
[Out] (-((d*(1 + 2*Cos[2*x]))/(c + d*x)) + 4*CosIntegral[2*(c/d + x)]*Sin[(2*c)/d
] - 4*Cos[(2*c)/d]*SinIntegral[2*(c/d + x)])/d^2
```

fricas [A] time = 0.48, size = 95, normalized size = 1.22

$$-\frac{4\,d\cos(x)^2+4\,(dx+c)\cos\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\left((dx+c)\,\text{Ci}\left(\frac{2\,(dx+c)}{d}\right)+(dx+c)\,\text{Ci}\left(-\frac{2\,(dx+c)}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)-d}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(x)^2 + 4*(d*x + c)*cos(2*c/d)*sin_integral(2*(d*x + c)/d) - 2*((d
*x + c)*cos_integral(2*(d*x + c)/d) + (d*x + c)*cos_integral(-2*(d*x + c)/d
))*sin(2*c/d) - d)/(d^3*x + c*d^2)
```

giac [A] time = 0.20, size = 111, normalized size = 1.42

$$\frac{4\,dx\,\text{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,dx\cos\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right)+4\,c\,\text{Ci}\left(\frac{2\,(dx+c)}{d}\right)\sin\left(\frac{2\,c}{d}\right)-4\,c\cos\left(\frac{2\,c}{d}\right)\text{Si}\left(\frac{2\,(dx+c)}{d}\right)-2\,d\,c}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (4*d*x*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*d*x*cos(2*c/d)*sin_integr
al(2*(d*x + c)/d) + 4*c*cos_integral(2*(d*x + c)/d)*sin(2*c/d) - 4*c*cos(2*
c/d)*sin_integral(2*(d*x + c)/d) - 2*d*cos(2*x) - d)/(d^3*x + c*d^2)
```

maple [A] time = 0.05, size = 82, normalized size = 1.05

$$-\frac{2\cos(2x)}{(dx+c)\,d}-\frac{2\left(\frac{2\,\text{Si}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}-\frac{2\,\text{Ci}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}\right)}{d}-\frac{1}{d\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^2,x)
```

```
[Out] -2*cos(2*x)/(d*x+c)/d-2*(2*Si(2*c/d+2*x)*cos(2*c/d)/d-2*Ci(2*c/d+2*x)*sin(2
*c/d)/d)/d-1/d/(d*x+c)
```

maxima [C] time = 0.40, size = 324, normalized size = 4.15

$$-\frac{\left(E_2\left(\frac{2i\,dx+2i\,c}{d}\right)+E_2\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3+\left(i\,E_2\left(\frac{2i\,dx+2i\,c}{d}\right)-i\,E_2\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+\left(\left(E_2\left(\frac{2i\,dx+2i\,c}{d}\right)+E_2\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + exp_integral_e(2, -(2*I*d*x
 + 2*I*c)/d))*cos(2*c/d)^3 + (I*exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I*
exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d)^3 + ((exp_integral_e(2,
 (2*I*d*x + 2*I*c)/d) + exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d)
 + 2)*sin(2*c/d)^2 + (exp_integral_e(2, (2*I*d*x + 2*I*c)/d) + exp_integral
_e(2, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((I*exp_integral
_e(2, (2*I*d*x + 2*I*c)/d) - I*exp_integral_e(2, -(2*I*d*x + 2*I*c)/d))*cos
(2*c/d)^2 + I*exp_integral_e(2, (2*I*d*x + 2*I*c)/d) - I*exp_integral_e(2,
-(2*I*d*x + 2*I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2*c/d)^2)*d^2*x + (
c*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,x)}{\sin(x)\,(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)^2),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(3x)\csc(x)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**2,x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x)**2, x)
```

3.367 $\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx$

Optimal. Leaf size=99

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{Ci}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

```
[Out] -4*Ci(2*c/d+2*x)*cos(2*c/d)/d^3-3/2*cos(x)^2/d/(d*x+c)^2-4*Si(2*c/d+2*x)*si
n(2*c/d)/d^3+4*cos(x)*sin(x)/d^2/(d*x+c)+1/2*sin(x)^2/d/(d*x+c)^2
```

Rubi [A] time = 0.33, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4\cos\left(\frac{2c}{d}\right)\text{CosIntegral}\left(\frac{2c}{d}+2x\right)}{d^3}-\frac{4\sin\left(\frac{2c}{d}\right)\text{Si}\left(\frac{2c}{d}+2x\right)}{d^3}+\frac{4\sin(x)\cos(x)}{d^2(c+dx)}+\frac{\sin^2(x)}{2d(c+dx)^2}-\frac{3\cos^2(x)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[x]^2)/(2*d*(c + d*x)^2) - (4*Cos[(2*c)/d]*CosIntegral[(2*c)/d + 2*x
])/d^3 + (4*Cos[x]*Sin[x])/(d^2*(c + d*x)) + Sin[x]^2/(2*d*(c + d*x)^2) - (
4*Sin[(2*c)/d]*SinIntegral[(2*c)/d + 2*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(x)\sin(3x)}{(c+dx)^3}\,dx &= \int \left(\frac{3\cos^2(x)}{(c+dx)^3} - \frac{\sin^2(x)}{(c+dx)^3}\right) dx \\
&= 3\int \frac{\cos^2(x)}{(c+dx)^3}\,dx - \int \frac{\sin^2(x)}{(c+dx)^3}\,dx \\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{1}{c+dx}\,dx}{d^2} + \frac{2\int \frac{\sin^2(x)}{c+dx}\,dx}{d^2} + \frac{3\int \frac{1}{c+dx}\,dx}{d^2} \\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{2\log(c+dx)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} + \frac{2\int \left(\frac{1}{2(c+dx)} - \frac{\cos(2x)}{2(c+dx)}\right)}{d^2} \\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\int \frac{\cos(2x)}{c+dx}\,dx}{d^2} - \frac{3\int \frac{\cos(2x)}{c+dx}\,dx}{d^2} \\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{\cos\left(\frac{2c}{d}\right)\int \frac{\cos\left(\frac{2c}{d}+2x\right)}{c+dx}\,dx}{d^2} - \left(3\cos\left(\frac{2c}{d}\right)\right) \\
&= -\frac{3\cos^2(x)}{2d(c+dx)^2} - \frac{4\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(\frac{2c}{d}+2x\right)}{d^3} + \frac{4\cos(x)\sin(x)}{d^2(c+dx)} + \frac{\sin^2(x)}{2d(c+dx)^2} - \frac{4\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\right.}{d^3}
\end{aligned}
$$

Mathematica [A] time = 0.24, size = 77, normalized size = 0.78

$$
\frac{-8\cos\left(\frac{2c}{d}\right)\operatorname{Ci}\left(2\left(\frac{c}{d}+x\right)\right) - 8\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(2\left(\frac{c}{d}+x\right)\right) + \frac{d(4\sin(2x)(c+dx)-2d\cos(2x)-d)}{(c+dx)^2}}{2d^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[x]*Sin[3*x])/(c + d*x)^3,x]
```

```
[Out] (-8*Cos[(2*c)/d]*CosIntegral[2*(c/d + x)] + (d*(-d - 2*d*Cos[2*x] + 4*(c +
d*x)*Sin[2*x]))/(c + d*x)^2 - 8*Sin[(2*c)/d]*SinIntegral[2*(c/d + x)])/(2*d
^3)
```

fricas [A] time = 0.44, size = 158, normalized size = 1.60

$$
-\frac{4d^2\cos(x)^2 - 8\left(d^2x + cd\right)\cos(x)\sin(x) + 8\left(d^2x^2 + 2cdx + c^2\right)\sin\left(\frac{2c}{d}\right)\operatorname{Si}\left(\frac{2(dx+c)}{d}\right) - d^2 + 4\left(\left(d^2x^2 + 2cdx +\right.\right.}{2\left(d^5x^2 + 2cd^4x + c^2d^3\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(x)^2 - 8*(d^2*x + c*d)*cos(x)*sin(x) + 8*(d^2*x^2 + 2*c*d*x
 + c^2)*sin(2*c/d)*sin_integral(2*(d*x + c)/d) - d^2 + 4*((d^2*x^2 + 2*c*d*
x + c^2)*cos_integral(2*(d*x + c)/d) + (d^2*x^2 + 2*c*d*x + c^2)*cos_integr
al(-2*(d*x + c)/d))*cos(2*c/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

giac [B] time = 2.73, size = 201, normalized size = 2.03

$$-\frac{8\,d^2x^2\cos\left(\frac{2\,c}{d}\right)\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+8\,d^2x^2\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\cos\left(\frac{2\,c}{d}\right)\mathrm{Ci}\left(\frac{2\,(dx+c)}{d}\right)+16\,cdx\sin\left(\frac{2\,c}{d}\right)\mathrm{Si}\left(\frac{2\,(d}{}\right.}{2\left(d^5x^2+2\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -1/2*(8*d^2*x^2*cos(2*c/d)*cos_integral(2*(d*x + c)/d) + 8*d^2*x^2*sin(2*c/
d)*sin_integral(2*(d*x + c)/d) + 16*c*d*x*cos(2*c/d)*cos_integral(2*(d*x +
c)/d) + 16*c*d*x*sin(2*c/d)*sin_integral(2*(d*x + c)/d) + 8*c^2*cos(2*c/d)*
cos_integral(2*(d*x + c)/d) - 4*d^2*x*sin(2*x) + 8*c^2*sin(2*c/d)*sin_integ
ral(2*(d*x + c)/d) + 2*d^2*cos(2*x) - 4*c*d*sin(2*x) + d^2)/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

maple [A] time = 0.06, size = 104, normalized size = 1.05

$$-\frac{\cos\left(2x\right)}{\left(dx+c\right)^{2}d}-\frac{-\frac{2\sin(2x)}{(dx+c)d}+\frac{\frac{4\,\mathrm{Si}\left(\frac{2c}{d}+2x\right)\sin\left(\frac{2c}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(\frac{2c}{d}+2x\right)\cos\left(\frac{2c}{d}\right)}{d}}{d}}{d}-\frac{1}{2d\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*sin(3*x)/(d*x+c)^3,x)
```

```
[Out] -cos(2*x)/(d*x+c)^2/d-(-2*sin(2*x)/(d*x+c)/d+2*(2*Si(2*c/d+2*x)*sin(2*c/d)/
d+2*Ci(2*c/d+2*x)*cos(2*c/d)/d)/d)/d-1/2/d/(d*x+c)^2
```

maxima [C] time = 0.42, size = 362, normalized size = 3.66

$$-\frac{2\left(E_3\left(\frac{2i\,dx+2i\,c}{d}\right)+E_3\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\cos\left(\frac{2\,c}{d}\right)^3+\left(2i\,E_3\left(\frac{2i\,dx+2i\,c}{d}\right)-2i\,E_3\left(-\frac{2i\,dx+2i\,c}{d}\right)\right)\sin\left(\frac{2\,c}{d}\right)^3+2\left(\left(E_3\left(\frac{2i\,dx+2i\,c}{d}\right.\right.\right.}{4\left(\left(\mathrm{c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + exp_integral_e(3, -(2*I*d
*x + 2*I*c)/d))*cos(2*c/d)^3 + (2*I*exp_integral_e(3, (2*I*d*x + 2*I*c)/d)
- 2*I*exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d)^3 + 2*((exp_integ
ral_e(3, (2*I*d*x + 2*I*c)/d) + exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*co
s(2*c/d) + 1)*sin(2*c/d)^2 + 2*(exp_integral_e(3, (2*I*d*x + 2*I*c)/d) + ex
p_integral_e(3, -(2*I*d*x + 2*I*c)/d))*cos(2*c/d) + 2*cos(2*c/d)^2 + ((2*I*
exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*exp_integral_e(3, -(2*I*d*x +
2*I*c)/d))*cos(2*c/d)^2 + 2*I*exp_integral_e(3, (2*I*d*x + 2*I*c)/d) - 2*I*
exp_integral_e(3, -(2*I*d*x + 2*I*c)/d))*sin(2*c/d))/((cos(2*c/d)^2 + sin(2
*c/d)^2)*d^3*x^2 + 2*(c*cos(2*c/d)^2 + c*sin(2*c/d)^2)*d^2*x + (c^2*cos(2*c
/d)^2 + c^2*sin(2*c/d)^2)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (3\,x)}{\sin(x)\,{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x)/(sin(x)*(c + d*x)^3),x)
```

```
[Out] int(sin(3*x)/(sin(x)*(c + d*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin (3x) \csc (x)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*sin(3*x)/(d*x+c)**3,x)
```

```
[Out] Integral(sin(3*x)*csc(x)/(c + d*x)**3, x)
```

3.368 $\int (c+dx)^4 \csc(a+bx) \sin(3a+3bx)\,dx$

Optimal. Leaf size=198

$$\frac{3d^4\sin(a+bx)\cos(a+bx)}{b^5}+\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a$$

```
[Out] 3/2*d^4*x/b^4-d*(d*x+c)^3/b^2+1/5*(d*x+c)^5/d-9/2*d^3*(d*x+c)*cos(b*x+a)^2/
b^4+3*d*(d*x+c)^3*cos(b*x+a)^2/b^2+3*d^4*cos(b*x+a)*sin(b*x+a)/b^5-6*d^2*(d
*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^3+2*(d*x+c)^4*cos(b*x+a)*sin(b*x+a)/b+3/2*d
^3*(d*x+c)*sin(b*x+a)^2/b^4-d*(d*x+c)^3*sin(b*x+a)^2/b^2
```

Rubi [A] time = 0.25, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3311, 32, 2635, 8}

$$\frac{3d^3(c+dx)\sin^2(a+bx)}{2b^4}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}-\frac{6d^2(c+dx)^2\sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)^3\sin^2(a+bx}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*d^4*x)/(2*b^4) - (d*(c + d*x)^3)/b^2 + (c + d*x)^5/(5*d) - (9*d^3*(c + d
*x)*Cos[a + b*x]^2)/(2*b^4) + (3*d*(c + d*x)^3*Cos[a + b*x]^2)/b^2 + (3*d^4
*Cos[a + b*x]*Sin[a + b*x])/b^5 - (6*d^2*(c + d*x)^2*Cos[a + b*x]*Sin[a + b
*x])/b^3 + (2*(c + d*x)^4*Cos[a + b*x]*Sin[a + b*x])/b + (3*d^3*(c + d*x)*S
in[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Sin[a + b*x]^2)/b^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4 \csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^4\cos^2(a+bx)-(c+dx)^4\sin^2(a+bx)\right)dx \\
&= 3\int (c+dx)^4\cos^2(a+bx)\,dx-\int (c+dx)^4\sin^2(a+bx)\,dx \\
&= \frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}+\frac{2(c+dx)^4\cos(a+bx)\sin(a+bx)}{b}-\frac{d(c
\\
&= \frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3\cos^2(a+bx)}{b^2}-
\\
&= -\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+dx)^3 c}{b^2}
\\
&= \frac{3d^4x}{2b^4}-\frac{d(c+dx)^3}{b^2}+\frac{(c+dx)^5}{5d}-\frac{9d^3(c+dx)\cos^2(a+bx)}{2b^4}+\frac{3d(c+}{}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 128, normalized size = 0.65

$$\frac{d(c+dx)\cos(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)}{b^4}+\frac{\sin(2(a+bx))\left(2b^4(c+dx)^4-6b^2d^2(c+dx)^2+3d^4\right)}{2b^5}+c^4x+2c^3d$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^4*x + 2*c^3*d*x^2 + 2*c^2*d^2*x^3 + c*d^3*x^4 + (d^4*x^5)/5 + (d*(c + d*x
)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Cos[2*(a + b*x)])/b^4 + ((3*d^4 - 6*b^2*d^2*
(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sin[2*(a + b*x)])/(2*b^5)
```

fricas [A] time = 0.45, size = 283, normalized size = 1.43

$$\frac{b^5d^4x^5+5\,b^5cd^3x^4+10\left(b^5c^2d^2-b^3d^4\right)x^3+10\left(b^5c^3d-3\,b^3cd^3\right)x^2+10\left(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d-3\,bcd^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/5*(b^5*d^4*x^5 + 5*b^5*c*d^3*x^4 + 10*(b^5*c^2*d^2 - b^3*d^4)*x^3 + 10*(b
^5*c^3*d - 3*b^3*c*d^3)*x^2 + 10*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c
^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*x)*cos(b*x + a)^2 + 5*(2*b^4*d
^4*x^4 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2
*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)*sin(b*x
 + a) + 5*(b^5*c^4 - 6*b^3*c^2*d^2 + 3*b*d^4)*x)/b^5
```

giac [B] time = 0.55, size = 4684, normalized size = 23.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/5*(b^5*d^4*x^5*tan(1/2*b*x)^4*tan(1/2*a)^4 + 5*b^5*c*d^3*x^4*tan(1/2*b*x)
^4*tan(1/2*a)^4 + 2*b^5*d^4*x^5*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^5*d^4*x^5
*tan(1/2*b*x)^2*tan(1/2*a)^4 + 10*b^5*c^2*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)
^4 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*d^4*x^4*tan(1/2*
b*x)^4*tan(1/2*a)^3 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^4 - 20*b^4
*d^4*x^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 10*b^5*c^3*d*x^2*tan(1/2*b*x)^4*tan(
1/2*a)^4 + b^5*d^4*x^5*tan(1/2*b*x)^4 + 4*b^5*d^4*x^5*tan(1/2*b*x)^2*tan(1/
2*a)^2 + 20*b^5*c^2*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c*d^3*x^3*
tan(1/2*b*x)^4*tan(1/2*a)^3 + b^5*d^4*x^5*tan(1/2*a)^4 + 20*b^5*c^2*d^2*x^3
*tan(1/2*b*x)^2*tan(1/2*a)^4 - 80*b^4*c*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^4
 + 5*b^5*c^4*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^3*d^4*x^3*tan(1/2*b*x)^4*
tan(1/2*a)^4 + 5*b^5*c*d^3*x^4*tan(1/2*b*x)^4 + 20*b^4*d^4*x^4*tan(1/2*b*x)
^4*tan(1/2*a) + 20*b^5*c*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^4*d^4*
x^4*tan(1/2*b*x)^3*tan(1/2*a)^2 + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^4*tan(1/2*a
)^2 + 120*b^4*d^4*x^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 120*b^4*c^2*d^2*x^2*tan
(1/2*b*x)^4*tan(1/2*a)^3 + 5*b^5*c*d^3*x^4*tan(1/2*a)^4 + 20*b^4*d^4*x^4*ta
n(1/2*b*x)*tan(1/2*a)^4 + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12
0*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 30*b^3*c*d^3*x^2*tan(1/2*b*
x)^4*tan(1/2*a)^4 + 2*b^5*d^4*x^5*tan(1/2*b*x)^2 + 10*b^5*c^2*d^2*x^3*tan(1
/2*b*x)^4 + 80*b^4*c*d^3*x^3*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^5*d^4*x^5*tan(
1/2*a)^2 + 40*b^5*c^2*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 480*b^4*c*d^3*x
^3*tan(1/2*b*x)^3*tan(1/2*a)^2 + 10*b^5*c^4*x*tan(1/2*b*x)^4*tan(1/2*a)^2 -
```

60*b^3*d^4*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 480*b^4*c*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a)^3 - 160*b^3*d^4*x^3*tan(1/2*b*x)^3*tan(1/2*a)^3 - 80*b^4*c^3*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^5*c^2*d^2*x^3*tan(1/2*a)^4 + 80*b^4*c*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^4 + 10*b^5*c^4*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 60*b^3*d^4*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 80*b^4*c^3*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 30*b^3*c^2*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^5*c*d^3*x^4*tan(1/2*b*x)^2 - 20*b^4*d^4*x^4*tan(1/2*b*x)^3 + 10*b^5*c^3*d*x^2*tan(1/2*b*x)^4 - 120*b^4*d^4*x^4*tan(1/2*b*x)^2*tan(1/2*a) + 120*b^4*c^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 10*b^5*c*d^3*x^4*tan(1/2*a)^2 - 120*b^4*d^4*x^4*tan(1/2*b*x)*tan(1/2*a)^2 + 40*b^5*c^3*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 180*b^3*c*d^3*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*d^4*x^4*tan(1/2*a)^3 + 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 480*b^3*c*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 20*b^4*c^4*tan(1/2*b*x)^4*tan(1/2*a)^3 + 60*b^2*d^4*x^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^5*c^3*d*x^2*tan(1/2*a)^4 + 120*b^4*c^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 180*b^3*c*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 20*b^4*c^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 60*b^2*d^4*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 10*b^3*c^3*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^5*d^4*x^5 + 20*b^5*c^2*d^2*x^3*tan(1/2*b*x)^2 - 80*b^4*c*d^3*x^3*tan(1/2*b*x)^3 + 5*b^5*c^4*x*tan(1/2*b*x)^4 + 10*b^3*d^4*x^3*tan(1/2*b*x)^4 - 480*b^4*c*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a) + 160*b^3*d^4*x^3*tan(1/2*b*x)^3*tan(1/2*a) + 80*b^4*c^3*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 20*b^5*c^2*d^2*x^3*tan(1/2*a)^2 - 480*b^4*c*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^2 + 20*b^5*c^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 360*b^3*d^4*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 480*b^4*c^3*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c*d^3*x^3*tan(1/2*a)^3 + 160*b^3*d^4*x^3*tan(1/2*b*x)*tan(1/2*a)^3 + 480*b^4*c^3*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 480*b^3*c^2*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^3 + 120*b^2*c*d^3*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 5*b^5*c^4*x*tan(1/2*a)^4 + 10*b^3*d^4*x^3*tan(1/2*a)^4 + 80*b^4*c^3*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 120*b^2*c*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^4 - 15*b*d^4*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 5*b^5*c*d^3*x^4 + 20*b^4*d^4*x^4*tan(1/2*b*x) + 20*b^5*c^3*d*x^2*tan(1/2*b*x)^2 - 120*b^4*c^2*d^2*x^2*tan(1/2*b*x)^3 + 30*b^3*c*d^3*x^2*tan(1/2*b*x)^4 + 20*b^4*d^4*x^4*tan(1/2*a) - 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 480*b^3*c*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a) + 20*b^4*c^4*tan(1/2*b*x)^4*tan(1/2*a) - 60*b^2*d^4*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 20*b^5*c^3*d*x^2*tan(1/2*a)^2 - 720*b^4*c^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 1080*b^3*c*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^4*c^4*tan(1/2*b*x)^3*tan(1/2*a)^2 - 360*b^2*d^4*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 60*b^3*c^3*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 120*b^4*c^2*d^2*x^2*tan(1/2*a)^3 + 480*b^3*c*d^3*x^2*tan(1/2*b*x)*tan(1/2*a)^3 + 120*b^4*c^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 360*b^2*d^4*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 160*b^3*c^3*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 60*b^2*c^2*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 30*b^3*c*d^3*x^2*tan(1/2*a)^4 + 20*b^4*c^4*tan(1/2*b*x)*tan(1/2*a)^4 - 60*b^2*d^4*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 60*b^3*c^3*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 60*b^2*c^2*d^2*tan(1/2* b*x)^3*tan(1/2*a)^4 - 15*b*c*d^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b^5*c^2*d^2*x^3 + 80*b^4*c*d^3*x^3*tan(1/2*b*x) + 10*b^5*c^4*x*tan(1/2*b*x)^2 - 60*b^3*d^4*x^3*tan(1/2*b*x)^2 - 80*b^4*c^3*d*x*tan(1/2*b*x)^3 + 30*b^3*c^2*d^2*x*tan(1/2*b*x)^4 + 80*b^4*c*d^3*x^3*tan(1/2*a) - 160*b^3*d^4*x^3*tan(1/2*b*x)*tan(1/2*a) - 480*b^4*c^3*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 480*b^3*c^2*d^2*x*tan(1/2*b*x)^3*tan(1/2*a) - 120*b^2*c*d^3*x*tan(1/2*b*x)^4*tan(1/2*a) + 10*b^5*c^4*x*tan(1/2*a)^2 - 60*b^3*d^4*x^3*tan(1/2*a)^2 - 480*b^4*c^3*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 1080*b^3*c^2*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 720*b^2*c*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 90*b*d^4*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 80*b^4*c^3*d*x*tan(1/2*a)^3 + 480*b^3*c^2*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3 - 720*b^2*c*d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 240*b*d^4*x*tan(1/2*b*x)^3*tan(1/2*a)^3 + 30*b^3*c^2*d^2*x*tan(1/2*a)^4 - 120*b^2*c*d^3*x*tan(1/2*b*x)*tan(1/2*a)^4 + 90*b*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 10*b^5*c^3*d*x^2 + 120*b^4*c^2*d^2*x^2*tan(1/2*b*x) - 180*b^3*c*d^3*x^2*tan(1/2*b*x)^2 - 20*b^4*c^4*tan(1/2*b*x)^3 + 60*b^2*d^4*x^2*tan(1/2*b*x)^3 + 10*b^3*c^3*d*tan(1/2*b*x)^4 + 120*b^4*c^2*d^2*x^2*tan(1/2*a) - 480*b^3*c*d^3*x^2*tan(1/2*b*x)*tan(1/2*a) - 120*b^4*c^4*tan(1/2*b*x)^2*tan(1/2*a) + 360*b^2*d^4*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 160*b^3*c^3*d*tan(1/2*b*x)^3*tan(1/2*a) - 60*b^2*c^2*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 180*b^3*c*d^3*x^2*tan(1/2*a)^2 - 120*b^4*c^4*tan(1/2*b*x)*tan(1/2*a)^2 + 360*b^2*d^4*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 360*b^3*c^3*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 360*b^2*c^2*d^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 90*b*c*d^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 20*b^4*c^4*tan(1/2*a)^3 + 60*b^2*d^4*x^2*tan(1/2*a)^3 + 160*b^3*c^3*d*tan(1/2*b*x)*tan(1/2*a)^3 - 360*b^2*c^2*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 240*b*c*d^3*tan(1/2*b*x)^3*tan(1/2*a)^3 - 30*d^4*tan(1/2*b*x)^4*tan(1/2*a)^3 + 10*b^3*c^3*d*tan(1/2*a)^4 - 60*b^2*c^2*d^2*tan(1/2*b*x)*tan(1/2*a)^4 + 90*b*c*d^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 30*d^4*tan(1/2*b*x)^3*tan(1/2*a)^4 + 5*b^5*c^4*x + 10*b^3*d^4*x^3 + 80*b^4*c^3*d*x*tan(1/2*b*x) - 180*b^3*c^2*d^2*x*tan(1/2*b*x)^2 + 120*b^2*c*d^3*x*tan(1/2*b*x)^3 - 15*b*d^4*x*tan(1/2*b*x)^4 + 80*b^4*c^3*d*x*tan(1/2*a) - 480*b^3*c^2*d^2*x*tan(1/2*b*x)*tan(1/2*a) + 720*b^2*c*d^3*x*tan(1/2*b*x)^2*tan(1/2*a) - 240*b*d^4*x*tan(1/2*b*x)^3*tan(1/2*a) - 180*b^3*c^2*d^2*x*tan(1/2*a)^2 + 720*b^2*c*d^3*x*tan(1/2*b*x)*tan(1/2*a)^2 - 540*b*d^4*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 120*b^2*c*d^3*x*tan(1/2*a)^3 - 240*b*d^4*x*tan(1/2*b*x)*tan(1/2*a)^3 - 15*b*d^4*x*tan(1/2*a)^4 + 30*b^3*c*d^3*x^2 + 20*b^4*c^4*tan(1/2*b*x) - 60*b^2*d^4*x^2*tan(1/2*b*x) - 60*b^3*c^3*d*tan(1/2*b*x)^2 + 60*b^2*c^2*d^2*tan(1/2*b*x)^3 - 15*b*c*d^3*tan(1/2*b*x)^4 + 20*b^4*c^4*tan(1/2*a) - 60*b^2*d^4*x^2*tan(1/2*a) - 160*b^3*c^3*d*tan(1/2*b*x)*tan(1/2*a) + 360*b^2*c^2*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 240*b*c*d^3*tan(1/2*b*x)^3*tan(1/2*a) + 30*d^4*tan(1/2*b*x)^4*tan(1/2*a) - 60*b^3*c^3*d*tan(1/2*a)^2 + 360*b^2*c^2*d^2*tan(1/2*b*x)*tan(1/2*a)^2 - 540*b*c*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 180*d^4*tan(1/2*b*x)^3*tan(1/2*a)^2 + 60*b^2*c^2*d^2*tan(1/2*a)^3 - 240*b*c*d^3*tan(1/2*b*x)*tan(1/2*a)^3 + 180*d^4*tan(1/2*b*x)^2*tan(1/2*a)^3 - 15*b*c*d^3*tan(1/2*a)^4 + 30*d^4*tan(1/2*b*x)*tan(1/2*a)^4 + 30*b^3*c^2*d^2*x - 120*b^2*c*d^3*x*tan(1/2*b*x) + 90*b*d^4*x*tan(1/2*b*x)^2 - 120*b^2*c*d^3*x*tan(1/2*a) + 240*b*d^4*x*tan(1/2*b*x)*tan

```
(1/2*a) + 90*b*d^4*x*tan(1/2*a)^2 + 10*b^3*c^3*d - 60*b^2*c^2*d^2*tan(1/2*b
*x) + 90*b*c*d^3*tan(1/2*b*x)^2 - 30*d^4*tan(1/2*b*x)^3 - 60*b^2*c^2*d^2*ta
n(1/2*a) + 240*b*c*d^3*tan(1/2*b*x)*tan(1/2*a) - 180*d^4*tan(1/2*b*x)^2*tan
(1/2*a) + 90*b*c*d^3*tan(1/2*a)^2 - 180*d^4*tan(1/2*b*x)*tan(1/2*a)^2 - 30*
d^4*tan(1/2*a)^3 - 15*b*d^4*x - 15*b*c*d^3 + 30*d^4*tan(1/2*b*x) + 30*d^4*t
an(1/2*a))/(b^5*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^5*tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*b^5*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^5*tan(1/2*b*x)^4 + 4*b^5*tan(1
/2*b*x)^2*tan(1/2*a)^2 + b^5*tan(1/2*a)^4 + 2*b^5*tan(1/2*b*x)^2 + 2*b^5*ta
n(1/2*a)^2 + b^5)
```

maple [B] time = 0.06, size = 1000, normalized size = 5.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^4*x-1/5*d^4*x^5+4*c^4/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-2*c^3*
d*x^2-2*c^2*d^2*x^3-c*d^3*x^4+4*d^4/b^5*((b*x+a)^4*(1/2*cos(b*x+a)*sin(b*x+
a)+1/2*b*x+1/2*a)+(b*x+a)^3*cos(b*x+a)^2-3*(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*
x+a)+1/2*b*x+1/2*a)-3/2*(b*x+a)*cos(b*x+a)^2+3/4*cos(b*x+a)*sin(b*x+a)+3/4*
b*x+3/4*a+(b*x+a)^3-2/5*(b*x+a)^5-4*a*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)
+1/2*b*x+1/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(
b*x+a)+1/2*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4)+6*a^2*(
(b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^
2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)-4*a^3*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^4*
(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+16*c^3*d/b^2*((b*x+a)*(1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b
*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+24*c^2*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a
)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b
*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/
2*b*x+1/2*a))+16*d^3*c/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/
2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/2
*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+
a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*s
in(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+a
)*sin(b*x+a)+1/2*b*x+1/2*a))
```

maxima [A] time = 0.40, size = 244, normalized size = 1.23

$$\frac{(bx + \sin(2\,bx + 2\,a))c^4}{b} + \frac{2\left(b^2x^2 + 2\,bx\sin(2\,bx + 2\,a) + \cos(2\,bx + 2\,a)\right)c^3d}{b^2} + \frac{\left(2\,b^3x^3 + 6\,bx\cos(2\,bx + 2\,a) + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^4/b + 2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*c^3*d/b^2 + (2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*c^2*d^2/b^3 + (b^4*x^4 + 3*(2*b^2*x^2 - 1)*cos(2*b
*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*c*d^3/b^4 + 1/10*(2*b^5
*x^5 + 10*(2*b^3*x^3 - 3*b*x)*cos(2*b*x + 2*a) + 5*(2*b^4*x^4 - 6*b^2*x^2 +
 3)*sin(2*b*x + 2*a))*d^4/b^5
```

mupad [B] time = 0.65, size = 344, normalized size = 1.74

$$\frac{\frac{3\,d^4\,\sin(2\,a+2\,b\,x)}{2}+b^5\,c^4\,x+b^4\,c^4\,\sin\left(2\,a+2\,b\,x\right)+\frac{b^5\,d^4\,x^5}{5}+2\,b^3\,c^3\,d\,\cos\left(2\,a+2\,b\,x\right)+2\,b^5\,c^3\,d\,x^2+b^5\,c\,d^3\,x^4-3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^4)/sin(a + b*x),x)
```

```
[Out] ((3*d^4*sin(2*a + 2*b*x))/2 + b^5*c^4*x + b^4*c^4*sin(2*a + 2*b*x) + (b^5*d
^4*x^5)/5 + 2*b^3*c^3*d*cos(2*a + 2*b*x) + 2*b^5*c^3*d*x^2 + b^5*c*d^3*x^4
- 3*b^2*c^2*d^2*sin(2*a + 2*b*x) + 2*b^3*d^4*x^3*cos(2*a + 2*b*x) + 2*b^5*c
^2*d^2*x^3 - 3*b^2*d^4*x^2*sin(2*a + 2*b*x) + b^4*d^4*x^4*sin(2*a + 2*b*x)
- 3*b*c*d^3*cos(2*a + 2*b*x) - 3*b*d^4*x*cos(2*a + 2*b*x) + 6*b^4*c^2*d^2*x
^2*sin(2*a + 2*b*x) - 6*b^2*c*d^3*x*sin(2*a + 2*b*x) + 4*b^4*c^3*d*x*sin(2*
a + 2*b*x) + 6*b^3*c^2*d^2*x*cos(2*a + 2*b*x) + 6*b^3*c*d^3*x^2*cos(2*a + 2
*b*x) + 4*b^4*c*d^3*x^3*sin(2*a + 2*b*x))/b^5
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.369 $\int (c+dx)^3 \csc(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=171

$$\frac{3d^3\sin^2(a+bx)}{8b^4}-\frac{9d^3\cos^2(a+bx)}{8b^4}-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2}{4}$$

```
[Out] -3/2*c*d^2*x/b^2-3/4*d^3*x^2/b^2+1/4*(d*x+c)^4/d-9/8*d^3*cos(b*x+a)^2/b^4+9
/4*d*(d*x+c)^2*cos(b*x+a)^2/b^2-3*d^2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^3+2*(
d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b+3/8*d^3*sin(b*x+a)^2/b^4-3/4*d*(d*x+c)^2*s
in(b*x+a)^2/b^2
```

Rubi [A] time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {4431, 3311, 32, 3310}

$$-\frac{3d^2(c+dx)\sin(a+bx)\cos(a+bx)}{b^3}-\frac{3d(c+dx)^2\sin^2(a+bx)}{4b^2}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{3d^3\sin^2(a+bx)}{8b^4}-9$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (-3*c*d^2*x)/(2*b^2) - (3*d^3*x^2)/(4*b^2) + (c + d*x)^4/(4*d) - (9*d^3*Cos
[a + b*x]^2)/(8*b^4) + (9*d*(c + d*x)^2*Cos[a + b*x]^2)/(4*b^2) - (3*d^2*(c
 + d*x)*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^3*Cos[a + b*x]*Sin[a
+ b*x])/b + (3*d^3*Sin[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Sin[a + b*x]^
2)/(4*b^2)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^3\cos^2(a+bx)-(c+dx)^3\sin^2(a+bx)\right)\,dx\\ &= 3\int (c+dx)^3\cos^2(a+bx)\,dx-\int (c+dx)^3\sin^2(a+bx)\,dx\\ &= \frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)^3\cos(a+bx)\sin(a+bx)}{b}-\frac{3d(c\\ &= \frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2\cos^2(a+bx)}{4b^2}-\frac{3d^2(c+dx\\ &= -\frac{3cd^2x}{2b^2}-\frac{3d^3x^2}{4b^2}+\frac{(c+dx)^4}{4d}-\frac{9d^3\cos^2(a+bx)}{8b^4}+\frac{9d(c+dx)^2\cos^2(a\\ &\;\;4b^2\end{aligned}$$

Mathematica [A] time = 0.42, size = 105, normalized size = 0.61

$$\frac{2b(c+dx)\sin(2(a+bx))\left(2b^2(c+dx)^2-3d^2\right)+3d\cos(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)+b^4x\left(4c^3+6c^2dx+4cd^2\right.}{4b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 3*d*(-d^2 + 2*b^2*(c +
 d*x)^2)*Cos[2*(a + b*x)] + 2*b*(c + d*x)*(-3*d^2 + 2*b^2*(c + d*x)^2)*Sin[
2*(a + b*x)])/(4*b^4)
```

fricas [A] time = 0.45, size = 188, normalized size = 1.10

$$\frac{b^4d^3x^4+4\,b^4cd^2x^3+6\left(b^4c^2d-b^2d^3\right)x^2+6\left(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d-d^3\right)\cos\left(bx+a\right)^2+4\left(2\,b^3d^3x^3+6\,b\right.}{4\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] 1/4*(b^4*d^3*x^4 + 4*b^4*c*d^2*x^3 + 6*(b^4*c^2*d - b^2*d^3)*x^2 + 6*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 + 4*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)*sin(b*x + a) + 4*(b^4*c^3 - 3*b^2*c*d^2)*x)/b^4

giac [B] time = 4.43, size = 3139, normalized size = 18.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out] 1/4*(b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*b*x)^4*tan(1/2*a)^3 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 - 16*b^3*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^4 + 4*b^4*c^3*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + b^4*d^3*x^4*tan(1/2*b*x)^4 + 4*b^4*d^3*x^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + 12*b^4*c^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^4*d^3*x^4*tan(1/2*a)^4 + 12*b^4*c^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b^2*d^3*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4*b^4*c*d^2*x^3*tan(1/2*b*x)^4 + 16*b^3*d^3*x^3*tan(1/2*b*x)^4*tan(1/2*a) + 16*b^4*c*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 96*b^3*d^3*x^3*tan(1/2*b*x)^3*tan(1/2*a)^2 + 8*b^4*c^3*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 96*b^3*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b^3*c^2*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^4*c*d^2*x^3*tan(1/2*a)^4 + 16*b^3*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^4 + 8*b^4*c^3*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 48*b^3*c^2*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 12*b^2*c*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^4*d^3*x^4*tan(1/2*b*x)^2 + 6*b^4*c^2*d*x^2*tan(1/2*b*x)^4 + 48*b^3*c*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^4*d^3*x^4*tan(1/2*a)^2 + 24*b^4*c^2*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 288*b^3*c*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 36*b^2*d^3*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 288*b^3*c*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 96*b^2*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - 16*b^3*c^3*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^4*c^2*d*x^2*tan(1/2*a)^4 + 48*b^3*c*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 36*b^2*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 16*b^3*c^3*tan(1/2*b*x)^3*tan(1/2*a)^4 + 6*b^2*c^2*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 8*b^4*c*d^2*x^3*tan(1/2*b*x)^2 - 16*b^3*d^3*x^3*tan(1/2*b*x)^3 + 4*b^4*c^3*x*tan(1/2*b*x)^4 - 96*b^3*d^3*x^3*tan(1/2*b*x)^2*tan(1/2*a) + 48*b^3*c^2*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^4*c*d^2*x^3*tan(1/2*a)^2 - 96*b^3*d^3*x^3*tan(1/2*b*x)*tan(1/2*a)^2 + 16*b^4*c^3*x*tan

(1/2*b*x)^2*tan(1/2*a)^2 + 288*b^3*c^2*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 72
*b^2*c*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*d^3*x^3*tan(1/2*a)^3 + 28
8*b^3*c^2*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 192*b^2*c*d^2*x*tan(1/2*b*x)^3*
tan(1/2*a)^3 + 24*b*d^3*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + 4*b^4*c^3*x*tan(1/2
*a)^4 + 48*b^3*c^2*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 72*b^2*c*d^2*x*tan(1/2*b
*x)^2*tan(1/2*a)^4 + 24*b*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^4*d^3*x^4 +
 12*b^4*c^2*d*x^2*tan(1/2*b*x)^2 - 48*b^3*c*d^2*x^2*tan(1/2*b*x)^3 + 6*b^2*
d^3*x^2*tan(1/2*b*x)^4 - 288*b^3*c*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 96*b
^2*d^3*x^2*tan(1/2*b*x)^3*tan(1/2*a) + 16*b^3*c^3*tan(1/2*b*x)^4*tan(1/2*a)
 + 12*b^4*c^2*d*x^2*tan(1/2*a)^2 - 288*b^3*c*d^2*x^2*tan(1/2*b*x)*tan(1/2*a
)^2 + 216*b^2*d^3*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 96*b^3*c^3*tan(1/2*b*x)
^3*tan(1/2*a)^2 - 36*b^2*c^2*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 48*b^3*c*d^2*x
^2*tan(1/2*a)^3 + 96*b^2*d^3*x^2*tan(1/2*b*x)*tan(1/2*a)^3 + 96*b^3*c^3*tan
(1/2*b*x)^2*tan(1/2*a)^3 - 96*b^2*c^2*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 24*b*
c*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^2*d^3*x^2*tan(1/2*a)^4 + 16*b^3*c^3
*tan(1/2*b*x)*tan(1/2*a)^4 - 36*b^2*c^2*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 24*
b*c*d^2*tan(1/2*b*x)^3*tan(1/2*a)^4 - 3*d^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 4
*b^4*c*d^2*x^3 + 16*b^3*d^3*x^3*tan(1/2*b*x) + 8*b^4*c^3*x*tan(1/2*b*x)^2 -
 48*b^3*c^2*d*x*tan(1/2*b*x)^3 + 12*b^2*c*d^2*x*tan(1/2*b*x)^4 + 16*b^3*d^3
*x^3*tan(1/2*a) - 288*b^3*c^2*d*x*tan(1/2*b*x)^2*tan(1/2*a) + 192*b^2*c*d^2
*x*tan(1/2*b*x)^3*tan(1/2*a) - 24*b*d^3*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^4
*c^3*x*tan(1/2*a)^2 - 288*b^3*c^2*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 432*b^2*c
*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 - 144*b*d^3*x*tan(1/2*b*x)^3*tan(1/2*a)^
2 - 48*b^3*c^2*d*x*tan(1/2*a)^3 + 192*b^2*c*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3
 - 144*b*d^3*x*tan(1/2*b*x)^2*tan(1/2*a)^3 + 12*b^2*c*d^2*x*tan(1/2*a)^4 -
24*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^4 + 6*b^4*c^2*d*x^2 + 48*b^3*c*d^2*x^2*t
an(1/2*b*x) - 36*b^2*d^3*x^2*tan(1/2*b*x)^2 - 16*b^3*c^3*tan(1/2*b*x)^3 + 6
*b^2*c^2*d*tan(1/2*b*x)^4 + 48*b^3*c*d^2*x^2*tan(1/2*a) - 96*b^2*d^3*x^2*ta
n(1/2*b*x)*tan(1/2*a) - 96*b^3*c^3*tan(1/2*b*x)^2*tan(1/2*a) + 96*b^2*c^2*d
*tan(1/2*b*x)^3*tan(1/2*a) - 24*b*c*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 36*b^2*
d^3*x^2*tan(1/2*a)^2 - 96*b^3*c^3*tan(1/2*b*x)*tan(1/2*a)^2 + 216*b^2*c^2*d
*tan(1/2*b*x)^2*tan(1/2*a)^2 - 144*b*c*d^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 18
*d^3*tan(1/2*b*x)^4*tan(1/2*a)^2 - 16*b^3*c^3*tan(1/2*a)^3 + 96*b^2*c^2*d*t
an(1/2*b*x)*tan(1/2*a)^3 - 144*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a)^3 + 48*d^3
*tan(1/2*b*x)^3*tan(1/2*a)^3 + 6*b^2*c^2*d*tan(1/2*a)^4 - 24*b*c*d^2*tan(1/
2*b*x)*tan(1/2*a)^4 + 18*d^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + 4*b^4*c^3*x + 48
*b^3*c^2*d*x*tan(1/2*b*x) - 72*b^2*c*d^2*x*tan(1/2*b*x)^2 + 24*b*d^3*x*tan(
1/2*b*x)^3 + 48*b^3*c^2*d*x*tan(1/2*a) - 192*b^2*c*d^2*x*tan(1/2*b*x)*tan(1
/2*a) + 144*b*d^3*x*tan(1/2*b*x)^2*tan(1/2*a) - 72*b^2*c*d^2*x*tan(1/2*a)^2
 + 144*b*d^3*x*tan(1/2*b*x)*tan(1/2*a)^2 + 24*b*d^3*x*tan(1/2*a)^3 + 6*b^2*
d^3*x^2 + 16*b^3*c^3*tan(1/2*b*x) - 36*b^2*c^2*d*tan(1/2*b*x)^2 + 24*b*c*d^
2*tan(1/2*b*x)^3 - 3*d^3*tan(1/2*b*x)^4 + 16*b^3*c^3*tan(1/2*a) - 96*b^2*c^
2*d*tan(1/2*b*x)*tan(1/2*a) + 144*b*c*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 48*d^
3*tan(1/2*b*x)^3*tan(1/2*a) - 36*b^2*c^2*d*tan(1/2*a)^2 + 144*b*c*d^2*tan(1
/2*b*x)*tan(1/2*a)^2 - 108*d^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 24*b*c*d^2*tan

```
(1/2*a)^3 - 48*d^3*tan(1/2*b*x)*tan(1/2*a)^3 - 3*d^3*tan(1/2*a)^4 + 12*b^2*
c*d^2*x - 24*b*d^3*x*tan(1/2*b*x) - 24*b*d^3*x*tan(1/2*a) + 6*b^2*c^2*d - 2
4*b*c*d^2*tan(1/2*b*x) + 18*d^3*tan(1/2*b*x)^2 - 24*b*c*d^2*tan(1/2*a) + 48
*d^3*tan(1/2*b*x)*tan(1/2*a) + 18*d^3*tan(1/2*a)^2 - 3*d^3)/(b^4*tan(1/2*b*
x)^4*tan(1/2*a)^4 + 2*b^4*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^4*tan(1/2*b*x)^
2*tan(1/2*a)^4 + b^4*tan(1/2*b*x)^4 + 4*b^4*tan(1/2*b*x)^2*tan(1/2*a)^2 + b
^4*tan(1/2*a)^4 + 2*b^4*tan(1/2*b*x)^2 + 2*b^4*tan(1/2*a)^2 + b^4)
```

maple [B] time = 0.04, size = 580, normalized size = 3.39

$$-c^3x-\frac{d^3x^4}{4}+\frac{4c^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-\frac{3c^2d\,x^2}{2}-c\,d^2x^3+\frac{4d^3\left((bx+a)^3\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{3(bx\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^3*x-1/4*d^3*x^4+4*c^3/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-3/2*c^
2*d*x^2-c*d^2*x^3+4*d^3/b^4*((b*x+a)^3*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1
/2*a)+3/4*(b*x+a)^2*cos(b*x+a)^2-3/2*(b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2
*b*x+1/2*a)+3/8*(b*x+a)^2+3/8*sin(b*x+a)^2-3/8*(b*x+a)^4-3*a*((b*x+a)^2*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x
+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3)+3*a^2*((b*x+a)*(1/2*cos(b*x+a)*
sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2)-a^3*(1/2*cos(b*x+
a)*sin(b*x+a)+1/2*b*x+1/2*a))+12*c^2*d/b^2*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/2*cos(b*x+a)*sin(b*x
+a)+1/2*b*x+1/2*a))+12*c*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*
b*x+1/2*a)+1/2*(b*x+a)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a
-1/3*(b*x+a)^3-2*a*((b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(
b*x+a)^2-1/4*sin(b*x+a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

maxima [A] time = 0.37, size = 173, normalized size = 1.01

$$\frac{(bx+\sin(2\,bx+2\,a))c^3}{b}+\frac{3\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)c^2d}{2\,b^2}+\frac{\left(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^3/b + 3/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + co
s(2*b*x + 2*a))*c^2*d/b^2 + 1/2*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*
b^2*x^2 - 1)*sin(2*b*x + 2*a))*c*d^2/b^3 + 1/4*(b^4*x^4 + 3*(2*b^2*x^2 - 1)
*cos(2*b*x + 2*a) + 2*(2*b^3*x^3 - 3*b*x)*sin(2*b*x + 2*a))*d^3/b^4
```

mupad [B] time = 2.18, size = 216, normalized size = 1.26

$$c^3\,x+\frac{d^3\,x^4}{4}+\frac{3\,c^2\,d\,x^2}{2}+c\,d^2\,x^3-\frac{3\,d^3\,\cos\left(2\,a+2\,b\,x\right)}{4\,b^4}+\frac{c^3\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c^2\,d\,\cos\left(2\,a+2\,b\,x\right)}{2\,b^2}-\frac{3\,c\,d^2\,\sin\left(2\,a+\right.}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x),x)
```

```
[Out] c^3*x + (d^3*x^4)/4 + (3*c^2*d*x^2)/2 + c*d^2*x^3 - (3*d^3*cos(2*a + 2*b*x)
)/(4*b^4) + (c^3*sin(2*a + 2*b*x))/b + (3*c^2*d*cos(2*a + 2*b*x))/(2*b^2) -
 (3*c*d^2*sin(2*a + 2*b*x))/(2*b^3) - (3*d^3*x*sin(2*a + 2*b*x))/(2*b^3) +
(3*d^3*x^2*cos(2*a + 2*b*x))/(2*b^2) + (d^3*x^3*sin(2*a + 2*b*x))/b + (3*c*
d^2*x*cos(2*a + 2*b*x))/b^2 + (3*c^2*d*x*sin(2*a + 2*b*x))/b + (3*c*d^2*x^2
*sin(2*a + 2*b*x))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.370 $\int (c+dx)^2 \csc(a+bx) \sin(3a+3bx)\,dx$

Optimal. Leaf size=112

$$-\frac{d^2 \sin(a+bx)\cos(a+bx)}{b^3}-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{2(c+dx)^2 \sin(a+bx)\cos(a+b}{b}$$

```
[Out] -1/2*d^2*x/b^2+1/3*(d*x+c)^3/d+3/2*d*(d*x+c)*cos(b*x+a)^2/b^2-d^2*cos(b*x+a
)*sin(b*x+a)/b^3+2*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b-1/2*d*(d*x+c)*sin(b*x+
a)^2/b^2
```

Rubi [A] time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3311, 32, 2635, 8}

$$-\frac{d(c+dx)\sin^2(a+bx)}{2b^2}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2 \sin(a+bx)\cos(a+bx)}{b^3}+\frac{2(c+dx)^2 \sin(a+bx)\cos(a+b}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -(d^2*x)/(2*b^2) + (c + d*x)^3/(3*d) + (3*d*(c + d*x)*Cos[a + b*x]^2)/(2*b^
2) - (d^2*Cos[a + b*x]*Sin[a + b*x])/b^3 + (2*(c + d*x)^2*Cos[a + b*x]*Sin[
a + b*x])/b - (d*(c + d*x)*Sin[a + b*x]^2)/(2*b^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
```

```
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^2\csc(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)^2\cos^2(a+bx)-(c+dx)^2\sin^2(a+bx)\right)dx\\
&= 3\int (c+dx)^2\cos^2(a+bx)\,dx-\int (c+dx)^2\sin^2(a+bx)\,dx\\
&= \frac{3d(c+dx)\cos^2(a+bx)}{2b^2}+\frac{2(c+dx)^2\cos(a+bx)\sin(a+bx)}{b}-\frac{d(c+\cdots}{\cdots}\\
&= \frac{(c+dx)^3}{3d}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+bx)}{b^3}+\frac{2\cdots}{\cdots}\\
&= -\frac{d^2x}{2b^2}+\frac{(c+dx)^3}{3d}+\frac{3d(c+dx)\cos^2(a+bx)}{2b^2}-\frac{d^2\cos(a+bx)\sin(a+\cdots}{b^3}
\end{aligned}$$

Mathematica [A] time = 0.40, size = 73, normalized size = 0.65

$$\frac{d(c+dx)\cos(2(a+bx))}{b^2}+\frac{\sin(2(a+bx))\left(2b^2(c+dx)^2-d^2\right)}{2b^3}+c^2x+cdx^2+\frac{d^2x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c^2*x + c*d*x^2 + (d^2*x^3)/3 + (d*(c + d*x)*Cos[2*(a + b*x)])/b^2 + ((-d^2
 + 2*b^2*(c + d*x)^2)*Sin[2*(a + b*x)])/(2*b^3)
```

fricas [A] time = 0.45, size = 111, normalized size = 0.99

$$\frac{b^3d^2x^3+3\,b^3cdx^2+6\left(bd^2x+bcd\right)\cos\left(bx+a\right)^2+3\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-d^2\right)\cos\left(bx+a\right)\sin\left(bx+a\right)\cdots}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")

[Out] 1/3*(b^3*d^2*x^3 + 3*b^3*c*d*x^2 + 6*(b*d^2*x + b*c*d)*cos(b*x + a)^2 + 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 - d^2)*cos(b*x + a)*sin(b*x + a) + 3*(b^3*c^2 - b*d^2)*x)/b^3

giac [B] time = 5.77, size = 1880, normalized size = 16.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")

[Out] 1/3*(b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*d^2*x^3*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + 3*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 6*b^3*c*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 6*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^4 - 12*b^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^3*d^2*x^3*tan(1/2*b*x)^4 + 4*b^3*d^2*x^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + 6*b^3*c^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 24*b^2*c*d*x*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^3*d^2*x^3*tan(1/2*a)^4 + 6*b^3*c^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 24*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^4 + 3*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 3*b^3*c*d*x^2*tan(1/2*b*x)^4 + 12*b^2*d^2*x^2*tan(1/2*b*x)^4*tan(1/2*a) + 12*b^3*c*d*x^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 72*b^2*d^2*x^2*tan(1/2*b*x)^3*tan(1/2*a)^2 + 72*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 3*b^3*c*d*x^2*tan(1/2*a)^4 + 12*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^4 - 12*b^2*c^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + 3*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*d^2*x^3*tan(1/2*b*x)^2 + 3*b^3*c^2*x*tan(1/2*b*x)^4 + 24*b^2*c*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 2*b^3*d^2*x^3*tan(1/2*a)^2 + 12*b^3*c^2*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 144*b^2*c*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 - 18*b*d^2*x*tan(1/2*b*x)^4*tan(1/2*a)^2 + 144*b^2*c*d*x*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a)^3 + 3*b^3*c^2*x*tan(1/2*a)^4 + 24*b^2*c*d*x*tan(1/2*b*x)*tan(1/2*a)^4 - 18*b*d^2*x*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*b^3*c*d*x^2*tan(1/2*b*x)^2 - 12*b^2*d^2*x^2*tan(1/2*b*x)^3 - 72*b^2*d^2*x^2*tan(1/2*b*x)^2*tan(1/2*a) + 12*b^2*c^2*tan(1/2*b*x)^4*tan(1/2*a) + 6*b^3*c*d*x^2*tan(1/2*a)^2 - 72*b^2*d^2*x^2*tan(1/2*b*x)*tan(1/2*a)^2 + 72*b^2*c^2*tan(1/2*b*x)^3*tan(1/2*a)^2 - 18*b*c*d*tan(1/2*b*x)^4*tan(1/2*a)^2 - 12*b^2*d^2*x^2*tan(1/2*a)^3 + 72*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a)^3 - 48*b*c*d*tan(1/2*b*x)^3*tan(1/2*a)^3 + 6*d^2*tan(1/2*b*x)^4*tan(1/2*a)^3 + 12*b^2*c^2*tan(1/2*b*x)*tan(1/2*a)^4 - 18*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^4 + 6*d^2*tan(1/2*b*x)^3*tan(1/2*a)^4 + b^3*d^2*x^3 + 6*b^3*c^2*x*tan(1/2*b*x)^2 - 24*b^2*c*d*x*tan(1/2*b*x)^3 + 3*b*d^2*x*tan(1/2*b*x)^4 - 144*b^2*c*d*x*tan(1/2*b*x)^2*ta

```
n(1/2*a) + 48*b*d^2*x*tan(1/2*b*x)^3*tan(1/2*a) + 6*b^3*c^2*x*tan(1/2*a)^2
- 144*b^2*c*d*x*tan(1/2*b*x)*tan(1/2*a)^2 + 108*b*d^2*x*tan(1/2*b*x)^2*tan(
1/2*a)^2 - 24*b^2*c*d*x*tan(1/2*a)^3 + 48*b*d^2*x*tan(1/2*b*x)*tan(1/2*a)^3
 + 3*b*d^2*x*tan(1/2*a)^4 + 3*b^3*c*d*x^2 + 12*b^2*d^2*x^2*tan(1/2*b*x) - 1
2*b^2*c^2*tan(1/2*b*x)^3 + 3*b*c*d*tan(1/2*b*x)^4 + 12*b^2*d^2*x^2*tan(1/2*
a) - 72*b^2*c^2*tan(1/2*b*x)^2*tan(1/2*a) + 48*b*c*d*tan(1/2*b*x)^3*tan(1/2
*a) - 6*d^2*tan(1/2*b*x)^4*tan(1/2*a) - 72*b^2*c^2*tan(1/2*b*x)*tan(1/2*a)^
2 + 108*b*c*d*tan(1/2*b*x)^2*tan(1/2*a)^2 - 36*d^2*tan(1/2*b*x)^3*tan(1/2*a
)^2 - 12*b^2*c^2*tan(1/2*a)^3 + 48*b*c*d*tan(1/2*b*x)*tan(1/2*a)^3 - 36*d^2
*tan(1/2*b*x)^2*tan(1/2*a)^3 + 3*b*c*d*tan(1/2*a)^4 - 6*d^2*tan(1/2*b*x)*ta
n(1/2*a)^4 + 3*b^3*c^2*x + 24*b^2*c*d*x*tan(1/2*b*x) - 18*b*d^2*x*tan(1/2*b
*x)^2 + 24*b^2*c*d*x*tan(1/2*a) - 48*b*d^2*x*tan(1/2*b*x)*tan(1/2*a) - 18*b
*d^2*x*tan(1/2*a)^2 + 12*b^2*c^2*tan(1/2*b*x) - 18*b*c*d*tan(1/2*b*x)^2 + 6
*d^2*tan(1/2*b*x)^3 + 12*b^2*c^2*tan(1/2*a) - 48*b*c*d*tan(1/2*b*x)*tan(1/2
*a) + 36*d^2*tan(1/2*b*x)^2*tan(1/2*a) - 18*b*c*d*tan(1/2*a)^2 + 36*d^2*tan
(1/2*b*x)*tan(1/2*a)^2 + 6*d^2*tan(1/2*a)^3 + 3*b*d^2*x + 3*b*c*d - 6*d^2*t
an(1/2*b*x) - 6*d^2*tan(1/2*a))/(b^3*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^3*ta
n(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^3*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^3*tan(1/2
*b*x)^4 + 4*b^3*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^3*tan(1/2*a)^4 + 2*b^3*tan(
1/2*b*x)^2 + 2*b^3*tan(1/2*a)^2 + b^3)
```

maple [B] time = 0.03, size = 294, normalized size = 2.62

$$-c^2x-\frac{d^2x^3}{3}+\frac{4c^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}-cd\,x^2+\frac{4d^2\left((bx+a)^2\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)+\frac{(bx+a)\left(\cos^2(bx+a)\right)}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c^2*x-1/3*d^2*x^3+4*c^2/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-c*d*x^
2+4*d^2/b^3*((b*x+a)^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+1/2*(b*x+a
)*cos(b*x+a)^2-1/4*cos(b*x+a)*sin(b*x+a)-1/4*b*x-1/4*a-1/3*(b*x+a)^3-2*a*((
b*x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+
a)^2)+a^2*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))+8*c*d/b^2*((b*x+a)*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)^2-a*(1/
2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

maxima [A] time = 0.35, size = 108, normalized size = 0.96

$$\frac{(bx+\sin(2\,bx+2\,a))c^2}{b}+\frac{\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)cd}{b^2}+\frac{\left(2\,b^3x^3+6\,bx\cos(2\,bx+2\,a)+3\,\left(\right.\right.}{6\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c^2/b + (b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(2*
b*x + 2*a))*c*d/b^2 + 1/6*(2*b^3*x^3 + 6*b*x*cos(2*b*x + 2*a) + 3*(2*b^2*x^
2 - 1)*sin(2*b*x + 2*a))*d^2/b^3
```

mupad [B] time = 0.31, size = 121, normalized size = 1.08

$$c^2\,x+\frac{d^2\,x^3}{3}+\frac{c^2\,\sin\left(2\,a+2\,b\,x\right)}{b}-\frac{d^2\,\sin\left(2\,a+2\,b\,x\right)}{2\,b^3}+c\,d\,x^2+\frac{d^2\,x\,\cos\left(2\,a+2\,b\,x\right)}{b^2}+\frac{d^2\,x^2\,\sin\left(2\,a+2\,b\,x\right)}{b}+\frac{c\,d\,\mathrm{c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x),x)
```

```
[Out] c^2*x + (d^2*x^3)/3 + (c^2*sin(2*a + 2*b*x))/b - (d^2*sin(2*a + 2*b*x))/(2*
b^3) + c*d*x^2 + (d^2*x*cos(2*a + 2*b*x))/b^2 + (d^2*x^2*sin(2*a + 2*b*x))/
b + (c*d*cos(2*a + 2*b*x))/b^2 + (2*c*d*x*sin(2*a + 2*b*x))/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.371 $\int (c + dx) \csc(a + bx) \sin(3a + 3bx) \, dx$

Optimal. Leaf size=66

$$-\frac{d \sin^2(a + bx)}{4b^2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \sin(a + bx) \cos(a + bx)}{b} + cx + \frac{dx^2}{2}$$

```
[Out] c*x+1/2*d*x^2+3/4*d*cos(b*x+a)^2/b^2+2*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b-1/4*
d*sin(b*x+a)^2/b^2
```

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} =$ 0.095, Rules used = {4431, 3310}

$$-\frac{d \sin^2(a + bx)}{4b^2} + \frac{3d \cos^2(a + bx)}{4b^2} + \frac{2(c + dx) \sin(a + bx) \cos(a + bx)}{b} + cx + \frac{dx^2}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] c*x + (d*x^2)/2 + (3*d*Cos[a + b*x]^2)/(4*b^2) + (2*(c + d*x)*Cos[a + b*x]*
Sin[a + b*x])/b - (d*Sin[a + b*x]^2)/(4*b^2)
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\csc(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)\cos^2(a+bx)-(c+dx)\sin^2(a+bx)\right)dx$$
$$= 3\int (c+dx)\cos^2(a+bx)\,dx-\int (c+dx)\sin^2(a+bx)\,dx$$
$$= \frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b}-\frac{d\sin^2(a+bx)}{4b^2}$$
$$= cx+\frac{dx^2}{2}+\frac{3d\cos^2(a+bx)}{4b^2}+\frac{2(c+dx)\cos(a+bx)\sin(a+bx)}{b}-\frac{d\,\text{s}}{}$$

Mathematica [A] time = 0.14, size = 46, normalized size = 0.70

$$\frac{b(2(c+dx)\sin(2(a+bx))+bx(2c+dx))+d\cos(2(a+bx))}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Cos[2*(a + b*x)] + b*(b*x*(2*c + d*x) + 2*(c + d*x)*Sin[2*(a + b*x)]))/(
2*b^2)
```

fricas [A] time = 0.44, size = 54, normalized size = 0.82

$$\frac{b^2dx^2+2\,b^2cx+2\,d\cos\left(bx+a\right)^2+4\left(bdx+bc\right)\cos\left(bx+a\right)\sin\left(bx+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*d*x^2 + 2*b^2*c*x + 2*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(b*x +
 a)*sin(b*x + a))/b^2
```

giac [B] time = 1.99, size = 920, normalized size = 13.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*c*x*tan(1/2*b*x)^4*tan(1
/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*b^2*d*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^4*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*
```

```
b*x)^4*tan(1/2*a)^3 + 4*b^2*c*x*tan(1/2*b*x)^2*tan(1/2*a)^4 - 8*b*d*x*tan(1
/2*b*x)^3*tan(1/2*a)^4 + b^2*d*x^2*tan(1/2*b*x)^4 + 4*b^2*d*x^2*tan(1/2*b*x
)^2*tan(1/2*a)^2 - 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)^3 + b^2*d*x^2*tan(1/2*a)
^4 - 8*b*c*tan(1/2*b*x)^3*tan(1/2*a)^4 + d*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*
b^2*c*x*tan(1/2*b*x)^4 + 8*b*d*x*tan(1/2*b*x)^4*tan(1/2*a) + 8*b^2*c*x*tan(
1/2*b*x)^2*tan(1/2*a)^2 + 48*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^2 + 48*b*d*x*t
an(1/2*b*x)^2*tan(1/2*a)^3 + 2*b^2*c*x*tan(1/2*a)^4 + 8*b*d*x*tan(1/2*b*x)*
tan(1/2*a)^4 + 2*b^2*d*x^2*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*b*x)^4*tan(1/2*a)
 + 2*b^2*d*x^2*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^3*tan(1/2*a)^2 - 6*d*tan(
1/2*b*x)^4*tan(1/2*a)^2 + 48*b*c*tan(1/2*b*x)^2*tan(1/2*a)^3 - 16*d*tan(1/2
*b*x)^3*tan(1/2*a)^3 + 8*b*c*tan(1/2*b*x)*tan(1/2*a)^4 - 6*d*tan(1/2*b*x)^2
*tan(1/2*a)^4 + 4*b^2*c*x*tan(1/2*b*x)^2 - 8*b*d*x*tan(1/2*b*x)^3 - 48*b*d*
x*tan(1/2*b*x)^2*tan(1/2*a) + 4*b^2*c*x*tan(1/2*a)^2 - 48*b*d*x*tan(1/2*b*x
)*tan(1/2*a)^2 - 8*b*d*x*tan(1/2*a)^3 + b^2*d*x^2 - 8*b*c*tan(1/2*b*x)^3 +
d*tan(1/2*b*x)^4 - 48*b*c*tan(1/2*b*x)^2*tan(1/2*a) + 16*d*tan(1/2*b*x)^3*t
an(1/2*a) - 48*b*c*tan(1/2*b*x)*tan(1/2*a)^2 + 36*d*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 8*b*c*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)*tan(1/2*a)^3 + d*tan(1/2*a)^4
 + 2*b^2*c*x + 8*b*d*x*tan(1/2*b*x) + 8*b*d*x*tan(1/2*a) + 8*b*c*tan(1/2*b*
x) - 6*d*tan(1/2*b*x)^2 + 8*b*c*tan(1/2*a) - 16*d*tan(1/2*b*x)*tan(1/2*a) -
 6*d*tan(1/2*a)^2 + d)/(b^2*tan(1/2*b*x)^4*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x
)^4*tan(1/2*a)^2 + 2*b^2*tan(1/2*b*x)^2*tan(1/2*a)^4 + b^2*tan(1/2*b*x)^4 +
 4*b^2*tan(1/2*b*x)^2*tan(1/2*a)^2 + b^2*tan(1/2*a)^4 + 2*b^2*tan(1/2*b*x)^
2 + 2*b^2*tan(1/2*a)^2 + b^2)
```

maple [A] time = 0.03, size = 119, normalized size = 1.80

$$-cx-\frac{d\,x^2}{2}+\frac{4c\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+\frac{4d\left((bx+a)\left(\frac{\cos(bx+a)\sin(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)-\frac{(bx+a)^2}{4}-\frac{\left(\sin^2(bx+a)\right)}{4}-a\left(\cos\cdots\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -c*x-1/2*d*x^2+4*c/b*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)+4*d/b^2*((b*
x+a)*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a)-1/4*(b*x+a)^2-1/4*sin(b*x+a)
^2-a*(1/2*cos(b*x+a)*sin(b*x+a)+1/2*b*x+1/2*a))
```

maxima [A] time = 0.34, size = 55, normalized size = 0.83

$$\frac{(bx+\sin(2\,bx+2\,a))c}{b}+\frac{\left(b^2x^2+2\,bx\sin(2\,bx+2\,a)+\cos(2\,bx+2\,a)\right)d}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] (b*x + sin(2*b*x + 2*a))*c/b + 1/2*(b^2*x^2 + 2*b*x*sin(2*b*x + 2*a) + cos(
2*b*x + 2*a))*d/b^2
```

mupad [B] time = 0.21, size = 53, normalized size = 0.80

$$c\,x+\frac{d\,x^2}{2}+\frac{\frac{d\,\cos\left(2\,a+2\,b\,x\right)}{2}+b\,\left(c\,\sin\left(2\,a+2\,b\,x\right)+d\,x\,\sin\left(2\,a+2\,b\,x\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x),x)
```

```
[Out] c*x + (d*x^2)/2 + ((d*cos(2*a + 2*b*x))/2 + b*(c*sin(2*a + 2*b*x) + d*x*sin
(2*a + 2*b*x)))/b^2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\sin(3a+3bx)\csc(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Integral((c + d*x)*sin(3*a + 3*b*x)*csc(a + b*x), x)
```

3.372 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=71

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

```
[Out] 2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d+ln(d*x+c)/d-2*Si(2*b*c/d+2*b*x)*sin(
2*a-2*b*c/d)/d
```

Rubi [A] time = 0.28, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {4431, 3312, 3303, 3299, 3302}

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d}-\frac{2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d + Log[c + d*x]/d
- (2*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos^2(a+bx)}{c+dx} - \frac{\sin^2(a+bx)}{c+dx}\right) dx \\
&= 3\int \frac{\cos^2(a+bx)}{c+dx}\,dx - \int \frac{\sin^2(a+bx)}{c+dx}\,dx \\
&= 3\int \left(\frac{1}{2(c+dx)} + \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx - \int \left(\frac{1}{2(c+dx)} - \frac{\cos(2a+2bx)}{2(c+dx)}\right) dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx + \frac{3}{2}\int \frac{\cos(2a+2bx)}{c+dx}\,dx \\
&= \frac{\log(c+dx)}{d} + \frac{1}{2}\cos\left(2a-\frac{2bc}{d}\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right) \right. \\
&= \frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d} + \frac{\log(c+dx)}{d} - \frac{2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2b\right.}{d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 63, normalized size = 0.89

$$\frac{2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) - 2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] - 2*S
in[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d
```

fricas [A] time = 0.46, size = 85, normalized size = 1.20

$$\frac{\left(\operatorname{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\operatorname{Ci}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-2\,\sin\left(-\frac{2\,(bc-ad)}{d}\right)\operatorname{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] ((cos_integral(2*(b*d*x + b*c)/d) + cos_integral(-2*(b*d*x + b*c)/d))*cos(-
2*(b*c - a*d)/d) - 2*sin(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d)
+ log(d*x + c))/d
```

giac [C] time = 0.32, size = 1118, normalized size = 15.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] (log(abs(d*x + c))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 2*imag_part(cos_integral(2*b*x + 2*b*c/
d))*tan(1/2*a)^4*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*t
an(1/2*a)^4*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan
(b*c/d) - 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d
)^2 + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2
 - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + log(abs(d*
x + c))*tan(1/2*a)^4 + real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4 + real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 8*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 8*real_part(cos_inte
gral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 2*log(abs(d*x + c))*tan(1
/2*a)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a
)^2*tan(b*c/d)^2 + 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d)^2 + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 - 4
*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 + 8*sin_integral(2*
(b*d*x + b*c)/d)*tan(1/2*a)^3 - 12*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(1/2*a)^2*tan(b*c/d) + 12*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(1/2*a)^2*tan(b*c/d) - 24*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(
b*c/d) + 4*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2
 - 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 + 2*log(abs(d*x + c
))*tan(1/2*a)^2 - 6*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 -
 6*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 8*real_part(cos
```

```
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 8*real_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + log(abs(d*x + c))*tan(b*c/d)^2
 - real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 - real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 4*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(1/2*a) + 4*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a
) - 8*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) + 2*imag_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*c/d) - 2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d) + 4*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + log(abs(d*x +
c)) + real_part(cos_integral(2*b*x + 2*b*c/d)) + real_part(cos_integral(-2*
b*x - 2*b*c/d)))/(d*tan(1/2*a)^4*tan(b*c/d)^2 + d*tan(1/2*a)^4 + 2*d*tan(1/
2*a)^2*tan(b*c/d)^2 + 2*d*tan(1/2*a)^2 + d*tan(b*c/d)^2 + d)
```

maple [A] time = 0.04, size = 116, normalized size = 1.63

$$-\frac{\ln(dx+c)}{d}+\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{2\ln((bx+a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] -ln(d*x+c)/d+2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d+2*Ci(2*b*
x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d+2*ln((b*x+a)*d-d*a+c*b)/d
```

maxima [C] time = 0.41, size = 117, normalized size = 1.65

$$-\frac{\left(E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(i\,E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)-\mathrm{l}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(1, -(2*I*b*d
*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(1, (2*I*b*d*x +
 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) - log(d*x + c))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

3.373 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

```
[Out] -3*cos(b*x+a)^2/d/(d*x+c)-4*b*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^2-4*b*Ci
(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^2+sin(b*x+a)^2/d/(d*x+c)
```

Rubi [A] time = 0.28, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4431, 3313, 12, 3303, 3299, 3302}

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{\sin^2(a+bx)}{d(c+dx)}-\frac{3\cos^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(d*(c + d*x)) - (4*b*CosIntegral[(2*b*c)/d + 2*b*x]*Sin
[2*a - (2*b*c)/d])/d^2 + Sin[a + b*x]^2/(d*(c + d*x)) - (4*b*Cos[2*a - (2*b
*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)}{(c+dx)^2}\right) dx$$

$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^2}\,dx$$

$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{(2b)\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d} + \frac{(6b)\int -\frac{\sin(2a+2bx)}{2(c+dx)}\,dx}{d}$$

$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{b\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d} - \frac{(3b)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d}$$

$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{\left(b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} - \frac{\left(3b\cos\right.}{}$$

$$= -\frac{3\cos^2(a+bx)}{d(c+dx)} - \frac{4b\,\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{\sin^2(a+bx)}{d(c+dx)} - \frac{4b\cos\left(2\right.}{}$$

Mathematica [A] time = 0.53, size = 81, normalized size = 0.79

$$-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2b(c+dx)}{d}\right)+4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2b(c+dx)}{d}\right)+\frac{d(2\cos(2(a+bx))+1)}{c+dx}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] -(((d*(1 + 2*Cos[2*(a + b*x)]))/(c + d*x) + 4*b*CosIntegral[(2*b*(c + d*x))
/d]*Sin[2*a - (2*b*c)/d] + 4*b*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d
*x))/d])/d^2)
```

fricas [A] time = 0.45, size = 131, normalized size = 1.28

$$-\frac{4\,d\cos\left(bx+a\right)^2+4\left(bdx+bc\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)\mathrm{Si}\left(\frac{2\,(bdx+bc)}{d}\right)+2\left(\left(bdx+bc\right)\mathrm{Ci}\left(\frac{2\,(bdx+bc)}{d}\right)+\left(bdx+bc\right)\mathrm{Ci}\left(-\frac{2}{}\right.\right.}{d^3x+cd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -(4*d*cos(b*x + a)^2 + 4*(b*d*x + b*c)*cos(-2*(b*c - a*d)/d)*sin_integral(2
*(b*d*x + b*c)/d) + 2*((b*d*x + b*c)*cos_integral(2*(b*d*x + b*c)/d) + (b*d
*x + b*c)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d) - d)/(d^3
*x + c*d^2)
```

giac [C] time = 10.37, size = 5381, normalized size = 52.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] (2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*t
an(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*
tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x
)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 4*b*d*x*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 8*b*d*x*real_part(co
s_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*d*x
*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/
d)^2 + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^4*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x
```

```
)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*
c/d))*tan(b*x)^2*tan(1/2*a)^4 + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b
*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*ta
n(b*x)^2*tan(1/2*a)^4 + 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*t
an(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 16*b*d*x*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 4*b*c*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 4*b*c*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 12*b
*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b
*c/d)^2 + 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^2*tan(b*c/d)^2 - 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^
2*tan(1/2*a)^2*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(-2*b*
x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 2*b*d*x*imag_part(cos_
integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - 2*b*d*x*imag_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*d*x*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 8*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 2*b*c*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 2*b*c*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b*c*sin_integral(2*(b*d*
x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^4 - 24*b*d*x*real_part(cos_integral(2*b*x
 + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 24*b*d*x*real_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 16*b*c*imag_
part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 16
*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(
b*c/d) + 32*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan
(b*c/d) + 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan
(b*c/d) + 4*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*ta
n(b*c/d) + 8*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 12*b*c*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 12*b*c*imag_part(cos_integral
(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b*c*sin_integ
ral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 8*b*d*x*real_
part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*d*x*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 2*b*c*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 - 2*b*c*im
ag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b*c*s
in_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d)^2 + d*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 + 12*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(1/2*a)^2 - 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(1/2*a)^2 + 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*
x)^2*tan(1/2*a)^2 + 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)
```

^2*tan(1/2*a)^3 + 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 + 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4 - 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 16*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 24*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 24*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 16*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d) + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 + 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 12*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 8*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a) - 8*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 12*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 12*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2 + 8*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 8*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 - 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 + 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4 + d*tan(b*x)^2*tan(1/2*a)^4 + 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 4*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 16*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 16*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 24*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 24*b*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 16*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 16*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d) + 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integra

```
l(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b*c*sin_integral(2*(b*d*x
+ b*c)/d)*tan(b*x)^2*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*b*d*x*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 12*b*c*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 12*b*c*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b*c*sin_integral(2*(b*d*x + b*c)
/d)*tan(1/2*a)^2*tan(b*c/d)^2 - 14*d*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 -
 16*d*tan(b*x)*tan(1/2*a)^3*tan(b*c/d)^2 - 3*d*tan(1/2*a)^4*tan(b*c/d)^2 -
2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b*d*x*imag_
part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 - 4*b*d*x*sin_integral(2*(b
*d*x + b*c)/d)*tan(b*x)^2 - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*
tan(b*x)^2*tan(1/2*a) - 8*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan
(b*x)^2*tan(1/2*a) + 12*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(
1/2*a)^2 - 12*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
+ 24*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2 + 8*b*c*real_part(c
os_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 8*b*c*real_part(cos_integral(-
2*b*x - 2*b*c/d))*tan(1/2*a)^3 + 4*b*c*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) - 16*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))
*tan(1/2*a)*tan(b*c/d) + 16*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(1/2*a)*tan(b*c/d) - 32*b*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a
)*tan(b*c/d) - 24*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d) - 24*b*c*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2
*tan(b*c/d) + 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2
 - 2*b*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 4*b*d*x
*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + 8*b*c*real_part(cos_integra
l(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 8*b*c*real_part(cos_integral(
-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(b*x)^2 + 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2 - 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2 - 8*b*d*x*
real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 8*b*d*x*real_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 12*b*c*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(1/2*a)^2 - 12*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(1/2*a)^2 + 24*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2 - 14
*d*tan(b*x)^2*tan(1/2*a)^2 - 16*d*tan(b*x)*tan(1/2*a)^3 - 3*d*tan(1/2*a)^4
+ 4*b*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*b*d*x*rea
l_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 16*b*c*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 16*b*c*imag_part(cos_integ
ral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b*c*sin_integral(2*(b*d*x
 + b*c)/d)*tan(1/2*a)*tan(b*c/d) + 2*b*c*imag_part(cos_integral(2*b*x + 2*b
*c/d))*tan(b*c/d)^2 - 2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b
*c/d)^2 + 4*b*c*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d)^2 + d*tan(b*x)^2
*tan(b*c/d)^2 + 16*d*tan(b*x)*tan(1/2*a)*tan(b*c/d)^2 + 10*d*tan(1/2*a)^2*t
an(b*c/d)^2 - 2*b*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)) + 2*b*d*x*im
ag_part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b*d*x*sin_integral(2*(b*d*x + b
```

```
*c)/d) - 8*b*c*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 8*b*c*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 4*b*c*real_part(cos_
integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 4*b*c*real_part(cos_integral(-2*b*x
 - 2*b*c/d))*tan(b*c/d) - 2*b*c*imag_part(cos_integral(2*b*x + 2*b*c/d)) +
2*b*c*imag_part(cos_integral(-2*b*x - 2*b*c/d)) - 4*b*c*sin_integral(2*(b*d
*x + b*c)/d) + d*tan(b*x)^2 + 16*d*tan(b*x)*tan(1/2*a) + 10*d*tan(1/2*a)^2
- 3*d*tan(b*c/d)^2 - 3*d)/(d^3*x*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + c*d
^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + d^3*x*tan(b*x)^2*tan(1/2*a)^4 + 2
*d^3*x*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^3*x*tan(1/2*a)^4*tan(b*c/d)
^2 + c*d^2*tan(b*x)^2*tan(1/2*a)^4 + 2*c*d^2*tan(b*x)^2*tan(1/2*a)^2*tan(b*
c/d)^2 + c*d^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*d^3*x*tan(b*x)^2*tan(1/2*a)^2
+ d^3*x*tan(1/2*a)^4 + d^3*x*tan(b*x)^2*tan(b*c/d)^2 + 2*d^3*x*tan(1/2*a)^2
*tan(b*c/d)^2 + 2*c*d^2*tan(b*x)^2*tan(1/2*a)^2 + c*d^2*tan(1/2*a)^4 + c*d^
2*tan(b*x)^2*tan(b*c/d)^2 + 2*c*d^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^3*x*tan(b
*x)^2 + 2*d^3*x*tan(1/2*a)^2 + d^3*x*tan(b*c/d)^2 + c*d^2*tan(b*x)^2 + 2*c*
d^2*tan(1/2*a)^2 + c*d^2*tan(b*c/d)^2 + d^3*x + c*d^2)
```

maple [A] time = 0.04, size = 169, normalized size = 1.66

$$\frac{1}{d\left(dx+c\right)}+\frac{b^{2}\left(-\frac{2\cos\left(2bx+2a\right)}{\left(\left(bx+a\right)d-da+cb\right)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}\right)}{d}\right)-\frac{2b^{2}}{\left(\left(bx+a\right)d-da+cb\right)d}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] 1/d/(d*x+c)+4/b*(1/4*b^2*(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d-2*(2*Si(2
*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/
d)*sin(2*(-a*d+b*c)/d)/d)/d)-1/2*b^2/((b*x+a)*d-d*a+c*b)/d)
```

maxima [C] time = 0.42, size = 118, normalized size = 1.16

$$-\frac{\left(E_{2}\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_{2}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\left(bc-ad\right)}{d}\right)-\left(i\,E_{2}\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_{2}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\left(bc-ad\right)}{d}\right)+1}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(2, -(2*I*b*d
*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (I*exp_integral_e(2, (2*I*b*d*x +
 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c -
 a*d)/d) + 1)/(d^2*x + c*d)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3a+3bx)}{\sin(a+bx)\,(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2),x)
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
[Out] Timed out
```

3.374 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=136

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\sin^2(a+bx)}{2d(c+dx)^2}$$

```
[Out] -4*b^2*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^3-3/2*cos(b*x+a)^2/d/(d*x+c)^2+
4*b^2*Si(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3+4*b*cos(b*x+a)*sin(b*x+a)/d^2/
(d*x+c)+1/2*sin(b*x+a)^2/d/(d*x+c)^2
```

Rubi [A] time = 0.37, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4431, 3314, 31, 3312, 3303, 3299, 3302}

$$-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{4b\sin(a+bx)\cos(a+bx)}{d^2(c+dx)}+\frac{\text{si}}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-3*Cos[a + b*x]^2)/(2*d*(c + d*x)^2) - (4*b^2*Cos[2*a - (2*b*c)/d]*CosInte
gral[(2*b*c)/d + 2*b*x])/d^3 + (4*b*Cos[a + b*x]*Sin[a + b*x])/(d^2*(c + d*
x)) + Sin[a + b*x]^2/(2*d*(c + d*x)^2) + (4*b^2*Sin[2*a - (2*b*c)/d]*SinInt
egral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3312

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)}{(c+dx)^3}\right)dx$$
$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^3}\,dx$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{1}{c+dx}\,dx}{d^2}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{2b^2\log(c+dx)}{d^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+}{2d(c+d}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{b^2\int \frac{\cos(2a+2b}{c+dx}}{d^2}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} + \frac{4b\cos(a+bx)\sin(a+bx)}{d^2(c+dx)} + \frac{\sin^2(a+bx)}{2d(c+dx)^2} - \frac{\left(b^2\cos\left(2a-\right.\right.}{}$$
$$= -\frac{3\cos^2(a+bx)}{2d(c+dx)^2} - \frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3} + \frac{4b\cos(a+bx)\sin(a}{d^2(c+dx)}$$

Mathematica [A] time = 0.99, size = 104, normalized size = 0.76

$$-\frac{8b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) - 8b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d(-4b(c+dx)\sin(2(a+bx))+2d\cos(2(a+bx))+d)}{(c+dx)^2}}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*(8*b^2*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*(c + d*x))/d] + (d*(d + 2
*d*Cos[2*(a + b*x)] - 4*b*(c + d*x)*Sin[2*(a + b*x)]))/(c + d*x)^2 - 8*b^2*
Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d])/d^3
```

fricas [A] time = 0.46, size = 225, normalized size = 1.65

$$-\frac{4\,d^2\cos\left(bx+a\right)^2 - 8\left(bd^2x+bcd\right)\cos\left(bx+a\right)\sin\left(bx+a\right) - 8\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)\text{Si}\left(\frac{2}{}\right.}{2\left(d^5x^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*d^2*cos(b*x + a)^2 - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a)
- 8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*sin(-2*(b*c - a*d)/d)*sin_integra
l(2*(b*d*x + b*c)/d) - d^2 + 4*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_i
ntegral(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*cos_inte
gral(-2*(b*d*x + b*c)/d))*cos(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2
*d^3)
```

giac [C] time = 10.03, size = 9416, normalized size = 69.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/2*(4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)^4*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 8*b^2*d^2*x^2*imag_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*d^2*x^2*
imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d
) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^4*
tan(b*c/d) + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*
x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b
*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_in
tegral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b^2*c*d*
x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/
d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan
(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 32*b^2*d^2*x^2*real_part(cos_integral
(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*d^2*x^2*real
_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) -
16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^4*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/
d)*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 24*b^2*d^2*x^2*real_part(cos_integr
al(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*d^2*x^2*
real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d
)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(
1/2*a)^3*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 64*b^2*c*d*x*sin_integral(2*(b*d
*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part
(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*d^2*x^2*r
eal_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*
```

c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 + 48*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) - 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d) - 8*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 + 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3 - 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^3 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^4 - 32*b^2*d^2*x^2*real_part(cos_inte

gral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b^2*d^2*x^2*re
al_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) +
96*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)
^2*tan(b*c/d) - 96*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(
b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 192*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)
/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 32*b^2*d^2*x^2*real_part(cos_integ
ral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*d^2*x^2*real_part(co
s_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b^2*c^2*real_par
t(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d) + 32*b^
2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^3*tan
(b*c/d) - 16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^
4*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1
/2*a)^4*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a
)^4*tan(b*c/d) + 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(b*c/d)^2 + 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/
d))*tan(b*x)^2*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 32*b^2*c*d*x*imag_part(cos_in
tegral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 64*b^2*c*d*x
sin_integral(2(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 24*b^
2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^
2 - 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*t
an(b*c/d)^2 - 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 32*b^2*c*d*x*imag_part(cos_i
ntegral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 32*b^2*c*d*x*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 64*b^2*c*d*x*
sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b*d^2*x*tan(
b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(2*b*x +
 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d)^2 + 8*b*d^2*x*tan(b*x)*tan(1/2*a)^4
*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan
(b*x)^2*tan(1/2*a) - 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d
))*tan(b*x)^2*tan(1/2*a) + 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(b*x)^2*tan(1/2*a) + 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*
b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b
*x + 2*b*c/d))*tan(1/2*a)^3 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x
- 2*b*c/d))*tan(1/2*a)^3 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*t
an(1/2*a)^3 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^
2*tan(1/2*a)^3 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b
*x)^2*tan(1/2*a)^3 - 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*
tan(1/2*a)^3 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2
*a)^4 - 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4
- 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c
/d) + 8*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*ta

n(b*c/d) - 16*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 48*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 48*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 64*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) - 8*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^4*tan(b*c/d) + 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 16*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*d^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d)^2 + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3*tan(b*c/d)^2 + 16*b*c*d*tan(b*x)^2*tan(1/2*a)^3*tan(b*c/d)^2 + 8*b*c*d*tan(b*x)*tan(1/2*a)^4*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 4*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a) - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(1/2*a) + 24*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2 + 24*b^2*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 + 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 + 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)^2 - 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3 + 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 - 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 + 16*b*d^2*x*tan(b*x)^2*tan(1/2*a)^3 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^4 - 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^4 + 8*b*d^2*x*t

an(b*x)*tan(1/2*a)^4 - 16*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d)
)*tan(b*x)^2*tan(b*c/d) + 16*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*
c/d))*tan(b*x)^2*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*
tan(b*x)^2*tan(b*c/d) - 32*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*d^2*x^2*real_part(cos_integral(-2*b*x -
 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(2*b*x
+ 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(1/2*a)*tan(b*c/d) + 96*b^2*c*d*x*im
ag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 96*b^2*c*d
*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 192*
b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 32*b^2*
c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) + 32*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3*tan(b*c/d) +
4*b^2*d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 4*b^2
*d^2*x^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 + 4*b^2*c^2
*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 + 4*b^2*c
^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d)^2 - 32*b
^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 +
 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/
d)^2 - 64*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2
 - 16*b*d^2*x*tan(b*x)^2*tan(1/2*a)*tan(b*c/d)^2 - 24*b^2*c^2*real_part(cos
_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 24*b^2*c^2*real_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d)^2 - 48*b*d^2*x*ta
n(b*x)*tan(1/2*a)^2*tan(b*c/d)^2 - 16*b*d^2*x*tan(1/2*a)^3*tan(b*c/d)^2 - 8
*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*x)^2 - 8*b^2*c*d*
x*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2 + 16*b^2*d^2*x^2*ima
g_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*a) - 16*b^2*d^2*x^2*imag_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 32*b^2*d^2*x^2*sin_integral(2
*(b*d*x + b*c)/d)*tan(1/2*a) + 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*
b*c/d))*tan(b*x)^2*tan(1/2*a) - 16*b^2*c^2*imag_part(cos_integral(-2*b*x -
2*b*c/d))*tan(b*x)^2*tan(1/2*a) + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d
)*tan(b*x)^2*tan(1/2*a) + 48*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c
/d))*tan(1/2*a)^2 + 48*b^2*c*d*x*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(1/2*a)^2 - 16*b^2*c^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*
a)^3 + 16*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^3 -
32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a)^3 + 16*b*c*d*tan(b*x)
^2*tan(1/2*a)^3 + 8*b*c*d*tan(b*x)*tan(1/2*a)^4 + d^2*tan(b*x)^2*tan(1/2*a)
^4 - 8*b^2*d^2*x^2*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 8*
b^2*d^2*x^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 16*b^2*d
^2*x^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) - 8*b^2*c^2*imag_part(cos
_integral(2*b*x + 2*b*c/d))*tan(b*x)^2*tan(b*c/d) + 8*b^2*c^2*imag_part(cos
_integral(-2*b*x - 2*b*c/d))*tan(b*x)^2*tan(b*c/d) - 16*b^2*c^2*sin_integra
l(2*(b*d*x + b*c)/d)*tan(b*x)^2*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_int
egral(2*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d) - 64*b^2*c*d*x*real_part(cos_
integral(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d) + 48*b^2*c^2*imag_part(co

s_integral(2*b*x + 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) - 48*b^2*c^2*imag_part
(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2*tan(b*c/d) + 96*b^2*c^2*sin_i
ntegral(2*(b*d*x + b*c)/d)*tan(1/2*a)^2*tan(b*c/d) + 8*b^2*c*d*x*real_part(
cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d)^2 + 8*b^2*c*d*x*real_part(cos_int
egral(-2*b*x - 2*b*c/d))*tan(b*c/d)^2 - 16*b^2*c^2*imag_part(cos_integral(2
*b*x + 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 + 16*b^2*c^2*imag_part(cos_integra
l(-2*b*x - 2*b*c/d))*tan(1/2*a)*tan(b*c/d)^2 - 32*b^2*c^2*sin_integral(2*(b
*d*x + b*c)/d)*tan(1/2*a)*tan(b*c/d)^2 - 16*b*c*d*tan(b*x)^2*tan(1/2*a)*tan
(b*c/d)^2 - 48*b*c*d*tan(b*x)*tan(1/2*a)^2*tan(b*c/d)^2 - 14*d^2*tan(b*x)^2
*tan(1/2*a)^2*tan(b*c/d)^2 - 16*b*c*d*tan(1/2*a)^3*tan(b*c/d)^2 - 16*d^2*ta
n(b*x)*tan(1/2*a)^3*tan(b*c/d)^2 - 3*d^2*tan(1/2*a)^4*tan(b*c/d)^2 - 4*b^2*
d^2*x^2*real_part(cos_integral(2*b*x + 2*b*c/d)) - 4*b^2*d^2*x^2*real_part(
cos_integral(-2*b*x - 2*b*c/d)) - 4*b^2*c^2*real_part(cos_integral(2*b*x +
2*b*c/d))*tan(b*x)^2 - 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*
tan(b*x)^2 + 32*b^2*c*d*x*imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2*
a) - 32*b^2*c*d*x*imag_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a) + 64
*b^2*c*d*x*sin_integral(2*(b*d*x + b*c)/d)*tan(1/2*a) - 16*b*d^2*x*tan(b*x)
^2*tan(1/2*a) + 24*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d))*tan(1/2
*a)^2 + 24*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))*tan(1/2*a)^2 -
48*b*d^2*x*tan(b*x)*tan(1/2*a)^2 - 16*b*d^2*x*tan(1/2*a)^3 - 16*b^2*c*d*x*
imag_part(cos_integral(2*b*x + 2*b*c/d))*tan(b*c/d) + 16*b^2*c*d*x*imag_par
t(cos_integral(-2*b*x - 2*b*c/d))*tan(b*c/d) - 32*b^2*c*d*x*sin_integral(2*
(b*d*x + b*c)/d)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(2*b*x + 2*b
*c/d))*tan(1/2*a)*tan(b*c/d) - 32*b^2*c^2*real_part(cos_integral(-2*b*x - 2
*b*c/d))*tan(1/2*a)*tan(b*c/d) + 4*b^2*c^2*real_part(cos_integral(2*b*x + 2
*b*c/d))*tan(b*c/d)^2 + 4*b^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d)^2 + 8*b*d^2*x*tan(b*x)*tan(b*c/d)^2 + 16*b*d^2*x*tan(1/2*a)*tan
(b*c/d)^2 - 8*b^2*c*d*x*real_part(cos_integral(2*b*x + 2*b*c/d)) - 8*b^2*c*
d*x*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 16*b^2*c^2*imag_part(cos_in
tegral(2*b*x + 2*b*c/d))*tan(1/2*a) - 16*b^2*c^2*imag_part(cos_integral(-2*
b*x - 2*b*c/d))*tan(1/2*a) + 32*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan
(1/2*a) - 16*b*c*d*tan(b*x)^2*tan(1/2*a) - 48*b*c*d*tan(b*x)*tan(1/2*a)^2 -
14*d^2*tan(b*x)^2*tan(1/2*a)^2 - 16*b*c*d*tan(1/2*a)^3 - 16*d^2*tan(b*x)*t
an(1/2*a)^3 - 3*d^2*tan(1/2*a)^4 - 8*b^2*c^2*imag_part(cos_integral(2*b*x +
2*b*c/d))*tan(b*c/d) + 8*b^2*c^2*imag_part(cos_integral(-2*b*x - 2*b*c/d))
*tan(b*c/d) - 16*b^2*c^2*sin_integral(2*(b*d*x + b*c)/d)*tan(b*c/d) + 8*b*c
*d*tan(b*x)*tan(b*c/d)^2 + d^2*tan(b*x)^2*tan(b*c/d)^2 + 16*b*c*d*tan(1/2*a
)*tan(b*c/d)^2 + 16*d^2*tan(b*x)*tan(1/2*a)*tan(b*c/d)^2 + 10*d^2*tan(1/2*a
)^2*tan(b*c/d)^2 - 4*b^2*c^2*real_part(cos_integral(2*b*x + 2*b*c/d)) - 4*b
^2*c^2*real_part(cos_integral(-2*b*x - 2*b*c/d)) + 8*b*d^2*x*tan(b*x) + 16*
b*d^2*x*tan(1/2*a) + 8*b*c*d*tan(b*x) + d^2*tan(b*x)^2 + 16*b*c*d*tan(1/2*a
) + 16*d^2*tan(b*x)*tan(1/2*a) + 10*d^2*tan(1/2*a)^2 - 3*d^2*tan(b*c/d)^2 -
3*d^2)/(d^5*x^2*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^
2*tan(1/2*a)^4*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2*tan(1/2*a)^4 + 2*d^5*x^2*t
an(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + d^5*x^2*tan(1/2*a)^4*tan(b*c/d)^2 + c

```
^2*d^3*tan(b*x)^2*tan(1/2*a)^4*tan(b*c/d)^2 + 2*c*d^4*x*tan(b*x)^2*tan(1/2*
a)^4 + 4*c*d^4*x*tan(b*x)^2*tan(1/2*a)^2*tan(b*c/d)^2 + 2*c*d^4*x*tan(1/2*a
)^4*tan(b*c/d)^2 + 2*d^5*x^2*tan(b*x)^2*tan(1/2*a)^2 + d^5*x^2*tan(1/2*a)^4
 + c^2*d^3*tan(b*x)^2*tan(1/2*a)^4 + d^5*x^2*tan(b*x)^2*tan(b*c/d)^2 + 2*d^
5*x^2*tan(1/2*a)^2*tan(b*c/d)^2 + 2*c^2*d^3*tan(b*x)^2*tan(1/2*a)^2*tan(b*c
/d)^2 + c^2*d^3*tan(1/2*a)^4*tan(b*c/d)^2 + 4*c*d^4*x*tan(b*x)^2*tan(1/2*a)
^2 + 2*c*d^4*x*tan(1/2*a)^4 + 2*c*d^4*x*tan(b*x)^2*tan(b*c/d)^2 + 4*c*d^4*x
*tan(1/2*a)^2*tan(b*c/d)^2 + d^5*x^2*tan(b*x)^2 + 2*d^5*x^2*tan(1/2*a)^2 +
2*c^2*d^3*tan(b*x)^2*tan(1/2*a)^2 + c^2*d^3*tan(1/2*a)^4 + d^5*x^2*tan(b*c/
d)^2 + c^2*d^3*tan(b*x)^2*tan(b*c/d)^2 + 2*c^2*d^3*tan(1/2*a)^2*tan(b*c/d)^
2 + 2*c*d^4*x*tan(b*x)^2 + 4*c*d^4*x*tan(1/2*a)^2 + 2*c*d^4*x*tan(b*c/d)^2
+ d^5*x^2 + c^2*d^3*tan(b*x)^2 + 2*c^2*d^3*tan(1/2*a)^2 + c^2*d^3*tan(b*c/d
)^2 + 2*c*d^4*x + c^2*d^3)
```

maple [A] time = 0.04, size = 207, normalized size = 1.52

$$\frac{1}{2d\left(dx+c\right)^{2}}+\frac{b^{3}\left(-\frac{\cos(2bx+2a)}{((bx+a)d-da+cb)^{2}d}-\frac{-\frac{2\sin(2bx+2a)}{((bx+a)d-da+cb)d}+\frac{\frac{4\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\sin\left(\frac{-2da+2cb}{d}\right)}{d}+\frac{4\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}}{d}}{d}\right)-\frac{\cdots}{((b\cdots}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] 1/2/d/(d*x+c)^2+4/b*(1/4*b^3*(-cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d-(-2*s
in(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d+2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2
*(-a*d+b*c)/d)/d+2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d)/d)/d
)-1/4*b^3/((b*x+a)*d-d*a+c*b)^2/d)
```

maxima [C] time = 0.43, size = 130, normalized size = 0.96

$$-\frac{2\left(E_{3}\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_{3}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(2i\,E_{3}\left(\frac{2i\,bdx+2i\,bc}{d}\right)-2i\,E_{3}\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)+\cdots}{2\left(d^{3}x^{2}+2\,cd^{2}x+c^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(3, -(2
*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (2*I*exp_integral_e(3, (2*I
*b*d*x + 2*I*b*c)/d) - 2*I*exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*sin
(-2*(b*c - a*d)/d) + 1)/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3a+3bx)}{\sin(a+bx)\,(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3),x)
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
[Out] Timed out
```

3.375 $\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=205

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+bx)}{d^3(c+dx)}+\frac{4b\sin(}{}$$

```
[Out] -2/3*b^2/d^3/(d*x+c)-cos(b*x+a)^2/d/(d*x+c)^3+2*b^2*cos(b*x+a)^2/d^3/(d*x+c
)+8/3*b^3*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^4+8/3*b^3*Ci(2*b*c/d+2*b*x)*
sin(2*a-2*b*c/d)/d^4+4/3*b*cos(b*x+a)*sin(b*x+a)/d^2/(d*x+c)^2+1/3*sin(b*x+
a)^2/d/(d*x+c)^3-2/3*b^2*sin(b*x+a)^2/d^3/(d*x+c)
```

Rubi [A] time = 0.38, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {4431, 3314, 32, 3313, 12, 3303, 3299, 3302}

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{CosIntegral}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\sin^2(a+bx)}{3d^3(c+dx)}+\frac{2b^2\cos^2(a+b}{d^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (-2*b^2)/(3*d^3*(c + d*x)) - Cos[a + b*x]^2/(d*(c + d*x)^3) + (2*b^2*Cos[a
+ b*x]^2)/(d^3*(c + d*x)) + (8*b^3*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a -
 (2*b*c)/d])/(3*d^4) + (4*b*Cos[a + b*x]*Sin[a + b*x])/(3*d^2*(c + d*x)^2)
+ Sin[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Sin[a + b*x]^2)/(3*d^3*(c + d*x
)) + (8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3299

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3302

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3303

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[(c*f)/d + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[(c*f)/d + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3313

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[((c + d*x)^(m + 1)*Sin[e + f*x]^n)/(d*(m + 1)), x] - Dist[(f*n)/(d*(m +
1)), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3314

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[((c + d*x)^(m + 1)*(b*Sin[e + f*x])^n)/(d*(m + 1)), x] + (Dist[(
b^2*f^2*n*(n - 1))/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[(f^2*n^2)/(d^2*(m + 1)*(m + 2)), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[(b*f*n*(c + d*x)^(m + 2)*Cos[e +
 f*x]*(b*Sin[e + f*x])^(n - 1))/(d^2*(m + 1)*(m + 2)), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int \frac{\csc(a+bx)\sin(3a+3bx)}{(c+dx)^4}\,dx = \int \left(\frac{3\cos^2(a+bx)}{(c+dx)^4} - \frac{\sin^2(a+bx)}{(c+dx)^4}\right) dx$$

$$= 3\int \frac{\cos^2(a+bx)}{(c+dx)^4}\,dx - \int \frac{\sin^2(a+bx)}{(c+dx)^4}\,dx$$

$$= -\frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2} + \frac{\sin^2(a+bx)}{3d(c+dx)^3} - \frac{b^2\int \frac{1}{(c+dx)^2}\,dx}{3d^2} +$$

$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}$$

$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}$$

$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{4b\cos(a+bx)\sin(a+bx)}{3d^2(c+dx)^2}$$

$$= -\frac{2b^2}{3d^3(c+dx)} - \frac{\cos^2(a+bx)}{d(c+dx)^3} + \frac{2b^2\cos^2(a+bx)}{d^3(c+dx)} + \frac{8b^3\text{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a\right.}{3d^4}$$

Mathematica [A] time = 1.14, size = 125, normalized size = 0.61

$$\frac{8b^3\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2b(c+dx)}{d}\right) + 8b^3\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2b(c+dx)}{d}\right) + \frac{d\left(\cos(2(a+bx))\left(4b^2(c+dx)^2-2d^2\right)+d(2b(c+dx)\sin(2(a+bx))-d\right.}{(c+dx)^3}}{3d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^4,x]
```

```
[Out] (8*b^3*CosIntegral[(2*b*(c + d*x))/d]*Sin[2*a - (2*b*c)/d] + (d*((-2*d^2 +
4*b^2*(c + d*x)^2)*Cos[2*(a + b*x)] + d*(-d + 2*b*(c + d*x)*Sin[2*(a + b*x)
])))/(c + d*x)^3 + 8*b^3*Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*(c + d*x))/d
])/(3*d^4)
```

fricas [A] time = 0.49, size = 343, normalized size = 1.67

$$-\frac{4\,b^2d^3x^2 + 8\,b^2cd^2x + 4\,b^2c^2d - d^3 - 4\left(2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d - d^3\right)\cos\left(bx+a\right)^2 - 4\left(bd^3x + bcd^2\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="fricas")

[Out] -1/3*(4*b^2*d^3*x^2 + 8*b^2*c*d^2*x + 4*b^2*c^2*d - d^3 - 4*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)^2 - 4*(b*d^3*x + b*c*d^2)*cos(b*x + a)*sin(b*x + a) - 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos(-2*(b*c - a*d)/d)*sin_integral(2*(b*d*x + b*c)/d) - 4*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*cos_integral(-2*(b*d*x + b*c)/d))*sin(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="giac")

[Out] Timed out

maple [A] time = 0.04, size = 243, normalized size = 1.19

$$\frac{1}{3d\,(dx+c)^3}+\frac{b^4\left(-\frac{2\cos(2bx+2a)}{3((bx+a)d-da+cb)^3 d}-\frac{2\left(-\frac{\sin(2bx+2a)}{((bx+a)d-da+cb)^2 d}+\frac{-\frac{2\cos(2bx+2a)}{((bx+a)d-da+cb)d}-\frac{2\left(\frac{2\,\mathrm{Si}\left(2bx+2a+\frac{-2da+2cb}{d}\right)\cos\left(\frac{-2da+2cb}{d}\right)}{d}-\frac{2\,\mathrm{Ci}\left(2bx+2a+\frac{-2da+}{d}\right.}{}\right.}{d}}{d}\right.}{3d}\right.}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x)

[Out] 1/3/d/(d*x+c)^3+4/b*(1/4*b^4*(-2/3*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^3/d-2/3*(-sin(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)^2/d+(-2*cos(2*b*x+2*a)/((b*x+a)*d-d*a+c*b)/d-2*(2*Si(2*b*x+2*a+2*(-a*d+b*c)/d)*cos(2*(-a*d+b*c)/d)/d-2*Ci(2*b*x+2*a+2*(-a*d+b*c)/d)*sin(2*(-a*d+b*c)/d)/d)/d)/d)/d)-1/6*b^4/((b*x+a)*d-d*a+c*b)^3/d)

maxima [C] time = 0.45, size = 141, normalized size = 0.69

$$-\frac{3\left(E_4\left(\frac{2i\,bdx+2i\,bc}{d}\right)+E_4\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)-\left(3i\,E_4\left(\frac{2i\,bdx+2i\,bc}{d}\right)-3i\,E_4\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\sin\left(-\frac{2\,(bc-ad)}{d}\right)}{3\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(exp_integral_e(4, (2*I*b*d*x + 2*I*b*c)/d) + exp_integral_e(4, -(2
*I*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) - (3*I*exp_integral_e(4, (2*I
*b*d*x + 2*I*b*c)/d) - 3*I*exp_integral_e(4, -(2*I*b*d*x + 2*I*b*c)/d))*sin
(-2*(b*c - a*d)/d) + 1)/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)\,(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)*(c + d*x)^4), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.376 $\int (c+dx)^3 \csc^2(a+bx) \sin(3a+3bx) \, dx$

Optimal. Leaf size=255

$$-\frac{18id^3\text{Li}_4\left(-e^{i(a+bx)}\right)}{b^4}+\frac{18id^3\text{Li}_4\left(e^{i(a+bx)}\right)}{b^4}+\frac{24d^3\sin(a+bx)}{b^4}-\frac{18d^2(c+dx)\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{Li}_3\left(e^{i(a+\right.}}{b^3}$$

```
[Out] -6*(d*x+c)^3*arctanh(exp(I*(b*x+a)))/b-24*d^2*(d*x+c)*cos(b*x+a)/b^3+4*(d*x
+c)^3*cos(b*x+a)/b+9*I*d*(d*x+c)^2*polylog(2,-exp(I*(b*x+a)))/b^2-9*I*d*(d*
x+c)^2*polylog(2,exp(I*(b*x+a)))/b^2-18*d^2*(d*x+c)*polylog(3,-exp(I*(b*x+a
)))/b^3+18*d^2*(d*x+c)*polylog(3,exp(I*(b*x+a)))/b^3-18*I*d^3*polylog(4,-ex
p(I*(b*x+a)))/b^4+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4+24*d^3*sin(b*x+a)/
b^4-12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A] time = 0.35, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4431, 4408, 3296, 2637, 4183, 2531, 6609, 2282, 6589}

$$-\frac{18d^2(c+dx)\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{18d^2(c+dx)\text{PolyLog}\left(3,e^{i(a+bx)}\right)}{b^3}+\frac{9id(c+dx)^2\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{9i}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)^3*ArcTanh[E^(I*(a + b*x))])/b - (24*d^2*(c + d*x)*Cos[a + b*x
])/b^3 + (4*(c + d*x)^3*Cos[a + b*x])/b + ((9*I)*d*(c + d*x)^2*PolyLog[2, -
E^(I*(a + b*x))])/b^2 - ((9*I)*d*(c + d*x)^2*PolyLog[2, E^(I*(a + b*x))])/b
^2 - (18*d^2*(c + d*x)*PolyLog[3, -E^(I*(a + b*x))])/b^3 + (18*d^2*(c + d*x
)*PolyLog[3, E^(I*(a + b*x))])/b^3 - ((18*I)*d^3*PolyLog[4, -E^(I*(a + b*x)
)])/b^4 + ((18*I)*d^3*PolyLog[4, E^(I*(a + b*x))])/b^4 + (24*d^3*Sin[a + b*
x])/b^4 - (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
```

```
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\cot(a+bx) - (c+dx)^3\sin(a+bx)\right)dx$$

$$= 3\int (c+dx)^3\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\,d$$

$$= \frac{(c+dx)^3\cos(a+bx)}{b} + 3\int (c+dx)^3\csc(a+bx)\,dx - 3\int (c+dx)^3$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{3d(c+dx)}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{6d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+dx)}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+d}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+d}{}$$

$$= -\frac{6(c+dx)^3\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{24d^2(c+dx)\cos(a+bx)}{b^3} + \frac{4(c+d}{}$$

Mathematica [A] time = 1.54, size = 459, normalized size = 1.80

$$\frac{4\cos(bx)\left(b^3c^3\cos(a) + 3b^3c^2dx\cos(a) + 3b^3cd^2x^2\cos(a) + b^3d^3x^3\cos(a) - 3b^2c^2d\sin(a) - 6b^2cd^2x\sin(a) - 3b\right.}{b^4}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*(-2*b^3*(c + d*x)^3*ArcTanh[Cos[a + b*x] + I*Sin[a + b*x]] + (3*I)*d*(b^
2*(c + d*x)^2*PolyLog[2, -Cos[a + b*x] - I*Sin[a + b*x]] + (2*I)*b*d*(c + d
*x)*PolyLog[3, -Cos[a + b*x] - I*Sin[a + b*x]] - 2*d^2*PolyLog[4, -Cos[a +
b*x] - I*Sin[a + b*x]]) - (3*I)*d*(b^2*(c + d*x)^2*PolyLog[2, Cos[a + b*x]
+ I*Sin[a + b*x]] + (2*I)*b*d*(c + d*x)*PolyLog[3, Cos[a + b*x] + I*Sin[a +
 b*x]] - 2*d^2*PolyLog[4, Cos[a + b*x] + I*Sin[a + b*x]])))/b^4 + (4*Cos[b*
```

```
x]*(b^3*c^3*Cos[a] - 6*b*c*d^2*Cos[a] + 3*b^3*c^2*d*x*Cos[a] - 6*b*d^3*x*Co
s[a] + 3*b^3*c*d^2*x^2*Cos[a] + b^3*d^3*x^3*Cos[a] - 3*b^2*c^2*d*Sin[a] + 6
*d^3*Sin[a] - 6*b^2*c*d^2*x*Sin[a] - 3*b^2*d^3*x^2*Sin[a]))/b^4 - (4*(3*b^2
*c^2*d*Cos[a] - 6*d^3*Cos[a] + 6*b^2*c*d^2*x*Cos[a] + 3*b^2*d^3*x^2*Cos[a]
+ b^3*c^3*Sin[a] - 6*b*c*d^2*Sin[a] + 3*b^3*c^2*d*x*Sin[a] - 6*b*d^3*x*Sin[
a] + 3*b^3*c*d^2*x^2*Sin[a] + b^3*d^3*x^3*Sin[a])*Sin[b*x])/b^4
```

fricas [C] time = 0.56, size = 925, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(18*I*d^3*polylog(4, cos(b*x + a) + I*sin(b*x + a)) - 18*I*d^3*polylog(
4, cos(b*x + a) - I*sin(b*x + a)) + 18*I*d^3*polylog(4, -cos(b*x + a) + I*s
in(b*x + a)) - 18*I*d^3*polylog(4, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*
x)*cos(b*x + a) + (-9*I*b^2*d^3*x^2 - 18*I*b^2*c*d^2*x - 9*I*b^2*c^2*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (9*I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9
*I*b^2*c^2*d)*dilog(cos(b*x + a) - I*sin(b*x + a)) + (-9*I*b^2*d^3*x^2 - 18
*I*b^2*c*d^2*x - 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) + I*sin(b*x + a)) + (9*
I*b^2*d^3*x^2 + 18*I*b^2*c*d^2*x + 9*I*b^2*c^2*d)*dilog(-cos(b*x + a) - I*s
in(b*x + a)) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*
log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + b^3*c^3)*log(cos(b*x + a) - I*sin(b*x + a) + 1) + 3*(b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-1/2*cos(b*x + a) + 1/2*I
*sin(b*x + a) + 1/2) + 3*(b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(
-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*
b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(-cos(b*x + a) -
I*sin(b*x + a) + 1) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) + I*si
n(b*x + a)) + 18*(b*d^3*x + b*c*d^2)*polylog(3, cos(b*x + a) - I*sin(b*x +
a)) - 18*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) + I*sin(b*x + a)) - 1
8*(b*d^3*x + b*c*d^2)*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) - 24*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*sin(b*x + a))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \csc (bx + a)^2 \sin (3\,bx + 3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

[Out] integrate((d*x + c)^3*csc(b*x + a)^2*sin(3*b*x + 3*a), x)

maple [B] time = 0.12, size = 849, normalized size = 3.33

$$-\frac{18ic\,d^2 \operatorname{polylog}\left(2, e^{i(bx+a)}\right) x}{b^2}+\frac{18ic\,d^2 \operatorname{polylog}\left(2, -e^{i(bx+a)}\right) x}{b^2}+\frac{3d^3 \ln\left(1-e^{i(bx+a)}\right) x^3}{b}+\frac{3d^3 \ln\left(1-e^{i(bx+a)}\right) a^3}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x)

[Out] -18*I*d^3*polylog(4,-exp(I*(b*x+a)))/b^4+18*I/b^2*c*d^2*polylog(2,-exp(I*(b*x+a)))*x-18*I/b^2*c*d^2*polylog(2,exp(I*(b*x+a)))*x+18*I*d^3*polylog(4,exp(I*(b*x+a)))/b^4-18/b^3*c*d^2*polylog(3,-exp(I*(b*x+a)))+18/b^3*c*d^2*polylog(3,exp(I*(b*x+a)))+18/b^3*d^3*polylog(3,exp(I*(b*x+a)))*x-18/b^3*d^3*polylog(3,-exp(I*(b*x+a)))*x+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))+2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-6/b*c^3*arctanh(exp(I*(b*x+a)))+6/b^4*d^3*a^3*arctanh(exp(I*(b*x+a)))-9/b^2*c^2*d*ln(exp(I*(b*x+a))+1)*a+9/b^3*c*d^2*a^2*ln(exp(I*(b*x+a))+1)-3/b^4*d^3*ln(exp(I*(b*x+a))+1)*a^3-18/b^3*c*d^2*a^2*arctanh(exp(I*(b*x+a)))+18/b^2*c^2*d*a*arctanh(exp(I*(b*x+a)))-9/b*c^2*d*ln(exp(I*(b*x+a))+1)*x+9/b*c^2*d*ln(1-exp(I*(b*x+a)))*x+9/b^2*c^2*d*ln(1-exp(I*(b*x+a)))*a-9/b^3*c*d^2*a^2*ln(1-exp(I*(b*x+a)))+9/b*c*d^2*ln(1-exp(I*(b*x+a)))*x^2-9/b*c*d^2*ln(exp(I*(b*x+a))+1)*x^2+3/b*d^3*ln(1-exp(I*(b*x+a)))*x^3+3/b^4*d^3*ln(1-exp(I*(b*x+a)))*a^3-3/b*d^3*ln(exp(I*(b*x+a))+1)*x^3-9*I/b^2*d^3*polylog(2,exp(I*(b*x+a)))*x^2+9*I/b^2*d^3*polylog(2,-exp(I*(b*x+a)))*x^2-9*I/b^2*c^2*d*polylog(2,exp(I*(b*x+a)))+9*I/b^2*c^2*d*polylog(2,-exp(I*(b*x+a)))
)

maxima [B] time = 0.54, size = 602, normalized size = 2.36

$$\frac{c^3\left(8\cos(bx+a)-3\log\left(\cos(bx)^2+2\cos(bx)\cos(a)+\cos(a)^2+\sin(bx)^2-2\sin(bx)\sin(a)+\sin(a)^2\right)+3\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")

[Out] 1/2*c^3*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(36*I*d^3*polylog(4, -e^(I*b*x + I*a)) - 36*I*d^3*polylog(4, e^(I*b*x + I*a)) + (6*I*b^3*d^3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*b^3*c^2*d*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) + (6*I*b^3*d^3*x^3 + 18*I*b^3*c*d^2*x^2 + 18*I*b^

```
3*c^2*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1) - 8*(b^3*d^3*x^3 + 3*b^
3*c*d^2*x^2 - 6*b*c*d^2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a) + (-18*I*
b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*dilog(-e^(I*b*x + I*a)) +
(18*I*b^2*d^3*x^2 + 36*I*b^2*c*d^2*x + 18*I*b^2*c^2*d)*dilog(e^(I*b*x + I*a
)) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x)*log(cos(b*x + a)^2 +
 sin(b*x + a)^2 + 2*cos(b*x + a) + 1) - 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
3*b^3*c^2*d*x)*log(cos(b*x + a)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) +
36*(b*d^3*x + b*c*d^2)*polylog(3, -e^(I*b*x + I*a)) - 36*(b*d^3*x + b*c*d^2
)*polylog(3, e^(I*b*x + I*a)) + 24*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d
 - 2*d^3)*sin(b*x + a))/b^4
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.377 $\int (c+dx)^2 \csc^2(a+bx) \sin(3a+3bx)\,dx$

Optimal. Leaf size=172

$$-\frac{6d^2\text{Li}_3\left(-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{Li}_3\left(e^{i(a+bx)}\right)}{b^3}-\frac{8d^2\cos(a+bx)}{b^3}+\frac{6id(c+dx)\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{8d}{}$$

```
[Out] -6*(d*x+c)^2*arctanh(exp(I*(b*x+a)))/b-8*d^2*cos(b*x+a)/b^3+4*(d*x+c)^2*cos
(b*x+a)/b+6*I*d*(d*x+c)*polylog(2,-exp(I*(b*x+a)))/b^2-6*I*d*(d*x+c)*polylo
g(2,exp(I*(b*x+a)))/b^2-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polylog(
3,exp(I*(b*x+a)))/b^3-8*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A] time = 0.23, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4431, 4408, 3296, 2638, 4183, 2531, 2282, 6589}

$$\frac{6id(c+dx)\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{6id(c+dx)\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{6d^2\text{PolyLog}\left(3,-e^{i(a+bx)}\right)}{b^3}+\frac{6d^2\text{PolyLog}\left(3\right.}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)^2*ArcTanh[E^(I*(a + b*x))])/b - (8*d^2*Cos[a + b*x])/b^3 + (4
*(c + d*x)^2*Cos[a + b*x])/b + ((6*I)*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x
))])/b^2 - ((6*I)*d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))])/b^2 - (6*d^2*Pol
yLog[3, -E^(I*(a + b*x))])/b^3 + (6*d^2*PolyLog[3, E^(I*(a + b*x))])/b^3 -
(8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \csc^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\cot(a+bx) - (c+dx)^2\sin(a+bx)\right)\,dx$$
$$= 3\int (c+dx)^2\cos(a+bx)\cot(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\,d$$
$$= \frac{(c+dx)^2\cos(a+bx)}{b} + 3\int (c+dx)^2\csc(a+bx)\,dx - 3\int (c+dx)^2$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} + \frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{2d(c+dx}{}$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{2d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a}{b}$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a}{b}$$
$$= -\frac{6(c+dx)^2\tanh^{-1}\left(e^{i(a+bx)}\right)}{b} - \frac{8d^2\cos(a+bx)}{b^3} + \frac{4(c+dx)^2\cos(a}{b}$$

Mathematica [A] time = 1.10, size = 223, normalized size = 1.30

$$\frac{4\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)-4\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a)(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*b^2*(c + d*x)^2*Log[1 - E^(I*(a + b*x))] - 3*b^2*(c + d*x)^2*Log[1 + E^(
I*(a + b*x))] + (6*I)*b*d*(c + d*x)*PolyLog[2, -E^(I*(a + b*x))] - (6*I)*b*
d*(c + d*x)*PolyLog[2, E^(I*(a + b*x))] - 6*d^2*PolyLog[3, -E^(I*(a + b*x))
] + 6*d^2*PolyLog[3, E^(I*(a + b*x))] + 4*Cos[b*x]*((-2*d^2 + b^2*(c + d*x)
^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) - 4*(2*b*d*(c + d*x)*Cos[a] + (-2*d^2
+ b^2*(c + d*x)^2)*Sin[a])*Sin[b*x])/b^3
```

fricas [C] time = 0.54, size = 562, normalized size = 3.27

$$\frac{6\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)+i\,\sin\left(bx+a\right)\right)+6\,d^2\mathrm{polylog}\left(3,\cos\left(bx+a\right)-i\,\sin\left(bx+a\right)\right)-6\,d^2\mathrm{polylog}\left(3,-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(6*d^2*polylog(3, cos(b*x + a) + I*sin(b*x + a)) + 6*d^2*polylog(3, cos
(b*x + a) - I*sin(b*x + a)) - 6*d^2*polylog(3, -cos(b*x + a) + I*sin(b*x +
a)) - 6*d^2*polylog(3, -cos(b*x + a) - I*sin(b*x + a)) + 8*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2 - 2*d^2)*cos(b*x + a) + (-6*I*b*d^2*x - 6*I*b*c*d)*dil
og(cos(b*x + a) + I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(cos(b*x
 + a) - I*sin(b*x + a)) + (-6*I*b*d^2*x - 6*I*b*c*d)*dilog(-cos(b*x + a) +
I*sin(b*x + a)) + (6*I*b*d^2*x + 6*I*b*c*d)*dilog(-cos(b*x + a) - I*sin(b*x
 + a)) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) + I*sin(b
*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(cos(b*x + a) - I
*sin(b*x + a) + 1) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*cos(b*x + a
) + 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-1/2*
cos(b*x + a) - 1/2*I*sin(b*x + a) + 1/2) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2
*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(b^2*d^2*x^
2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) +
 1) - 16*(b*d^2*x + b*c*d)*sin(b*x + a))/b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2 \csc(bx+a)^2 \sin(3\,bx+3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.20, size = 481, normalized size = 2.80

$$\frac{2\left(d^2x^2b^2+2b^2cdx+2ib\,d^2x+b^2c^2+2ibcd-2d^2\right)\mathrm{e}^{i(bx+a)}}{b^3}+\frac{2\left(d^2x^2b^2+2b^2cdx-2ib\,d^2x+b^2c^2-2ibcd-2d^2\right)\mathrm{e}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2+2*I*b*d^2*x-2*d^2+2*I*b*c*d)/b^3*exp(I*(
b*x+a))+2*(d^2*x^2*b^2+2*b^2*c*d*x+b^2*c^2-2*I*b*d^2*x-2*d^2-2*I*b*c*d)/b^3
*exp(-I*(b*x+a))+12/b^2*c*d*a*arctanh(exp(I*(b*x+a)))-6*I/b^2*c*d*polylog(2
,exp(I*(b*x+a)))-6*I/b^2*polylog(2,exp(I*(b*x+a)))*d^2*x-6/b*c^2*arctanh(ex
p(I*(b*x+a)))-6/b^3*d^2*a^2*arctanh(exp(I*(b*x+a)))-3/b*d^2*ln(exp(I*(b*x+a
))+1)*x^2+3/b^3*d^2*ln(exp(I*(b*x+a))+1)*a^2+6*I/b^2*c*d*polylog(2,-exp(I*(
b*x+a)))+3/b*d^2*ln(1-exp(I*(b*x+a)))*x^2-3/b^3*d^2*ln(1-exp(I*(b*x+a)))*a^
2+6*I/b^2*d^2*polylog(2,-exp(I*(b*x+a)))*x+6/b*c*d*ln(1-exp(I*(b*x+a)))*x+6
/b^2*c*d*ln(1-exp(I*(b*x+a)))*a-6/b*c*d*ln(exp(I*(b*x+a))+1)*x-6/b^2*c*d*ln
(exp(I*(b*x+a))+1)*a-6*d^2*polylog(3,-exp(I*(b*x+a)))/b^3+6*d^2*polylog(3,e
xp(I*(b*x+a)))/b^3
```

maxima [B] time = 0.49, size = 409, normalized size = 2.38

$$\frac{c^2\left(8\,\cos\left(bx+a\right)-3\,\log\left(\cos\left(bx\right)^2+2\,\cos\left(bx\right)\cos(a)+\cos(a)^2+\sin\left(bx\right)^2-2\,\sin\left(bx\right)\sin(a)+\sin(a)^2\right)+3\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 +
 sin(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)
*cos(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b - 1/2*(1
2*d^2*polylog(3, -e^(I*b*x + I*a)) - 12*d^2*polylog(3, e^(I*b*x + I*a)) + (
6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), cos(b*x + a) + 1) +
 (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(b*x + a), -cos(b*x + a) + 1
) - 8*(b^2*d^2*x^2 + 2*b^2*c*d*x - 2*d^2)*cos(b*x + a) + (-12*I*b*d^2*x - 1
2*I*b*c*d)*dilog(-e^(I*b*x + I*a)) + (12*I*b*d^2*x + 12*I*b*c*d)*dilog(e^(I
*b*x + I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a)^2 + sin(b*x +
 a)^2 + 2*cos(b*x + a) + 1) - 3*(b^2*d^2*x^2 + 2*b^2*c*d*x)*log(cos(b*x + a
)^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1) + 16*(b*d^2*x + b*c*d)*sin(b*x +
 a))/b^3
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.378 $\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=95

$$\frac{3id\text{Li}_2\left(-e^{i(a+bx)}\right)}{b^2}-\frac{3id\text{Li}_2\left(e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

```
[Out] -6*(d*x+c)*arctanh(exp(I*(b*x+a)))/b+4*(d*x+c)*cos(b*x+a)/b+3*I*d*polylog(2
,-exp(I*(b*x+a)))/b^2-3*I*d*polylog(2,exp(I*(b*x+a)))/b^2-4*d*sin(b*x+a)/b^
2
```

Rubi [A] time = 0.11, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {4431, 4408, 3296, 2637, 4183, 2279, 2391}

$$\frac{3id\text{PolyLog}\left(2,-e^{i(a+bx)}\right)}{b^2}-\frac{3id\text{PolyLog}\left(2,e^{i(a+bx)}\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (-6*(c + d*x)*ArcTanh[E^(I*(a + b*x))])/b + (4*(c + d*x)*Cos[a + b*x])/b +
((3*I)*d*PolyLog[2, -E^(I*(a + b*x))])/b^2 - ((3*I)*d*PolyLog[2, E^(I*(a +
b*x))])/b^2 - (4*d*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
```

```
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4183

```
Int[csc[(e_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp[(
-2*(c + d*x)^m*ArcTanh[E^(I*(e + f*x))])/f, x] + (-Dist[(d*m)/f, Int[(c + d
*x)^(m - 1)*Log[1 - E^(I*(e + f*x))], x], x] + Dist[(d*m)/f, Int[(c + d*x)^
(m - 1)*Log[1 + E^(I*(e + f*x))], x], x]) /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4408

```
Int[Cos[(a_.) + (b_.)*(x_)]^(n_.)*Cot[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d
_.)*(x_))^(m_.), x_Symbol] :> -Int[(c + d*x)^m*Cos[a + b*x]^n*Cot[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Cos[a + b*x]^(n - 2)*Cot[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)\csc^2(a+bx)\sin(3a+3bx)\,dx &= \int (3(c+dx)\cos(a+bx)\cot(a+bx)-(c+dx)\sin(a+bx))\,dx \\
&= 3\int (c+dx)\cos(a+bx)\cot(a+bx)\,dx-\int (c+dx)\sin(a+bx)\,dx \\
&= \frac{(c+dx)\cos(a+bx)}{b}+3\int (c+dx)\csc(a+bx)\,dx-3\int (c+dx)\sin \\
&= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{d\sin(a+bx)}{b^2} \\
&= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}-\frac{4d\sin(a+bx}{b^2} \\
&= -\frac{6(c+dx)\tanh^{-1}\left(e^{i(a+bx)}\right)}{b}+\frac{4(c+dx)\cos(a+bx)}{b}+\frac{3id\mathrm{Li}_2\left(-e^{i(a-}\right.}{b^2}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 171, normalized size = 1.80

$$\frac{3d\left(i\left(\text{Li}_2\left(-e^{i(a+bx)}\right)-\text{Li}_2\left(e^{i(a+bx)}\right)\right)+(a+bx)\left(\log\left(1-e^{i(a+bx)}\right)-\log\left(1+e^{i(a+bx)}\right)\right)\right)}{b^2}-\frac{4d\sin(a+bx)}{b^2}-\frac{3ad\log\left(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Csc[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*c*Cos[a + b*x])/b + (4*d*x*Cos[a + b*x])/b - (3*c*Log[Cos[(a + b*x)/2]])
/b + (3*c*Log[Sin[(a + b*x)/2]])/b - (3*a*d*Log[Tan[(a + b*x)/2]])/b^2 + (3
*d*((a + b*x)*(Log[1 - E^(I*(a + b*x))] - Log[1 + E^(I*(a + b*x))]) + I*(Po
lyLog[2, -E^(I*(a + b*x))] - PolyLog[2, E^(I*(a + b*x))])))/b^2 - (4*d*Sin[
a + b*x])/b^2
```

fricas [B] time = 0.48, size = 281, normalized size = 2.96

$$\frac{8\,(bdx+bc)\cos\,(bx+a)-3i\,d\text{Li}_2\,(\cos\,(bx+a)+i\,\sin\,(bx+a))+3i\,d\text{Li}_2\,(\cos\,(bx+a)-i\,\sin\,(bx+a))-3i\,d\text{Li}_2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(8*(b*d*x + b*c)*cos(b*x + a) - 3*I*d*dilog(cos(b*x + a) + I*sin(b*x +
a)) + 3*I*d*dilog(cos(b*x + a) - I*sin(b*x + a)) - 3*I*d*dilog(-cos(b*x + a
) + I*sin(b*x + a)) + 3*I*d*dilog(-cos(b*x + a) - I*sin(b*x + a)) - 3*(b*d*
x + b*c)*log(cos(b*x + a) + I*sin(b*x + a) + 1) - 3*(b*d*x + b*c)*log(cos(b
*x + a) - I*sin(b*x + a) + 1) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) + 1/2*I
*sin(b*x + a) + 1/2) + 3*(b*c - a*d)*log(-1/2*cos(b*x + a) - 1/2*I*sin(b*x
+ a) + 1/2) + 3*(b*d*x + a*d)*log(-cos(b*x + a) + I*sin(b*x + a) + 1) + 3*(
b*d*x + a*d)*log(-cos(b*x + a) - I*sin(b*x + a) + 1) - 8*d*sin(b*x + a))/b^
2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)\csc\,(bx+a)^2\sin\,(3\,bx+3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*csc(b*x + a)^2*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.07, size = 204, normalized size = 2.15

$$\frac{3c\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{b}+\frac{3d\ln\left(1-\mathrm{e}^{i(bx+a)}\right)x}{b}-\frac{3d\ln\left(\mathrm{e}^{i(bx+a)}+1\right)x}{b}-\frac{3da\ln\left(\csc\left(bx+a\right)-\cot\left(bx+a\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] 3/b*c*ln(csc(b*x+a)-cot(b*x+a))+3/b*d*ln(1-exp(I*(b*x+a)))*x-3/b*d*ln(exp(I
*(b*x+a))+1)*x-3/b^2*d*a*ln(csc(b*x+a)-cot(b*x+a))+3*I*d/b^2*dilog(exp(I*(b
*x+a))+1)-3*I*d/b^2*dilog(1-exp(I*(b*x+a)))+3/b^2*d*ln(1-exp(I*(b*x+a)))*a-
3/b^2*d*ln(exp(I*(b*x+a))+1)*a+4/b*c*cos(b*x+a)+4/b*d*cos(b*x+a)*x-4*d*sin(
b*x+a)/b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{c\left(8\cos\left(bx+a\right)-3\log\left(\cos\left(bx\right)^2+2\cos\left(bx\right)\cos(a)+\cos(a)^2+\sin\left(bx\right)^2-2\sin\left(bx\right)\sin(a)+\sin(a)^2\right)+3\ldots}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] 1/2*c*(8*cos(b*x + a) - 3*log(cos(b*x)^2 + 2*cos(b*x)*cos(a) + cos(a)^2 + s
in(b*x)^2 - 2*sin(b*x)*sin(a) + sin(a)^2) + 3*log(cos(b*x)^2 - 2*cos(b*x)*c
os(a) + cos(a)^2 + sin(b*x)^2 + 2*sin(b*x)*sin(a) + sin(a)^2))/b + (4*b*x*c
os(b*x + a) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)^2 + sin(b*x + a)
^2 + 2*cos(b*x + a) + 1), x) + 3*b^2*integrate(x*sin(b*x + a)/(cos(b*x + a)
^2 + sin(b*x + a)^2 - 2*cos(b*x + a) + 1), x) - 4*sin(b*x + a))*d/b^2
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/sin(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*csc(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.379 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=72

$$3\text{Int}\left(\frac{\csc(a+bx)}{c+dx},x\right)-\frac{4\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -4*cos(a-b*c/d)*Si(b*c/d+b*x)/d-4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d+3*Unintegrab
le(csc(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.21, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (-4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d - (4*Cos[a - (b*c)/d]*Si
nIntegral[(b*c)/d + b*x])/d + 3*Defer[Int][Csc[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{c+dx}-\frac{\sin(a+bx)}{c+dx}\right)dx\\ &= 3\int \frac{\cos(a+bx)\cot(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)}{c+dx}\,dx\\ &= 3\int \frac{\csc(a+bx)}{c+dx}\,dx-3\int \frac{\sin(a+bx)}{c+dx}\,dx-\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx\\ &= -\frac{\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+bx)}{c+dx}\,dx\\ &= -\frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}-\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+3\int \frac{\csc(a+bx)}{c+dx}\end{aligned}$$

Mathematica [A] time = 6.23, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

fricas [A] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^{2}\sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

maple [A] time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^{2}\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2i\,E_{1}\left(\frac{i\,bdx+i\,bc}{d}\right)-2i\,E_{1}\left(-\frac{i\,bdx+i\,bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+3\,d\int\frac{\sin(bx+a)}{(dx+c)\left(\cos(bx+a)^{2}+\sin(bx+a)^{2}+2\,\cos(bx+a)+1\right)}\,dx+3\,d\int\frac{}{(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((2*I*exp_integral_e(1, (I*b*d*x + I*b*c)/d) - 2*I*exp_integral_e(1, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 3*d*integrate(sin(b*x + a)/((d*x + c
)*cos(b*x + a)^2 + (d*x + c)*sin(b*x + a)^2 + d*x + 2*(d*x + c)*cos(b*x + a
) + c), x) + 3*d*integrate(sin(b*x + a)/((d*x + c)*cos(b*x + a)^2 + (d*x +
c)*sin(b*x + a)^2 + d*x - 2*(d*x + c)*cos(b*x + a) + c), x) + 2*(exp_integr
al_e(1, (I*b*d*x + I*b*c)/d) + exp_integral_e(1, -(I*b*d*x + I*b*c)/d))*sin
(-(b*c - a*d)/d))/d
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

3.380 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=92

$$3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^2},x\right)-\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{4\sin(a+bx)}{d(c+dx)}$$

```
[Out] -4*b*Ci(b*c/d+b*x)*cos(a-b*c/d)/d^2+4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2+4*si
n(b*x+a)/d/(d*x+c)+3*Unintegrable(csc(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.28, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (-4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 + (4*Sin[a + b*x])/(
d*(c + d*x)) + (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 + 3*De
fer[Int][Csc[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)}{(c+dx)^2}\right)dx$$
$$= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{\sin(a+bx)}{d(c+dx)} + 3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx - 3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx - \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d}$$
$$= \frac{4\sin(a+bx)}{d(c+dx)} + 3\int \frac{\csc(a+bx)}{(c+dx)^2}\,dx - \frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d} - \frac{\left(b\cos\left(a-\frac{bc}{d}\right)\right)\int}{d}$$
$$= -\frac{b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{4\sin(a+bx)}{d(c+dx)} + \frac{b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2} + 3$$
$$= -\frac{4b\cos\left(a-\frac{bc}{d}\right)\operatorname{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} + \frac{4\sin(a+bx)}{d(c+dx)} + \frac{4b\sin\left(a-\frac{bc}{d}\right)\operatorname{Si}\left(\frac{bc}{d}+bx\right)}{d^2}$$

Mathematica [A] time = 6.71, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

maple [A] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{\left(dx+c\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(2i\,E_2\left(\frac{i\,bdx+i\,bc}{d}\right)-2i\,E_2\left(-\frac{i\,bdx+i\,bc}{d}\right)\right)\cos\left(-\frac{bc-ad}{d}\right)+3\left(d^2x+cd\right)\int \frac{\sin(bx+a)}{(dx+c)^2\left(\cos(bx+a)^2+\sin(bx+a)^2+2\cos(bx+a)+1\right)}\,dx+3}{d^2x+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((2*I*exp_integral_e(2, (I*b*d*x + I*b*c)/d) - 2*I*exp_integral_e(2, -(I*b*
d*x + I*b*c)/d))*cos(-(b*c - a*d)/d) + 3*(d^2*x + c*d)*integrate(sin(b*x +
a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2
 + 2*c*d*x + c^2)*sin(b*x + a)^2 + c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*
x + a)), x) + 3*(d^2*x + c*d)*integrate(sin(b*x + a)/(d^2*x^2 + 2*c*d*x + (
d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(b*x
 + a)^2 + c^2 - 2*(d^2*x^2 + 2*c*d*x + c^2)*cos(b*x + a)), x) + 2*(exp_inte
gral_e(2, (I*b*d*x + I*b*c)/d) + exp_integral_e(2, -(I*b*d*x + I*b*c)/d))*s
in(-(b*c - a*d)/d))/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\sin\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.381 $\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=115

$$3\text{Int}\left(\frac{\csc(a+bx)}{(c+dx)^3},x\right)+\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^3}+\frac{2b\cos(a+bx)}{d^2(c+dx)}+\frac{2\sin(a+b}{d(c+dx)^2}$$

```
[Out] 2*b*cos(b*x+a)/d^2/(d*x+c)+2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3+2*b^2*Ci(b*
c/d+b*x)*sin(a-b*c/d)/d^3+2*sin(b*x+a)/d/(d*x+c)^2+3*Unintegrable(csc(b*x+a
)/(d*x+c)^3,x)
```

Rubi [A] time = 0.33, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (2*b*Cos[a + b*x])/(d^2*(c + d*x)) + (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin[
a - (b*c)/d])/d^3 + (2*Sin[a + b*x])/(d*(c + d*x)^2) + (2*b^2*Cos[a - (b*c)
/d]*SinIntegral[(b*c)/d + b*x])/d^3 + 3*Defer[Int][Csc[a + b*x]/(c + d*x)^3
, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx &= \int \left(\frac{3\cos(a+bx)\cot(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)}{(c+dx)^3}\right) dx \\
&= 3\int \frac{\cos(a+bx)\cot(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin(a+bx)}{(c+dx)^3}\,dx \\
&= \frac{\sin(a+bx)}{2d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx - 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d} \\
&= \frac{b\cos(a+bx)}{2d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} - \cdots \\
&= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2\sin(a+bx)}{d(c+dx)^2} + 3\int \frac{\csc(a+bx)}{(c+dx)^3}\,dx + \frac{(3b^2)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} + \cdots \\
&= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \cdots \\
&= \frac{2b\cos(a+bx)}{d^2(c+dx)} + \frac{2b^2\operatorname{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} + \frac{2\sin(a+bx)}{d(c+dx)^2} + \cdots
\end{aligned}
$$

Mathematica [A] time = 7.14, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Csc[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\frac{\csc(bx+a)^2\sin(3bx+3a)}{d^3x^3+3cd^2x^2+3c^2dx+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(bx+a)^2 \sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
[Out] integrate(csc(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

maple [A] time = 0.75, size = 0, normalized size = 0.00

$$\int \frac{\left(\csc^2(bx+a)\right)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
[Out] int(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\sin(a+b\,x)^2\,(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3),x)
[Out] int(sin(3*a + 3*b*x)/(sin(a + b*x)^2*(c + d*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.382 $\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=299

$$-\frac{3d^4\text{Li}_5\left(-e^{2i(a+bx)}\right)}{2b^5}+\frac{3d^4\sin^2(a+bx)}{b^5}+\frac{3id^3(c+dx)\text{Li}_4\left(-e^{2i(a+bx)}\right)}{b^4}-\frac{6d^3(c+dx)\sin(a+bx)\cos(a+bx)}{b^4}+\frac{3d^2(}{}$$

```
[Out] 6*c*d^3*x/b^3+3*d^4*x^2/b^3-(d*x+c)^4/b-1/5*I*(d*x+c)^5/d+(d*x+c)^4*ln(1+ex
p(2*I*(b*x+a)))/b-2*I*d*(d*x+c)^3*polylog(2,-exp(2*I*(b*x+a)))/b^2+3*d^2*(d
*x+c)^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+3*I*d^3*(d*x+c)*polylog(4,-exp(2*I
*(b*x+a)))/b^4-3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-6*d^3*(d*x+c)*cos(b
*x+a)*sin(b*x+a)/b^4+4*d*(d*x+c)^3*cos(b*x+a)*sin(b*x+a)/b^2+3*d^4*sin(b*x+
a)^2/b^5-6*d^2*(d*x+c)^2*sin(b*x+a)^2/b^3+2*(d*x+c)^4*sin(b*x+a)^2/b
```

Rubi [A] time = 0.50, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 12, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {4431, 4404, 3311, 32, 3310, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{b^3}+\frac{3id^3(c+dx)\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{b^4}-\frac{2id(c+dx)^3\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (6*c*d^3*x)/b^3 + (3*d^4*x^2)/b^3 - (c + d*x)^4/b - ((I/5)*(c + d*x)^5)/d +
 ((c + d*x)^4*Log[1 + E^((2*I)*(a + b*x))])/b - ((2*I)*d*(c + d*x)^3*PolyLo
g[2, -E^((2*I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)^2*PolyLog[3, -E^((2*I)*(
a + b*x))])/b^3 + ((3*I)*d^3*(c + d*x)*PolyLog[4, -E^((2*I)*(a + b*x))])/b^
4 - (3*d^4*PolyLog[5, -E^((2*I)*(a + b*x))])/(2*b^5) - (6*d^3*(c + d*x)*Cos
[a + b*x]*Sin[a + b*x])/b^4 + (4*d*(c + d*x)^3*Cos[a + b*x]*Sin[a + b*x])/b
^2 + (3*d^4*Sin[a + b*x]^2)/b^5 - (6*d^2*(c + d*x)^2*Sin[a + b*x]^2)/b^3 +
(2*(c + d*x)^4*Sin[a + b*x]^2)/b
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
```

```
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^4 \sec(a+bx)\sin(3a+3bx)\,dx = \int \Big(3(c+dx)^4\cos(a+bx)\sin(a+bx) - (c+dx)^4\sin^2(a+bx)\tan(a+$$

$$= 3\int (c+dx)^4\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^4\sin^2(a+bx)\tan$$

$$= \frac{3(c+dx)^4\sin^2(a+bx)}{2b} - \frac{(6d)\int (c+dx)^3\sin^2(a+bx)\,dx}{b} + \int (c+dx$$

$$= -\frac{i(c+dx)^5}{5d} + \frac{3d(c+dx)^3\cos(a+bx)\sin(a+bx)}{b^2} - \frac{9d^2(c+dx)^2\sin^2}{2b^3}$$

$$= -\frac{3(c+dx)^4}{4b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3(c+dx)}{}$$

$$= \frac{9cd^3x}{2b^3} + \frac{9d^4x^2}{4b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

$$= \frac{6cd^3x}{b^3} + \frac{3d^4x^2}{b^3} - \frac{(c+dx)^4}{b} - \frac{i(c+dx)^5}{5d} + \frac{(c+dx)^4\log\left(1+e^{2i(a+bx)}\right)}{b}$$

<u>Mathematica</u> [B] time = 6.75, size = 2482, normalized size = 8.30

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^4*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/2)*c^2*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I
)*(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))]
- (3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E
^(I*a)) + (I/2)*c*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*
a))*x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*
PolyLog[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*
x))] - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] - (d^4
*((-4*I)*x^5 - (10*(1 + E^((2*I)*a))*x^4*Log[1 + E^((-2*I)*(a + b*x))])/b +
 (5*(1 + E^((2*I)*a))*((-4*I)*b^3*x^3*PolyLog[2, -E^((-2*I)*(a + b*x))] - 6
*b^2*x^2*PolyLog[3, -E^((-2*I)*(a + b*x))] + (6*I)*b*x*PolyLog[4, -E^((-2*I
```

)*(a + b*x))] + 3*PolyLog[5, -E^((-2*I)*(a + b*x))]))/b^5)*Sec[a])/(20*E^(I
*a)) + (c^4*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin
[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (2*c^3*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Co
t[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x
)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + P
i*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyL
og[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*
Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(40*b^5) -
 ((I/40)*Sin[2*a + 2*b*x])/b^5)*(-20*b^4*c^4*Cos[a] + (40*I)*b^3*c^3*d*Cos[
a] + 60*b^2*c^2*d^2*Cos[a] - (60*I)*b*c*d^3*Cos[a] - 30*d^4*Cos[a] - 80*b^4
*c^3*d*x*Cos[a] + (120*I)*b^3*c^2*d^2*x*Cos[a] + 120*b^2*c*d^3*x*Cos[a] - (
60*I)*b*d^4*x*Cos[a] - 120*b^4*c^2*d^2*x^2*Cos[a] + (120*I)*b^3*c*d^3*x^2*C
os[a] + 60*b^2*d^4*x^2*Cos[a] - 80*b^4*c*d^3*x^3*Cos[a] + (40*I)*b^3*d^4*x^
3*Cos[a] - 20*b^4*d^4*x^4*Cos[a] - (20*I)*b^5*c^4*x*Cos[a + 2*b*x] - (40*I)
*b^5*c^3*d*x^2*Cos[a + 2*b*x] - (40*I)*b^5*c^2*d^2*x^3*Cos[a + 2*b*x] - (20
*I)*b^5*c*d^3*x^4*Cos[a + 2*b*x] - (4*I)*b^5*d^4*x^5*Cos[a + 2*b*x] + (20*I
)*b^5*c^4*x*Cos[3*a + 2*b*x] + (40*I)*b^5*c^3*d*x^2*Cos[3*a + 2*b*x] + (40*
I)*b^5*c^2*d^2*x^3*Cos[3*a + 2*b*x] + (20*I)*b^5*c*d^3*x^4*Cos[3*a + 2*b*x]
 + (4*I)*b^5*d^4*x^5*Cos[3*a + 2*b*x] - 10*b^4*c^4*Cos[3*a + 4*b*x] - (20*I
)*b^3*c^3*d*Cos[3*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[3*a + 4*b*x] + (30*I)*b*c
*d^3*Cos[3*a + 4*b*x] - 15*d^4*Cos[3*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[3*a +
4*b*x] - (60*I)*b^3*c^2*d^2*x*Cos[3*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[3*a + 4
*b*x] + (30*I)*b*d^4*x*Cos[3*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[3*a + 4*b*
x] - (60*I)*b^3*c*d^3*x^2*Cos[3*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[3*a + 4*b*x
] - 40*b^4*c*d^3*x^3*Cos[3*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[3*a + 4*b*x]
 - 10*b^4*d^4*x^4*Cos[3*a + 4*b*x] - 10*b^4*c^4*Cos[5*a + 4*b*x] - (20*I)*b
^3*c^3*d*Cos[5*a + 4*b*x] + 30*b^2*c^2*d^2*Cos[5*a + 4*b*x] + (30*I)*b*c*d^
3*Cos[5*a + 4*b*x] - 15*d^4*Cos[5*a + 4*b*x] - 40*b^4*c^3*d*x*Cos[5*a + 4*b
*x] - (60*I)*b^3*c^2*d^2*x*Cos[5*a + 4*b*x] + 60*b^2*c*d^3*x*Cos[5*a + 4*b*
x] + (30*I)*b*d^4*x*Cos[5*a + 4*b*x] - 60*b^4*c^2*d^2*x^2*Cos[5*a + 4*b*x]
- (60*I)*b^3*c*d^3*x^2*Cos[5*a + 4*b*x] + 30*b^2*d^4*x^2*Cos[5*a + 4*b*x] -
 40*b^4*c*d^3*x^3*Cos[5*a + 4*b*x] - (20*I)*b^3*d^4*x^3*Cos[5*a + 4*b*x] -
10*b^4*d^4*x^4*Cos[5*a + 4*b*x] + 20*b^5*c^4*x*Sin[a + 2*b*x] + 40*b^5*c^3*
d*x^2*Sin[a + 2*b*x] + 40*b^5*c^2*d^2*x^3*Sin[a + 2*b*x] + 20*b^5*c*d^3*x^4
*Sin[a + 2*b*x] + 4*b^5*d^4*x^5*Sin[a + 2*b*x] - 20*b^5*c^4*x*Sin[3*a + 2*b
*x] - 40*b^5*c^3*d*x^2*Sin[3*a + 2*b*x] - 40*b^5*c^2*d^2*x^3*Sin[3*a + 2*b*
x] - 20*b^5*c*d^3*x^4*Sin[3*a + 2*b*x] - 4*b^5*d^4*x^5*Sin[3*a + 2*b*x] - (
10*I)*b^4*c^4*Sin[3*a + 4*b*x] + 20*b^3*c^3*d*Sin[3*a + 4*b*x] + (30*I)*b^2
*c^2*d^2*Sin[3*a + 4*b*x] - 30*b*c*d^3*Sin[3*a + 4*b*x] - (15*I)*d^4*Sin[3*
a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[3*a + 4*b*x] + 60*b^3*c^2*d^2*x*Sin[3*a
 + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[3*a + 4*b*x] - 30*b*d^4*x*Sin[3*a + 4*b*
x] - (60*I)*b^4*c^2*d^2*x^2*Sin[3*a + 4*b*x] + 60*b^3*c*d^3*x^2*Sin[3*a + 4
*b*x] + (30*I)*b^2*d^4*x^2*Sin[3*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3*Sin[3*a
+ 4*b*x] + 20*b^3*d^4*x^3*Sin[3*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Sin[3*a + 4
*b*x] - (10*I)*b^4*c^4*Sin[5*a + 4*b*x] + 20*b^3*c^3*d*Sin[5*a + 4*b*x] + (

```
30*I)*b^2*c^2*d^2*Sin[5*a + 4*b*x] - 30*b*c*d^3*Sin[5*a + 4*b*x] - (15*I)*d
^4*Sin[5*a + 4*b*x] - (40*I)*b^4*c^3*d*x*Sin[5*a + 4*b*x] + 60*b^3*c^2*d^2*
x*Sin[5*a + 4*b*x] + (60*I)*b^2*c*d^3*x*Sin[5*a + 4*b*x] - 30*b*d^4*x*Sin[5
*a + 4*b*x] - (60*I)*b^4*c^2*d^2*x^2*Sin[5*a + 4*b*x] + 60*b^3*c*d^3*x^2*Si
n[5*a + 4*b*x] + (30*I)*b^2*d^4*x^2*Sin[5*a + 4*b*x] - (40*I)*b^4*c*d^3*x^3
*Sin[5*a + 4*b*x] + 20*b^3*d^4*x^3*Sin[5*a + 4*b*x] - (10*I)*b^4*d^4*x^4*Si
n[5*a + 4*b*x])
```

fricas [C] time = 0.64, size = 1644, normalized size = 5.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 24*d^4*polylog(5, I*cos(b*x + a) + s
in(b*x + a)) - 24*d^4*polylog(5, I*cos(b*x + a) - sin(b*x + a)) - 24*d^4*po
lylog(5, -I*cos(b*x + a) + sin(b*x + a)) - 24*d^4*polylog(5, -I*cos(b*x + a
) - sin(b*x + a)) + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 - 2*(2*b^4*d^4*x^4 + 8*
b^4*c*d^3*x^3 + 2*b^4*c^4 - 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*
d^4)*x^2 + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x)*cos(b*x + a)^2 + 4*(2*b^3*d^4*x
^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 - b*d^4)*
x)*cos(b*x + a)*sin(b*x + a) + 4*(2*b^4*c^3*d - 3*b^2*c*d^3)*x + (4*I*b^3*d
^4*x^3 + 12*I*b^3*c*d^3*x^2 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(I*c
os(b*x + a) + sin(b*x + a)) + (-4*I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I
*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-4*
I*b^3*d^4*x^3 - 12*I*b^3*c*d^3*x^2 - 12*I*b^3*c^2*d^2*x - 4*I*b^3*c^3*d)*di
log(-I*cos(b*x + a) + sin(b*x + a)) + (4*I*b^3*d^4*x^3 + 12*I*b^3*c*d^3*x^2
 + 12*I*b^3*c^2*d^2*x + 4*I*b^3*c^3*d)*dilog(-I*cos(b*x + a) - sin(b*x + a)
) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)
*log(cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*
b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(cos(b*x + a) - I*sin(b*x + a) +
I) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4
*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^
2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4
*d^4)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^4*d^4*x^4 + 4*b^4*c*d^3*x
^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*d - 6*a^2*b^2*c^2*d^2
+ 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^4*d
^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^4*c^3*d*x + 4*a*b^3*c^3*
d - 6*a^2*b^2*c^2*d^2 + 4*a^3*b*c*d^3 - a^4*d^4)*log(-I*cos(b*x + a) - sin(
b*x + a) + 1) + (b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^
3 + a^4*d^4)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*log(-cos(b*x + a) - I*s
in(b*x + a) + I) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a)
```

```
 + sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, I*cos(b*x + a)
- sin(b*x + a)) + (24*I*b*d^4*x + 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a)
+ sin(b*x + a)) + (-24*I*b*d^4*x - 24*I*b*c*d^3)*polylog(4, -I*cos(b*x + a)
 - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3
, I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^
2*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 12*(b^2*d^4*x^2 + 2*b^2*
c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 12*(b^2
*d^4*x^2 + 2*b^2*c*d^3*x + b^2*c^2*d^2)*polylog(3, -I*cos(b*x + a) - sin(b*
x + a)))/b^5
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^4 \sec(bx+a)\sin(3\,bx+3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^4*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.13, size = 956, normalized size = 3.20

$$\frac{3c^2d^2\operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)}{b^3}+\frac{3d^4\operatorname{polylog}\left(3,-e^{2i(bx+a)}\right)x^2}{b^3}+\frac{d^4\ln\left(1+e^{2i(bx+a)}\right)x^4}{b}+\frac{2id^4a^4x}{b^4}-\frac{4ic^3d\,a^2}{b^2}+\frac{8ic^2d^2}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -3/2*d^4*polylog(5,-exp(2*I*(b*x+a)))/b^5-2/b^5*d^4*a^4*ln(exp(I*(b*x+a)))+
8/5*I/b^5*d^4*a^5-I*c*d^3*x^4-2*I*c^2*d^2*x^3-2*I*c^3*d*x^2+3/b^3*c^2*d^2*p
olylog(3,-exp(2*I*(b*x+a)))+3/b^3*d^4*polylog(3,-exp(2*I*(b*x+a)))*x^2+4/b*
c*d^3*ln(1+exp(2*I*(b*x+a)))*x^3+I*c^4*x+1/b*c^4*ln(1+exp(2*I*(b*x+a)))-1/4
*(2*d^4*x^4*b^4+4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3+12*I*b^3*c*d^3*x^2+12*b^4*c
^2*d^2*x^2+12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x+4*I*b^3*c^3*d+2*b^4*c^4-6*b^2*d
^4*x^2-6*I*b*d^4*x-12*b^2*c*d^3*x-6*I*b*c*d^3-6*c^2*d^2*b^2+3*d^4)/b^5*exp(
2*I*(b*x+a))-1/4*(2*d^4*x^4*b^4-4*I*b^3*d^4*x^3+8*b^4*c*d^3*x^3-12*I*b^3*c*
d^3*x^2+12*b^4*c^2*d^2*x^2-12*I*b^3*c^2*d^2*x+8*b^4*c^3*d*x-4*I*b^3*c^3*d+2
*b^4*c^4-6*b^2*d^4*x^2+6*I*b*d^4*x-12*b^2*c*d^3*x+6*I*b*c*d^3-6*c^2*d^2*b^2
+3*d^4)/b^5*exp(-2*I*(b*x+a))-2/b*c^4*ln(exp(I*(b*x+a)))-1/5*I*d^4*x^5+1/b*
d^4*ln(1+exp(2*I*(b*x+a)))*x^4+2*I/b^4*d^4*a^4*x-4*I/b^2*c^3*d*a^2+8*I/b^3*
c^2*d^2*a^3-6*I/b^4*c*d^3*a^4+8/b^2*c^3*d*a*ln(exp(I*(b*x+a)))+8/b^4*c*d^3*
a^3*ln(exp(I*(b*x+a)))-12/b^3*c^2*d^2*a^2*ln(exp(I*(b*x+a)))-8*I/b^3*c*d^3*
a^3*x+12*I/b^2*c^2*d^2*a^2*x-8*I/b*c^3*d*a*x+3*I/b^4*c*d^3*polylog(4,-exp(2
*I*(b*x+a)))+3*I/b^4*d^4*polylog(4,-exp(2*I*(b*x+a)))*x-2*I/b^2*d^4*polylog
(2,-exp(2*I*(b*x+a)))*x^3-2*I/b^2*c^3*d*polylog(2,-exp(2*I*(b*x+a)))+4/b*c^
```

```
3*d*ln(1+exp(2*I*(b*x+a)))*x+6/b^3*c*d^3*polylog(3,-exp(2*I*(b*x+a)))*x+6/b
*c^2*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-6*I/b^2*c^2*d^2*polylog(2,-exp(2*I*(b*x
+a)))*x-6*I/b^2*c*d^3*polylog(2,-exp(2*I*(b*x+a)))*x^2
```

maxima [B] time = 0.52, size = 607, normalized size = 2.03

$$-\frac{c^4\left(2\cos\left(2bx+2a\right)-\log\left(\cos\left(2bx\right)^2+2\cos\left(2bx\right)\cos\left(2a\right)+\cos\left(2a\right)^2+\sin\left(2bx\right)^2-2\sin\left(2bx\right)\sin\left(2a\right)\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^4*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/30*(-
6*I*b^5*d^4*x^5 - 30*I*b^5*c*d^3*x^4 - 60*I*b^5*c^2*d^2*x^3 - 60*I*b^5*c^3*
d*x^2 - 90*d^4*polylog(5, -e^(2*I*b*x + 2*I*a)) + (60*I*b^4*d^4*x^4 + 160*I
*b^4*c*d^3*x^3 + 180*I*b^4*c^2*d^2*x^2 + 120*I*b^4*c^3*d*x)*arctan2(sin(2*b
*x + 2*a), cos(2*b*x + 2*a) + 1) - 15*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 - 6*
b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 - b^2*d^4)*x^2 + 4*(2*b^4*c^3*d - 3*
b^2*c*d^3)*x)*cos(2*b*x + 2*a) + (-120*I*b^3*d^4*x^3 - 240*I*b^3*c*d^3*x^2
- 180*I*b^3*c^2*d^2*x - 60*I*b^3*c^3*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 10*(3
*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 9*b^4*c^2*d^2*x^2 + 6*b^4*c^3*d*x)*log(cos
(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + (180*I*b*d
^4*x + 120*I*b*c*d^3)*polylog(4, -e^(2*I*b*x + 2*I*a)) + 30*(6*b^2*d^4*x^2
+ 8*b^2*c*d^3*x + 3*b^2*c^2*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 30*(2*b
^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d - 3*b*c*d^3 + 3*(2*b^3*c^2*d^2 -
 b*d^4)*x)*sin(2*b*x + 2*a))/b^5
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)\,{\left(c+d\,x\right)}^4}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^4)/cos(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.383 $\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=242

$$\frac{3id^3\text{Li}_4\left(-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^3\sin(a+bx)\cos(a+bx)}{2b^4}+\frac{3d^2(c+dx)\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{3d^2(c+dx)\sin^2(a+bx)}{b^3}-\frac{3id(c+$$

```
[Out] 3/2*d^3*x/b^3-(d*x+c)^3/b-1/4*I*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*I*(b*x+a))
)/b-3/2*I*d*(d*x+c)^2*polylog(2,-exp(2*I*(b*x+a)))/b^2+3/2*d^2*(d*x+c)*poly
log(3,-exp(2*I*(b*x+a)))/b^3+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/b^4-3/2
*d^3*cos(b*x+a)*sin(b*x+a)/b^4+3*d*(d*x+c)^2*cos(b*x+a)*sin(b*x+a)/b^2-3*d^
2*(d*x+c)*sin(b*x+a)^2/b^3+2*(d*x+c)^3*sin(b*x+a)^2/b
```

Rubi [A] time = 0.45, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {4431, 4404, 3311, 32, 2635, 8, 4407, 3719, 2190, 2531, 6609, 2282, 6589}

$$\frac{3d^2(c+dx)\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}-\frac{3id(c+dx)^2\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{3id^3\text{PolyLog}\left(4,-e^{2i(a+bx)}\right)}{4b^4}-\frac{3d^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (3*d^3*x)/(2*b^3) - (c + d*x)^3/b - ((I/4)*(c + d*x)^4)/d + ((c + d*x)^3*Lo
g[1 + E^((2*I)*(a + b*x))])/b - (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, -E^((2*
I)*(a + b*x))])/b^2 + (3*d^2*(c + d*x)*PolyLog[3, -E^((2*I)*(a + b*x))])/(2
*b^3) + (((3*I)/4)*d^3*PolyLog[4, -E^((2*I)*(a + b*x))])/b^4 - (3*d^3*Cos[a
 + b*x]*Sin[a + b*x])/(2*b^4) + (3*d*(c + d*x)^2*Cos[a + b*x]*Sin[a + b*x])
/b^2 - (3*d^2*(c + d*x)*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^3*Sin[a + b*x]^2
)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
```

```
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3311

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(d*m*(c + d*x)^(m - 1)*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist
[(b^2*(n - 1))/n, Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[(
d^2*m*(m - 1))/(f^2*n^2), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[(b*(c + d*x)^m*Cos[e + f*x]*(b*Sin[e + f*x])^(n - 1))/(f*n), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6609

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[((e + f*x)^m*PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p])/(b*c*p*Log[F]), x] - Dist[(f*m)/(b*c*p*Log[F]), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\cos(a+bx)\sin(a+bx) - (c+dx)^3\sin^2(a+bx)\tan(a+\right.$$

$$= 3\int (c+dx)^3\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)^3\sin^2(a+bx)\tan$$

$$= \frac{3(c+dx)^3\sin^2(a+bx)}{2b} - \frac{(9d)\int (c+dx)^2\sin^2(a+bx)\,dx}{2b} + \int (c+dx$$

$$= -\frac{i(c+dx)^4}{4d} + \frac{9d(c+dx)^2\cos(a+bx)\sin(a+bx)}{4b^2} - \frac{9d^2(c+dx)\sin^2(}{4b^3}$$

$$= -\frac{3(c+dx)^3}{4b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{9d^3\cos(a+}{}$$

$$= \frac{9d^3x}{8b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

$$= \frac{3d^3x}{2b^3} - \frac{(c+dx)^3}{b} - \frac{i(c+dx)^4}{4d} + \frac{(c+dx)^3\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{3id(c+}{}$$

Mathematica [B] time = 6.64, size = 1719, normalized size = 7.10

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/4)*c*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*
(a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] -
(3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(
I*a)) + (I/8)*d^3*E^(I*a)*((2*x^4)/E^((2*I)*a) - ((4*I)*(1 + E^((-2*I)*a))*
x^3*Log[1 + E^((-2*I)*(a + b*x))])/b + (3*(1 + E^((2*I)*a))*(2*b^2*x^2*Poly
Log[2, -E^((-2*I)*(a + b*x))] - (2*I)*b*x*PolyLog[3, -E^((-2*I)*(a + b*x))]
 - PolyLog[4, -E^((-2*I)*(a + b*x))]))/(b^4*E^((2*I)*a)))*Sec[a] + (c^3*Sec
[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin[a]))/(b*(Cos[a
]^2 + Sin[a]^2)) + (3*c^2*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a
]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - A
rcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]]
 - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*
```

```
(b*x - ArcTan[Cot[a]]))])))/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2
*(Cos[a]^2 + Sin[a]^2)]) + Sec[a]*(Cos[2*a + 2*b*x]/(16*b^4) - ((I/16)*Sin[
2*a + 2*b*x])/b^4)*(-8*b^3*c^3*Cos[a] + (12*I)*b^2*c^2*d*Cos[a] + 12*b*c*d^
2*Cos[a] - (6*I)*d^3*Cos[a] - 24*b^3*c^2*d*x*Cos[a] + (24*I)*b^2*c*d^2*x*Co
s[a] + 12*b*d^3*x*Cos[a] - 24*b^3*c*d^2*x^2*Cos[a] + (12*I)*b^2*d^3*x^2*Cos
[a] - 8*b^3*d^3*x^3*Cos[a] - (8*I)*b^4*c^3*x*Cos[a + 2*b*x] - (12*I)*b^4*c^
2*d*x^2*Cos[a + 2*b*x] - (8*I)*b^4*c*d^2*x^3*Cos[a + 2*b*x] - (2*I)*b^4*d^3
*x^4*Cos[a + 2*b*x] + (8*I)*b^4*c^3*x*Cos[3*a + 2*b*x] + (12*I)*b^4*c^2*d*x
^2*Cos[3*a + 2*b*x] + (8*I)*b^4*c*d^2*x^3*Cos[3*a + 2*b*x] + (2*I)*b^4*d^3*
x^4*Cos[3*a + 2*b*x] - 4*b^3*c^3*Cos[3*a + 4*b*x] - (6*I)*b^2*c^2*d*Cos[3*a
 + 4*b*x] + 6*b*c*d^2*Cos[3*a + 4*b*x] + (3*I)*d^3*Cos[3*a + 4*b*x] - 12*b^
3*c^2*d*x*Cos[3*a + 4*b*x] - (12*I)*b^2*c*d^2*x*Cos[3*a + 4*b*x] + 6*b*d^3*
x*Cos[3*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[3*a + 4*b*x] - (6*I)*b^2*d^3*x^2*
Cos[3*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[3*a + 4*b*x] - 4*b^3*c^3*Cos[5*a + 4*b
*x] - (6*I)*b^2*c^2*d*Cos[5*a + 4*b*x] + 6*b*c*d^2*Cos[5*a + 4*b*x] + (3*I)
*d^3*Cos[5*a + 4*b*x] - 12*b^3*c^2*d*x*Cos[5*a + 4*b*x] - (12*I)*b^2*c*d^2*
x*Cos[5*a + 4*b*x] + 6*b*d^3*x*Cos[5*a + 4*b*x] - 12*b^3*c*d^2*x^2*Cos[5*a
+ 4*b*x] - (6*I)*b^2*d^3*x^2*Cos[5*a + 4*b*x] - 4*b^3*d^3*x^3*Cos[5*a + 4*b
*x] + 8*b^4*c^3*x*Sin[a + 2*b*x] + 12*b^4*c^2*d*x^2*Sin[a + 2*b*x] + 8*b^4*
c*d^2*x^3*Sin[a + 2*b*x] + 2*b^4*d^3*x^4*Sin[a + 2*b*x] - 8*b^4*c^3*x*Sin[3
*a + 2*b*x] - 12*b^4*c^2*d*x^2*Sin[3*a + 2*b*x] - 8*b^4*c*d^2*x^3*Sin[3*a +
 2*b*x] - 2*b^4*d^3*x^4*Sin[3*a + 2*b*x] - (4*I)*b^3*c^3*Sin[3*a + 4*b*x] +
 6*b^2*c^2*d*Sin[3*a + 4*b*x] + (6*I)*b*c*d^2*Sin[3*a + 4*b*x] - 3*d^3*Sin[
3*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[3*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[3*a
 + 4*b*x] + (6*I)*b*d^3*x*Sin[3*a + 4*b*x] - (12*I)*b^3*c*d^2*x^2*Sin[3*a +
 4*b*x] + 6*b^2*d^3*x^2*Sin[3*a + 4*b*x] - (4*I)*b^3*d^3*x^3*Sin[3*a + 4*b*
x] - (4*I)*b^3*c^3*Sin[5*a + 4*b*x] + 6*b^2*c^2*d*Sin[5*a + 4*b*x] + (6*I)*
b*c*d^2*Sin[5*a + 4*b*x] - 3*d^3*Sin[5*a + 4*b*x] - (12*I)*b^3*c^2*d*x*Sin[
5*a + 4*b*x] + 12*b^2*c*d^2*x*Sin[5*a + 4*b*x] + (6*I)*b*d^3*x*Sin[5*a + 4*
b*x] - (12*I)*b^3*c*d^2*x^2*Sin[5*a + 4*b*x] + 6*b^2*d^3*x^2*Sin[5*a + 4*b*
x] - (4*I)*b^3*d^3*x^3*Sin[5*a + 4*b*x])
```

fricas [C] time = 0.60, size = 1122, normalized size = 4.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 - 6*I*d^3*polylog(4, I*cos(b*x + a) +
sin(b*x + a)) + 6*I*d^3*polylog(4, I*cos(b*x + a) - sin(b*x + a)) + 6*I*d^3
*polylog(4, -I*cos(b*x + a) + sin(b*x + a)) - 6*I*d^3*polylog(4, -I*cos(b*x
 + a) - sin(b*x + a)) - 2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 - 3*
b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(b*x + a)^2 + 3*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*cos(b*x + a)*sin(b*x + a) + 3*(2*b^3*c^2*d
```

```
 - b*d^3)*x + (3*I*b^2*d^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(I*c
os(b*x + a) + sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*c*d^2*x - 3*I*b^2
*c^2*d)*dilog(I*cos(b*x + a) - sin(b*x + a)) + (-3*I*b^2*d^3*x^2 - 6*I*b^2*
c*d^2*x - 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) + sin(b*x + a)) + (3*I*b^2*d
^3*x^2 + 6*I*b^2*c*d^2*x + 3*I*b^2*c^2*d)*dilog(-I*cos(b*x + a) - sin(b*x +
 a)) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(cos(b*x + a)
 + I*sin(b*x + a) + I) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 +
 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a
) + sin(b*x + a) + 1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*
a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*log(I*cos(b*x + a) - sin(b*x + a) +
1) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2
*b*c*d^2 + a^3*d^3)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^3*d^3*x^3
+ 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b^3*c^3 - 3*a*b^2*c^2*d + 3*a^
2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) + I*sin(b*x + a) + I) + (b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*log(-cos(b*x + a) - I*sin(b*x + a)
+ I) + 6*(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) + 6*
(b*d^3*x + b*c*d^2)*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*(b*d^3*x
+ b*c*d^2)*polylog(3, -I*cos(b*x + a) + sin(b*x + a)) + 6*(b*d^3*x + b*c*d^
2)*polylog(3, -I*cos(b*x + a) - sin(b*x + a)))/b^4
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \sec(bx+a)\sin(3\,bx+3\,a)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.12, size = 639, normalized size = 2.64

$$\frac{3d^3\,\mathrm{polylog}\left(3,-e^{2i(bx+a)}\right)x}{2b^3}+\frac{d^3\ln\left(1+e^{2i(bx+a)}\right)x^3}{b}+\frac{6ic\,d^2a^2x}{b^2}-\frac{6ic^2dax}{b}+\frac{2d^3a^3\ln\left(e^{i(bx+a)}\right)}{b^4}-\frac{3ia^4d^3}{2b^4}-ic\,d^2x^3-\frac{3i}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] 6*I/b^2*c*d^2*a^2*x-6*I/b*c^2*d*a*x+3/4*I*d^3*polylog(4,-exp(2*I*(b*x+a)))/
b^4+I*c^3*x+1/b*c^3*ln(1+exp(2*I*(b*x+a)))+2/b^4*d^3*a^3*ln(exp(I*(b*x+a)))
-3/2*I/b^4*a^4*d^3-I*c*d^2*x^3-3/2*I*c^2*d*x^2-1/8*(4*d^3*x^3*b^3+6*I*b^2*d
^3*x^2+12*b^3*c*d^2*x^2+12*I*b^2*c*d^2*x+12*b^3*c^2*d*x+6*I*b^2*c^2*d+4*b^3
*c^3-6*b*d^3*x-3*I*d^3-6*c*d^2*b)/b^4*exp(2*I*(b*x+a))-1/8*(4*d^3*x^3*b^3-6
```

```
*I*b^2*d^3*x^2+12*b^3*c*d^2*x^2-12*I*b^2*c*d^2*x+12*b^3*c^2*d*x-6*I*b^2*c^2
*d+4*b^3*c^3-6*b*d^3*x+3*I*d^3-6*c*d^2*b)/b^4*exp(-2*I*(b*x+a))+3/2/b^3*c*d
^2*polylog(3,-exp(2*I*(b*x+a)))+3/2/b^3*d^3*polylog(3,-exp(2*I*(b*x+a)))*x-
1/4*I*d^3*x^4-2/b*c^3*ln(exp(I*(b*x+a)))-3/2*I/b^2*d^3*polylog(2,-exp(2*I*(
b*x+a)))*x^2-3/2*I/b^2*c^2*d*polylog(2,-exp(2*I*(b*x+a)))-6/b^3*c*d^2*a^2*l
n(exp(I*(b*x+a)))-3*I/b^2*c^2*d*a^2-2*I/b^3*a^3*d^3*x+4*I/b^3*c*d^2*a^3+6/b
^2*c^2*d*a*ln(exp(I*(b*x+a)))+1/b*d^3*ln(1+exp(2*I*(b*x+a)))*x^3-3*I/b^2*po
lylog(2,-exp(2*I*(b*x+a)))*c*d^2*x+3/b*c^2*d*ln(1+exp(2*I*(b*x+a)))*x+3/b*c
*d^2*ln(1+exp(2*I*(b*x+a)))*x^2
```

maxima [B] time = 0.48, size = 442, normalized size = 1.83

$$-\frac{c^3\left(2\cos(2bx+2a)-\log\left(\cos(2bx)^2+2\cos(2bx)\cos(2a)+\cos(2a)^2+\sin(2bx)^2-2\sin(2bx)\sin(2a)\right.\right.}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^3*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/12*(-
3*I*b^4*d^3*x^4 - 12*I*b^4*c*d^2*x^3 - 18*I*b^4*c^2*d*x^2 + 12*I*d^3*polylo
g(4, -e^(2*I*b*x + 2*I*a)) + (16*I*b^3*d^3*x^3 + 36*I*b^3*c*d^2*x^2 + 36*I*
b^3*c^2*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2*a) + 1) - 6*(2*b^3*d^3
*x^3 + 6*b^3*c*d^2*x^2 - 3*b*c*d^2 + 3*(2*b^3*c^2*d - b*d^3)*x)*cos(2*b*x +
 2*a) + (-24*I*b^2*d^3*x^2 - 36*I*b^2*c*d^2*x - 18*I*b^2*c^2*d)*dilog(-e^(2
*I*b*x + 2*I*a)) + 2*(4*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 9*b^3*c^2*d*x)*log(
cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1) + 6*(4*b*
d^3*x + 3*b*c*d^2)*polylog(3, -e^(2*I*b*x + 2*I*a)) + 9*(2*b^2*d^3*x^2 + 4*
b^2*c*d^2*x + 2*b^2*c^2*d - d^3)*sin(2*b*x + 2*a))/b^4
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(3a+3bx)\,(c+dx)^3}{\cos(a+bx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.384 $\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=173

$$\frac{d^2\text{Li}_3\left(-e^{2i(a+bx)}\right)}{2b^3}-\frac{d^2\sin^2(a+bx)}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-e^{2i(a+bx)}\right)}{b^2}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)^2\log}{}$$

```
[Out] -2*c*d*x/b-d^2*x^2/b-1/3*I*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*I*(b*x+a)))/b-I
*d*(d*x+c)*polylog(2,-exp(2*I*(b*x+a)))/b^2+1/2*d^2*polylog(3,-exp(2*I*(b*x
+a)))/b^3+2*d*(d*x+c)*cos(b*x+a)*sin(b*x+a)/b^2-d^2*sin(b*x+a)^2/b^3+2*(d*x
+c)^2*sin(b*x+a)^2/b
```

Rubi [A] time = 0.33, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {4431, 4404, 3310, 4407, 3719, 2190, 2531, 2282, 6589}

$$-\frac{id(c+dx)\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{b^2}+\frac{d^2\text{PolyLog}\left(3,-e^{2i(a+bx)}\right)}{2b^3}+\frac{2d(c+dx)\sin(a+bx)\cos(a+bx)}{b^2}-\frac{d^2\sin^2(a+}{b^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (-2*c*d*x)/b - (d^2*x^2)/b - ((I/3)*(c + d*x)^3)/d + ((c + d*x)^2*Log[1 + E
^((2*I)*(a + b*x))])/b - (I*d*(c + d*x)*PolyLog[2, -E^((2*I)*(a + b*x))])/b
^2 + (d^2*PolyLog[3, -E^((2*I)*(a + b*x))])/(2*b^3) + (2*d*(c + d*x)*Cos[a
+ b*x]*Sin[a + b*x])/b^2 - (d^2*Sin[a + b*x]^2)/b^3 + (2*(c + d*x)^2*Sin[a
+ b*x]^2)/b
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
, g, n}, x] && GtQ[m, 0]
```

Rule 3310

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(d*(b*Sin[e + f*x])^n)/(f^2*n^2), x] + (Dist[(b^2*(n - 1))/n, Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[(b*(c + d*x)*Cos[e + f*x]*(b
*Sin[e + f*x])^(n - 1))/(f*n), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^2 \sec(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\cos(a+bx)\sin(a+bx)-(c+dx)^2\sin^2(a+bx)\tan(a\right.$$

$$= 3\int (c+dx)^2\cos(a+bx)\sin(a+bx)\,dx-\int (c+dx)^2\sin^2(a+bx)\,t$$

$$= \frac{3(c+dx)^2\sin^2(a+bx)}{2b}-\frac{(3d)\int (c+dx)\sin^2(a+bx)\,dx}{b}+\int (c+d$$

$$= -\frac{i(c+dx)^3}{3d}+\frac{3d(c+dx)\cos(a+bx)\sin(a+bx)}{2b^2}-\frac{3d^2\sin^2(a+bx)}{4b^3}$$

$$= -\frac{3cdx}{2b}-\frac{3d^2x^2}{4b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2d(c+}{}$$

$$= -\frac{2cdx}{b}-\frac{d^2x^2}{b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id(c+d}{}$$

$$= -\frac{2cdx}{b}-\frac{d^2x^2}{b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id(c+d}{}$$

$$= -\frac{2cdx}{b}-\frac{d^2x^2}{b}-\frac{i(c+dx)^3}{3d}+\frac{(c+dx)^2\log\left(1+e^{2i(a+bx)}\right)}{b}-\frac{id(c+d}{}$$

Mathematica [B] time = 6.56, size = 516, normalized size = 2.98

$$\frac{cd\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))}-\frac{\cot(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] ((I/12)*d^2*(2*b^2*x^2*(2*b*x - (3*I)*(1 + E^((2*I)*a))*Log[1 + E^((-2*I)*(
a + b*x))]) + 6*b*(1 + E^((2*I)*a))*x*PolyLog[2, -E^((-2*I)*(a + b*x))] - (
3*I)*(1 + E^((2*I)*a))*PolyLog[3, -E^((-2*I)*(a + b*x))])*Sec[a])/(b^3*E^(I
```

```
*a)) + (c^2*Sec[a]*(Cos[a]*Log[Cos[a]*Cos[b*x] - Sin[a]*Sin[b*x]] + b*x*Sin
[a]))/(b*(Cos[a]^2 + Sin[a]^2)) + (c*d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]
]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] -
 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Lo
g[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[Sin[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2
, E^((2*I)*(b*x - ArcTan[Cot[a]]))]))/Sqrt[1 + Cot[a]^2])*Sec[a])/(b^2*Sqrt
[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)]) - (Cos[2*b*x]*(2*b^2*c^2*Cos[2*a] - d^2*C
os[2*a] + 4*b^2*c*d*x*Cos[2*a] + 2*b^2*d^2*x^2*Cos[2*a] - 2*b*c*d*Sin[2*a]
- 2*b*d^2*x*Sin[2*a]))/(2*b^3) + ((2*b*c*d*Cos[2*a] + 2*b*d^2*x*Cos[2*a] +
2*b^2*c^2*Sin[2*a] - d^2*Sin[2*a] + 4*b^2*c*d*x*Sin[2*a] + 2*b^2*d^2*x^2*Si
n[2*a])*Sin[2*b*x])/(2*b^3) - (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tan[a])/3
```

fricas [C] time = 0.55, size = 677, normalized size = 3.91

$$\frac{2\,b^2d^2x^2 + 4\,b^2cdx - 2\left(2\,b^2d^2x^2 + 4\,b^2cdx + 2\,b^2c^2 - d^2\right)\cos\left(bx + a\right)^2 + 2\,d^2\mathrm{polylog}\left(3, i\,\cos\left(bx + a\right) + \sin\left(bx \right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - 2*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c
^2 - d^2)*cos(b*x + a)^2 + 2*d^2*polylog(3, I*cos(b*x + a) + sin(b*x + a))
+ 2*d^2*polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 2*d^2*polylog(3, -I*cos
(b*x + a) + sin(b*x + a)) + 2*d^2*polylog(3, -I*cos(b*x + a) - sin(b*x + a)
) + 4*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a) + (2*I*b*d^2*x + 2*I*b*c*
d)*dilog(I*cos(b*x + a) + sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(
I*cos(b*x + a) - sin(b*x + a)) + (-2*I*b*d^2*x - 2*I*b*c*d)*dilog(-I*cos(b*
x + a) + sin(b*x + a)) + (2*I*b*d^2*x + 2*I*b*c*d)*dilog(-I*cos(b*x + a) -
sin(b*x + a)) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(cos(b*x + a) - I*sin(b*x
+ a) + I) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(I*cos(b*x
 + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^
2)*log(I*cos(b*x + a) - sin(b*x + a) + 1) + (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*
a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + (b^2*d^2*x^2 +
 2*b^2*c*d*x + 2*a*b*c*d - a^2*d^2)*log(-I*cos(b*x + a) - sin(b*x + a) + 1)
 + (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) + I*sin(b*x + a) + I)
+ (b^2*c^2 - 2*a*b*c*d + a^2*d^2)*log(-cos(b*x + a) - I*sin(b*x + a) + I))/
b^3
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \sec\left(bx + a\right)\sin\left(3\,bx + 3\,a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.13, size = 377, normalized size = 2.18

$$-\frac{id^2x^3}{3}+\frac{2id^2a^2x}{b^2}+ic^2x-\frac{\left(2d^2x^2b^2+4b^2cdx+2ib\,d^2x+2b^2c^2+2ibcd-d^2\right)e^{2i(bx+a)}}{4b^3}-\frac{\left(2d^2x^2b^2+4b^2cdx-2ib\,d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -1/3*I*d^2*x^3-I/b^2*d^2*polylog(2,-exp(2*I*(b*x+a)))*x+2*I/b^2*d^2*a^2*x-1
/4*(2*d^2*x^2*b^2+2*I*b*d^2*x+4*b^2*c*d*x+2*I*b*c*d+2*b^2*c^2-d^2)/b^3*exp(
2*I*(b*x+a))-1/4*(2*d^2*x^2*b^2-2*I*b*d^2*x+4*b^2*c*d*x-2*I*b*c*d+2*b^2*c^2
-d^2)/b^3*exp(-2*I*(b*x+a))+1/b*c^2*ln(1+exp(2*I*(b*x+a)))-2/b*c^2*ln(exp(I
*(b*x+a)))-2/b^3*d^2*a^2*ln(exp(I*(b*x+a)))-I/b^2*c*d*polylog(2,-exp(2*I*(b
*x+a)))+I*c^2*x+4/3*I/b^3*d^2*a^3+1/b*d^2*ln(1+exp(2*I*(b*x+a)))*x^2-I*c*d*
x^2+1/2*d^2*polylog(3,-exp(2*I*(b*x+a)))/b^3+4/b^2*c*d*a*ln(exp(I*(b*x+a)))
+2/b*c*d*ln(1+exp(2*I*(b*x+a)))*x-2*I/b^2*c*d*a^2-4*I/b*c*d*a*x
```

maxima [A] time = 0.45, size = 301, normalized size = 1.74

$$-\frac{c^2\left(2\cos\left(2\,bx+2\,a\right)-\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right.\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + c
os(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b + 1/6*(-2
*I*b^3*d^2*x^3 - 6*I*b^3*c*d*x^2 + 3*d^2*polylog(3, -e^(2*I*b*x + 2*I*a)) +
 (6*I*b^2*d^2*x^2 + 12*I*b^2*c*d*x)*arctan2(sin(2*b*x + 2*a), cos(2*b*x + 2
*a) + 1) - 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x - d^2)*cos(2*b*x + 2*a) + (-6*I*b
*d^2*x - 6*I*b*c*d)*dilog(-e^(2*I*b*x + 2*I*a)) + 3*(b^2*d^2*x^2 + 2*b^2*c*
d*x)*log(cos(2*b*x + 2*a)^2 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1)
+ 6*(b*d^2*x + b*c*d)*sin(2*b*x + 2*a))/b^3
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)\,{\left(c+d\,x\right)}^2}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.385 $\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=107

$$-\frac{id\text{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-\frac{i(c+dx}{2d}$$

```
[Out] -d*x/b-1/2*I*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*I*(b*x+a)))/b-1/2*I*d*polylog(2
,-exp(2*I*(b*x+a)))/b^2+d*cos(b*x+a)*sin(b*x+a)/b^2+2*(d*x+c)*sin(b*x+a)^2/
b
```

Rubi [A] time = 0.18, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {4431, 4404, 2635, 8, 4407, 3719, 2190, 2279, 2391}

$$-\frac{id\text{PolyLog}\left(2,-e^{2i(a+bx)}\right)}{2b^2}+\frac{d\sin(a+bx)\cos(a+bx)}{b^2}+\frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b}+\frac{2(c+dx)\sin^2(a+bx)}{b}-\frac{dx}{b}-$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] -((d*x)/b) - ((I/2)*(c + d*x)^2)/d + ((c + d*x)*Log[1 + E^((2*I)*(a + b*x))
])/b - ((I/2)*d*PolyLog[2, -E^((2*I)*(a + b*x))])/b^2 + (d*Cos[a + b*x]*Sin
[a + b*x])/b^2 + (2*(c + d*x)*Sin[a + b*x]^2)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2190

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m*Log[1 + (b*(F^(g*(e + f*x)))^n)/a])/(b*f*g*n*Log[F]), x] - Di
st[(d*m)/(b*f*g*n*Log[F]), Int[(c + d*x)^(m - 1)*Log[1 + (b*(F^(g*(e + f*x)
))^n)/a], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3719

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
I*(c + d*x)^(m + 1))/(d*(m + 1)), x] - Dist[2*I, Int[((c + d*x)^m*E^(2*I*(e
 + f*x)))/(1 + E^(2*I*(e + f*x))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4404

```
Int[Cos[(a_.) + (b_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x
_)]^(n_.), x_Symbol] :> Simp[((c + d*x)^m*Sin[a + b*x]^(n + 1))/(b*(n + 1))
, x] - Dist[(d*m)/(b*(n + 1)), Int[(c + d*x)^(m - 1)*Sin[a + b*x]^(n + 1),
x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)\sec(a+bx)\sin(3a+3bx)\,dx = \int \Big(3(c+dx)\cos(a+bx)\sin(a+bx) - (c+dx)\sin^2(a+bx)\tan(a+$$
$$= 3\int (c+dx)\cos(a+bx)\sin(a+bx)\,dx - \int (c+dx)\sin^2(a+bx)\tan($$
$$= \frac{3(c+dx)\sin^2(a+bx)}{2b} - \frac{(3d)\int \sin^2(a+bx)\,dx}{2b} + \int (c+dx)\cos(a+$$
$$= -\frac{i(c+dx)^2}{2d} + \frac{3d\cos(a+bx)\sin(a+bx)}{4b^2} + \frac{2(c+dx)\sin^2(a+bx)}{b} +$$
$$= -\frac{3dx}{4b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\sin(a}{b^2}$$
$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} + \frac{d\cos(a+bx)\sin(a}{b^2}$$
$$= -\frac{dx}{b} - \frac{i(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2i(a+bx)}\right)}{b} - \frac{id\mathrm{Li}_2\left(-e^{2i(a+bx)}\right)}{2b^2} +$$

Mathematica [B] time = 5.57, size = 254, normalized size = 2.37

$$\frac{d\csc(a)\sec(a)\left(b^2x^2e^{-i\tan^{-1}(\cot(a))} - \frac{\cot(a)\left(i\mathrm{Li}_2\left(e^{2i\left(bx-\tan^{-1}(\cot(a))\right)}\right)+ibx\left(-2\tan^{-1}(\cot(a))-\pi\right)-2\left(bx-\tan^{-1}(\cot(a))\right)\log\left(1-e^{2i\left(bx-\tan^{-1}(c\right.}\right.\right.}{\sqrt{\cot^2(a)+1}}\right.}{2b^2\sqrt{\csc^2(a)\left(\sin^2(a)+\cos^2(a)\right)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*Csc[a]*((b^2*x^2)/E^(I*ArcTan[Cot[a]]) - (Cot[a]*(I*b*x*(-Pi - 2*ArcTan[
Cot[a]]) - Pi*Log[1 + E^((-2*I)*b*x)] - 2*(b*x - ArcTan[Cot[a]])*Log[1 - E^
((2*I)*(b*x - ArcTan[Cot[a]]))] + Pi*Log[Cos[b*x]] - 2*ArcTan[Cot[a]]*Log[S
in[b*x - ArcTan[Cot[a]]]] + I*PolyLog[2, E^((2*I)*(b*x - ArcTan[Cot[a]]))])
)/Sqrt[1 + Cot[a]^2])*Sec[a])/(2*b^2*Sqrt[Csc[a]^2*(Cos[a]^2 + Sin[a]^2)])
- (d*Cos[2*b*x]*(2*b*x*Cos[2*a] - Sin[2*a]))/(2*b^2) + (d*(Cos[2*a] + 2*b*x
*Sin[2*a])*Sin[2*b*x])/(2*b^2) + (c*(Log[Cos[a + b*x]] + 2*Sin[a + b*x]^2))
/b - (d*x^2*Tan[a])/2
```

fricas [B] time = 0.53, size = 340, normalized size = 3.18

$$\frac{2\,bdx - 4\,(bdx + bc)\cos(bx+a)^2 + 2\,d\cos(bx+a)\sin(bx+a) + i\,d\mathrm{Li}_2\left(i\,\cos(bx+a) + \sin(bx+a)\right) - i\,d\mathrm{Li}_2\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x - 4*(b*d*x + b*c)*cos(b*x + a)^2 + 2*d*cos(b*x + a)*sin(b*x +
a) + I*d*dilog(I*cos(b*x + a) + sin(b*x + a)) - I*d*dilog(I*cos(b*x + a) -
sin(b*x + a)) - I*d*dilog(-I*cos(b*x + a) + sin(b*x + a)) + I*d*dilog(-I*co
s(b*x + a) - sin(b*x + a)) + (b*c - a*d)*log(cos(b*x + a) + I*sin(b*x + a)
+ I) + (b*c - a*d)*log(cos(b*x + a) - I*sin(b*x + a) + I) + (b*d*x + a*d)*l
og(I*cos(b*x + a) + sin(b*x + a) + 1) + (b*d*x + a*d)*log(I*cos(b*x + a) -
sin(b*x + a) + 1) + (b*d*x + a*d)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) +
 (b*d*x + a*d)*log(-I*cos(b*x + a) - sin(b*x + a) + 1) + (b*c - a*d)*log(-c
os(b*x + a) + I*sin(b*x + a) + I) + (b*c - a*d)*log(-cos(b*x + a) - I*sin(b
*x + a) + I))/b^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c) \sec (bx + a) \sin (3\,bx + 3\,a)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sec(b*x + a)*sin(3*b*x + 3*a), x)
```

maple [A] time = 0.31, size = 177, normalized size = 1.65

$$-\frac{id\,x^2}{2}+icx-\frac{(2bdx+2cb+id)\,\mathrm{e}^{2i(bx+a)}}{4b^2}-\frac{(2bdx+2cb-id)\,\mathrm{e}^{-2i(bx+a)}}{4b^2}+\frac{c\ln\left(1+\mathrm{e}^{2i(bx+a)}\right)}{b}-\frac{2c\ln\left(\mathrm{e}^{i(bx+a)}\right)}{b}-\frac{2idax}{b}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] -1/2*I*d*x^2+I*c*x-1/4*(2*b*d*x+I*d+2*c*b)/b^2*exp(2*I*(b*x+a))-1/4*(2*b*d*
x-I*d+2*c*b)/b^2*exp(-2*I*(b*x+a))+1/b*c*ln(1+exp(2*I*(b*x+a)))-2/b*c*ln(ex
p(I*(b*x+a)))-2*I/b*d*a*x-I/b^2*d*a^2+1/b*d*ln(1+exp(2*I*(b*x+a)))*x-1/2*I*
d*polylog(2,-exp(2*I*(b*x+a)))/b^2+2/b^2*d*a*ln(exp(I*(b*x+a)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{c\left(2\,\cos\left(2\,bx+2\,a\right)-\log\left(\cos\left(2\,bx\right)^2+2\,\cos\left(2\,bx\right)\cos\left(2\,a\right)+\cos\left(2\,a\right)^2+\sin\left(2\,bx\right)^2-2\,\sin\left(2\,bx\right)\sin\left(2\,a\right)+\right.\right.}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -1/2*c*(2*cos(2*b*x + 2*a) - log(cos(2*b*x)^2 + 2*cos(2*b*x)*cos(2*a) + cos
(2*a)^2 + sin(2*b*x)^2 - 2*sin(2*b*x)*sin(2*a) + sin(2*a)^2))/b - 1/2*(2*b*
x*cos(2*b*x + 2*a) + 4*b^2*integrate(x*sin(2*b*x + 2*a)/(cos(2*b*x + 2*a)^2
 + sin(2*b*x + 2*a)^2 + 2*cos(2*b*x + 2*a) + 1), x) - sin(2*b*x + 2*a))*d/b
^2
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)\,\left(c+d\,x\right)}{\cos\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x),x)
```

```
[Out] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)*sin(3*b*x+3*a),x)
```

```
[Out] Timed out
```

3.386 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=80

$$-\mathrm{Int}\left(\frac{\tan(a+bx)}{c+dx},x\right)+\frac{2\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d}$$

```
[Out] 2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d+2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)
/d-Unintegrable(tan(b*x+a)/(d*x+c),x)
```

Rubi [A] time = 0.30, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (2*CosIntegral[(2*b*c)/d + 2*b*x]*Sin[2*a - (2*b*c)/d])/d + (2*Cos[2*a - (2
*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d - Defer[Int][Tan[a + b*x]/(c + d
*x), x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx = \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{c+dx} - \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\right)dx$$
$$= 3\int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{c+dx}\,dx$$
$$= 3\int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{c+dx}\,dx - \int \frac{\tan(a+bx)}{c+dx}\,dx$$
$$= \frac{3}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)}\,dx - \int \frac{\tan(a+bx)}{c+dx}\,dx$$
$$= \frac{1}{2}\int \frac{\sin(2a+2bx)}{c+dx}\,dx + \frac{1}{2}\left(3\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sin\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx + \frac{1}{2}\left(3\,\mathrm{s}\right.$$
$$= \frac{3\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{2d} + \frac{3\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{2d} + \frac{1}{2}\cos\left(2a\right.$$
$$= \frac{2\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d} + \frac{2\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d} - \int \frac{\tan(a\ldots}{c+\ldots}$$

Mathematica [A] time = 3.23, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x), x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c), x)
```

maple [A] time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{\sec (bx + a) \sin (3bx + 3a)}{dx + c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(i E_1\left(\frac{2i\,bdx+2i\,bc}{d}\right) - i E_1\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right) + 2\,d\int \frac{\sin(2\,bx+2\,a)}{(dx+c)\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\,a)+1\right)}\,dx +}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(1, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(1, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*d*integrate(sin(2*b*x + 2*a
)/((d*x + c)*cos(2*b*x + 2*a)^2 + (d*x + c)*sin(2*b*x + 2*a)^2 + d*x + 2*(d
*x + c)*cos(2*b*x + 2*a) + c), x) + (exp_integral_e(1, (2*I*b*d*x + 2*I*b*c
)/d) + exp_integral_e(1, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))/
d
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(3\,a+3\,b\,x\right)}{\cos\left(a+b\,x\right)\,\left(c+d\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.387 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=103

$$-\mathrm{Int}\left(\frac{\tan(a+bx)}{(c+dx)^2},x\right)+\frac{4b\cos\left(2a-\frac{2bc}{d}\right)\mathrm{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{4b\sin\left(2a-\frac{2bc}{d}\right)\mathrm{Si}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{2\sin(2a+2bx)}{d(c+dx)}$$

```
[Out] 4*b*Ci(2*b*c/d+2*b*x)*cos(2*a-2*b*c/d)/d^2-4*b*Si(2*b*c/d+2*b*x)*sin(2*a-2*
b*c/d)/d^2-2*sin(2*b*x+2*a)/d/(d*x+c)-Unintegrable(tan(b*x+a)/(d*x+c)^2,x)
```

Rubi [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[2*a - (2*b*c)/d]*CosIntegral[(2*b*c)/d + 2*b*x])/d^2 - (2*Sin[2*a
+ 2*b*x])/(d*(c + d*x)) - (4*b*Sin[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d +
 2*b*x])/d^2 - Defer[Int][Tan[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^2} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\right) dx$$
$$= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^2}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^2}\,dx$$
$$= -\frac{3\sin(2a+2bx)}{2d(c+dx)} + \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^2}\,dx + \frac{(3b)\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} - \int \frac{\tan(a}{(c+}$$
$$= -\frac{2\sin(2a+2bx)}{d(c+dx)} + \frac{b\int \frac{\cos(2a+2bx)}{c+dx}\,dx}{d} + \frac{\left(3b\cos\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cos\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d}$$
$$= \frac{3b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{3b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b}{d}\right.}{d^2}$$
$$= \frac{4b\cos\left(2a-\frac{2bc}{d}\right)\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^2} - \frac{2\sin(2a+2bx)}{d(c+dx)} - \frac{4b\sin\left(2a-\frac{2bc}{d}\right)\operatorname{Si}\left(\frac{2b}{d}\right.}{d^2}$$

Mathematica [A] time = 4.21, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3\,bx+3\,a)}{d^2x^2+2\,cdx+c^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

maple [A] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(iE_2\left(\frac{2ibdx+2ibc}{d}\right)-iE_2\left(-\frac{2ibdx+2ibc}{d}\right)\right)\cos\left(-\frac{2(bc-ad)}{d}\right)+2\left(d^2x+cd\right)\int\frac{\sin(2\,bx+2\,a)}{(dx+c)^2\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2+2\,\cos(2\,bx+2\right.}}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(2, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(2, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^2*x + c*d)*integrate(sin
(2*b*x + 2*a)/(d^2*x^2 + 2*c*d*x + (d^2*x^2 + 2*c*d*x + c^2)*cos(2*b*x + 2*
a)^2 + (d^2*x^2 + 2*c*d*x + c^2)*sin(2*b*x + 2*a)^2 + c^2 + 2*(d^2*x^2 + 2*
c*d*x + c^2)*cos(2*b*x + 2*a)), x) + (exp_integral_e(2, (2*I*b*d*x + 2*I*b*
c)/d) + exp_integral_e(2, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d)/d))
/(d^2*x + c*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,{(c+d\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^2), x)
```

sympy [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**2,x)
[Out] Exception raised: HeuristicGCDFailed
```

3.388 $\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=129

$$-\text{Int}\left(\frac{\tan(a+bx)}{(c+dx)^3},x\right)-\frac{4b^2\sin\left(2a-\frac{2bc}{d}\right)\text{Ci}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{4b^2\cos\left(2a-\frac{2bc}{d}\right)\text{Si}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{2b\cos(2a+2bx)}{d^2(c+dx)}-\frac{\text{si}}{}$$

```
[Out] -2*b*cos(2*b*x+2*a)/d^2/(d*x+c)-4*b^2*cos(2*a-2*b*c/d)*Si(2*b*c/d+2*b*x)/d^
3-4*b^2*Ci(2*b*c/d+2*b*x)*sin(2*a-2*b*c/d)/d^3-sin(2*b*x+2*a)/d/(d*x+c)^2-U
nintegrable(tan(b*x+a)/(d*x+c)^3,x)
```

Rubi [A] time = 0.40, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[2*a + 2*b*x])/(d^2*(c + d*x)) - (4*b^2*CosIntegral[(2*b*c)/d + 2*
b*x]*Sin[2*a - (2*b*c)/d])/d^3 - Sin[2*a + 2*b*x]/(d*(c + d*x)^2) - (4*b^2*
Cos[2*a - (2*b*c)/d]*SinIntegral[(2*b*c)/d + 2*b*x])/d^3 - Defer[Int][Tan[a
 + b*x]/(c + d*x)^3, x]
```

Rubi steps

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\cos(a+bx)\sin(a+bx)}{(c+dx)^3} - \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\right) dx$$
$$= 3\int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin^2(a+bx)\tan(a+bx)}{(c+dx)^3}\,dx$$
$$= 3\int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx + \int \frac{\cos(a+bx)\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{3}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx + \int \frac{\sin(2a+2bx)}{2(c+dx)^3}\,dx - \int \frac{\tan(a+bx)}{(c+dx)^3}\,dx$$
$$= -\frac{3\sin(2a+2bx)}{4d(c+dx)^2} + \frac{1}{2}\int \frac{\sin(2a+2bx)}{(c+dx)^3}\,dx + \frac{(3b)\int \frac{\cos(2a+2bx)}{(c+dx)^2}\,dx}{2d} - \int \frac{\tan(a \cdots}{(c + \cdots}$$
$$= -\frac{3b\cos(2a+2bx)}{2d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{\left(3b^2\right)\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2} + \frac{b\int \frac{\cos(2a+2bx)}{(c+dx)^2} \cdots}{2d}$$
$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{b^2\int \frac{\sin(2a+2bx)}{c+dx}\,dx}{d^2} - \frac{\left(3b^2\cos\left(2a-\frac{2bc}{d} \cdots\right.\right.}{\cdots}$$
$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{3b^2\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{3b \cdots}{\cdots}$$
$$= -\frac{2b\cos(2a+2bx)}{d^2(c+dx)} - \frac{4b^2\operatorname{Ci}\left(\frac{2bc}{d}+2bx\right)\sin\left(2a-\frac{2bc}{d}\right)}{d^3} - \frac{\sin(2a+2bx)}{d(c+dx)^2} - \frac{4b \cdots}{\cdots}$$

Mathematica [A] time = 6.02, size = 0, normalized size = 0.00

$$\int \frac{\sec(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Sec[a + b*x]*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

fricas [A] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)\sin(3\,bx+3\,a)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x +
 c^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3\,bx+3\,a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate(sec(b*x + a)*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

maple [A] time = 0.64, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)\sin(3bx+3a)}{(dx+c)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

```
[Out] int(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\left(i\,E_3\left(\frac{2i\,bdx+2i\,bc}{d}\right)-i\,E_3\left(-\frac{2i\,bdx+2i\,bc}{d}\right)\right)\cos\left(-\frac{2\,(bc-ad)}{d}\right)+2\left(d^3x^2+2\,cd^2x+c^2d\right)\int\frac{\sin(2\,bx+2\,a)}{(dx+c)^3\left(\cos(2\,bx+2\,a)^2+\sin(2\,bx+2\,a)^2\right.}}{d^3x^2+2\,cd^2x+c^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -((I*exp_integral_e(3, (2*I*b*d*x + 2*I*b*c)/d) - I*exp_integral_e(3, -(2*I
*b*d*x + 2*I*b*c)/d))*cos(-2*(b*c - a*d)/d) + 2*(d^3*x^2 + 2*c*d^2*x + c^2*
d)*integrate(sin(2*b*x + 2*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3 + (d
^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)^2 + (d^3*x^3 + 3*c
*d^2*x^2 + 3*c^2*d*x + c^3)*sin(2*b*x + 2*a)^2 + 2*(d^3*x^3 + 3*c*d^2*x^2 +
 3*c^2*d*x + c^3)*cos(2*b*x + 2*a)), x) + (exp_integral_e(3, (2*I*b*d*x + 2
*I*b*c)/d) + exp_integral_e(3, -(2*I*b*d*x + 2*I*b*c)/d))*sin(-2*(b*c - a*d
)/d))/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3\,a+3\,b\,x)}{\cos(a+b\,x)\,(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)*(c + d*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)*sin(3*b*x+3*a)/(d*x+c)**3,x)
[Out] Timed out
```

3.389 $\int (c+dx)^3 \sec^2(a+bx) \sin(3a+3bx)\,dx$

Optimal. Leaf size=230

$$-\frac{6d^3\text{Li}_3\left(-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{Li}_3\left(ie^{i(a+bx)}\right)}{b^4}-\frac{24d^3\sin(a+bx)}{b^4}+\frac{6id^2(c+dx)\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}$$

```
[Out] -6*I*d*(d*x+c)^2*arctan(exp(I*(b*x+a)))/b^2+24*d^2*(d*x+c)*cos(b*x+a)/b^3-4
*(d*x+c)^3*cos(b*x+a)/b+6*I*d^2*(d*x+c)*polylog(2,-I*exp(I*(b*x+a)))/b^3-6*
I*d^2*(d*x+c)*polylog(2,I*exp(I*(b*x+a)))/b^3-6*d^3*polylog(3,-I*exp(I*(b*x
+a)))/b^4+6*d^3*polylog(3,I*exp(I*(b*x+a)))/b^4-(d*x+c)^3*sec(b*x+a)/b-24*d
^3*sin(b*x+a)/b^4+12*d*(d*x+c)^2*sin(b*x+a)/b^2
```

Rubi [A] time = 0.33, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4431, 3296, 2637, 4407, 4409, 4181, 2531, 2282, 6589}

$$\frac{6id^2(c+dx)\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{6id^2(c+dx)\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}-\frac{6d^3\text{PolyLog}\left(3,-ie^{i(a+bx)}\right)}{b^4}+\frac{6d^3\text{PolyLog}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] ((-6*I)*d*(c + d*x)^2*ArcTan[E^(I*(a + b*x))])/b^2 + (24*d^2*(c + d*x)*Cos[
a + b*x])/b^3 - (4*(c + d*x)^3*Cos[a + b*x])/b + ((6*I)*d^2*(c + d*x)*PolyL
og[2, (-I)*E^(I*(a + b*x))])/b^3 - ((6*I)*d^2*(c + d*x)*PolyLog[2, I*E^(I*(
a + b*x))])/b^3 - (6*d^3*PolyLog[3, (-I)*E^(I*(a + b*x))])/b^4 + (6*d^3*Pol
yLog[3, I*E^(I*(a + b*x))])/b^4 - ((c + d*x)^3*Sec[a + b*x])/b - (24*d^3*Si
n[a + b*x])/b^4 + (12*d*(c + d*x)^2*Sin[a + b*x])/b^2
```

Rule 2282

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2531

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> -Simp[((f + g*x)^m*PolyLog[2, -(e*(F^(c*(a + b*x
)))^n)])/(b*c*n*Log[F]), x] + Dist[(g*m)/(b*c*n*Log[F]), Int[(f + g*x)^(m -
 1)*PolyLog[2, -(e*(F^(c*(a + b*x)))^n)], x], x] /; FreeQ[{F, a, b, c, e, f
```

```
, g, n}, x] && GtQ[m, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rule 6589

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int (c+dx)^3 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^3\sin(a+bx) - (c+dx)^3\sin(a+bx)\tan^2(a+bx)\right)\,dx$$

$$= 3\int (c+dx)^3\sin(a+bx)\,dx - \int (c+dx)^3\sin(a+bx)\tan^2(a+bx)\,dx$$

$$= -\frac{3(c+dx)^3\cos(a+bx)}{b} + \frac{(9d)\int (c+dx)^2\cos(a+bx)\,dx}{b} + \int (c+$$

$$= -\frac{4(c+dx)^3\cos(a+bx)}{b} - \frac{(c+dx)^3\sec(a+bx)}{b} + \frac{9d(c+dx)^2\sin(a}{b^2}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{18d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx}{}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx}{}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx}{}$$

$$= -\frac{6id(c+dx)^2\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{24d^2(c+dx)\cos(a+bx)}{b^3} - \frac{4(c+dx}{}$$

Mathematica [B] time = 2.60, size = 607, normalized size = 2.64

$$-\frac{\sec(a+bx)\left(2b^3c^3\cos(2(a+bx)) + 6b^3c^2dx\cos(2(a+bx)) + 6b^3cd^2x^2\cos(2(a+bx)) + ib^3d^3x^3\cos(a+bx) + 2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b^3*c^3 - 12*b*c*d^2 + 9*b^3*c^2*d*x - 12*b*d^3*x + 9*b^
3*c*d^2*x^2 + 3*b^3*d^3*x^3 + I*b^3*d^3*x^3*Cos[a + b*x] + (6*I)*b^2*c^2*d*
ArcTan[Cos[a + b*x] + I*Sin[a + b*x]]*Cos[a + b*x] + 2*b^3*c^3*Cos[2*(a + b
*x)] - 12*b*c*d^2*Cos[2*(a + b*x)] + 6*b^3*c^2*d*x*Cos[2*(a + b*x)] - 12*b*
d^3*x*Cos[2*(a + b*x)] + 6*b^3*c*d^2*x^2*Cos[2*(a + b*x)] + 2*b^3*d^3*x^3*C
os[2*(a + b*x)] + 3*b^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*Cos[a + b*x] - Sin[a
 + b*x]] + 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 + I*Cos[a + b*x] - Sin[a + b*x]
] - 6*b^2*c*d^2*x*Cos[a + b*x]*Log[1 - I*Cos[a + b*x] + Sin[a + b*x]] - 3*b
```

```
^2*d^3*x^2*Cos[a + b*x]*Log[1 - I*Cos[a + b*x] + Sin[a + b*x]] + (6*I)*b*d^
2*(c + d*x)*Cos[a + b*x]*PolyLog[2, I*Cos[a + b*x] - Sin[a + b*x]] - (6*I)*
b*c*d^2*Cos[a + b*x]*PolyLog[2, (-I)*Cos[a + b*x] + Sin[a + b*x]] + (6*I)*b
*d^3*x*Cos[a + b*x]*PolyLog[2, I*Cos[a + b*x] + Sin[a + b*x]] - 6*d^3*Cos[a
 + b*x]*PolyLog[3, I*Cos[a + b*x] - Sin[a + b*x]] + 6*d^3*Cos[a + b*x]*Poly
Log[3, I*Cos[a + b*x] + Sin[a + b*x]] - 6*b^2*c^2*d*Sin[2*(a + b*x)] + 12*d
^3*Sin[2*(a + b*x)] - 12*b^2*c*d^2*x*Sin[2*(a + b*x)] - 6*b^2*d^3*x^2*Sin[2
*(a + b*x)]))/b^4)
```

fricas [C] time = 0.56, size = 896, normalized size = 3.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 6*b^3*c^2*d*x + 2*b^3*c^3 + 6*d^3*c
os(b*x + a)*polylog(3, I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*
polylog(3, I*cos(b*x + a) - sin(b*x + a)) + 6*d^3*cos(b*x + a)*polylog(3, -
I*cos(b*x + a) + sin(b*x + a)) - 6*d^3*cos(b*x + a)*polylog(3, -I*cos(b*x +
 a) - sin(b*x + a)) + 8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 - 6*b*c*d^
2 + 3*(b^3*c^2*d - 2*b*d^3)*x)*cos(b*x + a)^2 - (-6*I*b*d^3*x - 6*I*b*c*d^2
)*cos(b*x + a)*dilog(I*cos(b*x + a) + sin(b*x + a)) - (-6*I*b*d^3*x - 6*I*b
*c*d^2)*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x + a)) - (6*I*b*d^3*x +
6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - (6*I*b*d^
3*x + 6*I*b*c*d^2)*cos(b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) - 3*(
b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*x + a) + I*sin(b*
x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*log(cos(b*
x + a) - I*sin(b*x + a) + I) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2
 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^
3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(I*cos(b*x +
 a) - sin(b*x + a) + 1) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^
2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a) + sin(b*x + a) + 1) + 3*(b^2*d^3*x^
2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cos(b*x + a)*log(-I*cos(b*x + a)
 - sin(b*x + a) + 1) - 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cos(b*x + a)*l
og(-cos(b*x + a) + I*sin(b*x + a) + I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d
^3)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x + a) + I) - 24*(b^2*d^3*x^2
+ 2*b^2*c*d^2*x + b^2*c^2*d - 2*d^3)*cos(b*x + a)*sin(b*x + a))/(b^4*cos(b*
x + a))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^3 \sec(bx + a)^2 \sin(3\,bx + 3\,a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.18, size = 677, normalized size = 2.94

$$-\frac{2\left(d^3x^3b^3+3b^3c\,d^2x^2+3ib^2d^3x^2+3b^3c^2dx+6ib^2c\,d^2x+b^3c^3+3ib^2c^2d-6b\,d^3x-6c\,d^2b-6id^3\right)\mathrm{e}^{i(bx+a)}}{b^4}-\frac{2\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -2*(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3+3*I*b^2*d^3*x^2-6*b*d
^3*x+6*I*b^2*c*d^2*x-6*c*d^2*b+3*I*b^2*c^2*d-6*I*d^3)/b^4*exp(I*(b*x+a))-2*
(d^3*x^3*b^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3-3*I*b^2*d^3*x^2-6*b*d^3*
x-6*I*b^2*c*d^2*x-6*c*d^2*b-3*I*b^2*c^2*d+6*I*d^3)/b^4*exp(-I*(b*x+a))-2*ex
p(I*(b*x+a))*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/b/(1+exp(2*I*(b*x+a)))-3/b
^4*d^3*a^2*ln(1-I*exp(I*(b*x+a)))-6/b^3*d^2*c*ln(1+I*exp(I*(b*x+a)))*a-6/b^
2*d^2*c*ln(1+I*exp(I*(b*x+a)))*x-6*d^3*polylog(3,-I*exp(I*(b*x+a)))/b^4-6*I
/b^2*d*c^2*arctan(exp(I*(b*x+a)))+12*I/b^3*d^2*c*a*arctan(exp(I*(b*x+a)))-6
*I/b^3*d^2*c*polylog(2,I*exp(I*(b*x+a)))+6*I/b^3*d^3*polylog(2,-I*exp(I*(b*
x+a)))*x+3/b^4*d^3*a^2*ln(1+I*exp(I*(b*x+a)))+3/b^2*d^3*ln(1-I*exp(I*(b*x+a
)))*x^2-6*I/b^4*d^3*a^2*arctan(exp(I*(b*x+a)))+6*d^3*polylog(3,I*exp(I*(b*x
+a)))/b^4+6/b^3*d^2*c*ln(1-I*exp(I*(b*x+a)))*a+6/b^2*d^2*c*ln(1-I*exp(I*(b*
x+a)))*x-3/b^2*d^3*ln(1+I*exp(I*(b*x+a)))*x^2+6*I/b^3*d^2*c*polylog(2,-I*ex
p(I*(b*x+a)))-6*I/b^3*d^3*polylog(2,I*exp(I*(b*x+a)))*x
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c^3/(b*cos(3*b*x + 3*a)^2
+ 2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)
^2 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 3/2*(4*(cos(a)
^2 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) +
 b*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(
b*x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))
*cos(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x
```

```
 + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*s
in(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x +
 3*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a)
)*cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*
x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*
x*cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) +
(b*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b
*x + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*
(b*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + s
in(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x
+ a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*
x + a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x +
3*a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(
b*x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x +
 a)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*
b*x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b
*x + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^
2*cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2
 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x
+ 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x +
a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*
x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*
sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 +
sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 +
 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a
) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a
) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^
2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*
sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*
sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^
2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)
^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*
x + a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 +
 sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a
)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + si
n(a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) +
cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3
*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos
(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*c
os(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*c
os(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*c
os(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b
*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*
sin(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x
 + a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(co
```

```
s(b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x
+ a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) +
b*x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) -
cos(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3
*a) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2
*(b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) -
cos(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x
 + a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin
(a) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a
) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x +
 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*
sin(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b*x + a) - cos(a))*cos(2*
b*x + 3*a) - cos(2*b*x + 3*a)^2 - cos(a)^2 + 2*(3*b*x*sin(b*x + 2*a)*sin(b*
x + a) - sin(a))*sin(2*b*x + 3*a) - sin(2*b*x + 3*a)^2 - sin(a)^2)*sin(3*b*
x + 3*a) + 4*(2*b*x*cos(b*x + a)*sin(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b
*x + a)^2)*sin(b*x + 2*a) - 2*sin(b*x + a)*sin(a))*sin(2*b*x + 3*a) + 12*(b
*x*cos(b*x + a)^2*sin(a) + b*x*sin(b*x + a)^2*sin(a))*sin(b*x + 2*a) - 4*(c
os(a)^2 + sin(a)^2)*sin(b*x + a))*c^2*d/((cos(a)^2 + sin(a)^2)*b^2*cos(b*x
+ a)^2 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a)^2 + (b^2*cos(2*b*x + 3*a)^2
 + 2*b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x
 + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*cos(3*b*x + 3*a)^2 + (b^2*cos(b
*x + a)^2 + b^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^2*cos(2*b*x + 3*a)^
2 + 2*b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*
x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*sin(3*b*x + 3*a)^2 + (b^2*cos(
b*x + a)^2 + b^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^2*cos(2*b*x + 3*
a)^2*cos(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^2*cos(b*
x + a)*sin(2*b*x + 3*a)^2 + 2*b^2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (c
os(a)^2 + sin(a)^2)*b^2*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^2*cos(b*x + a
)^2*cos(a) + b^2*cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^2*cos(2*b*x
 + 3*a)^2*sin(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^2*s
in(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a)
 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^2*cos(b*
x + a)^2*sin(a) + b^2*sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a)) - (6*((b^3*d
^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(2*b*x + 3*a)*cos(b*x
+ 2*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(2*b*x
+ 3*a)*sin(b*x + 2*a) + (b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*
d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(b*x + 2*a) + (b^3*d^3*x^3*sin(a) + 3*b
^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin(b*x + 2*a))*
cos(3*b*x + 3*a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*
d^2)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 6*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d
^2)*cos(2*b*x + 3*a)*cos(b*x + 2*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*
d^3*x - 4*b*c*d^2)*sin(2*b*x + 3*a)*sin(b*x + 2*a) + (b^3*d^3*x^3*cos(a) +
3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(b*x + 2*a
) + (b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c
```

```
*d^2*sin(a))*sin(b*x + 2*a))*sin(3*b*x + 3*a)^2 + 2*((b^3*d^3*x^3 + 3*b^3*c
*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a) - 3*(b^2*d^3*x^2 + 2*b^2*c*d
^2*x - 2*d^3)*sin(b*x + a))*sin(2*b*x + 3*a)^2 + 2*((b^3*d^3*x^3*cos(a) - 6
*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2
+ 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 -
6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^3*d^3*x^3*cos(a) - 6*b*c*d
^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b
^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*cos(b*x + a)^2 + (b^3*d^3*x^3*cos(a) - 6
*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2
+ 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 -
6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
2*d^3)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a)^2 + (b^3*d^3*x^3*cos(a) - 6*b*c*d
^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b
^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*sin(b*x + a)^2 + 2*((b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a)*cos(b*x + a) + 3*(b^2*
d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)*sin(2*b*x + 3*a) + (b^3*d^3*x
^3*cos(a) - 6*b*c*d^2*cos(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3
*sin(a))*x^2 + 6*(b^2*c*d^2*sin(a) - b*d^3*cos(a))*x)*cos(b*x + a))*cos(3*b
*x + 3*a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*
x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*sin(b*x
+ a)^2)*cos(2*b*x + 3*a) + 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x -
6*b*c*d^2)*cos(2*b*x + 3*a)*sin(b*x + a) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x -
 2*d^3)*sin(2*b*x + 3*a)*sin(b*x + a) + (b^3*d^3*x^3*cos(a) - 6*b*c*d^2*cos
(a) - 6*d^3*sin(a) + 3*(b^3*c*d^2*cos(a) + b^2*d^3*sin(a))*x^2 + 6*(b^2*c*d
^2*sin(a) - b*d^3*cos(a))*x)*sin(b*x + a))*sin(3*b*x + 3*a) + 3*((b^2*d^3*x
^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x -
 2*d^3)*sin(b*x + a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 2*((cos(a)^2 +
 sin(a)^2)*b^3*d^3*x^3 + 3*(cos(a)^2 + sin(a)^2)*b^3*c*d^2*x^2 - 6*(cos(a)^
2 + sin(a)^2)*b*d^3*x - 6*(cos(a)^2 + sin(a)^2)*b*c*d^2 + (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(2*b*x + 3*a)^2 + 6*(b^3*d^3*x^3*
cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*cos(
b*x + 2*a)*cos(b*x + a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*
c*d^2)*sin(2*b*x + 3*a)^2 + 6*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a)
- 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*cos(b*x + a)*sin(b*x + 2*a) + 2*(b^3
*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 6*b*d^3*x*cos(a) - 6*b*c*d^2*cos
(a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + 2
*a)*cos(b*x + a))*cos(2*b*x + 3*a) + 2*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^
2*sin(a) - 6*b*d^3*x*sin(a) - 6*b*c*d^2*sin(a) + 3*(b^3*d^3*x^3 + 3*b^3*c*d
^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + a)*sin(b*x + 2*a))*sin(2*b*x + 3*
a))*cos(3*b*x + 3*a) + 2*(3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4
*b*c*d^2)*cos(b*x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b
*c*d^2)*sin(b*x + a)^2)*cos(b*x + 2*a) + 2*(b^3*d^3*x^3*cos(a) + 3*b^3*c*d^
2*x^2*cos(a) - 6*b*d^3*x*cos(a) - 6*b*c*d^2*cos(a))*cos(b*x + a) - 6*(b^2*d
^3*x^2*cos(a) + 2*b^2*c*d^2*x*cos(a) - 2*d^3*cos(a))*sin(b*x + a))*cos(2*b*
```

```
x + 3*a) + 6*((b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(
a) - 4*b*c*d^2*cos(a))*cos(b*x + a)^2 + (b^3*d^3*x^3*cos(a) + 3*b^3*c*d^2*x
^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*sin(b*x + a)^2)*cos(b*x +
2*a) + 2*((cos(a)^2 + sin(a)^2)*b^3*d^3*x^3 + 3*(cos(a)^2 + sin(a)^2)*b^3*c
*d^2*x^2 - 6*(cos(a)^2 + sin(a)^2)*b*d^3*x - 6*(cos(a)^2 + sin(a)^2)*b*c*d^
2)*cos(b*x + a) - ((cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a)^2 + (cos(a)^2 + s
in(a)^2)*b^4*sin(b*x + a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3
*a)*cos(a) + b^4*sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(
a)^2 + sin(a)^2)*b^4)*cos(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*
x + a)^2)*cos(2*b*x + 3*a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x +
3*a)*cos(a) + b^4*sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos
(a)^2 + sin(a)^2)*b^4)*sin(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b
*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^4*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*
b^4*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^4*cos(b*x + a)*sin(2*b*x + 3*a
)^2 + 2*b^4*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^
4*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*cos(a) + b^4*cos(a
)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^4*cos(2*b*x + 3*a)^2*sin(b*x + a)
 + 2*b^4*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^4*sin(2*b*x + 3*a)^2*sin(
b*x + a) + 2*b^4*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^
2)*b^4*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*sin(a) + b^4*
sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*integrate(6*((d^3*x^2 + 2*c*d^2*x)
*cos(2*b*x + 2*a)*cos(b*x + a) + (d^3*x^2 + 2*c*d^2*x)*sin(2*b*x + 2*a)*sin
(b*x + a) + (d^3*x^2 + 2*c*d^2*x)*cos(b*x + a))/(b*cos(2*b*x + 2*a)^2 + b*s
in(2*b*x + 2*a)^2 + 2*b*cos(2*b*x + 2*a) + b), x) + 2*((b^3*d^3*x^3*sin(a)
- 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x
^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 - 2*d^3)*cos(2*b*x + 3*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6
*b*c*d^2)*sin(2*b*x + 3*a))*cos(3*b*x + 3*a)^2 + (b^3*d^3*x^3*sin(a) - 6*b*
c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6
*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*cos(b*x + a)^2 + (b^3*d^3*x^3*sin(a)
- 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x
^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x - 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x
 - 2*d^3)*cos(2*b*x + 3*a) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6
*b*c*d^2)*sin(2*b*x + 3*a))*sin(3*b*x + 3*a)^2 + (b^3*d^3*x^3*sin(a) - 6*b*
c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6
*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*sin(b*x + a)^2 - 2*(3*(b^2*d^3*x^2 +
2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)*cos(b*x + a) - (b^3*d^3*x^3 + 3*b^3
*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*cos(b*x + a)*sin(2*b*x + 3*a) - (b^3*d^
3*x^3*sin(a) - 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d^2*sin(a) - b^2*
d^3*cos(a))*x^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*cos(b*x + a))*cos(
3*b*x + 3*a) - 3*((b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(b*x + a)^2 + (b
^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(b*x + a)^2)*cos(2*b*x + 3*a) - 2*(3
*(b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)*sin(b*x + a) - (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x - 6*b*c*d^2)*sin(2*b*x + 3*a)*sin(b*
x + a) - (b^3*d^3*x^3*sin(a) - 6*b*c*d^2*sin(a) + 6*d^3*cos(a) + 3*(b^3*c*d
```

```
^2*sin(a) - b^2*d^3*cos(a))*x^2 - 6*(b^2*c*d^2*cos(a) + b*d^3*sin(a))*x)*si
n(b*x + a))*sin(3*b*x + 3*a) + ((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x
- 6*b*c*d^2)*cos(b*x + a)^2 + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 6*b*d^3*x -
6*b*c*d^2)*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4*a) - 6*((cos(a)^
2 + sin(a)^2)*b^2*d^3*x^2 + 2*(cos(a)^2 + sin(a)^2)*b^2*c*d^2*x - 2*(cos(a)
^2 + sin(a)^2)*d^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*cos(2*b*x + 3*a)
^2 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x - 2*d^3)*sin(2*b*x + 3*a)^2 - 2*(b^3*d^3*
x^3*cos(a) + 3*b^3*c*d^2*x^2*cos(a) - 4*b*d^3*x*cos(a) - 4*b*c*d^2*cos(a))*
cos(b*x + 2*a)*sin(b*x + a) - 2*(b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*sin(a
) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin(b*x + 2*a)*sin(b*x + a) + 2*(b
^2*d^3*x^2*cos(a) + 2*b^2*c*d^2*x*cos(a) - 2*d^3*cos(a) - (b^3*d^3*x^3 + 3*
b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + 2*a)*sin(b*x + a))*cos(2*b
*x + 3*a) + 2*(b^2*d^3*x^2*sin(a) + 2*b^2*c*d^2*x*sin(a) - 2*d^3*sin(a) - (
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(b*x + 2*a)*sin(b
*x + a))*sin(2*b*x + 3*a))*sin(3*b*x + 3*a) + 2*(2*(b^3*d^3*x^3*sin(a) + 3*
b^3*c*d^2*x^2*sin(a) - 6*b*d^3*x*sin(a) - 6*b*c*d^2*sin(a))*cos(b*x + a) +
3*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*cos(b*x + a)^2 +
 (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 - 4*b*d^3*x - 4*b*c*d^2)*sin(b*x + a)^2)*si
n(b*x + 2*a) - 6*(b^2*d^3*x^2*sin(a) + 2*b^2*c*d^2*x*sin(a) - 2*d^3*sin(a))
*sin(b*x + a))*sin(2*b*x + 3*a) + 6*((b^3*d^3*x^3*sin(a) + 3*b^3*c*d^2*x^2*
sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*cos(b*x + a)^2 + (b^3*d^3*x^3
*sin(a) + 3*b^3*c*d^2*x^2*sin(a) - 4*b*d^3*x*sin(a) - 4*b*c*d^2*sin(a))*sin
(b*x + a)^2)*sin(b*x + 2*a) - 6*((cos(a)^2 + sin(a)^2)*b^2*d^3*x^2 + 2*(cos
(a)^2 + sin(a)^2)*b^2*c*d^2*x - 2*(cos(a)^2 + sin(a)^2)*d^3)*sin(b*x + a))/
((cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a)^2 + (cos(a)^2 + sin(a)^2)*b^4*sin(b
*x + a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3*a)*cos(a) + b^4*s
in(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b
^4)*cos(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*x + a)^2)*cos(2*b*
x + 3*a)^2 + (b^4*cos(2*b*x + 3*a)^2 + 2*b^4*cos(2*b*x + 3*a)*cos(a) + b^4*
sin(2*b*x + 3*a)^2 + 2*b^4*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*
b^4)*sin(3*b*x + 3*a)^2 + (b^4*cos(b*x + a)^2 + b^4*sin(b*x + a)^2)*sin(2*b
*x + 3*a)^2 + 2*(b^4*cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*b^4*cos(2*b*x + 3*
a)*cos(b*x + a)*cos(a) + b^4*cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*b^4*cos(b*
x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^4*cos(b*x + a))*co
s(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*cos(a) + b^4*cos(a)*sin(b*x + a)^2)*
cos(2*b*x + 3*a) + 2*(b^4*cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^4*cos(2*b*x
 + 3*a)*cos(a)*sin(b*x + a) + b^4*sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*b^4*s
in(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^4*sin(b*x + a
))*sin(3*b*x + 3*a) + 2*(b^4*cos(b*x + a)^2*sin(a) + b^4*sin(b*x + a)^2*sin
(a))*sin(2*b*x + 3*a))
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^3)/cos(a + b*x)^2,x)
```

```
[Out] \text{Hanged}
```

sympy **[F(-2)]** time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.390 $\int (c+dx)^2 \sec^2(a+bx) \sin(3a+3bx) \, dx$

Optimal. Leaf size=147

$$\frac{2id^2\text{Li}_2\left(-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{Li}_2\left(ie^{i(a+bx)}\right)}{b^3}+\frac{8d^2\cos(a+bx)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}-\frac{4}{}$$

```
[Out] -4*I*d*(d*x+c)*arctan(exp(I*(b*x+a)))/b^2+8*d^2*cos(b*x+a)/b^3-4*(d*x+c)^2*
cos(b*x+a)/b+2*I*d^2*polylog(2,-I*exp(I*(b*x+a)))/b^3-2*I*d^2*polylog(2,I*e
xp(I*(b*x+a)))/b^3-(d*x+c)^2*sec(b*x+a)/b+8*d*(d*x+c)*sin(b*x+a)/b^2
```

Rubi [A] time = 0.21, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {4431, 3296, 2638, 4407, 4409, 4181, 2279, 2391}

$$\frac{2id^2\text{PolyLog}\left(2,-ie^{i(a+bx)}\right)}{b^3}-\frac{2id^2\text{PolyLog}\left(2,ie^{i(a+bx)}\right)}{b^3}+\frac{8d(c+dx)\sin(a+bx)}{b^2}-\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2}+\frac{8d}{}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] ((-4*I)*d*(c + d*x)*ArcTan[E^(I*(a + b*x))])/b^2 + (8*d^2*Cos[a + b*x])/b^3
 - (4*(c + d*x)^2*Cos[a + b*x])/b + ((2*I)*d^2*PolyLog[2, (-I)*E^(I*(a + b*
x))])/b^3 - ((2*I)*d^2*PolyLog[2, I*E^(I*(a + b*x))])/b^3 - ((c + d*x)^2*Se
c[a + b*x])/b + (8*d*(c + d*x)*Sin[a + b*x])/b^2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\int (c+dx)^2 \sec^2(a+bx)\sin(3a+3bx)\,dx = \int \left(3(c+dx)^2\sin(a+bx) - (c+dx)^2\sin(a+bx)\tan^2(a+bx)\right)\,dx$$

$$= 3\int (c+dx)^2\sin(a+bx)\,dx - \int (c+dx)^2\sin(a+bx)\tan^2(a+bx)$$

$$= -\frac{3(c+dx)^2\cos(a+bx)}{b} + \frac{(6d)\int (c+dx)\cos(a+bx)\,dx}{b} + \int (c+$$

$$= -\frac{4(c+dx)^2\cos(a+bx)}{b} - \frac{(c+dx)^2\sec(a+bx)}{b} + \frac{6d(c+dx)\sin(a}{b^2}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{6d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a}{b}$$

$$= -\frac{4id(c+dx)\tan^{-1}\left(e^{i(a+bx)}\right)}{b^2} + \frac{8d^2\cos(a+bx)}{b^3} - \frac{4(c+dx)^2\cos(a}{b}$$

Mathematica [B] time = 3.83, size = 364, normalized size = 2.48

$$\frac{-4\cos(bx)\left(\cos(a)\left(b^2(c+dx)^2-2d^2\right)-2bd\sin(a)(c+dx)\right)+4\sin(bx)\left(\sin(a)\left(b^2(c+dx)^2-2d^2\right)+2bd\cos(a\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (4*b*c*d*ArcTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] + 2*d^2*(2*ArcTan[Cot[a]]*Ar
cTanh[Sin[a] + Cos[a]*Tan[(b*x)/2]] - (Csc[a]*((b*x - ArcTan[Cot[a]])*(Log[
1 - E^(I*(b*x - ArcTan[Cot[a]]))] - Log[1 + E^(I*(b*x - ArcTan[Cot[a]]))])
+ I*PolyLog[2, -E^(I*(b*x - ArcTan[Cot[a]]))] - I*PolyLog[2, E^(I*(b*x - Ar
cTan[Cot[a]]))]))/Sqrt[Csc[a]^2]) - b^2*(c + d*x)^2*Sec[a] - 4*Cos[b*x]*((-
2*d^2 + b^2*(c + d*x)^2)*Cos[a] - 2*b*d*(c + d*x)*Sin[a]) + 4*(2*b*d*(c + d
*x)*Cos[a] + (-2*d^2 + b^2*(c + d*x)^2)*Sin[a])*Sin[b*x] - (b^2*(c + d*x)^2
*Sin[(b*x)/2])/((Cos[a/2] - Sin[a/2])*(Cos[(a + b*x)/2] - Sin[(a + b*x)/2])
) + (b^2*(c + d*x)^2*Sin[(b*x)/2])/((Cos[a/2] + Sin[a/2])*(Cos[(a + b*x)/2]
 + Sin[(a + b*x)/2])))/b^3
```

fricas [B] time = 0.54, size = 513, normalized size = 3.49

$$-\frac{b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + i\,d^2\cos(bx+a)\,\mathrm{Li}_2\left(i\,\cos(bx+a)+\sin(bx+a)\right) + i\,d^2\cos(bx+a)\,\mathrm{Li}_2\left(i\,\cos(bx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + I*d^2*cos(b*x + a)*dilog(I*cos(b*x
+ a) + sin(b*x + a)) + I*d^2*cos(b*x + a)*dilog(I*cos(b*x + a) - sin(b*x +
a)) - I*d^2*cos(b*x + a)*dilog(-I*cos(b*x + a) + sin(b*x + a)) - I*d^2*cos(
b*x + a)*dilog(-I*cos(b*x + a) - sin(b*x + a)) + 4*(b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2 - 2*d^2)*cos(b*x + a)^2 - (b*c*d - a*d^2)*cos(b*x + a)*log(cos
(b*x + a) + I*sin(b*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(cos(b*x
+ a) - I*sin(b*x + a) + I) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x +
 a) + sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(I*cos(b*x + a)
 - sin(b*x + a) + 1) - (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) +
 sin(b*x + a) + 1) + (b*d^2*x + a*d^2)*cos(b*x + a)*log(-I*cos(b*x + a) - s
in(b*x + a) + 1) - (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) + I*sin(b
*x + a) + I) + (b*c*d - a*d^2)*cos(b*x + a)*log(-cos(b*x + a) - I*sin(b*x +
 a) + I) - 8*(b*d^2*x + b*c*d)*cos(b*x + a)*sin(b*x + a))/(b^3*cos(b*x + a)
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (dx + c)^2 \sec (bx + a)^2 \sin (3\,bx + 3\,a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2*sec(b*x + a)^2*sin(3*b*x + 3*a), x)
```

maple [B] time = 0.07, size = 328, normalized size = 2.23

$$-\frac{4c^2\cos(bx+a)}{b}-\frac{c^2}{b\cos(bx+a)}-\frac{4d^2\cos(bx+a)\,x^2}{b}+\frac{8d^2\sin(bx+a)\,x}{b^2}+\frac{8d^2\cos(bx+a)}{b^3}-\frac{d^2x^2}{b\cos(bx+a)}-\frac{2d^2\ \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -4*c^2/b*cos(b*x+a)-1/b/cos(b*x+a)*c^2-4*d^2/b*cos(b*x+a)*x^2+8*d^2/b^2*sin
(b*x+a)*x+8*d^2*cos(b*x+a)/b^3-1/b*d^2/cos(b*x+a)*x^2-2/b^2*d^2*ln(1+I*exp(
I*(b*x+a)))*x-2/b^3*d^2*ln(1+I*exp(I*(b*x+a)))*a+2/b^2*d^2*ln(1-I*exp(I*(b*
x+a)))*x+2/b^3*d^2*ln(1-I*exp(I*(b*x+a)))*a-2*I*d^2/b^3*dilog(1-I*exp(I*(b*
x+a)))+2*I*d^2/b^3*dilog(1+I*exp(I*(b*x+a)))-2/b^3*a*d^2*ln(sec(b*x+a)+tan(
b*x+a))-8*c*d/b*cos(b*x+a)*x+8*c*d/b^2*sin(b*x+a)-2/b*c*d/cos(b*x+a)*x+2/b^
2*c*d*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
[Out] Timed out
```

mupad [**F(-1)**] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x)^2)/cos(a + b*x)^2,x)
[Out] \text{Hanged}
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sec(b*x+a)**2*sin(3*b*x+3*a),x)
[Out] Exception raised: HeuristicGCDFailed
```

3.391 $\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx$

Optimal. Leaf size=57

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

```
[Out] d*arctanh(sin(b*x+a))/b^2-4*(d*x+c)*cos(b*x+a)/b-(d*x+c)*sec(b*x+a)/b+4*d*s
in(b*x+a)/b^2
```

Rubi [A] time = 0.09, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {4431, 3296, 2637, 4407, 4409, 3770}

$$\frac{4d\sin(a+bx)}{b^2}+\frac{d\tanh^{-1}(\sin(a+bx))}{b^2}-\frac{4(c+dx)\cos(a+bx)}{b}-\frac{(c+dx)\sec(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] (d*ArcTanh[Sin[a + b*x]])/b^2 - (4*(c + d*x)*Cos[a + b*x])/b - ((c + d*x)*S
ec[a + b*x])/b + (4*d*Sin[a + b*x])/b^2
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
```

```
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4409

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> Simp[((c + d*x)^m*Sec[a + b*x]^n)/(b*n), x] -
 Dist[(d*m)/(b*n), Int[(c + d*x)^(m - 1)*Sec[a + b*x]^n, x], x] /; FreeQ[{a
, b, c, d, n}, x] && EqQ[p, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)\sec^2(a+bx)\sin(3a+3bx)\,dx &= \int \left(3(c+dx)\sin(a+bx)-(c+dx)\sin(a+bx)\tan^2(a+bx)\right)\,dx \\
&= 3\int (c+dx)\sin(a+bx)\,dx - \int (c+dx)\sin(a+bx)\tan^2(a+bx)\,dx \\
&= -\frac{3(c+dx)\cos(a+bx)}{b} + \frac{(3d)\int \cos(a+bx)\,dx}{b} + \int (c+dx)\sin(a \\
&= -\frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+bx)}{b} + \frac{3d\sin(a+bx)}{b^2} + \frac{d}{} \\
&= \frac{d\tanh^{-1}(\sin(a+bx))}{b^2} - \frac{4(c+dx)\cos(a+bx)}{b} - \frac{(c+dx)\sec(a+bx}{b}
\end{aligned}
$$

Mathematica [A] time = 0.48, size = 105, normalized size = 1.84

$$
-\frac{\sec(a+bx)\left(2b(c+dx)\cos(2(a+bx))-2d\sin(2(a+bx))+d\cos(a+bx)\left(\log\left(\cos\left(\frac{1}{2}(a+bx)\right)-\sin\left(\frac{1}{2}(a+\right.\right.\right.\right.}{b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sec[a + b*x]^2*Sin[3*a + 3*b*x],x]
```

```
[Out] -((Sec[a + b*x]*(3*b*c + 3*b*d*x + 2*b*(c + d*x)*Cos[2*(a + b*x)] + d*Cos[a
 + b*x]*(Log[Cos[(a + b*x)/2] - Sin[(a + b*x)/2]] - Log[Cos[(a + b*x)/2] +
Sin[(a + b*x)/2]]) - 2*d*Sin[2*(a + b*x)]))/b^2)
```

fricas [A] time = 0.44, size = 93, normalized size = 1.63

$$-\frac{2\,bdx+8\,(bdx+bc)\cos{(bx+a)}^2-d\cos{(bx+a)}\log{(\sin{(bx+a)}+1)}+d\cos{(bx+a)}\log{(-\sin{(bx+a)}+1)}}{2\,b^2\cos{(bx+a)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*x + 8*(b*d*x + b*c)*cos(b*x + a)^2 - d*cos(b*x + a)*log(sin(b*x
 + a) + 1) + d*cos(b*x + a)*log(-sin(b*x + a) + 1) - 8*d*cos(b*x + a)*sin(b
*x + a) + 2*b*c)/(b^2*cos(b*x + a))
```

giac [B] time = 1.79, size = 2876, normalized size = 50.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="giac")
```

```
[Out] -1/2*(10*b*d*x*tan(1/2*b*x)^4*tan(1/2*a)^4 + 10*b*c*tan(1/2*b*x)^4*tan(1/2*
a)^4 + d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) +
 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*
a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 +
tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1
/2*b*x)^4*tan(1/2*a)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b
*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1
/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2
*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/
2*a)^2 + 1))*tan(1/2*b*x)^4*tan(1/2*a)^4 - 12*b*d*x*tan(1/2*b*x)^4*tan(1/2*
a)^2 - 64*b*d*x*tan(1/2*b*x)^3*tan(1/2*a)^3 - 12*b*d*x*tan(1/2*b*x)^2*tan(1
/2*a)^4 - 12*b*c*tan(1/2*b*x)^4*tan(1/2*a)^2 - 64*b*c*tan(1/2*b*x)^3*tan(1/
2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*
a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(
1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^
2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*t
an(1/2*b*x)^3*tan(1/2*a)^3 + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*tan
(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2
*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2*a)
^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1)/(
tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a)^3 + 16*d*tan(1/2*b*x)^4*tan(1/
2*a)^3 - 12*b*c*tan(1/2*b*x)^2*tan(1/2*a)^4 + 16*d*tan(1/2*b*x)^3*tan(1/2*a
)^4 + 10*b*d*x*tan(1/2*b*x)^4 + 64*b*d*x*tan(1/2*b*x)^3*tan(1/2*a) + 168*b*
d*x*tan(1/2*b*x)^2*tan(1/2*a)^2 + 64*b*d*x*tan(1/2*b*x)*tan(1/2*a)^3 + 10*b
*d*x*tan(1/2*a)^4 + 10*b*c*tan(1/2*b*x)^4 - d*log(2*(tan(1/2*b*x)^4*tan(1/2
```

*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1
/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*
x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(
1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + d*log(2*(tan(1/2*b*x)^4*ta
n(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 +
tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1
/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2
*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^4 + 64*b*c*tan(1/2*b*x)^3
*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan
(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2
*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*
b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 +
1))*tan(1/2*b*x)^3*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 - 2*
tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)^4
+ 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(1/2
*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a) + 1
)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)^3*tan(1/2*a) - 16*d*tan(1/2*b*x)^4*tan(1
/2*a) + 168*b*c*tan(1/2*b*x)^2*tan(1/2*a)^2 - 96*d*tan(1/2*b*x)^3*tan(1/2*a
)^2 + 64*b*c*tan(1/2*b*x)*tan(1/2*a)^3 - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*
a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/
2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x
)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1
/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3 + 4*d*log(2*(tan(1
/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan
(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)
^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(
1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a)^3
- 96*d*tan(1/2*b*x)^2*tan(1/2*a)^3 + 10*b*c*tan(1/2*a)^4 - d*log(2*(tan(1/2
*b*x)^4*tan(1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1
/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3
 + 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/
2*b*x) - 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 + d*log(2*(tan(
1/2*b*x)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*ta
n(1/2*a)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x
)^3 - 2*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan
(1/2*b*x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*a)^4 - 16*d*tan(1
/2*b*x)*tan(1/2*a)^4 - 12*b*d*x*tan(1/2*b*x)^2 - 64*b*d*x*tan(1/2*b*x)*tan(
1/2*a) - 12*b*d*x*tan(1/2*a)^2 - 12*b*c*tan(1/2*b*x)^2 + 16*d*tan(1/2*b*x)^
3 - 64*b*c*tan(1/2*b*x)*tan(1/2*a) - 4*d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2
 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*
x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2*b*x)*ta
n(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*tan(1/2*a
) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 4*d*log(2*(tan(1/2*b*x
)^4*tan(1/2*a)^2 - 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a
)^2 + tan(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2

```
*tan(1/2*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*
x) + 2*tan(1/2*a) + 1)/(tan(1/2*a)^2 + 1))*tan(1/2*b*x)*tan(1/2*a) + 96*d*t
an(1/2*b*x)^2*tan(1/2*a) - 12*b*c*tan(1/2*a)^2 + 96*d*tan(1/2*b*x)*tan(1/2*
a)^2 + 16*d*tan(1/2*a)^3 + 10*b*d*x + 10*b*c + d*log(2*(tan(1/2*b*x)^4*tan(
1/2*a)^2 + 2*tan(1/2*b*x)^4*tan(1/2*a) + 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + ta
n(1/2*b*x)^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 - 2*tan(1/2*b*x)^3 + 2*tan(1/2
*b*x)*tan(1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 - 2*tan(1/2*b*x) - 2*t
an(1/2*a) + 1)/(tan(1/2*a)^2 + 1)) - d*log(2*(tan(1/2*b*x)^4*tan(1/2*a)^2 -
 2*tan(1/2*b*x)^4*tan(1/2*a) - 2*tan(1/2*b*x)^3*tan(1/2*a)^2 + tan(1/2*b*x)
^4 + 2*tan(1/2*b*x)^2*tan(1/2*a)^2 + 2*tan(1/2*b*x)^3 - 2*tan(1/2*b*x)*tan(
1/2*a)^2 + 2*tan(1/2*b*x)^2 + tan(1/2*a)^2 + 2*tan(1/2*b*x) + 2*tan(1/2*a)
+ 1)/(tan(1/2*a)^2 + 1)) - 16*d*tan(1/2*b*x) - 16*d*tan(1/2*a))/(b^2*tan(1/
2*b*x)^4*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a)^3 - b^2*tan(1/2*b*x
)^4 - 4*b^2*tan(1/2*b*x)^3*tan(1/2*a) - 4*b^2*tan(1/2*b*x)*tan(1/2*a)^3 - b
^2*tan(1/2*a)^4 - 4*b^2*tan(1/2*b*x)*tan(1/2*a) + b^2)
```

maple [A] time = 0.04, size = 87, normalized size = 1.53

$$-\frac{4c\cos(bx+a)}{b}-\frac{c}{b\cos(bx+a)}-\frac{4d\cos(bx+a)\,x}{b}+\frac{4d\sin(bx+a)}{b^2}-\frac{dx}{b\cos(bx+a)}+\frac{d\ln(\sec(bx+a)+\tan(bx}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x)
```

```
[Out] -4/b*c*cos(b*x+a)-1/b*c/cos(b*x+a)-4/b*d*cos(b*x+a)*x+4*d*sin(b*x+a)/b^2-1/
b*d/cos(b*x+a)*x+1/b^2*d*ln(sec(b*x+a)+tan(b*x+a))
```

maxima [B] time = 1.12, size = 3330, normalized size = 58.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)^2*sin(3*b*x+3*a),x, algorithm="maxima")
```

```
[Out] -2*((cos(3*b*x + 3*a) + cos(b*x + a))*cos(4*b*x + 4*a) + (3*cos(2*b*x + 2*a
) + 1)*cos(3*b*x + 3*a) + 3*cos(2*b*x + 2*a)*cos(b*x + a) + (sin(3*b*x + 3*
a) + sin(b*x + a))*sin(4*b*x + 4*a) + 3*sin(3*b*x + 3*a)*sin(2*b*x + 2*a) +
 3*sin(2*b*x + 2*a)*sin(b*x + a) + cos(b*x + a))*c/(b*cos(3*b*x + 3*a)^2 +
2*b*cos(3*b*x + 3*a)*cos(b*x + a) + b*cos(b*x + a)^2 + b*sin(3*b*x + 3*a)^2
 + 2*b*sin(3*b*x + 3*a)*sin(b*x + a) + b*sin(b*x + a)^2) - 1/2*(4*(cos(a)^2
 + sin(a)^2)*b*x*cos(b*x + a) + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b
*x*cos(b*x + 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*
x + 2*a)*sin(a))*cos(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*c
os(2*b*x + 3*a)^2 + 12*(b*x*cos(2*b*x + 3*a)*cos(b*x + 2*a) + b*x*cos(b*x +
 2*a)*cos(a) + b*x*sin(2*b*x + 3*a)*sin(b*x + 2*a) + b*x*sin(b*x + 2*a)*sin
```

```
(a))*sin(3*b*x + 3*a)^2 + 4*(b*x*cos(b*x + a) - sin(b*x + a))*sin(2*b*x + 3
*a)^2 + 4*((b*x*cos(2*b*x + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*
cos(3*b*x + 3*a)^2 + (b*x*cos(a) + sin(a))*cos(b*x + a)^2 + (b*x*cos(2*b*x
 + 3*a) + b*x*cos(a) + sin(2*b*x + 3*a) + sin(a))*sin(3*b*x + 3*a)^2 + (b*x*
cos(a) + sin(a))*sin(b*x + a)^2 + 2*(b*x*cos(2*b*x + 3*a)*cos(b*x + a) + (b
*x*cos(a) + sin(a))*cos(b*x + a) + cos(b*x + a)*sin(2*b*x + 3*a))*cos(3*b*x
 + 3*a) + (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b
*x*cos(2*b*x + 3*a)*sin(b*x + a) + (b*x*cos(a) + sin(a))*sin(b*x + a) + sin
(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a) + (cos(b*x + a)^2 + sin(b*x +
a)^2)*sin(2*b*x + 3*a))*cos(3*b*x + 4*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(b*x
 + a)*cos(a) + 6*b*x*cos(b*x + a)*sin(b*x + 2*a)*sin(a) + b*x*cos(2*b*x + 3*
a)^2 + b*x*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*b*x + 2*(3*b*x*cos(b*
x + 2*a)*cos(b*x + a) + b*x*cos(a))*cos(2*b*x + 3*a) + 2*(3*b*x*cos(b*x + a
)*sin(b*x + 2*a) + b*x*sin(a))*sin(2*b*x + 3*a))*cos(3*b*x + 3*a) + 4*(2*b*
x*cos(b*x + a)*cos(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*cos(b*x
 + 2*a) - 2*cos(a)*sin(b*x + a))*cos(2*b*x + 3*a) + 12*(b*x*cos(b*x + a)^2*
cos(a) + b*x*cos(a)*sin(b*x + a)^2)*cos(b*x + 2*a) - ((cos(2*b*x + 3*a)^2 +
 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x +
3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)
^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*cos(b*x + a)^2 + (cos(2*b*x
 + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + sin(2*b*x + 3*a)^2 + 2*si
n(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b*x + 3*a)^2 + (cos(b*x + a)^2 + si
n(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a)^2 + sin(a)^2)*sin(b*x + a)^2 + 2
*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a)
 + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a)
 + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(cos(b*x + a)^2*
cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(cos(2*b*x + 3*a)^2*si
n(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + sin(2*b*x + 3*a)^2*si
n(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (cos(a)^2 + sin(a)^2)
*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(b*x + a)^2*sin(a) + sin(b*x + a)^2
*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x + a)^2 + sin(b*x + a)^2 + 2*sin(b*x
 + a) + 1) + ((cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + cos(a)^2 + s
in(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*cos(3*b*x + 3*a)^
2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (cos(a)^2 + sin(
a)^2)*cos(b*x + a)^2 + (cos(2*b*x + 3*a)^2 + 2*cos(2*b*x + 3*a)*cos(a) + co
s(a)^2 + sin(2*b*x + 3*a)^2 + 2*sin(2*b*x + 3*a)*sin(a) + sin(a)^2)*sin(3*b
*x + 3*a)^2 + (cos(b*x + a)^2 + sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + (cos(a
)^2 + sin(a)^2)*sin(b*x + a)^2 + 2*(cos(2*b*x + 3*a)^2*cos(b*x + a) + 2*cos
(2*b*x + 3*a)*cos(b*x + a)*cos(a) + cos(b*x + a)*sin(2*b*x + 3*a)^2 + 2*cos
(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^2 + sin(a)^2)*cos(b*x + a))*cos
(3*b*x + 3*a) + 2*(cos(b*x + a)^2*cos(a) + cos(a)*sin(b*x + a)^2)*cos(2*b*x
 + 3*a) + 2*(cos(2*b*x + 3*a)^2*sin(b*x + a) + 2*cos(2*b*x + 3*a)*cos(a)*si
n(b*x + a) + sin(2*b*x + 3*a)^2*sin(b*x + a) + 2*sin(2*b*x + 3*a)*sin(b*x +
 a)*sin(a) + (cos(a)^2 + sin(a)^2)*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(cos(
b*x + a)^2*sin(a) + sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))*log(cos(b*x +
```

```
a)^2 + sin(b*x + a)^2 - 2*sin(b*x + a) + 1) + 4*((b*x*sin(2*b*x + 3*a) + b*
x*sin(a) - cos(2*b*x + 3*a) - cos(a))*cos(3*b*x + 3*a)^2 + (b*x*sin(a) - co
s(a))*cos(b*x + a)^2 + (b*x*sin(2*b*x + 3*a) + b*x*sin(a) - cos(2*b*x + 3*a
) - cos(a))*sin(3*b*x + 3*a)^2 + (b*x*sin(a) - cos(a))*sin(b*x + a)^2 + 2*(
b*x*cos(b*x + a)*sin(2*b*x + 3*a) + (b*x*sin(a) - cos(a))*cos(b*x + a) - co
s(2*b*x + 3*a)*cos(b*x + a))*cos(3*b*x + 3*a) - (cos(b*x + a)^2 + sin(b*x +
 a)^2)*cos(2*b*x + 3*a) + 2*(b*x*sin(2*b*x + 3*a)*sin(b*x + a) + (b*x*sin(a
) - cos(a))*sin(b*x + a) - cos(2*b*x + 3*a)*sin(b*x + a))*sin(3*b*x + 3*a)
+ (b*x*cos(b*x + a)^2 + b*x*sin(b*x + a)^2)*sin(2*b*x + 3*a))*sin(3*b*x + 4
*a) + 4*(6*b*x*cos(b*x + 2*a)*cos(a)*sin(b*x + a) + 6*b*x*sin(b*x + 2*a)*si
n(b*x + a)*sin(a) + 2*(3*b*x*cos(b*x + 2*a)*sin(b*x + a) - cos(a))*cos(2*b*
x + 3*a) - cos(2*b*x + 3*a)^2 - cos(a)^2 + 2*(3*b*x*sin(b*x + 2*a)*sin(b*x
+ a) - sin(a))*sin(2*b*x + 3*a) - sin(2*b*x + 3*a)^2 - sin(a)^2)*sin(3*b*x
+ 3*a) + 4*(2*b*x*cos(b*x + a)*sin(a) + 3*(b*x*cos(b*x + a)^2 + b*x*sin(b*x
 + a)^2)*sin(b*x + 2*a) - 2*sin(b*x + a)*sin(a))*sin(2*b*x + 3*a) + 12*(b*x
*cos(b*x + a)^2*sin(a) + b*x*sin(b*x + a)^2*sin(a))*sin(b*x + 2*a) - 4*(cos
(a)^2 + sin(a)^2)*sin(b*x + a))*d/((cos(a)^2 + sin(a)^2)*b^2*cos(b*x + a)^2
 + (cos(a)^2 + sin(a)^2)*b^2*sin(b*x + a)^2 + (b^2*cos(2*b*x + 3*a)^2 + 2*b
^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x + 3*a
)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*cos(3*b*x + 3*a)^2 + (b^2*cos(b*x + a
)^2 + b^2*sin(b*x + a)^2)*cos(2*b*x + 3*a)^2 + (b^2*cos(2*b*x + 3*a)^2 + 2*
b^2*cos(2*b*x + 3*a)*cos(a) + b^2*sin(2*b*x + 3*a)^2 + 2*b^2*sin(2*b*x + 3*
a)*sin(a) + (cos(a)^2 + sin(a)^2)*b^2)*sin(3*b*x + 3*a)^2 + (b^2*cos(b*x +
a)^2 + b^2*sin(b*x + a)^2)*sin(2*b*x + 3*a)^2 + 2*(b^2*cos(2*b*x + 3*a)^2*c
os(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(b*x + a)*cos(a) + b^2*cos(b*x + a)
*sin(2*b*x + 3*a)^2 + 2*b^2*cos(b*x + a)*sin(2*b*x + 3*a)*sin(a) + (cos(a)^
2 + sin(a)^2)*b^2*cos(b*x + a))*cos(3*b*x + 3*a) + 2*(b^2*cos(b*x + a)^2*co
s(a) + b^2*cos(a)*sin(b*x + a)^2)*cos(2*b*x + 3*a) + 2*(b^2*cos(2*b*x + 3*a
)^2*sin(b*x + a) + 2*b^2*cos(2*b*x + 3*a)*cos(a)*sin(b*x + a) + b^2*sin(2*b
*x + 3*a)^2*sin(b*x + a) + 2*b^2*sin(2*b*x + 3*a)*sin(b*x + a)*sin(a) + (co
s(a)^2 + sin(a)^2)*b^2*sin(b*x + a))*sin(3*b*x + 3*a) + 2*(b^2*cos(b*x + a)
^2*sin(a) + b^2*sin(b*x + a)^2*sin(a))*sin(2*b*x + 3*a))
```

mupad [B] time = 1.31, size = 150, normalized size = 2.63

$$e^{-a\,1i-b\,x\,1i}\left(\frac{-2\,b\,c+d\,2i}{b^2}-\frac{2\,d\,x}{b}\right)-e^{a\,1i+b\,x\,1i}\left(\frac{2\,b\,c+d\,2i}{b^2}+\frac{2\,d\,x}{b}\right)-\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}-i\right)}{b^2}+\frac{d\,\ln\left(e^{a\,1i+b\,x\,1i}+1i\right)}{b^2}-\frac{e^{a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((sin(3*a + 3*b*x)*(c + d*x))/cos(a + b*x)^2,x)
```

```
[Out] exp(- a*1i - b*x*1i)*((d*2i - 2*b*c)/b^2 - (2*d*x)/b) - exp(a*1i + b*x*1i)*
((d*2i + 2*b*c)/b^2 + (2*d*x)/b) - (d*log(exp(a*1i + b*x*1i) - 1i))/b^2 + (
```

```
d*log(exp(a*1i + b*x*1i) + 1i))/b^2 - (exp(a*1i + b*x*1i)*(c + d*x)*2i)/(b*
(exp(a*2i + b*x*2i)*1i + 1i))
```

sympy [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: HeuristicGCDFailed

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sec(b*x+a)**2*sin(3*b*x+3*a),x)
```

```
[Out] Exception raised: HeuristicGCDFailed
```

3.392 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$

Optimal. Leaf size=78

$$-\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{c+dx},x\right)+\frac{4\sin\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c),x)+4*cos(a-b*c/d)*Si(b*c/d+b
*x)/d+4*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
```

Rubi [A] time = 0.27, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
```

```
[Out] (4*CosIntegral[(b*c)/d + b*x]*Sin[a - (b*c)/d])/d + (4*Cos[a - (b*c)/d]*Sin
Integral[(b*c)/d + b*x])/d - Defer[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*
x), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx &= \int \left(\frac{3\sin(a+bx)}{c+dx}-\frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\right)dx\\
&= 3\int \frac{\sin(a+bx)}{c+dx}\,dx-\int \frac{\sin(a+bx)\tan^2(a+bx)}{c+dx}\,dx\\
&= \left(3\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\sin\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\left(3\sin\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx+\\
&= \frac{3\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{3\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}+\cos\left(a-\frac{bc}{d}\right)\int \frac{\sin}{}\\
&= \frac{4\text{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d}+\frac{4\cos\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d}-\int \frac{\sec(a+bx)\tan(}{c+dx}
\end{aligned}$$

Mathematica [A] time = 13.73, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{c+dx}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x),x]
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x), x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{dx+c},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c), x)
```

maple [A] time = 0.34, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2\left(bx+a\right)\right)\sin\left(3bx+3a\right)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(3a+3bx)}{\cos(a+bx)^2\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c),x)
```

```
[Out] Timed out
```

3.393 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=98

$$-\text{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^2},x\right)+\frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{4\sin(a+bx)}{d(c+dx)}$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^2,x)+4*b*Ci(b*c/d+b*x)*cos(a
-b*c/d)/d^2-4*b*Si(b*c/d+b*x)*sin(a-b*c/d)/d^2-4*sin(b*x+a)/d/(d*x+c)
```

Rubi [A] time = 0.34, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
```

```
[Out] (4*b*Cos[a - (b*c)/d]*CosIntegral[(b*c)/d + b*x])/d^2 - (4*Sin[a + b*x])/(d
*(c + d*x)) - (4*b*Sin[a - (b*c)/d]*SinIntegral[(b*c)/d + b*x])/d^2 - Defer
[Int][(Sec[a + b*x]*Tan[a + b*x])/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx = \int \left(\frac{3\sin(a+bx)}{(c+dx)^2} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\right) dx$$
$$= 3\int \frac{\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^2}\,dx$$
$$= -\frac{3\sin(a+bx)}{d(c+dx)} + \frac{(3b)\int \frac{\cos(a+bx)}{c+dx}\,dx}{d} + \int \frac{\sin(a+bx)}{(c+dx)^2}\,dx - \int \frac{\sec(a+bx)\tan}{(c+dx)}$$
$$= -\frac{4\sin(a+bx)}{d(c+dx)} + \frac{b\int \frac{\cos(a+bx)}{c+dx}\,dx}{d} + \frac{\left(3b\cos\left(a-\frac{bc}{d}\right)\right)\int \frac{\cos\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} - \frac{(3b\sin}{}$$
$$= \frac{3b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)} - \frac{3b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} +$$
$$= \frac{4b\cos\left(a-\frac{bc}{d}\right)\text{Ci}\left(\frac{bc}{d}+bx\right)}{d^2} - \frac{4\sin(a+bx)}{d(c+dx)} - \frac{4b\sin\left(a-\frac{bc}{d}\right)\text{Si}\left(\frac{bc}{d}+bx\right)}{d^2} -$$

Mathematica [A] time = 16.58, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2,x]
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^2, x]
```

fricas [A] time = 1.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{d^2x^2+2\,cdx+c^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(bx+a)^2\sin(3\,bx+3\,a)}{(dx+c)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^2, x)
```

maple [A] time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2 (bx + a)\right) \sin (3bx + 3a)}{(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (3\,a + 3\,b\,x)}{\cos (a + b\,x)^2\,(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**2,x)
[Out] Timed out
```

3.394 $\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=121

$$-\mathrm{Int}\left(\frac{\tan(a+bx)\sec(a+bx)}{(c+dx)^3},x\right)-\frac{2b^2\sin\left(a-\frac{bc}{d}\right)\mathrm{Ci}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b^2\cos\left(a-\frac{bc}{d}\right)\mathrm{Si}\left(\frac{bc}{d}+bx\right)}{d^3}-\frac{2b\cos(a+bx)}{d^2(c+dx)}-2$$

```
[Out] -CannotIntegrate(sec(b*x+a)*tan(b*x+a)/(d*x+c)^3,x)-2*b*cos(b*x+a)/d^2/(d*x
+c)-2*b^2*cos(a-b*c/d)*Si(b*c/d+b*x)/d^3-2*b^2*Ci(b*c/d+b*x)*sin(a-b*c/d)/d
^3-2*sin(b*x+a)/d/(d*x+c)^2
```

Rubi [A] time = 0.44, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] (-2*b*Cos[a + b*x])/(d^2*(c + d*x)) - (2*b^2*CosIntegral[(b*c)/d + b*x]*Sin
[a - (b*c)/d])/d^3 - (2*Sin[a + b*x])/(d*(c + d*x)^2) - (2*b^2*Cos[a - (b*c
)/d]*SinIntegral[(b*c)/d + b*x])/d^3 - Defer[Int][(Sec[a + b*x]*Tan[a + b*x
])/(c + d*x)^3, x]
```

Rubi steps

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx = \int \left(\frac{3\sin(a+bx)}{(c+dx)^3} - \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\right)dx$$

$$= 3\int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sin(a+bx)\tan^2(a+bx)}{(c+dx)^3}\,dx$$

$$= -\frac{3\sin(a+bx)}{2d(c+dx)^2} + \frac{(3b)\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d} + \int \frac{\sin(a+bx)}{(c+dx)^3}\,dx - \int \frac{\sec(a+bx)\,\mathrm{ta}}{(c+d}$$

$$= -\frac{3b\cos(a+bx)}{2d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{\left(3b^2\right)\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} + \frac{b\int \frac{\cos(a+bx)}{(c+dx)^2}\,dx}{2d} -$$

$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{b^2\int \frac{\sin(a+bx)}{c+dx}\,dx}{2d^2} - \frac{\left(3b^2\cos\left(a-\frac{bc}{d}\right)\right)\int}{2d^2}$$

$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{3b^2\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{2d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{3b^2\cos\left(\right.}{}$$

$$= -\frac{2b\cos(a+bx)}{d^2(c+dx)} - \frac{2b^2\mathrm{Ci}\left(\frac{bc}{d}+bx\right)\sin\left(a-\frac{bc}{d}\right)}{d^3} - \frac{2\sin(a+bx)}{d(c+dx)^2} - \frac{2b^2\cos\left(\right.}{}$$

Mathematica [A] time = 19.13, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(a+bx)\sin(3a+3bx)}{(c+dx)^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3,x]
```

```
[Out] Integrate[(Sec[a + b*x]^2*Sin[3*a + 3*b*x])/(c + d*x)^3, x]
```

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sec\left(bx+a\right)^2\sin\left(3\,bx+3\,a\right)}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x
 + c^3), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (bx + a)^2 \sin (3\, bx + 3\, a)}{(dx + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="giac")
[Out] integrate(sec(b*x + a)^2*sin(3*b*x + 3*a)/(d*x + c)^3, x)
```

maple [A] time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2 (bx + a)\right) \sin (3bx + 3a)}{(dx + c)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
[Out] int(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)^2*sin(3*b*x+3*a)/(d*x+c)^3,x, algorithm="maxima")
[Out] Timed out
```

mupad [A] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin (3\, a + 3\, b\, x)}{\cos (a + b\, x)^2\, (c + d\, x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3),x)
[Out] int(sin(3*a + 3*b*x)/(cos(a + b*x)^2*(c + d*x)^3), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(b*x+a)**2*sin(3*b*x+3*a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.395 $\int x\cos(2x)\sec(x)\,dx$

Optimal. Leaf size=57

$$-i\mathrm{Li}_2\left(-ie^{ix}\right)+i\mathrm{Li}_2\left(ie^{ix}\right)+2x\sin(x)+2\cos(x)+2ix\tan^{-1}\left(e^{ix}\right)$$

```
[Out] 2*I*x*arctan(exp(I*x))+2*cos(x)-I*polylog(2,-I*exp(I*x))+I*polylog(2,I*exp(
I*x))+2*x*sin(x)
```

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4431, 3296, 2638, 4407, 4181, 2279, 2391}

$$-i\text{PolyLog}\left(2,-ie^{ix}\right)+i\text{PolyLog}\left(2,ie^{ix}\right)+2x\sin(x)+2\cos(x)+2ix\tan^{-1}\left(e^{ix}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x],x]
```

```
[Out] (2*I)*x*ArcTan[E^(I*x)] + 2*Cos[x] - I*PolyLog[2, (-I)*E^(I*x)] + I*PolyLog
[2, I*E^(I*x)] + 2*x*Sin[x]
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3296

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> -Simp[
((c + d*x)^m*Cos[e + f*x])/f, x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Cos[
e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4407

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b_.)*(x_)]^(n_.)*Tan[(a_.) + (b
_.)*(x_)]^(p_.), x_Symbol] :> -Int[(c + d*x)^m*Sin[a + b*x]^n*Tan[a + b*x]^
(p - 2), x] + Int[(c + d*x)^m*Sin[a + b*x]^(n - 2)*Tan[a + b*x]^p, x] /; Fr
eeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(2x)\sec(x)\,dx &= \int (x\cos(x)-x\sin(x)\tan(x))\,dx\\
&= \int x\cos(x)\,dx-\int x\sin(x)\tan(x)\,dx\\
&= x\sin(x)+\int x\cos(x)\,dx-\int x\sec(x)\,dx-\int \sin(x)\,dx\\
&= 2ix\tan^{-1}\left(e^{ix}\right)+\cos(x)+2x\sin(x)+\int \log\left(1-ie^{ix}\right)\,dx-\int \log\left(1+ie^{ix}\right)\,dx-\int \sin\\
&= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)+2x\sin(x)-i\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx,x,e^{ix}\right)+i\,\mathrm{Subst}\left(\int\right.\\
&= 2ix\tan^{-1}\left(e^{ix}\right)+2\cos(x)-i\mathrm{Li}_2\left(-ie^{ix}\right)+i\mathrm{Li}_2\left(ie^{ix}\right)+2x\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.35

$$-i\left(\mathrm{Li}_2\left(-ie^{ix}\right)-\mathrm{Li}_2\left(ie^{ix}\right)\right)-x\left(\log\left(1-ie^{ix}\right)-\log\left(1+ie^{ix}\right)\right)+2x\sin(x)+2\cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x],x]
```

[Out] 2*Cos[x] - x*(Log[1 - I*E^(I*x)] - Log[1 + I*E^(I*x)]) - I*(PolyLog[2, (-I)*E^(I*x)] - PolyLog[2, I*E^(I*x)]) + 2*x*Sin[x]

fricas [B] time = 1.23, size = 106, normalized size = 1.86

$$-\frac{1}{2}\,x\log\left(i\,\cos(x)+\sin(x)+1\right)+\frac{1}{2}\,x\log\left(i\,\cos(x)-\sin(x)+1\right)-\frac{1}{2}\,x\log\left(-i\,\cos(x)+\sin(x)+1\right)+\frac{1}{2}\,x\log\left(-i\,\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x, algorithm="fricas")

[Out] -1/2*x*log(I*cos(x) + sin(x) + 1) + 1/2*x*log(I*cos(x) - sin(x) + 1) - 1/2*x*log(-I*cos(x) + sin(x) + 1) + 1/2*x*log(-I*cos(x) - sin(x) + 1) + 2*x*sin(x) + 2*cos(x) + 1/2*I*dilog(I*cos(x) + sin(x)) + 1/2*I*dilog(I*cos(x) - sin(x)) - 1/2*I*dilog(-I*cos(x) + sin(x)) - 1/2*I*dilog(-I*cos(x) - sin(x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(2\,x\right)\sec\left(x\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x, algorithm="giac")

[Out] integrate(x*cos(2*x)*sec(x), x)

maple [A] time = 0.06, size = 66, normalized size = 1.16

$$x\ln\left(1+i e^{ix}\right)-x\ln\left(1-i e^{ix}\right)-i\,\mathrm{dilog}\left(1+i e^{ix}\right)+i\,\mathrm{dilog}\left(1-i e^{ix}\right)+2\cos(x)+2x\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*cos(2*x)*sec(x),x)

[Out] x*ln(1+I*exp(I*x))-x*ln(1-I*exp(I*x))-I*dilog(1+I*exp(I*x))+I*dilog(1-I*exp(I*x))+2*cos(x)+2*x*sin(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\,x\sin\left(x\right)+2\,\cos\left(x\right)-2\int\frac{x\cos\left(2\,x\right)\cos\left(x\right)+x\sin\left(2\,x\right)\sin\left(x\right)+x\cos\left(x\right)}{\cos\left(2\,x\right)^{2}+\sin\left(2\,x\right)^{2}+2\,\cos\left(2\,x\right)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*cos(2*x)*sec(x),x, algorithm="maxima")

```
[Out] 2*x*sin(x) + 2*cos(x) - 2*integrate((x*cos(2*x)*cos(x) + x*sin(2*x)*sin(x)
+ x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x)
```

mupad [B] time = 2.21, size = 46, normalized size = 0.81

$$2\cos(x) + 2x\sin(x) - \operatorname{polylog}\left(2, -e^{x\,1i}\,1i\right)1i + \operatorname{polylog}\left(2, e^{x\,1i}\,1i\right)1i + x\operatorname{atan}\left(e^{x\,1i}\right)2i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x),x)
```

```
[Out] 2*cos(x) - polylog(2, -exp(x*1i)*1i)*1i + polylog(2, exp(x*1i)*1i)*1i + x*a
tan(exp(x*1i))*2i + 2*x*sin(x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(2x)\sec(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x),x)
```

```
[Out] Integral(x*cos(2*x)*sec(x), x)
```

3.396 $\int x\cos(2x)\sec^2(x)\,dx$

Optimal. Leaf size=14

$$x^2 - x\tan(x) - \log(\cos(x))$$

```
[Out] x^2-ln(cos(x))-x*tan(x)
```

Rubi [A] time = 0.03, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4431, 3720, 3475, 30}

$$x^2 - x\tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3475

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3720

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(b*(c + d*x)^m*(b*Tan[e + f*x])^(n - 1))/(f*(n - 1)), x] + (-Di
st[(b*d*m)/(f*(n - 1)), Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x],
 x] - Dist[b^2, Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; FreeQ[
{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}\int x\cos(2x)\sec^2(x)\,dx &= \int \left(x - x\tan^2(x)\right)\,dx\\ &= \frac{x^2}{2} - \int x\tan^2(x)\,dx\\ &= \frac{x^2}{2} - x\tan(x) + \int x\,dx + \int \tan(x)\,dx\\ &= x^2 - \log(\cos(x)) - x\tan(x)\end{aligned}$$

Mathematica [A] time = 0.02, size = 14, normalized size = 1.00

$$x^2 - x\tan(x) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^2,x]
```

```
[Out] x^2 - Log[Cos[x]] - x*Tan[x]
```

fricas [A] time = 0.56, size = 26, normalized size = 1.86

$$\frac{x^2\cos(x) - \cos(x)\log\left(-\cos(x)\right) - x\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="fricas")
```

```
[Out] (x^2*cos(x) - cos(x)*log(-cos(x)) - x*sin(x))/cos(x)
```

giac [B] time = 1.55, size = 118, normalized size = 8.43

$$\frac{2\,x^2\tan\left(\frac{1}{2}\,x\right)^2 - \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)\tan\left(\frac{1}{2}\,x\right)^2 - 2\,x^2 + 4\,x\tan\left(\frac{1}{2}\,x\right) + \log\left(\frac{4\left(\tan\left(\frac{1}{2}\,x\right)^4 - 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1\right)}{\tan\left(\frac{1}{2}\,x\right)^4 + 2\,\tan\left(\frac{1}{2}\,x\right)^2 + 1}\right)}{2\left(\tan\left(\frac{1}{2}\,x\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*x^2*tan(1/2*x)^2 - log(4*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/
2*x)^4 + 2*tan(1/2*x)^2 + 1))*tan(1/2*x)^2 - 2*x^2 + 4*x*tan(1/2*x) + log(4
*(tan(1/2*x)^4 - 2*tan(1/2*x)^2 + 1)/(tan(1/2*x)^4 + 2*tan(1/2*x)^2 + 1)))/
(tan(1/2*x)^2 - 1)
```

maple [A] time = 0.06, size = 15, normalized size = 1.07

$$x^2 - \ln(\cos(x)) - x\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^2,x)
```

```
[Out] x^2-ln(cos(x))-x*tan(x)
```

maxima [B] time = 0.42, size = 111, normalized size = 7.93

$$\frac{2\,x^2\cos(2\,x)^2 + 2\,x^2\sin(2\,x)^2 + 4\,x^2\cos(2\,x) + 2\,x^2 - \left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)\log\left(\cos(2\,x)^2 + \right.}{2\left(\cos(2\,x)^2 + \sin(2\,x)^2 + 2\,\cos(2\,x) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*cos(2*x)^2 + 2*x^2*sin(2*x)^2 + 4*x^2*cos(2*x) + 2*x^2 - (cos(2*
x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*log(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x
) + 1) - 4*x*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 2.35, size = 31, normalized size = 2.21

$$x^2 - \ln\left(e^{x\,2i} + 1\right) + x\,2i - \frac{x\,2i}{e^{x\,2i} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x)^2,x)
```

```
[Out] x*2i - log(exp(x*2i) + 1) - (x*2i)/(exp(x*2i) + 1) + x^2
```

sympy [B] time = 5.74, size = 144, normalized size = 10.29

$$x^2 + \frac{2x\tan\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tan\left(\frac{x}{2}\right) - 1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \frac{\log\left(\tan\left(\frac{x}{2}\right) - 1\right)}{\tan^2\left(\frac{x}{2}\right) - 1} - \frac{\log\left(\tan\left(\frac{x}{2}\right) + 1\right)\tan^2\left(\frac{x}{2}\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \frac{\log\left(\tan\left(\frac{x}{2}\right) + 1\right)}{\tan^2\left(\frac{x}{2}\right) - 1} + \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**2,x)
```

```
[Out] x**2 + 2*x*tan(x/2)/(tan(x/2)**2 - 1) - log(tan(x/2) - 1)*tan(x/2)**2/(tan(
x/2)**2 - 1) + log(tan(x/2) - 1)/(tan(x/2)**2 - 1) - log(tan(x/2) + 1)*tan(
x/2)**2/(tan(x/2)**2 - 1) + log(tan(x/2) + 1)/(tan(x/2)**2 - 1) + log(tan(x
/2)**2 + 1)*tan(x/2)**2/(tan(x/2)**2 - 1) - log(tan(x/2)**2 + 1)/(tan(x/2)*
*2 - 1)
```

3.397 $\int x\cos(2x)\sec^3(x)\,dx$

Optimal. Leaf size=67

$$\frac{3}{2}i\text{Li}_2\left(-ie^{ix}\right)-\frac{3}{2}i\text{Li}_2\left(ie^{ix}\right)-3ix\tan^{-1}\left(e^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\tan(x)\sec(x)$$

```
[Out] -3*I*x*arctan(exp(I*x))+3/2*I*polylog(2,-I*exp(I*x))-3/2*I*polylog(2,I*exp(
I*x))+1/2*sec(x)-1/2*x*sec(x)*tan(x)
```

Rubi [A] time = 0.14, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4431, 4181, 2279, 2391, 4413, 4185}

$$\frac{3}{2}i\text{PolyLog}\left(2,-ie^{ix}\right)-\frac{3}{2}i\text{PolyLog}\left(2,ie^{ix}\right)-3ix\tan^{-1}\left(e^{ix}\right)+\frac{\sec(x)}{2}-\frac{1}{2}x\tan(x)\sec(x)$$

Antiderivative was successfully verified.

```
[In] Int[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (-3*I)*x*ArcTan[E^(I*x)] + ((3*I)/2)*PolyLog[2, (-I)*E^(I*x)] - ((3*I)/2)*P
olyLog[2, I*E^(I*x)] + Sec[x]/2 - (x*Sec[x]*Tan[x])/2
```

Rule 2279

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2391

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> -Simp[PolyLog[2
, -(c*e*x^n)]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4181

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(-2*(c + d*x)^m*ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))])/f, x] + (-Di
st[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[(d*m)/f, Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4185

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 -Simp[(b^2*(c + d*x)*Cot[e + f*x]*(b*Csc[e + f*x])^(n - 2))/(f*(n - 1)), x
```

```
] + (Dist[(b^2*(n - 2))/(n - 1), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x]
, x] - Simp[(b^2*d*(b*Csc[e + f*x])^(n - 2))/(f^2*(n - 1)*(n - 2)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rule 4413

```
Int[((c_.) + (d_.)*(x_))^(m_.)*Sec[(a_.) + (b_.)*(x_)]*Tan[(a_.) + (b_.)*(x
_)]^(p_), x_Symbol] :> -Int[(c + d*x)^m*Sec[a + b*x]*Tan[a + b*x]^(p - 2),
x] + Int[(c + d*x)^m*Sec[a + b*x]^3*Tan[a + b*x]^(p - 2), x] /; FreeQ[{a, b
, c, d, m}, x] && IGtQ[p/2, 0]
```

Rule 4431

```
Int[((e_.) + (f_.)*(x_))^(m_.)*(F_)[(a_.) + (b_.)*(x_)]^(p_.)*(G_)[(c_.) +
(d_.)*(x_)]^(q_.), x_Symbol] :> Int[ExpandTrigExpand[(e + f*x)^m*G[c + d*x]
^q, F, c + d*x, p, b/d, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && Member
Q[{Sin, Cos}, F] && MemberQ[{Sec, Csc}, G] && IGtQ[p, 0] && IGtQ[q, 0] && E
qQ[b*c - a*d, 0] && IGtQ[b/d, 1]
```

Rubi steps

$$\begin{aligned}
\int x\cos(2x)\sec^3(x)\,dx &= \int \left(x\sec(x) - x\sec(x)\tan^2(x)\right)\,dx \\
&= \int x\sec(x)\,dx - \int x\sec(x)\tan^2(x)\,dx \\
&= -2ix\tan^{-1}\left(e^{ix}\right) - \int \log\left(1-ie^{ix}\right)\,dx + \int \log\left(1+ie^{ix}\right)\,dx + \int x\sec(x)\,dx - \int x\,\mathrm{se} \\
&= -4ix\tan^{-1}\left(e^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x) + i\,\mathrm{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{ix}\right) - i\,\mathrm{Su} \\
&= -3ix\tan^{-1}\left(e^{ix}\right) + i\mathrm{Li}_2\left(-ie^{ix}\right) - i\mathrm{Li}_2\left(ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x) + i\,\mathrm{Subst}\left(\int \right. \\
&= -3ix\tan^{-1}\left(e^{ix}\right) + 2i\mathrm{Li}_2\left(-ie^{ix}\right) - 2i\mathrm{Li}_2\left(ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x) - \frac{1}{2}i\,\mathrm{Subst}\left(\right. \\
&= -3ix\tan^{-1}\left(e^{ix}\right) + \frac{3}{2}i\mathrm{Li}_2\left(-ie^{ix}\right) - \frac{3}{2}i\mathrm{Li}_2\left(ie^{ix}\right) + \frac{\sec(x)}{2} - \frac{1}{2}x\sec(x)\tan(x)
\end{aligned}$$

Mathematica [B] time = 0.28, size = 146, normalized size = 2.18

$$\frac{1}{4}\left(6i\mathrm{Li}_2\left(-ie^{ix}\right) - 6i\mathrm{Li}_2\left(ie^{ix}\right) + 6x\log\left(1-ie^{ix}\right) - 6x\log\left(1+ie^{ix}\right) + \frac{x}{\sin(x)-1} + \frac{x}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} + \frac{2}{\cos\left(\frac{x}{2}\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cos[2*x]*Sec[x]^3,x]
```

```
[Out] (6*x*Log[1 - I*E^(I*x)] - 6*x*Log[1 + I*E^(I*x)] + (6*I)*PolyLog[2, (-I)*E^
(I*x)] - (6*I)*PolyLog[2, I*E^(I*x)] + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2]) +
 x/(Cos[x/2] + Sin[x/2])^2 - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + x/(-1 + S
in[x]))/4
```

fricas [B] time = 2.07, size = 144, normalized size = 2.15

$$\frac{3\,x\cos(x)^2\log\left(i\,\cos(x)+\sin(x)+1\right)-3\,x\cos(x)^2\log\left(i\,\cos(x)-\sin(x)+1\right)+3\,x\cos(x)^2\log\left(-i\,\cos(x)+\sin(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*x*cos(x)^2*log(I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(I*cos(x) -
sin(x) + 1) + 3*x*cos(x)^2*log(-I*cos(x) + sin(x) + 1) - 3*x*cos(x)^2*log(-
I*cos(x) - sin(x) + 1) - 3*I*cos(x)^2*dilog(I*cos(x) + sin(x)) - 3*I*cos(x)
^2*dilog(I*cos(x) - sin(x)) + 3*I*cos(x)^2*dilog(-I*cos(x) + sin(x)) + 3*I*
cos(x)^2*dilog(-I*cos(x) - sin(x)) - 2*x*sin(x) + 2*cos(x))/cos(x)^2
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos\left(2\,x\right)\sec(x)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="giac")
```

```
[Out] integrate(x*cos(2*x)*sec(x)^3, x)
```

maple [B] time = 0.21, size = 102, normalized size = 1.52

$$\frac{i\left(x\,\mathrm{e}^{3ix}-x\,\mathrm{e}^{ix}-i\mathrm{e}^{3ix}-i\mathrm{e}^{ix}\right)}{\left(\mathrm{e}^{2ix}+1\right)^2}-\frac{3x\ln\left(1+i\mathrm{e}^{ix}\right)}{2}+\frac{3x\ln\left(1-i\mathrm{e}^{ix}\right)}{2}+\frac{3i\,\mathrm{dilog}\left(1+i\mathrm{e}^{ix}\right)}{2}-\frac{3i\,\mathrm{dilog}\left(1-i\mathrm{e}^{ix}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cos(2*x)*sec(x)^3,x)
```

```
[Out] I/(exp(2*I*x)+1)^2*(x*exp(3*I*x)-x*exp(I*x)-I*exp(3*I*x)-I*exp(I*x))-3/2*x*
ln(1+I*exp(I*x))+3/2*x*ln(1-I*exp(I*x))+3/2*I*dilog(1+I*exp(I*x))-3/2*I*dil
og(1-I*exp(I*x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{(x\sin(3x) - x\sin(x) - \cos(3x) - \cos(x))\cos(4x) - (2x\sin(2x) + 2\cos(2x) + 1)\cos(3x) - 2(x\sin(x) + c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)^3,x, algorithm="maxima")
```

```
[Out] -((x*sin(3*x) - x*sin(x) - cos(3*x) - cos(x))*cos(4*x) - (2*x*sin(2*x) + 2*
cos(2*x) + 1)*cos(3*x) - 2*(x*sin(x) + cos(x))*cos(2*x) - 3*(2*(2*cos(2*x)
+ 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x
) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)*integrate((x*cos(2*x)*cos(x) + x*sin(2*x
)*sin(x) + x*cos(x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1), x) - (x*co
s(3*x) - x*cos(x) + sin(3*x) + sin(x))*sin(4*x) + (2*x*cos(2*x) + x - 2*sin
(2*x))*sin(3*x) + 2*(x*cos(x) - sin(x))*sin(2*x) - x*sin(x) - cos(x))/(2*(2
*cos(2*x) + 1)*cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*
x)*sin(2*x) + 4*sin(2*x)^2 + 4*cos(2*x) + 1)
```

mupad [B] time = 2.32, size = 63, normalized size = 0.94

$$\frac{1}{2\cos(x)} + x\,\mathrm{atanh}\left(e^{x\,1i}\,1i\right) - \frac{x\sin(x)}{2\cos(x)^2} + \frac{\mathrm{polylog}\left(2, -e^{x\,1i}\,1i\right)\,3i}{2} - \frac{\mathrm{polylog}\left(2, e^{x\,1i}\,1i\right)\,3i}{2} - x\,\mathrm{atan}\left(e^{x\,1i}\right)\,4i$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*cos(2*x))/cos(x)^3,x)
```

```
[Out] (polylog(2, -exp(x*1i)*1i)*3i)/2 - (polylog(2, exp(x*1i)*1i)*3i)/2 + 1/(2*c
os(x)) - x*atan(exp(x*1i))*4i + x*atanh(exp(x*1i)*1i) - (x*sin(x))/(2*cos(x
)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\cos(2x)\sec^3(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cos(2*x)*sec(x)**3,x)
```

```
[Out] Integral(x*cos(2*x)*sec(x)**3, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
```

```
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not
    as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
```

```
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:
```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)
   ))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
   (expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
```

```
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
   Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
```

```
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e',
            'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
    ']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
```

```
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.
    func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
```

```
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
        else: #result contains complex but optimal is not
            return "C"
    else: # result do not contain complex, this assumes optimal do not as
well
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
else:
    return "C"
```